

Assignment 5

KDSMIL001 MAM2000W 2IA

20 October 2020

5. (a) False. Row a could contain e in, for example, column b , meaning that $a * b = e$. However it is not necessarily true that $b * a = e$ so the condition for an inverse of a is not necessarily satisfied, so a could not have an inverse.
- (b) False. $\mathbb{N} \cup \{0\}$ is a set which is closed under addition, which is a binary operation. The set has identity $e = 0$. Every element n in this set has an inverse $-n$, but this $-n$ is not in the set, except when $n = 0$. So the set is not a group.
- (c) False. A group must have that every element in the group has an inverse which is itself in the group. In the case of the odd integers, any element is some $z \in \mathbb{Z}_{\text{odd}}$, so any multiplicative inverse of z must be $z^{-1} = \frac{1}{z}$, which is not an integer, save for when $z = 1$. Thus it is not a group.
- (d) False. Any group must be closed under its binary operator. The odd integers can be written as $2n + 1$, $n \in \mathbb{Z}$, so adding two odd integers together we see

$$(2n + 1) + (2n' + 1) = 2(\underbrace{n + n' + 1}_{\in \mathbb{Z}})$$

which is even, thus the sum of any two odd integers is an even integer and thus not in the set, so the set is not closed under addition and thus isn't a group.

- (e) True. Starting with S_1 , we see that

$$(0)(0) = (0) = (0)(0)$$

So the operation is commutative and S_1 is abelian. For S_2 , we can see that

$$(0, 1)(1, 0) = (0, 1) = (1, 0)(0, 1)$$

Again, the operation is commutative over all elements of S_2 , so it is abelian. For $n \geq 3$, we can show that S_n is non-abelian by first looking at $n = 3$:

$$\begin{aligned}(1, 0, 2)(2, 0, 1) &= (2, 1, 0) \\ (2, 0, 1)(1, 0, 2) &= (0, 2, 1)\end{aligned}$$

We can see that the operation is not commutative for every element in the set. It is simple to see that, given any value of n we can construct permutations which are the identity aside from the first 3 element, which we can set to the two permutations used above. If we then perform the operation on these two permutations we will find the same result, that it is not commutative for any $n \geq 3$, thus any S_n is non-abelian for $n \geq 3$.