The Magnetic Field of a Circular Coil: Induction and Inductance

KDSMIL001 PHY2004

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Contents

1 Introduction and Aim

In this practical we investigated the behaviour of the magnetic field produced due to an alternating current in a circular coil. This was done primarily by examining the induced voltage in a search coil placed near the primary coil.

2 Apparatus

The following equipment was used:

- Signal generator
- Power amplifier
- Ammeter
- Primary coil with 120 winds and diameter $(6.8 \pm 0.1) \times 10^{-2}$ m
- Secondary "search" coil with 175 winds and diameter $(1.3 \pm 0.1) \times 10^{-2}$ m
- Oscilloscope

The current in the primary coil was supplied by the amplifier, which was driven with a 2 V_{pp} sinusoidal signal from the signal generator. The ammeter was connected in series with the coil in order to monitor the current in the circuit. This ammeter displayed in rms, not amplitude, so we multiply by $\sqrt{2}$ in order to get the amplitude. Below is the set-up of the circuit.

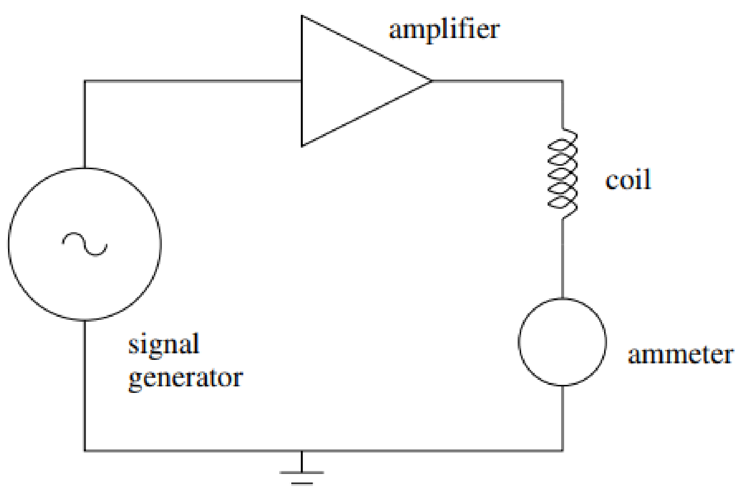

Figure 2.1: The primary circuit

Additionally, we had our secondary coil connected to an oscilloscope in order to monitor the induced emf ϵ. This search coil was placed on a contraption that allowed us to hold it at set distances from the primary coil, along the primary coil's axis.

3 Experiment

3.1 Field on the axis of a circular coil

In this section we looked specifically at the relationship between magnetic field $\vec{B}(\vec{r},t)$ and induced ϵ. First we look at Faraday's law, which says

$$\epsilon = -N_1 \frac{d}{dt} \int \vec{B} \cdot d\vec{A} \tag{3.1}$$

In our case, we are aligning things in a way to have the magnetic field dependent only on the distance z from the primary coil. We also say that the magnetic field is directed along the z-axis, approximately, allowing us to consider amplitudes only. So we have

$$\begin{aligned} B(z,t) &\approx B(z)\cos(\omega t) \\ \implies \epsilon &\approx -N_a A \frac{d}{dt}(B(z)\cos(\omega t)) \\ &= N_a A B(z) \omega \sin(\omega t) \\ \implies B(z) &\approx \frac{\epsilon}{N_a A \omega} \end{aligned}$$

where N_a is the number of winds in the search coil (175), A is the cross-sectional area of the search coil, ω is the angular frequency that the primary coil is being driven at $(2\pi f)$, and we have taken $\sin(\omega t)$ to be 1 as we are only interested in amplitudes. We now have a kind of "calibration factor", so when we measure the emf induced in the search coil, we can immediately know the approximate value of the magnetic field that induced it, i.e. the magnetic field produced by the primary coil.
We have a way of determining the magnetic field from the induced voltage, but we also want to know how well that method agrees with what we would expect from the primary coil. The magnitude of the magnetic field on the axis of a circular coil of radius a is

$$B(z,t) = \frac{\mu_0 N I(t)}{2} \frac{a^2}{(a^2+z^2)^{\frac{3}{2}}} \tag{3.2}$$

where $\mu_0 = 4\pi \times 10^{-7}$ is the permeability of free space, N is the number of winds on the coil (120), and $I(t) = I_0 \cos(\omega t)$. Again we can take the cos term to be 1 as we're looking at amplitudes, which leaves us with

$$B(z) = \frac{\mu_0 N I_0}{2} \frac{a^2}{(a^2+z^2)^{\frac{3}{2}}}$$

Finally we collected some data:
We ran the signal generator at $1000\,\mathrm{Hz} = 2000\pi\,\mathrm{rads}^{-1}$ and $2V_{pp}$, with the amplifier setting the current to $I_{rms} = (0.35300 \pm 0.02020)\,\mathrm{A} = (0.495 \pm 0.029)\,\mathrm{A}$. This uncertainty comes from reading the current off of our ammeter, which displayed 0.35 A rms, so we use a digital pdf with uncertainty $\frac{a}{2\sqrt{3}}$ and $a = 0.01$ as well as the 2% uncertainty rating on the ammeter to find $u(I_{rms}) = \sqrt{0.02^2 + \frac{0.01}{2\sqrt{3}}} = 0.02020$, so $u(I_0) = 0.02020\sqrt{2} = 0.029$.
The cross-sectional area of the search coil is $A = (6.5 \times 10^{-3})^2\pi = (1.3273 \pm 0.0204) \times 10^{-4}\,\mathrm{m}^2$. This uncertainty comes from the uncertainty on the measurement of the diameter of the search coil, using the formula $u(x^n) = |n|x^{n-1}u(x)$.
The uncertainty on any experimentally determined B comes from the equation

$$u(B) = u(\frac{\epsilon}{N_a A\omega}) = \frac{\epsilon}{N_a A\omega}\sqrt{\left(\frac{u(\epsilon)}{\epsilon}\right)^2 + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(\omega)}{\omega}\right)^2}$$

where $u(\omega)$ is 2% of the scale used on the display of the signal generator, which was $1kHz$, so $u(\omega) = 0.02 \cdot 2000\pi = 40\pi$, and $u(\epsilon)$ is determined from the 2% uncertainty of the display of the oscilloscope combined with the digital measurement uncertainty

$$\begin{aligned} u(\epsilon) &= \sqrt{0.02^2 + \left(\frac{1 \times 10^{-5}}{2\sqrt{3}}\right)} \\ &= 0.01 \end{aligned}$$

The data and the theoretical model are in Figure 3.1.

3.2 Frequency Dependence of Induced Voltage

This section focuses on the effect of varying the frequency of alternating current on the system we're investigating. We suspect that the relationship is linear. For simplicity's sake we moved the coil to position $z = 0$ and varied the frequency from 100 Hz to 2 kHz.

We didn't operate below 100 Hz as that would result in an increase in current through the coil as the impedance would decrease, so the coil would heat up.

The current was kept at $I_{rms} = (0.5700 \pm 0.0202)\,\mathrm{A}$ so $I_0 = (0.806 \pm 0.029)\,\mathrm{A}$ using the same methods as before. This is done to ensure that the magnetic field remains at the same magnitude throughout the experiment. The current needs to be adjusted as we change frequency to keep it the same as $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2+L^2\omega^2}}$, so the current changes with respect to frequency.
Now onto the experiment: Altering the current at each frequency and finding the induced ϵ at each ω, we found the plot in Figure 3.2.

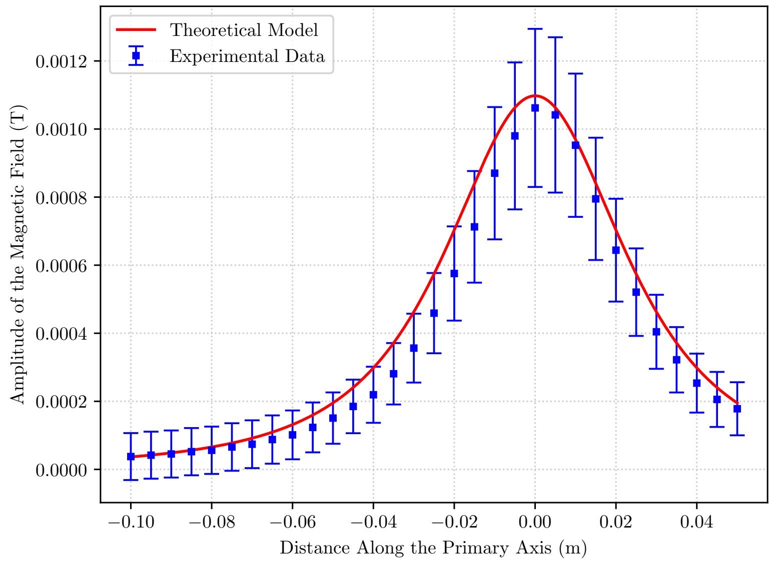

Figure 3.1: Magnetic field determined using the calibration factor and ϵ induced in a search coil due to a large primary coil, along with the theoretical prediction made using Equation 3.2

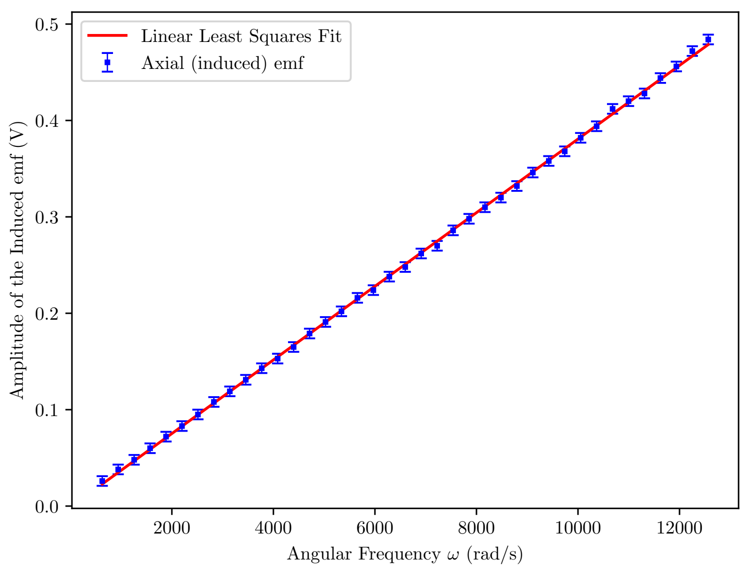

Figure 3.2: The emf induced in the axial coil due to a changing frequency of alternating current through the primary coil, kept at the same current to preserve the magnitude of the magnetic field.

The data is clearly linear. This confirms our earlier suspicion, but now let us check if our gradient calculated from the line of best fit agrees with what we would expect. Our linear least squares fit gave us a value of $m = (3.8188 \pm 0.0104) \times 10^{-5}$. To verify this we will use two methods. We notice that Equation 3.1 gives us a relation between induced ϵ and ω:

$$\epsilon = N_a A \omega B(z) \tag{3.3}$$

Since we are keeping B constant, we should have a gradient of $m = N_a AB$. N_a and A are the number of winds in the axial coil and the cross-sectional area of the axial coil respectively and are constants. B is also a constant but we have two ways of determining it. Firstly we can find it from the induced ϵ, using our calibration factor from the previous section on each ϵ and averaging it to find a B. Secondly we can use Equation 3.2 where $z = 0$, so

$$B = \frac{\mu_0 N I_0}{2a} \tag{3.4}$$

Using the first method we found $B_1 = (1.640 \pm 0.056) \times 10^{-3}\,\mathrm{T}$, so $m_1 = (3.81 \pm 0.14) \times 10^{-5}$. The uncertainty on B_1 is found with

$$u(B_1) = \frac{1}{n} u(\sum_i B_i) = \frac{1}{n}\sqrt{\sum_i u(B_i)^2}$$

$$u(B_i) = B_i \sqrt{\left(\frac{u(\epsilon)}{\epsilon}\right)^2 + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(\omega)}{\omega}\right)^2}$$

Using the second method we found $B_2 = (1.788 \pm 0.028) \times 10^{-3}\,\mathrm{T}$, so $m_2 = (4.15 \pm 0.64) \times 10^{-5}$. The uncertainty on B_2 is found with

$$u(B_2) = B_2 * \sqrt{\left(\frac{u(I_0)}{I_0}\right)^2 + \left(\frac{u(a)}{a}\right)^2}$$

where $u(I_0)$ is as given above and $u(a) = \frac{u(d_{primary})}{2} = 0.05 \times 10^{-2}\,\mathrm{m}$.

Experimental:	$m_E = (3.8188 \pm 0.0104) \times 10^{-5}$
Theoretical:	$m_1 = (3.81 \pm 0.14) \times 10^{-5}$
	$m_2 = (4.15 \pm 0.64) \times 10^{-5}$

Table 3.1: The experimental and two theoretical determinations of the gradient of induced voltage ϵ with respect to ω, the angular frequency of the AC voltage

3.3 Resistance and Inductance of the Primary Coil

This section focuses in on the primary coil as we aimed to determine the resistance and inductance of the primary coil. We used the same set-up as the previous section.

To do this we look at the equation

$$\begin{aligned} V &= IZ \\ &= I\sqrt{R^2 + L^2\omega^2} \end{aligned}$$

and see that, since we have kept the current constant, we have V as a function of ω, with R and L as parameters that we can optimise using `scipy.optimize.curve_fit`. Using the data from before, but this time with the voltage across the primary coil, we have Figure 3.3.

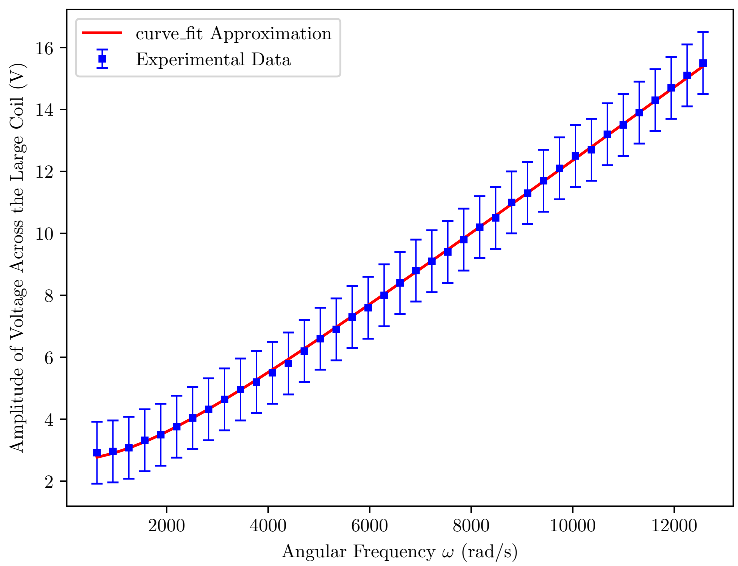

Figure 3.3: The amplitude of the voltage across the primary coil as a function of the angular frequency ω, with the `curve_fit` approximation of a line of best fit using the Jackknife method.

The error on the experimental data is a result of the limitations of the oscilloscope display and the uncertainty associated with digital measurement, given by

$$u(V_{\text{primary}}) = \frac{\sqrt{\left(\frac{0.01}{2\sqrt{3}}\right)^2 + 2^2}}{2}$$

Note the division by 2 that's necessary as the data was recorded as a V_{pp}, so we divide it by 2 to find the amplitude, and thus divide the uncertainty by 2.
`curve_fit` gives us the following values for R and L, with the uncertainties coming

from the Jackknife method of error approximation.

$$R = (3.3326 \pm 0.0085)\,\Omega$$
$$L = (1.50820 \pm 0.00037) \times 10^{-3}\,\mathrm{H}$$