PHY2004W: PHYLAB 2
Experimental Physics Lab Session
The Magnetic Field of a Circular Coil: Induction and Inductance

1 Introduction

We will investigate the magnetic field due to alternating current in a circular coil. We will do this by measuring the induced voltage in a search coil.

2 References

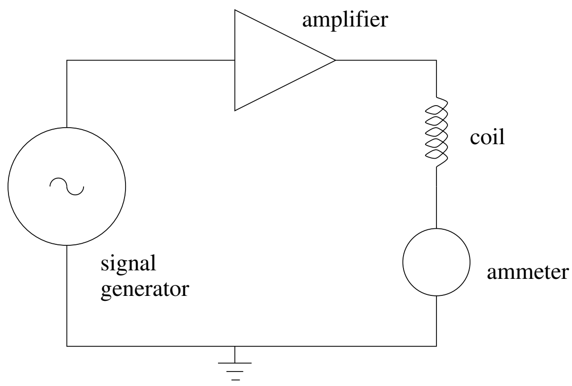
Most books on electromagnetism have something on this subject.
See e.g. Griffiths, *Introduction to Electrodynamics*.
Pollack and Stump, *Electromagnetism*, Addison-Wesley (2002): 10.2, 8.3, 8.7.

3 Emphasis

Basic concept (field of coil), connection of circuits, manipulation of measured data to infer magnetic field, graphical analysis.

4 Equipment

The large circular coil provided has 120 turns. This is used to produce the magnetic field. Current for the coil is supplied by the power amplifier, which is a simple audio amplifier, that is driven by the signal generator at a known frequency. The magnetic field is thus an alternating field, at the frequency determined by the signal generator. A digital ammeter (on 2.0 A scale) has been included in the circuit to monitor the current, such that we do not allow more than 1.0 A through the coil. High current will heat up the coil and distort the sine wave. Note that the meter gives the rms reading for the signal — to obtain the amplitude of the signal you must multiply the reading by $\sqrt{2}$. Note also that the meter is connected in series in the circuit — connecting it in parallel across the coil may damage the meter. (More precisely: it will blow a rather expensive fuse, and the meter will not work until it has been repaired.)



There is a "search coil" for measuring the resulting magnetic field. This *axial* coil is on the end of the rod that can be slid along the axis of the large coil. The axial search coil has 175 turns.

The time varying magnetic field produced by the large coil induces voltages in the coils which may be measured with the oscilloscope. The free coil is only intended for qualitative measurements.

Watch the following video **Setup** IND01

5 Instructions

5.1 Field on the axis of a circular coil.

We will first determine the field of the main coil on the central axis of the coil using the axial search coil. We adjust the signal generator to about 1 kHz 2.0V_{pp} and use the master volume of the amplifier to set the current to around 500 mA which is 353 mA RMS.
We use the axial search coil to measure the induced emf at points along the axis of the circular coil and plot a graph of the amplitude of the emf in arbitrary units against the distance from the centre of the coil.
We take measurements on both sides of the coil (i.e. for both positive and negative values of z) to ensure that the maximum can be precisely located.

Watch the following video to see how the data is taken **Axial Field Measurement** IND02.

The data to be analysed is given here: **Axial Field Data** DATA01

The varying magnetic field induces an emf in the axial coil. The amplitude of this emf can be used to determine the amplitude of the magnetic field (by Faraday's law). The magnetic field on the axis is directed along the $z-$axis; this is approximately true over the cross-sectional area of the search coil. The field is thus approximately $B(z,t) = B(z)\cos\omega t$ where z is measured from the centre of the large coil. The emf induced in the axial coil is given by

$$\epsilon = -N_a \frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} \approx -N_a A \frac{dB(z,t)}{dt} \tag{1}$$

where N_a is the number of turns in the coil, A is the cross-sectional area and B is the component of the field normal to the area. The field (and hence the emf) vary sinusoidally at the frequency set on the signal generator.
From the above expression (1) you can find a relation between the amplitude of the emf from the data and amplitude of the magnetic field at the position of the axial coil (averaged over the area of the coil). This provides a calibration factor for the apparatus that permits you to determine

the field from the emf from the data: in other words, a measurement of the emf is a measurement of the magnetic field.
Now, the theoretical expression for the magnitude of the magnetic field on the axis of a circular coil of radius a is

$$B(z,t) = \frac{\mu_0 N I(t)}{2} \frac{a^2}{(a^2+z^2)^{3/2}} \tag{2}$$

where z is the distance to the centre of the coil, N is the number of turns in the coil and $I = I_0 \cos \omega t$ is the current in the circuit.

Compare the measured value for the (amplitude of) the field (i.e. the experimental data) with the field as calculated from the above theoretical expression. Note that the multimeter reads the rms current. Plot a graph of the theoretical expression on the same axes as your experimental graph, normalising the theoretical expression so that the maxima of the two graphs have the same value. Compare the two graphs.

You will need to know the diameters of the main coil $d_{mc} = (6.8 \pm 0.1)\,\text{cm}$ and the search coil $d_{cc} = (1.3 \pm 0.1)\,\text{cm}$ in order to apply the expression.

Replot your experimental data together with the calculated induced voltage. How well do they agree?

5.2 Induction.

The axial coil is now moved to to $z = 0$ where the magnetic field is at a maximum. We vary the frequency over the range 100 Hz - 2 kHz.

Exercise 1 - If we operate the coil below this frequency range this will lead to the coil heating up. Think of why this would be the case.

We measure the amplitude of the voltage across the large coil, and the amplitude of the emf induced in the axial coil. At each frequency we have to adjust the current to a fixed value of 800 mA which is around 570 mA RMS, to ensure that the magnetic field produced by the large coil is remains constant.

Watch the following video to see how the data is taken **Induction Measurement** IND03.

The data to be analysed is given here: **Induction Data** DATA02

Exercise 2 - Write down reasons as to why the current needs to be adjusted as the frequency is changed.

By making use of expression (1) with the data obtained verify that the induced voltage is proportional to the frequency, and compare the slope of the graph to the calculated value. Take note that the calculated value is determined from the constant of proportionality between the emf ϵ and $\frac{dB(z,t)}{dt}$ as given in expression (1).

You will need to Evaluate expression (1) with the explicit form of the magnetic field given by: $B(z,t) = B \cos \omega t$.

Remember to include the magnitude B of the magnetic field from the large coil, the amplitude I_0 which you kept fixed as you swept the frequency. You can do this using expression (2) to determine the amplitude as well as using the calibration factor you determined earlier relating the field to the emf. Use both methods and compare your final result.
Make use of the variation of the voltage across the large coil with frequency to determine the resistance and inductance of the coil.

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