The Magnetic Field of a Circular Coil: Induction and Inductance

KDSMIL001 PHY2004

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1 Introduction and Aim

In this practical we investigated the behaviour of the magnetic field produced due to an alternating current in a circular coil. This was done primarily by examining the induced voltage in a search coil placed near the primary coil.

2 Apparatus

The following equipment was used:

- Signal generator
- Power amplifier
- Ammeter
- Primary coil with 120 winds and diameter $(6.8 \pm 0.1) \times 10^{-2}\,\mathrm{m}$
- Secondary "search" coil with 175 winds and diameter $(1.3 \pm 0.1) \times 10^{-2}\,\mathrm{m}$
- Oscilloscope

The current in the primary coil was supplied by the amplifier, which was driven with a 2 V_{pp} sinusoidal signal from the signal generator. The ammeter was connected in series with the coil in order to monitor the current in the circuit. This ammeter displayed in rms, not amplitude, so we multiply by $\sqrt{2}$ in order to get the amplitude. Below is the set-up of the circuit.

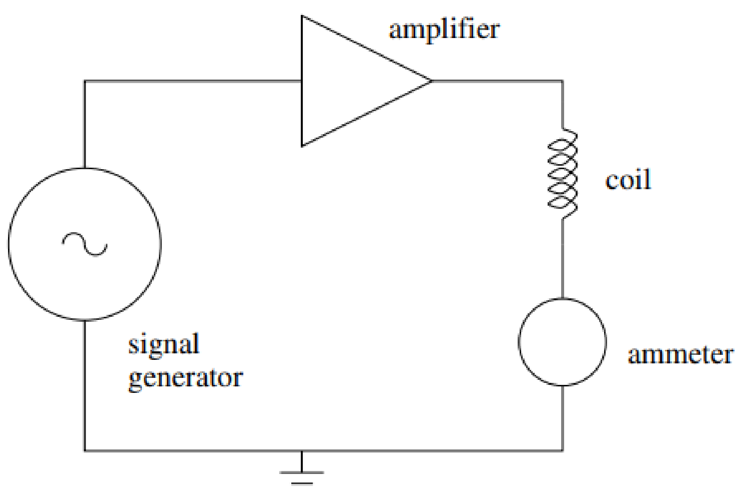

Figure 2.1: The primary circuit

Additionally, we had our secondary coil connected to an oscilloscope in order to monitor the induced emf ϵ. This search coil was placed on a contraption that allowed us to hold it at set distances from the primary coil, along the primary coil's axis.

3 Experiment

3.1 Field on the axis of a circular coil

In this section we looked specifically at the relationship between magnetic field $\vec{B}(\vec{r}, t)$ and induced ϵ. First we look at Faraday's law, which says

$$\epsilon = -N_1 \frac{d}{dt} \int \vec{B} \cdot d\vec{A} \tag{3.1}$$

In our case, we are aligning things in a way to have the magnetic field dependent only on the distance z from the primary coil. We also say that the magnetic field is directed along the z-axis, approximately, allowing us to consider amplitudes only. So we have

$$\begin{aligned} B(z,t) &\approx B(z)\cos(\omega t) \\ \implies \epsilon &\approx -N_a A \frac{d}{dt}(B(z)\cos(\omega t)) \\ &= N_a A B(z) \omega \sin(\omega t) \\ \implies B(z) &\approx \frac{\epsilon}{N_a A \omega} \end{aligned}$$

where N_a is the number of winds in the search coil (175), A is the cross-sectional area of the search coil, ω is the angular frequency that the primary coil is being driven at ($2\pi f$), and we have taken $\sin(\omega t)$ to be 1 as we are only interested in amplitudes. We now have a kind of "calibration factor", so when we measure the emf induced in the search coil, we can immediately know the approximate value of the magnetic field that induced it, i.e. the magnetic field produced by the primary coil.

We have a way of determining the magnetic field from the induced voltage, but we also want to know how well that method agrees with what we would expect from the primary coil. The magnitude of the magnetic field on the axis of a circular coil of radius a is

$$B(z,t) = \frac{\mu_0 N I(t)}{2} \frac{a^2}{(a^2 + z^2)^{\frac{3}{2}}} \tag{3.2}$$

where $\mu_0 = 4\pi \times 10^{-7}$ is the permeability of free space, N is the number of winds on the coil (120), and $I(t) = I_0 \cos(\omega t)$. Again we can take the cos term to be 1 as we're looking at amplitudes, which leaves us with

$$B(z) = \frac{\mu_0 N I_0}{2} \frac{a^2}{(a^2 + z^2)^{\frac{3}{2}}}$$

Finally we collected some data:
We ran the signal generator at $1000\,\mathrm{Hz} = 2000\pi\,\mathrm{rads}^{-1}$ and $2V_{pp}$, with the amplifier setting the current to $I_{rms} = (0.35300 \pm 0.00289)\,\mathrm{A} = (0.49498 \pm 0.00409)\,\mathrm{A}$. This uncertainty comes from reading the current off of our ammeter, which displayed 0.35 A rms, so we use a digital pdf with uncertainty $\frac{a}{2\sqrt{3}}$ and $a = 0.01$ to find $u(I_{rms}) = 0.00289$, so $u(I_0) = 0.00289\sqrt{2} = 0.00409$.
The area of the search coil is $A = (6.5 \times 10^{-3})^2\pi = (1.3273 \pm 0.0204) \times 10^{-4}\,\mathrm{m}^2$. This uncertainty comes from the uncertainty on the measurement of the diameter of the search coil, using the formula $u(x^n) = |n|x^{n-1}u(x)$.
The uncertainty on any experimentally determined B comes from the equation

$$u(B) = u(\frac{\epsilon}{N_a A\omega}) = \frac{\epsilon}{N_a A\omega}\sqrt{\left(\frac{u(\epsilon)}{\epsilon}\right)^2 + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(\omega)}{\omega}\right)^2}$$

where $u(\omega)$ is 2% of the scale used on the display of the signal generator, which was $1kHz$, so $u(\omega) = 0.02 \cdot 2000\pi = 40\pi$, and $u(\epsilon)$ is determined from the 2% uncertainty of the display of the oscilloscope combined with the digital measurement uncertainty

$$\begin{aligned} u(\epsilon) &= \sqrt{0.02^2 + \left(\frac{1 \times 10^{-5}}{2\sqrt{3}}\right)} \\ &= 0.01 \end{aligned}$$

Below is the data and the theoretical model

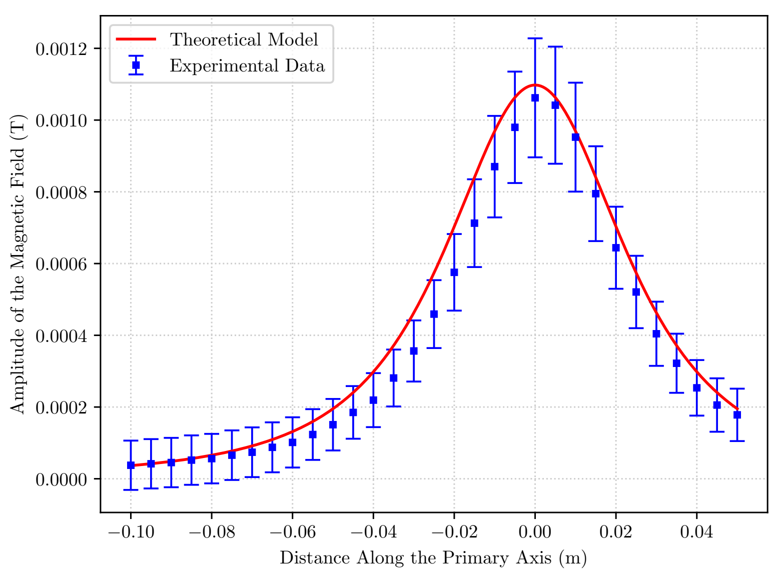

Figure 3.1: Magnetic field determined using the calibration factor and ϵ induced in a search coil due to a large primary coil, along with the theoretical prediction