Poisson Lab Prelim Q's

wikipedia → 1. A charged particle or photon enters the gas chamber of a geiger counter. The gas is inert and there are an anode and cathode inside the gas chamber, that do not touch, but across which a large voltage is applied. When the ~~~~ particle or photon enter the gas chamber they have a chance of ionising the gas, making it conductive for a moment. The circuit is then complete and the counter can detect a current pulse. ~~~~ The pulses are independent since radiation can only ionise the gas once.

2. Let $\mu = zp$ be the average number of events per trial.

$\Rightarrow p = \frac{\mu}{z}$

Now $$\lim_{z\to\infty} P(x;p,z) = \lim_{z\to\infty} \frac{z!}{x!(z-x)!}\left(\frac{\mu}{z}\right)^x\left(1-\frac{\mu}{z}\right)^{z-x}$$

$$= \frac{\mu^x}{x!}\lim_{z\to\infty}\frac{z!}{(z-x)!}\frac{1}{z^x}\left(1-\frac{\mu}{z}\right)^z\left(1-\frac{\mu}{z}\right)^{-x}$$

$$= \frac{\mu^x}{x!}(1)\cdot e^{-\mu}(1) = \frac{\mu^x}{x!}e^{-\mu}$$

a) $$\langle x\rangle = \sum_{x=0}^{\infty} x\frac{\mu^x}{x!}e^{-\mu} = \sum_{x=1}^{\infty}\frac{\mu^x}{(x-1)!}e^{-\mu} = \mu\sum_{x=1}^{\infty}\frac{\mu^{(x-1)}}{(x-1)!}e^{-\mu}$$
$$= e^{-\mu}\mu\sum_{x=0}^{\infty}\frac{\mu^x}{x!} = e^{-\mu}\mu e^{\mu} = \mu$$

b) $$\langle x^2\rangle = \langle x(x-1)+x\rangle = \langle x(x-1)\rangle + \langle x\rangle = \mu + \langle x(x-1)\rangle$$
$$= \mu + \sum_{x=0}^{\infty}(x(x-1))\frac{\mu^x e^{-\mu}}{x!} = \mu + \sum_{x=1}^{\infty}(x(x-1))\frac{\mu^x e^{-\mu}}{x!} = \mu + \mu e^{-\mu}\sum_{x=1}^{\infty}(x-1)\frac{\mu^{x-1}}{(x-1)!}$$
$$= \mu + \mu e^{-\mu}\sum_{x=2}^{\infty}(x-1)\frac{\mu^{x-1}}{(x-1)!} = \mu + \mu^2 e^{-\mu}\sum_{x=2}^{\infty}\frac{\mu^{x-2}}{(x-2)!} = \mu + e^{-\mu}\mu^2\sum_{x=0}^{\infty}\frac{\mu^x}{x!}$$
$$= \mu + e^{-\mu}\mu^2 e^{\mu} = \mu(1+\mu)$$

c) $$\langle (x-\mu)^2\rangle = \langle x^2 - 2\mu x + \mu^2\rangle = \langle x^2\rangle - \langle 2\mu x\rangle + \langle \mu^2\rangle = \langle x^2\rangle - 2\mu\langle x\rangle + \mu^2$$
$$= \mu(1+\mu) - 2\mu^2 + \mu^2 = \mu + \mu^2 - \mu^2 = \mu$$

3. Let X and Y be independent random variables distributed according to the Poisson distribution with μ_x and μ_y.

Then let $Z = X + Y$, $\mu = \mu_x + \mu_y$

Now

$$P(Z=z) = \sum_{i=0}^{z} P(X=i, Y=z-i) \qquad \text{so that } X+Y=z$$

$$= \sum_{i=0}^{z} P(X=i)\,P(Y=z-i) \qquad X, Y \text{ independent.}$$

$$= \sum_{i=0}^{z} \frac{e^{-\mu_x}\mu_x^i}{i!} \cdot \frac{e^{-\mu_y}\mu_y^{z-i}}{(z-i)!}$$

$$= \sum_{i=0}^{\infty} \frac{1}{i!(z-i)!} e^{-\mu_x}\mu_x^i e^{-\mu_y}\mu_y^{z-i}$$

$$= \sum_{i=0}^{\infty} \frac{z!}{i!(z-i)!} \frac{e^{-\mu_x}\mu_x^i e^{-\mu_y}\mu_y^{z-i}}{z!} \qquad \left(\cdot \frac{z!}{z!}\right)$$

$$= \sum_{i=0}^{\infty} \binom{z}{i} \frac{e^{-\mu}}{z!} \mu_x^i \mu_y^{z-i} \qquad (\mu_x + \mu_y = \mu)$$

$$= \frac{e^{-\mu}}{z!} (\mu_x + \mu_y)^z \qquad \text{(binomial expansion)}$$

$$= \frac{e^{-\mu}}{z!} \mu^z$$, which is a Poisson distribution.

4. $\Gamma(1) = \int_0^\infty x^{1-1} e^{-x}\,dx = \int_0^\infty e^{-x}\,dx = -e^{-x}\Big|_0^\infty = -e^{-\infty} + 1 = 1$

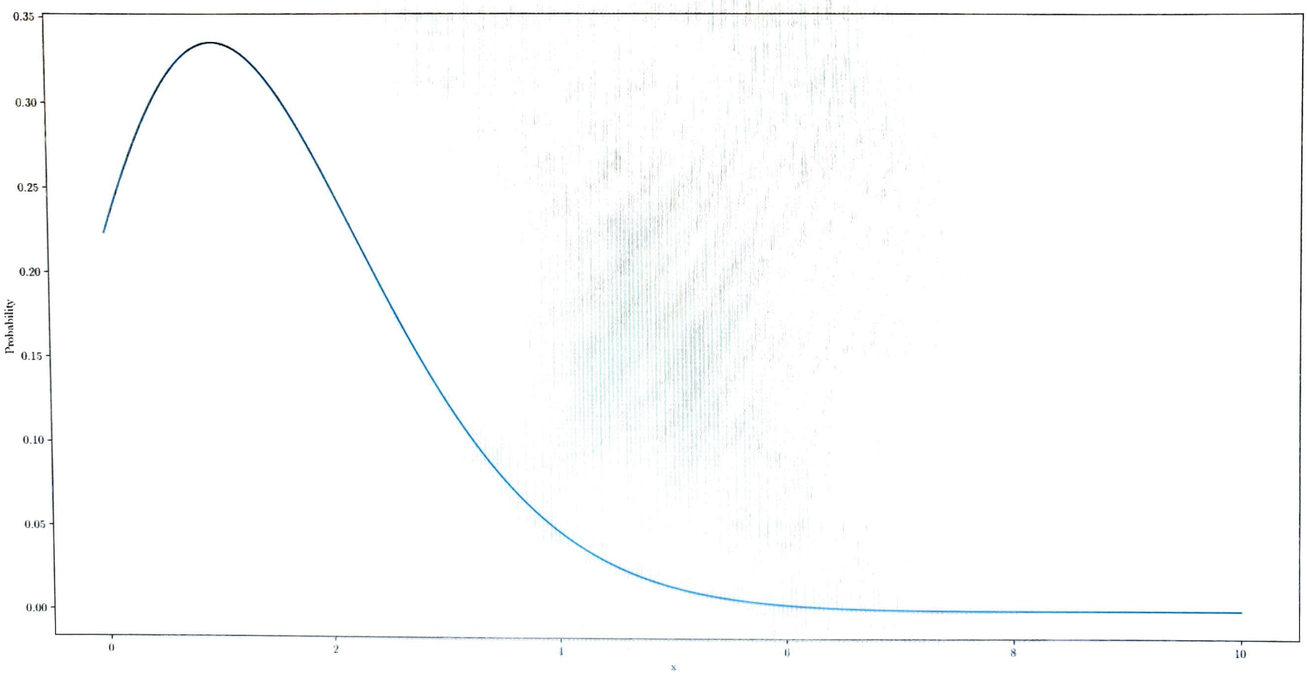
$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\,dx = -e^{-x}x^{n-1}\Big|_0^\infty - \int_0^\infty -e^{-x}(n-1)x^{n-2}\,dx$$

$$= [-e^{-\infty}\infty^{n-1} + (1\cdot 0)] + (n-1)\int_0^\infty e^{-x}x^{(n-1)-1}\,dx$$

$$= (n-1)\Gamma(n-1)$$

by the PMI... $\Gamma(n) = (n-1)!$ i guess

5.



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Probability
0
2
4
6
8
10
x

6. $\mu = 2.1$. so $P(0;2.1) = \frac{2.1^0}{0!} e^{-2.1} = 0.1224$

The mean interval between counts is $\frac{1}{2.1}$ s

7. The mean counts per trial is 3, given the ~~one~~ trial of the control group. So the probability of having fewer than 3 sick people in a following trial is $P(2;3) + P(1;3) + P(0;3) = \frac{3^2}{2!} e^{-3} + \frac{3^1}{1!} e^{-3} + \frac{3^0}{0!} e^{-3} = 0.4232$.

8. I don't have the tools to answer this.