Hall Effect

KDSMIL001 PHY2004W PHyLAB 2

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Contents

1 Introduction

The Hall Effect is a well known effect that tells us a lot about the way that electric charge conducts in materials. We will use it to determine some properties of the material we're using, which is the p-type semiconductor germanium.

2 Theory

If we have a block of material that conducts electricity and we put some current through it there will be some kind of charge carrier moving through the material. Depending on the material this could be either electrons, which are negatively charged, or positively charged "holes". If the material in question is placed within a magnetic field these moving charges will feel a force in a direction perpendicular to their movement, dictated by

$$\vec{F} = q\vec{v} \times \vec{B} \tag{2.1}$$

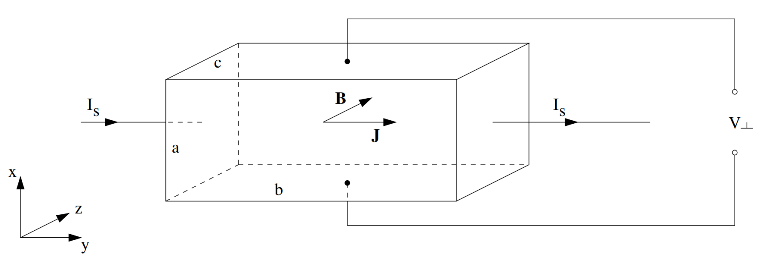
Because this force is perpendicular to the direction of movement of the charge carriers, we will see some charge build-up on one side of the material, and a lack of charge on the other. This means we can measure the voltage drop across the sides of the material. This voltage is governed by the equation

$$V_{\perp} = R_H I_S B/c \tag{2.2}$$

where B is the strength of the magnetic field, I_S is the applied current, and $R_H \equiv \frac{1}{nq}$ is known as the Hall coefficient, with n being the number of charge carriers in the material and q being the amount of charge each carrier can carry.
We can also measure the voltage drop across the length of the material, in the direction the current is flowing. That voltage is given by

$$V_{\parallel} = \frac{I_s b}{\sigma a c} \tag{2.3}$$

where σ is the electrical conductivity of the material. a, b, and c as well as the general set-up of the material is shown below.



By knowing all the other values, we can find values for R_H and σ.
Since we are using germanium, which is a semiconductor, the definition for R_H is

slightly different, it even depends on what type of charge carrier the semiconductor has. For p-type semiconductors, which germanium is, the Hall coefficient is given by

$$R_H = \frac{1.4}{ne} \tag{2.4}$$

with e being the charge on an electron, $e = 1.60217662 \times 10^{-19}$ C.

Now all this theory is based on the idea that the material in question stays at the same temperature, since in semiconductors both the number of charges and their mobility within the material are dependent on temperature. The problem is that as we drive a current through the germanium, it is going to heat up thanks to Joule heating. Equation 2.2 and Equation 2.3 are both linear, but since we expect values to change depending on the temperature of the material, we can expect to see some deviation from linearity.

3 Apparatus

- DC power supply.
- Offset potentiometer.
- A 500 Ω resistor.
- Multimeter set to 200 mA scale to measure the current through the material.
- Multimeter set to 200 mV scale to measure the Hall voltage.
- Multimeter set to 20 V scale to measure the voltage along the sample. (All multimeters have ±1% error.)
- A piece of germanium. (See Figure 3.1.)

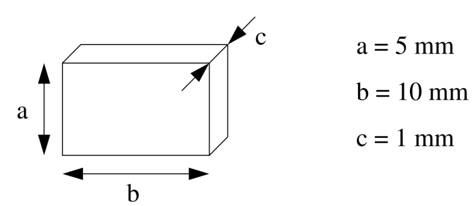


Figure 3.1: The dimensions of the sample of germanium used

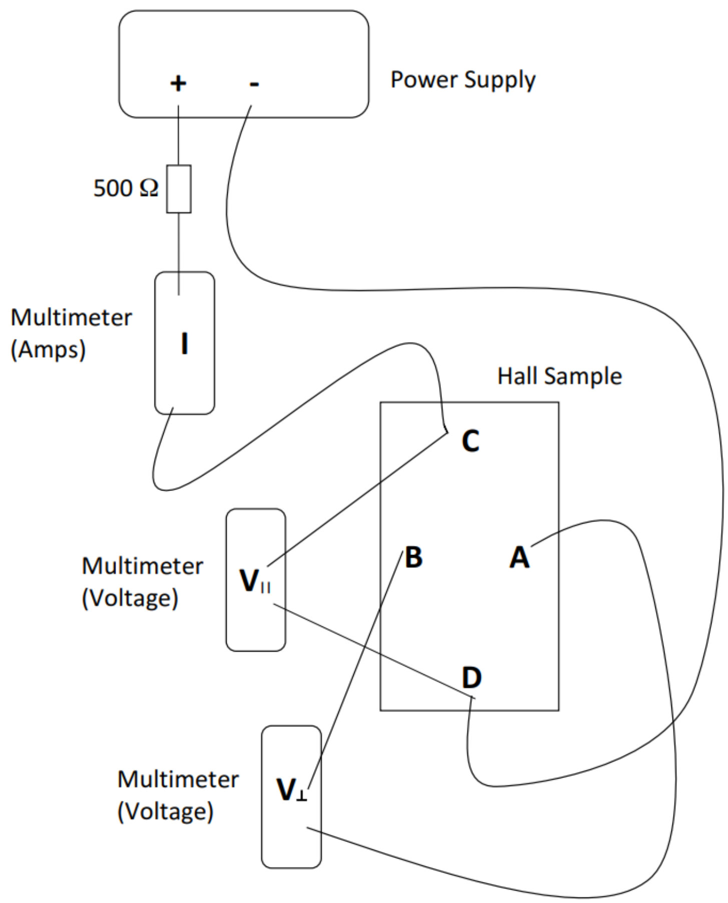


Figure 3.2: The set-up of the circuit

The potentiometer is used so we can adjust things to make sure that the Hall voltage is 0 when there is no magnetic field present, as a sort of calibration.

4 Method

The order of the steps taken is very important. They are as follows:

1. We connected the circuit as shown in Figure 3.2. It is important that the current through the sample does not exceed 50 mA so the 500 Ω resistor must be in place before switching the power supply on.

2. Before switching the power supply on, we set the current controls to their maximum and the voltage controls to their minimum, then turned the power supply on.

3. We then adjusted the voltage to 22 V on the power supply display, making sure that the current measured by our multimeter doesn't exceed 40 mA.

4. We then set the current in the sample to about 38 mA with no magnetic field near to the sample, varying our potentiometer to obtain a Hall voltage of 0 V.

5. We then measured the Hall voltage, $v_\perp$, and the voltage along the sample, $V_\parallel$, as a function of the current I over the range 0 to 40 mA. We made sure not to exceed 22 V on the power supply display. We call this the forward data.

6. We then left the circuit running at 40 mA for 10 minutes, allowing the sample to heat up, and then performed the same measurements, this time going backwards.

Some important things to note: We measured the temperature of the sample before we started and found it to be 25.4 C, as well as after the 10 minutes at 40 mA, finding that to be 31.3 C. This change is as a result of the Joule heating mentioned earlier.

5 Results

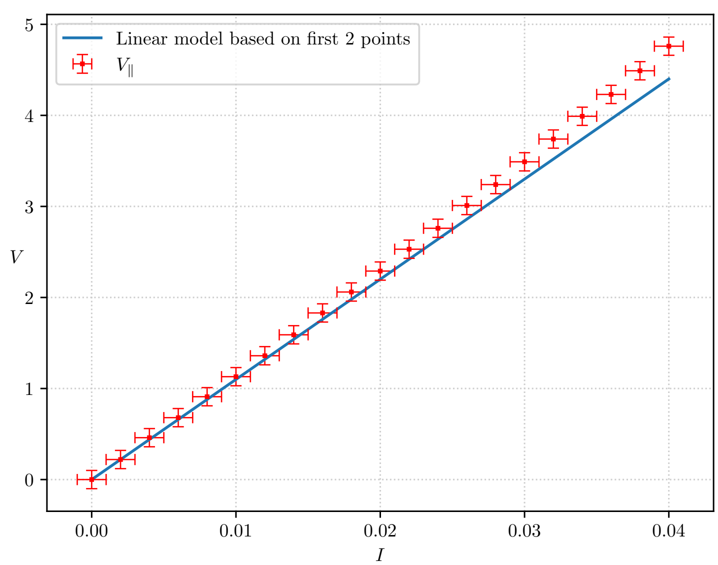


Figure 5.1: $V_\parallel$ plotted against I_S for the data collected going forward.

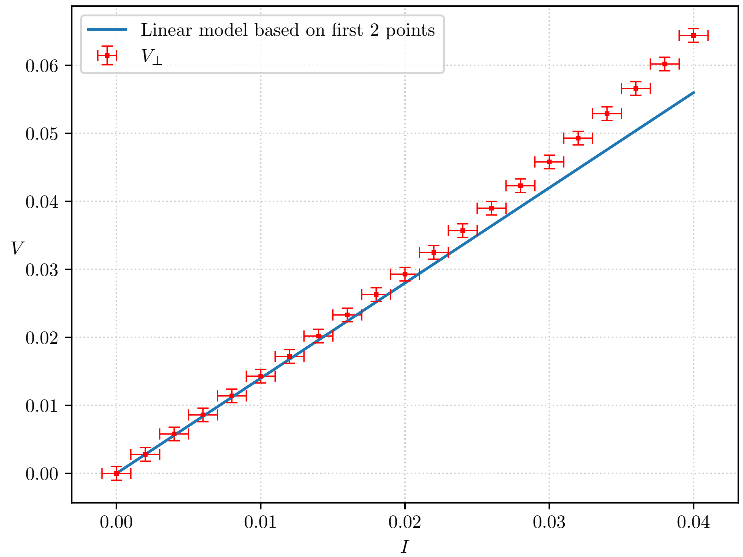


Figure 5.2: $V_{\perp}$ plotted against I_S for the data collected going forward.

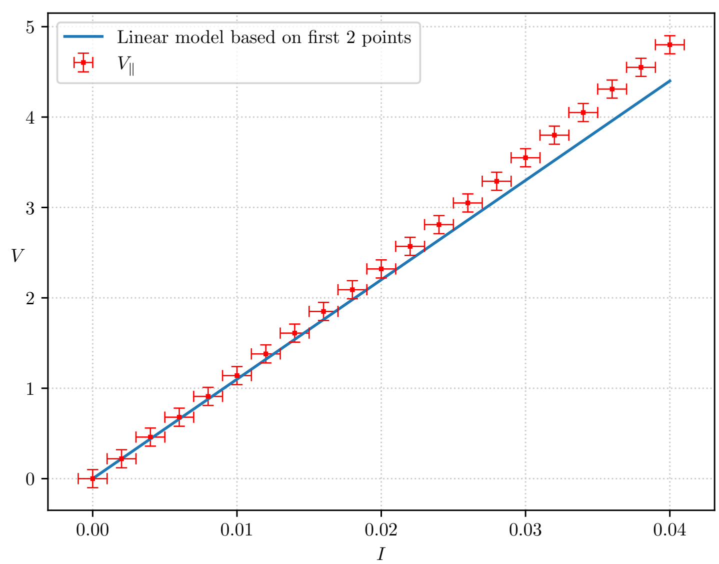


Figure 5.3: $V_{\parallel}$ plotted against I_S for the data collected going backwards.

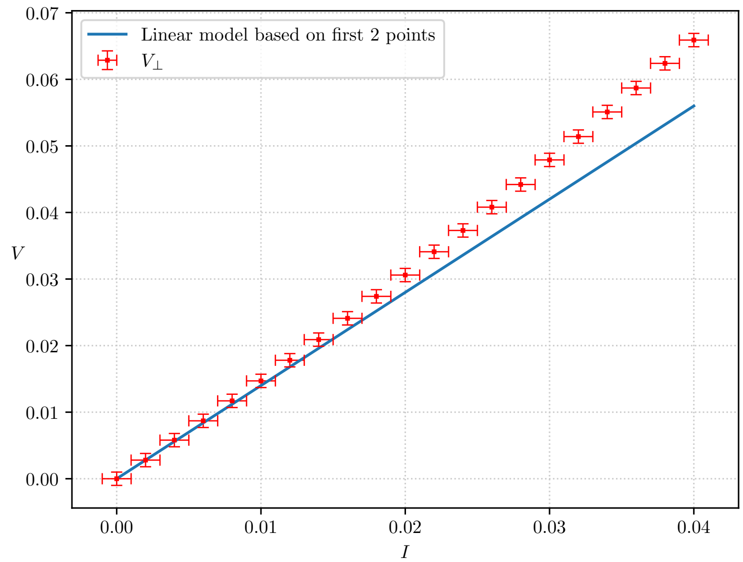


Figure 5.4: $V_\perp$ plotted against I_S for the data collected going backwards.

As a check of linearity we have calculated $V_\perp/I_S$ and $V_\perp/V_\parallel$ for each set of data and below are the approximate means and variances of each set.

	Forward	Backwards
$V_\perp/I_S$		
Mean	1.488	1.538
Variance	0.00338	0.00503
$V_\perp/V_\parallel$		
Mean	0.0129	0.0132
Variance	8.494×10^{-8}	1.149×10^{-7}

Table 5.1: Means and Variances for $V_\perp/I_S$ and $V_\perp/V_\parallel$ for both the forward and backwards data sets.

We also want to find values for R_H, σ, and n. To do this we use Equation 2.2, Equation 2.3, and Equation 2.4. We could look at these equations and see that, if the data was linear we could find the gradients and get very accurate estimations for these values, but our data isn't linear, so we must select the "best values" and just work with that. We have decided that the best values would be those measured at the beginning of the experiment when the germanium hadn't heated up that much. So that would be the first non-zero values in the forward data.

So we have

$$
\begin{aligned}
R_H &= \frac{V_\perp c}{I_s B} \\
&= \frac{0.0028 \cdot 0.001}{0.002 \cdot 0.125} \\
&= 0.0112 \\
u(R_H) &= R_H \sqrt{\left(\frac{u(V_\perp)}{V_\perp}\right)^2 + \left(\frac{u(I_S)}{I_S}\right)^2 + \left(\frac{u(B)}{B}\right)^2} \\
u(V_\perp) &= \sqrt{\left(\frac{0.1 \times 10^{-3}}{2\sqrt{3}}\right)^2 + (1\% \cdot 100 \times 10^{-3})^2} \\
&= 1 \times 10^{-3}\,\mathrm{V} \\
u(I_S) &= \sqrt{\left(\frac{0.1 \times 10^{-3}}{2\sqrt{3}}\right)^2 + (1\% \cdot 100 \times 10^{-3})^2} \\
&= 1 \times 10^{-3}\,\mathrm{A} \\
u(B) &= 0.005 \\
\implies u(R_H) &= 5.632113635 \times 10^{-3}
\end{aligned}
$$

$$
\begin{aligned}
\sigma &= \frac{I_s b}{a c V_\parallel} \\
&= \frac{0.002 \cdot 0.01}{0.005 \cdot 0.001 \cdot 0.22} \\
&= 18.1818\overline{18} \\
u(\sigma) &= \sigma \sqrt{\left(\frac{u(I_S)}{I_S}\right)^2 + \left(\frac{u(V_\parallel)}{V_\parallel}\right)^2} \\
u(V_\parallel) &= \sqrt{\left(\frac{0.01}{2\sqrt{3}}\right)^2 + (1\% \cdot 10)^2} \\
&= 0.1001\,\mathrm{V} \\
\implies u(\sigma) &= 12.28600723
\end{aligned}
$$

$$
\begin{aligned}
n &= \frac{1.4}{R_H e} \\
&= \frac{1.4}{0.0112 \cdot 1.60217662 \times 10^{-19}} \\
&= 7.801886411 \times 10^{20} \\
u(n) &= \frac{1.4}{e} \cdot u(1/R_H) \\
&= 3.92331 \times 10^{21}
\end{aligned}
$$

We can also find μ, the mobility of the charge carriers in the superconductor. It is given by

$$\begin{aligned}
\mu &= \frac{\sigma}{ne} \\
&= \frac{18.181818}{7.801886411 \times 10^{20} \cdot 1.60217662 \times 10^{-19}} \\
&= 0.1454\overline{545} \\
u(\mu) &= \mu\sqrt{\left(\frac{u(\sigma)}{\sigma}\right)^2 + \left(\frac{u(n)}{n}\right)^2} \\
&= 0.739269
\end{aligned}$$

So we have

R_H	$(0.0112 \pm 0.0056)\,\mathrm{m^3C^{-1}}$
σ	$(18 \pm 12)\,\mathrm{Sm^{-1}}$
n	$(0.78 \pm 3.90) \times 10^{21}$
μ	$(0.14 \pm 0.74)\,\mathrm{m^2V^{-1}s^{-1}}$

Table 5.2: Values for room temperature germanium

Exercises: Regarding the change in conductivity of the sample with respect to temperature, we can calculate the values calculated above from the reading taken after 10 minutes at 40 mA:

1.

$$\begin{aligned}
\sigma &= \frac{I_s b}{acV_{\parallel}} \\
&= \frac{0.040 \cdot 0.01}{0.005 \cdot 0.001 \cdot 4.81} \\
&= 16.63201663 \\
u(\sigma) &= 12.28600723
\end{aligned}$$

2.

$$
\begin{aligned}
R_H &= \frac{V_\perp c}{I_s B} \\
&= \frac{0.0661 \cdot 0.001}{0.04 \cdot 0.125} \\
&= 0.01322 \\
u(R_H) &= 5.632113635 \times 10^{-3} \\
n &= \frac{1.4}{R_H e} \\
&= \frac{1.4}{0.01322 \cdot 1.60217662 \times 10^{-19}} \\
&= 6.60976761 \times 10^{20} \\
u(n) &= 2.815957812 \times 10^{21}
\end{aligned}
$$

3.

$$
\begin{aligned}
\mu &= \frac{\sigma}{ne} \\
&= \frac{16.63201663}{6.60976761 \times 10^{20} \cdot 1.60217662 \times 10^{-19}} \\
&= 0.157053757 \\
u(\mu) &= 0.679079252
\end{aligned}
$$

We have

R_H	$(0.0132 \pm 0.0056)\,\mathrm{m^3C^{-1}}$
σ	$(16 \pm 12)\,\mathrm{Sm^{-1}}$
n	$(0.66 \pm 3.90) \times 10^{21}$
μ	$(0.15 \pm 0.68)\,\mathrm{m^2V^{-1}s^{-1}}$

Table 5.3: Values for higher than room temperature germanium

6 Discussion

Looking at the figures in section 5, we see that if we take a gradient from the first two points and extend that line along the whole data set, we see that the data deviates from linearity without exception. It deviates the most for the Hall voltage, which makes sense as the values of the Hall voltage are much smaller than those for $V_\parallel$. The backwards data also seems to be slightly more nonlinear than the forwards data. We are not entirely sure why this is, but it might be that after the 10 minutes at 40 mA, the germanium reached its peak temperature and then as the current decreased, it cooled down faster than it heated up when taking the forwards data, but this is just

speculation.
Regarding Table 5.1, we can see that while the variances are small, they are still big enough relative to the mean values, which shows that there is some deviation in the data sets, further indicating nonlinearity. **Exercises:**

1. Our calculated value for σ decreases as the temperature increases, but since the uncertainty on both the room temperature and higher temperature values is so large, it is hard to say for sure whether the conductivity increases or decreases with temperature. What we expect is that the conductivity increases with temperature as more charge carriers are released from the atoms of the material. Our results are inconclusive.

2. Again we see the opposite of what we expect, since we should see more charge carriers, but instead we see a decrease. Again, however, our results are inconclusive since the uncertainty on both values is greater than the value itself.

3. Lastly, we see a slight increase in mobility as temperature increases. Both values agree within experimental uncertainty with the provided value of $0.18\,\mathrm{m^2V^{-1}s^{-1}}$, but not with the idea that mobility should decrease with temperature.

We are not sure why we seem to observe the opposite of what we expect when increasing the temperature of the sample. Perhaps the sample is not a pure semiconductor.

7 Conclusion

To conclude, we were able to find values for R_H, σ, n, and μ for a sample of germanium, with varying levels of accuracy. When trying to investigate the effect of temperature on these values, we observed the opposite to what we expect for a this material. We are not sure why this has happened.