PHYLAB 2
Experimental Physics Lab
Capacitors

1 Introduction

A capacitor is a passive device that is used in a multitude of electronic circuits. Understanding exactly how it operates and how it can be used is essential for any physicist and engineer.
In this lab you will learn how a capacitor charges and discharges. You will learn what a time constant is, in the context of an RC circuit and you will investigate how the capacitor responds to a change in frequency of an applied AC voltage signal. You will also investigate the relationship between current and voltage.

2 Theory

Work through the theory on Capacitance below and complete all the exercises contained in the text boxes. Watch the video posted on Vula to give you extra assistance explaining the key concepts **Capacitance Theory** CAP01

2.1 Capacitance

As shown from Figure 1a on page 2, a capacitor is essentially two parallel plates with area A, separated by some distance d. The distance between the plates can be made up of a vacuum (vacuum permittivity ϵ_0) or another material with a specified dielectric constant ϵ. Figure 1b on page 2 shows the electric fields associated with the charge on the plates. Notice the fringe effects on the sides of the capacitor.
The amount of charge q, that is placed on the plates is proportional to the voltage V that is applied to the plates. The expression for this is given by $q = CV$ where C is the proportionality constant known as the capacitance.
The capacitance is determined by three parameters, the area of the plates A, the distance between the plates d and the dielectric constant of the material ϵ between the plates.

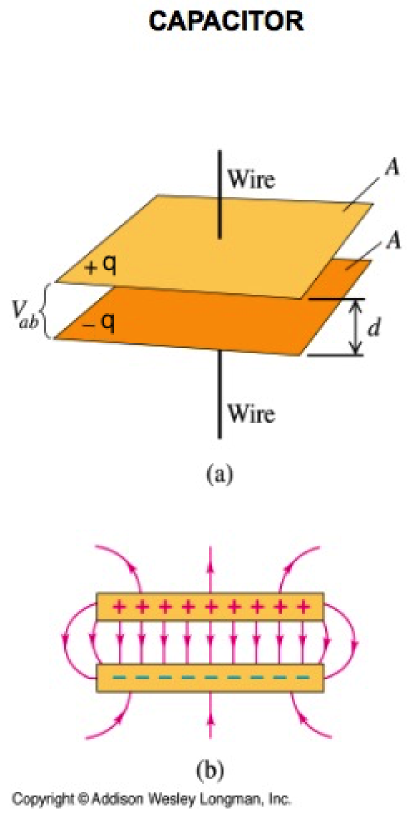

Figure 1:

Exercise 1 - Include full answer in final submitted lab report.

How are A, d and ϵ related. Think about what affect increasing A or d would have on the charge placed on the plates. Use your knowledge of electric fields to explain the effect. Don't forget to include this expression and a description of it in your final lab report, marks will be allocated to this.

2.2 Charging and Discharging

To study the charging and discharging properties of a capacitor we will make use of an RC circuit as shown in Figure 2a and Figure 2b on page 3

2.2.1 Charging

An RC circuit comprises a resistor with resistance R connected in series with a capacitor of capacitance C. The circuit is connected to a power supply with a DC voltage V_ϵ and a switch, as shown in Figure 2. If the switch is initially open we start with the charge q on the plates equal to zero. When the switch is closed, current will flow from the power supply in the direction as indicated towards the plates of the capacitor. The plates will begin to charge. The time it takes to charge the plates depends both on C and R of the circuit.

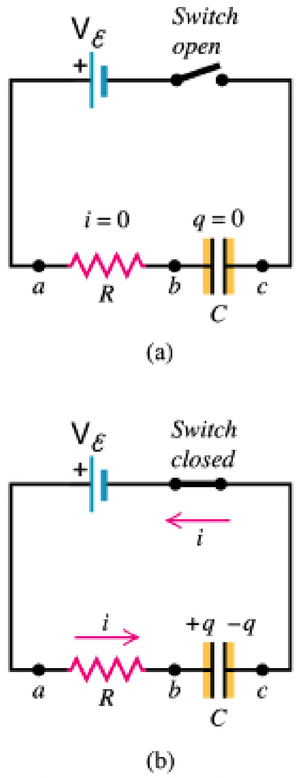

Figure 2:

Exercise 2 - Include full answer in your final submitted lab report.

By making use of dimensional analysis show that the units of the charging time of a capacitor τ as given by:

$$\tau = RC \tag{1}$$

is seconds (s).

As current flows, the instantaneous voltage drop across the resistor plus the instantaneous voltage drop across the capacitor must equal the voltage applied by the power supply such that:

$$V_\epsilon = V_R(t) + V_C(t) = i(t)R + \frac{q(t)}{C} = R\frac{dq(t)}{dt} + \frac{q(t)}{C} \tag{2}$$

If we express the applied voltage from the voltage source V_ϵ in terms of q_0 the maximum charge that can be stored on the plates with the given capacitance C and substitute into equation (2), we obtain:

$$\frac{q_0}{C} = R\frac{dq(t)}{dt} + \frac{q(t)}{C} \tag{3}$$

After some rearrangement we obtain:

$$0 = q(t) + RC\frac{dq(t)}{dt} - q_0 \tag{4}$$

The solution of this differential equation is given by:

$$q(t) = q_0\left[1 - e^{-\frac{t}{RC}}\right] \tag{5}$$

The time constant $\tau = RC$ gives the time period within which the capacitor is charged to $\left[1 - e^{-1}\right]$ of its maximum charge q_0.
The solution can easily be expressed in terms of voltage using the relationship $q = CV$ such that the solution of this differential equation is given by:

$$V_C(t) = V_\epsilon\left[1 - e^{-\frac{t}{RC}}\right] \tag{6}$$

2.2.2 Discharging

If the switch of the circuit is now opened, the charge on the capacitor is going to decay away at a rate once again determined by the time constant τ of the circuit.
The movement of charge away from the capacitor results in a current:

$$I(t) = -\frac{dq(t)}{dt} \tag{7}$$

The voltage drop across the resistor and capacitor must sum to zero as there is no applied voltage source, such that:

$$-R\frac{dq(t)}{dt} = \frac{q(t)}{C} \tag{8}$$

After some rearrangement we obtain:

$$q(t) = -RC\frac{dq(t)}{dt} \tag{9}$$

The solution of this differential equation with the initial condition $q(t = 0) = q_0$ (the capacitor is fully charged) is given by:

$$q(t) = q_0 e^{-\frac{t}{RC}} \tag{10}$$

The solution can easily be expressed in terms of voltage using the relationship $q = CV$ such that the solution of this differential equation is given by:

$$V_C(t) = V_\epsilon e^{-\frac{t}{RC}} \tag{11}$$

2.3 AC excitation of a RC Circuit

We will now discuss the reactance of a RC circuit to an alternating voltage signal given by:

$$V_\epsilon(t) = V_0 \cos(\omega t) \tag{12}$$

Once again the voltage across the resistor and capacitor must sum to the applied voltage such that:

$$V_\epsilon(t) = V_0 \cos(\omega t) = V_R(t) + V_C(t) = i(t)R + \frac{q(t)}{C} = R\frac{dq(t)}{dt} + \frac{q(t)}{C} \tag{13}$$

It follows that:

$$q(t) + RC\frac{dq(t)}{dt} - CV_0 \cos(\omega t) = 0 \tag{14}$$

We know from the charging and discharging section that there is a delay given by the time constant τ with which a capacitor charges and discharges. We therefore expect there to be a phase shift of ϕ between the applied voltage $V_\epsilon(t)$ and the charge build up $q(t)$ on the capacitor such that:

$$q(t) = q_0 \cos(\omega t + \phi) \tag{15}$$

We now proceed by expressing equation (12) and (15) in complex form such that:

$$V_\epsilon(t) = V_0 e^{i\omega t} \tag{16}$$

and

$$q(t) = q_0 e^{i(\omega t + \phi)} \tag{17}$$

Inserting both the above equations into equation (13) and dividing through by $e^{i\omega t}$ we obtain:

$$V_0 = i\omega R q_0 e^{i\phi} + \frac{1}{C} q_0 e^{i\phi} \tag{18}$$

It thus follows:

$$q_0 e^{i\phi} = \frac{V_0}{\frac{1}{C} + i\omega R} \tag{19}$$

We now need to determine the unknown quantities q_0 and ϕ.

$q_0 e^{i\phi}$ is a common form of a complex number $z = |z|e^{i\phi}$ such that $|z| = q_0$. Recall that the modulus of z is given by:

$$|z| = \sqrt{zz^*} \tag{20}$$

We can therefore express q_0 as:

$$q_0 = \sqrt{\frac{V_0}{\left(\frac{1}{C} + i\omega R\right)} \frac{V_0}{\left(\frac{1}{C} - i\omega R\right)}} = \frac{V_0 C}{\sqrt{1 + (\omega RC)^2}} \tag{21}$$

To calculate the phase angle ϕ we recall that the complex number z can be expressed as:

$$z = \alpha + i\beta \tag{22}$$

where α is the real part and β the imaginary part of z.

We first need to express equation (19) in a form as given by equation (22), to this end we need to separate (19) into real and imaginary parts. We proceed by multiplying the right hand side of (19) by:

$$\frac{\left(\frac{1}{C} - i\omega R\right)}{\left(\frac{1}{C} - i\omega R\right)} \tag{23}$$

such that:

$$q_0 e^{i\phi} = \frac{V_0\left(\frac{1}{C} - i\omega R\right)}{\left(\frac{1}{C} + i\omega R\right)\left(\frac{1}{C} - i\omega R\right)} = \frac{\frac{V_0}{C}}{\frac{1}{C^2} + \omega^2 R^2} - i\frac{V_0 \omega R}{\frac{1}{C^2} + \omega^2 R^2} \tag{24}$$

We therefore have:

$$\alpha = \frac{\frac{V_0}{C}}{\frac{1}{C^2} + \omega^2 R^2} \text{ and } \beta = -\frac{V_0 \omega R}{\frac{1}{C^2} + \omega^2 R^2} \tag{25}$$

From complex number theory, the phase angle ϕ is thus given by:

$$\phi = \arctan\left(\frac{\beta}{\alpha}\right) = \arctan\Big(-\omega RC\Big) \tag{26}$$

From the above expression, it is clear that ϕ is always negative.

From equation (26) we have $\tan\phi = -\omega RC$. Making us of the trig identity $\cos\phi = \frac{1}{\sqrt{\tan^2\phi + 1}}$, we can define:

$$\cos\phi = \frac{1}{\sqrt{(\omega RC)^2 + 1}} \tag{27}$$

Substituting equation (27) into (21) we obtain:

$$q_0 = CV_0 \cos\phi \tag{28}$$

Making use of equation (15) and $I(t) = \frac{dq}{dt}$ we obtain:

$$I(t) = -\omega q_0 \sin(\omega t + \phi) = \omega q_0 \cos\left(\omega t + \phi + \frac{\pi}{2}\right) = I_0 \cos(\omega t + \theta) \tag{29}$$

where:

$$I_0 = \omega q_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}} \tag{30}$$

and the phase shift between the current $I(t)$ and the voltage $V_\epsilon(t)$ given by:

$$\theta = \phi + \frac{\pi}{2} \tag{31}$$

2.4 Impedance

The impedance $Z(\omega)$ of an AC circuit is defined as the total resistance a circuit poses to an alternating voltage with angular frequency ω. The unit of impedance is the Ohm [Ω]. The impedance of a circuit will thus have an effect on the amplitude and phase of the current and thus should be represented using complex notation, such that:

$$Z(\omega) = R + iX(\omega) \tag{32}$$

R is the normal Ohmic resistance associated with an electrical circuit and $X(\omega)$, is the angular frequency dependent reactance of the circuit. In the case of a circuit with just a resistor, $X(\omega) = 0$ and $Z(\omega) = R$ and is angular frequency independent.

The magnitude of $Z(\omega)$ is given by:

$$|Z(\omega)| = \sqrt{R^2 + X^2(\omega)} \tag{33}$$

and the phase, is given by:

$$\phi = \arctan\left(\frac{X(\omega)}{R}\right) \tag{34}$$

The impedance Z can be represented on the complex plane as shown in Figure 3 on page 8.
In analogy to Ohm's law $|Z|$ is defined by the ratio of the voltage amplitude V_0 to the current amplitude I_0(equation (30)) such that:

$$|Z(\omega)| = \frac{V_0}{I_0} = \sqrt{R^2 + \frac{1}{(\omega C)^2}} \tag{35}$$

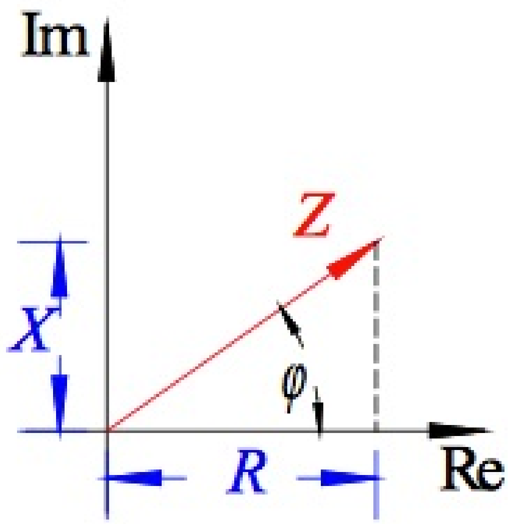

Figure 3:

On comparison with equation (32) we can define the capacitive reactance $X(\omega)$ such that:

$$X(\omega) = \frac{1}{\omega C} \tag{36}$$

Exercise 3 - Include full answer in your final submitted lab report.

Explain what effect an increase and decrease in frequency ω will have on the reactance and therefore the impedance of the RC circuit.

3 Experiment

3.1 Setup

Under normal circumstances, you would be expected to carryout the experimental setup yourself as well as collect all the relevant data. Instead watch the following two videos: **Breadboard Configuration** CAP02 and **Setup Video** CAP03 to see how to use a breadboard and how the experiment is setup. Answer then the questions that follow. All answers must be labelled and included in your final submitted lab report.

The following list of items are required.

- 1 5.6 kΩ resistor.
- 1 8.2 kΩ resistor.
- 1 10 kΩ resistor.
- 1 15 kΩ resistor.
- 1 100 nF capacitor.
- 1 myDAQ.

- 1 screwdriver.
- 1 function generator.
- 1 digital oscilloscope.
- 1 breadboard.
- wire

Exercise 4 - Make sure have first watched the setup video before answering the question.

Given the three photos below which one shows the correct way to insert a capacitor in a breadboard? You need to explain your answer for full credit.

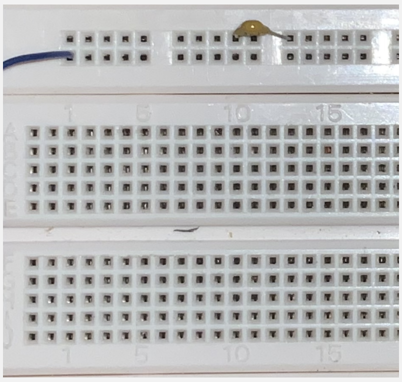

Figure 4: Photo A

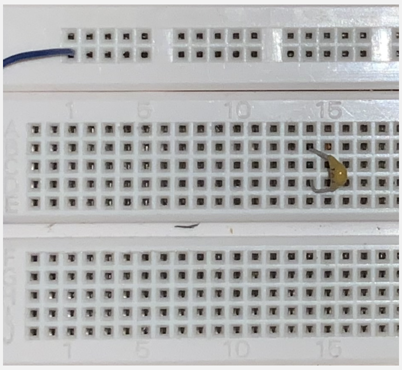

Figure 5: Photo B

Figure 6: Photo C

3.2 Data Collecting

Your task is to determine the time constant τ of the RC circuit. That is how long it takes to charge and discharge the capacitor through the resistor. You will also verify the capacitance of the capacitor. We will experiment with the different resistors as well as vary the frequency of the applied signal to see what effect this has on the signal we measure across the capacitor

3.2.1 Resistance and Frequency Invetigation

We drive the circuit with a square wave and set the peak to peak voltage of the signal from the generator to $8\,\mathrm{V}_{pp}$. We take several measurements at different frequencies and different R values. We need to monitor how the amplitude of the measured signal changes as a function of applied voltage and different R. Click on the data folder **Data Frequency and Resistance** DATA01 to download the data for this part of the experiment. All the data files have been labelled with the resistor value and frequency used.

3.2.2 Decay Time from Oscilloscope

Using an oscilloscope to take measurements is a vital skill needed for all physicists and engineers. The photograph contained in the link **PhotosDecay Time** Photo2 is the signal of the capacitor discharging through the $5.6\,\mathrm{k}\Omega$ resistor. From the photo determine how long it takes for capacitor to fully discharge. The signal across the capacitor is given in blue and the applied signal from the signal generator is given in yellow. Include a full uncertainty budget in your final answer.

3.2.3 Current Measurement

We are also going to measure the current that passes through the resistor. We will make use of the 15 kΩ resistor and measure the signal across the resistor. For this part of the experiment, we drive the circuit with a sinusoidal signal from the function generator.

Click on the data folder **Current** DATA02 to download the data for this part of the experiment.

Exercise 5 - Include full answer in final submitted lab report.

The photograph contained below is the signal of the capacitor discharging and discharging through a 15 kΩ resistor. From the photo the blue circles show that the voltage drop across the capacitor (in white) does not reach the same maximum and minimum as the applied square wave signal from the signal generator (in red). Explain why you think that is the case. What could you change in the setup to ensure that the voltage across the capacitor does indeed reach the same maximum and minimum as the applied signal? Explain carefully your answer.

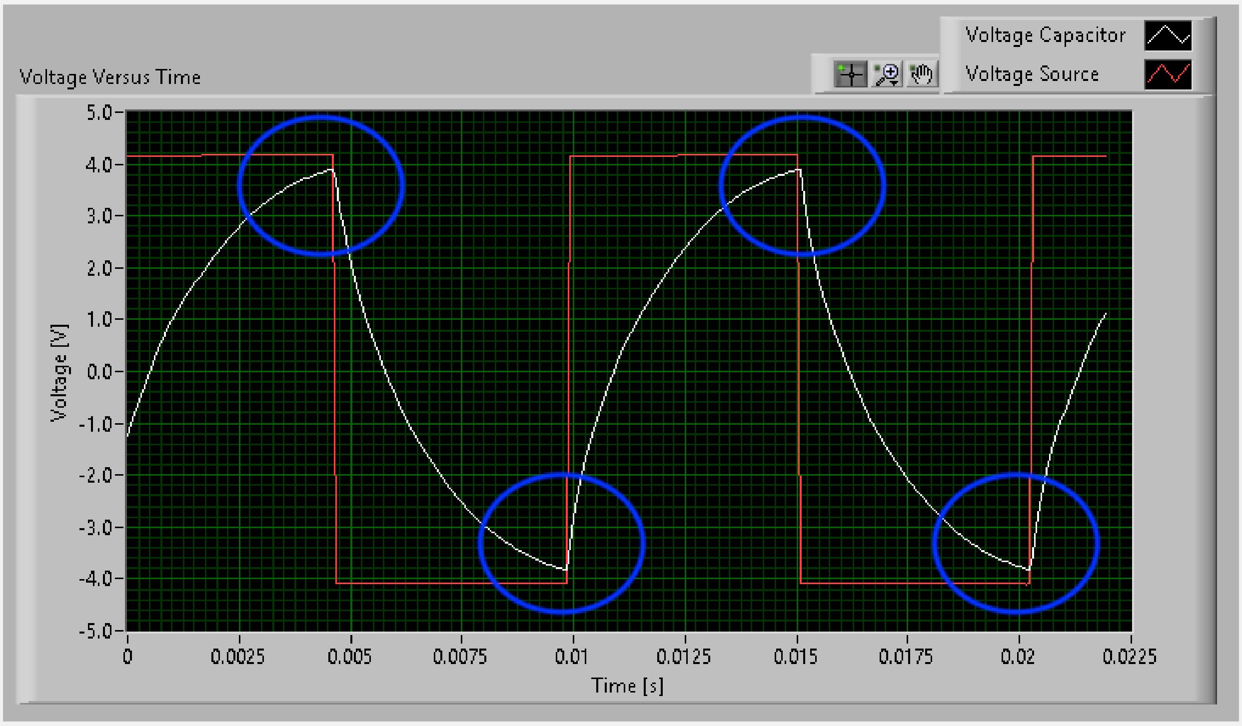

Figure 7: Voltage signal (white) accross 100 nF capacitor in series with 15 kΩ resistor.

4 File Format

All data has been saved in two formats:
A simple text file (txt) as well as in a Excel spreadsheet (xlsx).
The file names are of the following format:
Files starting with Cap have the voltage drop across the capacitor as well as the voltage from the signal generator.
Files starting with Res have the voltage drop across the resistor as well as the voltage from the signal generator.
The frequency set on the signal generator is given by the Hz value in the file name.

Resistor values used are also indicated in the file name:
5k6 is $5.6\,\mathrm{k}\Omega$, 8k2 is $8.2\,\mathrm{k}\Omega$, 15k is $15\,\mathrm{k}\Omega$

5 Analysis

Once you have saved all the data from the experiment you will need to plot all the relevant data using Python (during the first CP session you were shown how to read in from a file and plot data). You should find that some of your data sets will resemble Figure 8 on page 11.

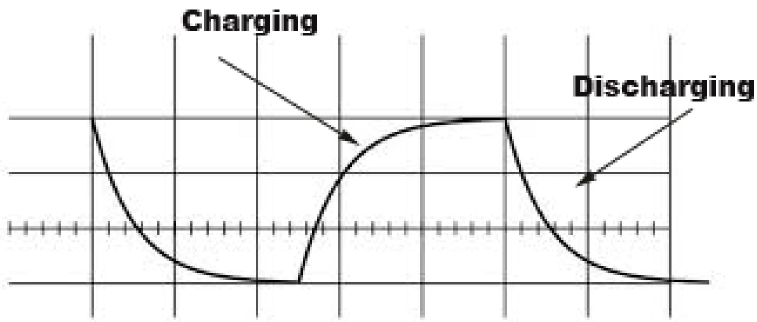

Figure 8:

5.1 Frequency and Resistor Response

Describe in detail with the aid of plots, how the output signal across the capacitor changes as the frequency is varied. Discuss what effect increasing or decreasing the resistor has on the output signal across the capacitor and how the effect on the time constant τ is evident from the behaviour of the data.

5.2 Time Constant $\tau = RC$

To determine the time constant τ from your plots, make sure you carry out a fitting on a data set where the signal measured across the capacitor has reached the maximum peak to peak voltage of $V_{pp} = 8\,\mathrm{V}$ as supplied by the signal generator. Determine the time constant τ from data for both the $5.6\mathrm{k}\,\Omega$ resistor and $15\,\mathrm{k}\Omega$ resistor. Since you are only able to perform unweighted linear least squares fits at present, you will need to linearise equations (6) and (11) for the charging and discharging phase of the plots and determine $\tau = RC$ from the fitting parameters. Compare the results for τ for both the charging and discharging stages of the data and compare it to the results expected with the given values of the resistor R and capacitor C used ($\tau = RC$). Include all uncertainty analysis in your fitting and calculations, and interpret your results.

5.3 Phase Difference Between Current I and Source Voltage V_ϵ in a RC circuit

To determine the current through the resistor make use of the voltage data you took across the resistor. Divide the voltage drop from across the resistor data by the resistor value to get the current through the resistor. Plot this current data and the voltage source data on the same set of axes. From the plot determine the phase difference between the two plots. As usual include all uncertainty analysis and comment on your result.

5.4 Capacitance C

You are expected from your data to verify the capacitance C of the capacitor used in this experiment. Once again select a data set where the signal measured across the capacitor has reached the maximum peak to peak voltage of $V_{pp} = 8\,\mathrm{V}$ as supplied by the signal generator. Make use of the discharge side of your data and equation (11).
Select two sets of points on the discharge curve as indicated in Figure 9 on page 12.

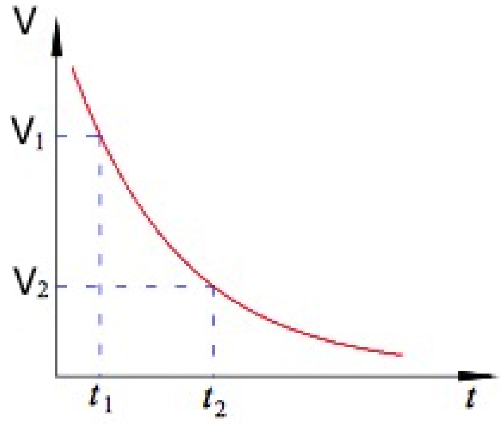

Figure 9:

You will need to set up two simultaneous equations with the data points taken from your curve and equation (11) and solve for C. Include all uncertainty analysis and compare to the expected capacitance of $100\,\mathrm{nF}$.

6 Report

You are required to hand in a full typed up report that deals with the experimental setup, method, observation and analysis. Also include all answers to the exercises contained in this manual. If you are unsure of anything, just ask. It is essential that you understand everything from this prac as the ideas learned here will be heavily utilised in future laboratories. You do not want to fall behind so early on in the course!

MDB 10/08/2020