Assignment 6

KDSMIL001 MAM2000W 2IA

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3. We can, in fact, drop the first condition. Provided H is non-empty, we can choose some $h \in H$. Since H is closed under inversion we have that $h^{-1} \in H$. Since H is also closed under the group operation, we have that $e = hh^{-1} = h^{-1}h \in H$.

4. For H to be a subgroup of G, it must pass the subgroup test.

   - Take the identity $e \in G$. $e^2 = ee = e$ and thus e satisfies the condition allowing $e \in H$.
   - If we take some $h_1, h_2 \in H$, given that G is abelian and $h_1, h_2 \in G$, we have
     $$h_1h_2h_1h_2 = h_1h_2h_2h_1 = h_1eh_1 = h_1h_1 = e \tag{0.1}$$
     so $h_1h_2 \in H$, i.e. H is closed under the group operation of G.
   - Take some $h \in H$. We have that $h^2 = hh = e = hh^{-1}$ under G, and by left cancellation law, we have that $h = h^{-1}$, so $h^{-1} \in H$.

   Thus H passes the subgroup test.

5. For $H \cap K$ to be a subgroup of G, it must pass the subgroup test.

   - H and K are subgroups of G, so we know that if e is the identity of G then $e \in H$ and $e \in K$. It is simple to see then that $e \in H \cap K$.
   - We know that H and K are closed under the group operation, which we will call $*$. Thus, given some $q_1, q_2 \in H \cap K$, we know that $q_1, q_2 \in H$ and $q_1, q_2 \in K$. Since H and K are subgroups of G, they must be closed under $*$ and so it must be that $q_1 * q_2 \in H$ and $q_1 * q_2 \in K$, and so clearly $q_1 * q_2 \in H \cap K$. So $*$ is closed under $H \cap K$.
   - Given some $q \in H \cap K$, we know then that $q \in H$ and $q \in K$. Since H and K are subgroups of G, they must be closed under inversion, so $q^{-1} \in H$ and $q^{-1} \in K$. Clearly this means $q^{-1} \in H \cap K$, so $H \cap K$ is closed under inversion.

   Thus $H \cap K$ passes the subgroup test.