Assignment 1

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MAM2000W 2IA KDSMIL001

1. Let P_n be the statement $3|n^3 - 4n$ for all $n \in \mathbb{N}$. Another way of saying this is $n^3 - 4n = 3t$ where $t \in \mathbb{Z}$.

   Base Case:
   $$P_n : 1^3 - 4(1) = 1 - 4 = -3$$
   It's simple to see that $3|-3$, so our base case is true.

   Inductive Hypothesis:
   We assume that the statement P_k is true, that is
   $$\begin{aligned} &3|k^3 - 4k \\ \implies &k^3 - 4k = 3p, p \in \mathbb{Z} \end{aligned}$$
   We now look at the case P_{k+1}:
   $$\begin{aligned} (k+1)^3 - 4(k+1) &= k^3 + 3k^2 + 3k + 1 - 4k - 4 \\ &= (k^3 - 4k) + (3k^2 + 3k - 3) \end{aligned}$$
   By the Inductive Hypothesis, we know that $k^3 - 4k = 3p$. We can also easily see that
   $$3k^2 + 3k - 3 = 3(k^2 + k - 1)$$
   We know that k is a natural number, so $k^2 + k - 1$ is an integer and therefore we can say $3k^2 + 3k - 3 = 3q, q \in \mathbb{Z}$. Finally we can say
   $$(k^3 - 4k) + (3k^2 + 3k - 3) = 3p + 3q = 3(p + q)$$
   The sum of two integers is an integer and so we can say that P_{k+1} is true and by the principle of mathematical induction P_n is true for all $n \in \mathbb{N}$.

   $\square$