The Magnetic Field of a Circular Coil: Induction and Inductance

KDSMIL001 PHY2004

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Contents

1 Introduction and Aim

In this practical we investigated the behaviour of the magnetic field produced due to an alternating current in a circular coil. This was done primarily by examining the induced voltage in a search coil placed near the primary coil.

2 Apparatus

The following equipment was used:

- Signal generator
- Power amplifier
- Ammeter
- Primary coil with 120 winds and diameter $(6.8 \pm 0.1) \times 10^{-2}\,\mathrm{m}$
- Secondary "search" coil with 175 winds and diameter $(1.3 \pm 0.1) \times 10^{-2}\,\mathrm{m}$
- Oscilloscope

The current in the primary coil was supplied by the amplifier, which was driven with a 2 V_{pp} sinusoidal signal from the signal generator. The ammeter was connected in series with the coil in order to monitor the current in the circuit. This ammeter displayed in rms, not amplitude, so we multiply by $\sqrt{2}$ in order to get the amplitude. Below is the set-up of the circuit.

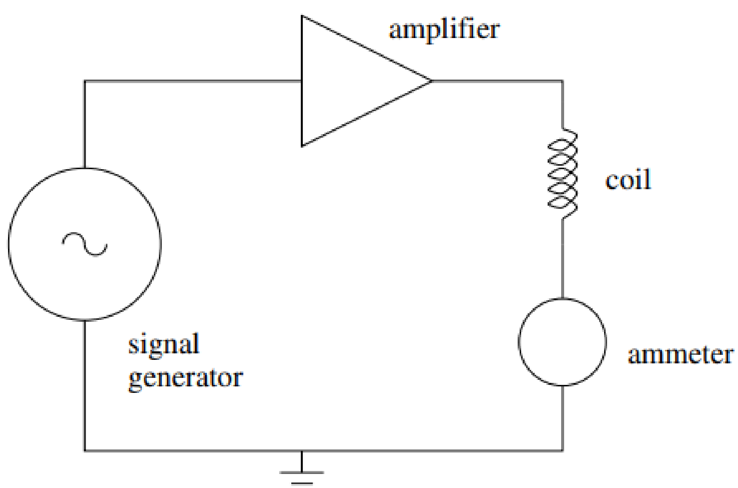

Figure 2.1: The primary circuit

Additionally, we had our secondary coil connected to an oscilloscope in order to monitor the induced emf ϵ. This search coil was placed on a contraption that allowed us to hold it at set distances from the primary coil, along the primary coil's axis.

3 Experiment

3.1 Field on the Axis of a Circular Coil

In this section we looked specifically at the relationship between magnetic field $\vec{B}(\vec{r}, t)$ and induced ϵ. First we look at Faraday's law, which says

$$\epsilon = -N_1 \frac{d}{dt} \int \vec{B} \cdot d\vec{A} \tag{3.1}$$

In our case, we are aligning things in a way to have the magnetic field dependent only on the distance z from the primary coil. We also say that the magnetic field is directed along the z-axis, approximately, allowing us to consider amplitudes only. So we have

$$\begin{aligned} B(z,t) &\approx B(z)\cos(\omega t) \\ \implies \epsilon &\approx -N_a A \frac{d}{dt}(B(z)\cos(\omega t)) \\ &= N_a A B(z) \omega \sin(\omega t) \\ \implies B(z) &\approx \frac{\epsilon}{N_a A \omega} \end{aligned}$$

where N_a is the number of winds in the search coil (175), A is the cross-sectional area of the search coil, ω is the angular frequency that the primary coil is being driven at ($2\pi f$), and we have taken $\sin(\omega t)$ to be 1 as we are only interested in amplitudes. We now have a kind of "calibration factor", so when we measure the emf induced in the search coil, we can immediately know the approximate value of the magnetic field that induced it, i.e. the magnetic field produced by the primary coil.

We have a way of determining the magnetic field from the induced voltage, but we also want to know how well that method agrees with what we would expect from the primary coil. The magnitude of the magnetic field on the axis of a circular coil of radius a is

$$B(z,t) = \frac{\mu_0 N I(t)}{2} \frac{a^2}{(a^2+z^2)^{\frac{3}{2}}} \tag{3.2}$$

where $\mu_0 = 4\pi \times 10^{-7}$ is the permeability of free space, N is the number of winds on the coil (120), and $I(t) = I_0 \cos(\omega t)$. Again we can take the cos term to be 1 as we're looking at amplitudes, which leaves us with

$$B(z) = \frac{\mu_0 N I_0}{2} \frac{a^2}{(a^2+z^2)^{\frac{3}{2}}}$$

Finally we collected some data:
We ran the signal generator at $1000\,\mathrm{Hz} = 2000\pi\,\mathrm{rads}^{-1}$ and $2V_{pp}$, with the amplifier setting the current to $I_{rms} = (0.35300 \pm 0.02020)\,\mathrm{A} = (0.495 \pm 0.029)\,\mathrm{A}$. This uncertainty comes from reading the current off of our ammeter, which displayed 0.35 A rms, so we use a digital pdf with uncertainty $\frac{a}{2\sqrt{3}}$ and $a = 0.01$ as well as the 2% uncertainty rating on the ammeter to find $u(I_{rms}) = \sqrt{0.02^2 + \frac{0.01}{2\sqrt{3}}} = 0.02020$, so $u(I_0) = 0.02020\sqrt{2} = 0.029$.
The cross-sectional area of the search coil is $A = (6.5 \times 10^{-3})^2\pi = (1.3273 \pm 0.0204) \times 10^{-4}\,\mathrm{m}^2$. This uncertainty comes from the uncertainty on the measurement of the diameter of the search coil, using the formula $u(x^n) = |n|x^{n-1}u(x)$.
The uncertainty on any experimentally determined B comes from the equation

$$u(B) = u(\frac{\epsilon}{N_a A\omega}) = \frac{\epsilon}{N_a A\omega}\sqrt{\left(\frac{u(\epsilon)}{\epsilon}\right)^2 + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(\omega)}{\omega}\right)^2}$$

where $u(\omega)$ is 2% of the scale used on the display of the signal generator, which was $1kHz$, so $u(\omega) = 0.02 \cdot 2000\pi = 40\pi$, and $u(\epsilon)$ is determined from the 2% uncertainty of the display of the oscilloscope combined with the digital measurement uncertainty

$$\begin{aligned} u(\epsilon) &= \sqrt{0.02^2 + \left(\frac{1 \times 10^{-5}}{2\sqrt{3}}\right)} \\ &= 0.01 \end{aligned}$$

The data and the theoretical model are in Figure 3.1.

3.2 Frequency Dependence of Induced Voltage

This section focuses on the effect of varying the frequency of alternating current on the system we're investigating. We suspect that the relationship is linear. For simplicity's sake we moved the coil to position $z = 0$ and varied the frequency from 100 Hz to 2 kHz. We did not operate below 100 Hz as that would lead to the inductive reactance $X_R = L\omega$ to decrease, effectively causing the coil to become a short circuit. The current would spike and the coil would heat up rapidly. The current was kept at $I_{rms} = (0.5700 \pm 0.0202)\,\mathrm{A}$ so $I_0 = (0.806 \pm 0.029)\,\mathrm{A}$ using the same methods as before. This is done to ensure that the magnetic field remains at the same magnitude throughout the experiment. The current needs to be adjusted as we change frequency to keep it the same as $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2+L^2\omega^2}}$, so the current changes with respect to frequency.
Now onto the experiment: Altering the current at each frequency and finding the induced ϵ at each ω, we found the plot in Figure 3.2.

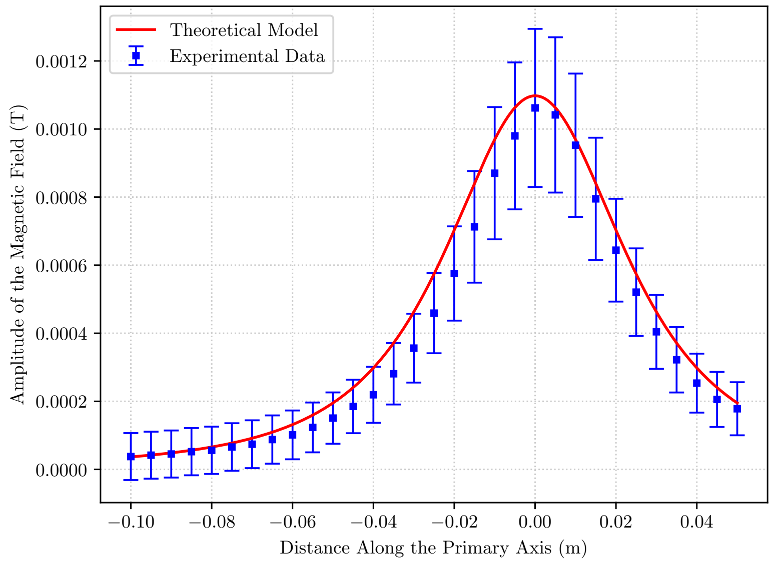

Figure 3.1: Magnetic field determined using the calibration factor and ϵ induced in a search coil due to a large primary coil, along with the theoretical prediction made using Equation 3.2

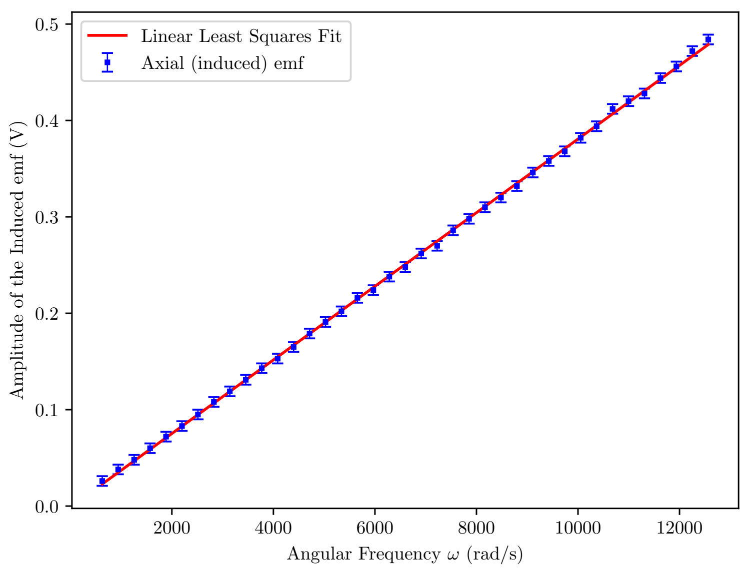

Figure 3.2: The emf induced in the axial coil due to a changing frequency of alternating current through the primary coil, kept at the same current to preserve the magnitude of the magnetic field.

The data is clearly linear. This confirms our earlier suspicion, but now let us check if our gradient calculated from the line of best fit agrees with what we would expect. Our linear least squares fit gave us a value of $m = (3.8188 \pm 0.0104) \times 10^{-5}$. To verify this we will use two methods. We notice that Equation 3.1 gives us a relation between induced ϵ and ω:

$$\epsilon = N_a A \omega B(z) \tag{3.3}$$

Since we are keeping B constant, we should have a gradient of $m = N_a AB$. N_a and A are the number of winds in the axial coil and the cross-sectional area of the axial coil respectively and are constants. B is also a constant but we have two ways of determining it. Firstly we can find it from the induced ϵ, using our calibration factor from the previous section on each ϵ and averaging it to find a B. Secondly we can use Equation 3.2 where $z = 0$, so

$$B = \frac{\mu_0 N I_0}{2a} \tag{3.4}$$

Using the first method we found $B_1 = (1.640 \pm 0.056) \times 10^{-3}\,\mathrm{T}$, so $m_1 = (3.81 \pm 0.14) \times 10^{-5}$. The uncertainty on B_1 is found with

$$u(B_1) = \frac{1}{n} u\Big(\sum_i B_i\Big) = \frac{1}{n}\sqrt{\sum_i u(B_i)^2}$$

$$u(B_i) = B_i \sqrt{\left(\frac{u(\epsilon)}{\epsilon}\right)^2 + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(\omega)}{\omega}\right)^2}$$

Using the second method we found $B_2 = (1.788 \pm 0.028) \times 10^{-3}\,\mathrm{T}$, so $m_2 = (4.15 \pm 0.64) \times 10^{-5}$. The uncertainty on B_2 is found with

$$u(B_2) = B_2 * \sqrt{\left(\frac{u(I_0)}{I_0}\right)^2 + \left(\frac{u(a)}{a}\right)^2}$$

where $u(I_0)$ is as given above and $u(a) = \frac{u(d_{primary})}{2} = 0.05 \times 10^{-2}\,\mathrm{m}$.

Experimental:	$m_E = (3.8188 \pm 0.0104) \times 10^{-5}$
Expected:	$m_1 = (3.81 \pm 0.14) \times 10^{-5}$
	$m_2 = (4.15 \pm 0.64) \times 10^{-5}$

Table 3.1: The experimental and two theoretical determinations of the gradient of induced voltage ϵ with respect to ω, the angular frequency of the AC voltage

3.3 Resistance and Inductance of the Primary Coil

This section focuses in on the primary coil as we aimed to determine the resistance and inductance of the primary coil. We used the same set-up as the previous section.

To do this we look at the equation

$$\begin{aligned} V &= IZ \\ &= I\sqrt{R^2 + L^2\omega^2} \end{aligned}$$

and see that, since we have kept the current constant, we have V as a function of ω, with R and L as parameters that we can optimise using `scipy.optimize.curve_fit`. Using the data from before, but this time with the voltage across the primary coil, we have Figure 3.3.

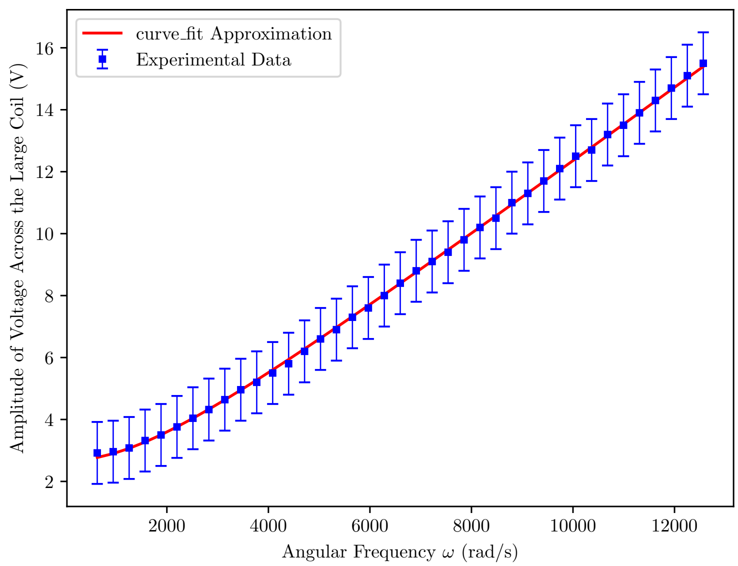

Figure 3.3: The amplitude of the voltage across the primary coil as a function of the angular frequency ω, with the `curve_fit` approximation of a line of best fit using the Jackknife method.

The error on the experimental data is a result of the limitations of the oscilloscope display and the uncertainty associated with digital measurement, given by

$$u(V_{\text{primary}}) = \frac{\sqrt{\left(\frac{0.01}{2\sqrt{3}}\right)^2 + 2^2}}{2}$$

Note the division by 2 that's necessary as the data was recorded as a V_{pp}, so we divide it by 2 to find the amplitude, and thus divide the uncertainty by 2.
`curve_fit` gives us the following values for R and L, with the uncertainties coming from the Jackknife method of error approximation.

Resistance:	$(3.3326 \pm 0.0085)\,\Omega$
Inductance:	$(1.50820 \pm 0.00037) \times 10^{-3}\,\mathrm{H}$

Table 3.2: Resistance and inductance of the primary coil determined using `curve_fit` on the emf across the primary coil when varying the frequency of AC voltage input, keeping current the same.

4 Discussion and Recommendations

4.1 Field on the Axis of a Circular Coil

Examining Figure 3.1 we see that for the most part our experimental data agrees comfortably with the theoretical prediction within experimental uncertainty. Most importantly, they have a very similar shape and both have a maximum at $z = 0$, which makes sense considering magnetic field is inversely proportional to distance, in the most basic form.
To improve the results of this experiment, we would recommend using a larger axial coil as it would then cover more of the magnetic field, as well as not making approximations when modifying Faraday's Law. This should lead to more accurate predictions of the fringe effects of magnetic field, which can be prominent.

4.2 Frequency Dependence of Induced Voltage

The linear least squares fit in Figure 3.2 fits well to the data, the uncertainty on the experimentally determined gradient being $\sim 0.2\%$ of the value. Regarding the expected values in Table 3.1, m_1 agrees with the m_E to an almost suspicious degree, but this makes sense since we used the calibration factor in the calculation of both the experimental and expected values. What's really interesting is that m_2 agrees with m_E within experimental uncertainty. This confirms that the methods we used to determine these values experimentally were valid.
The same improvements as in the previous section would improve these results. It's clear that the theoretical prediction of m is greater than that of m found using the approximation in Faraday's Law, meaning that our approximation underestimates the magnitude of the magnetic field.

4.3 Resistance and Inductance of the Primary Coil

The results in Table 3.2 seem quite reasonable for a coil of wire. We don't expect a large resistance value, and inductance is usually on the scale of 1×10^{-3}. Our confidence in these values is confirmed by the small uncertainties.
There is not much that can be done to improve this section of the experiment except for an improvement in the accuracy of the tools used to measure the values, which should give us lower uncertainty values.

5 Conclusion

To conclude, this experiment produced values which agreed with their theoretical counterparts, when there were predictions for the values at least, and values which make sense considering the physical attributes of the apparatus used. We accurately determined the relationship between the magnetic field produced by a coil of wire driven by and AC voltage and the voltage induced in a second coil oriented in the same direction as the primary coil. We confirmed that the relationship between induced voltage and frequency of the AC supply current in that circuit is linear, and found values for the resistance and inductance of the primary coil.

6 Appendix

Appendix 1: Code for subsection 3.1

```
from matplotlib import pyplot as plt
import numpy as np
from scipy.optimize import curve_fit
from numpy import cos, pi, sin, sqrt, exp, random
import matplotlib
# matplotlib.use('pgf')
matplotlib.rcParams.update({
    'pgf.texsystem': 'pdflatex',
    'font.family': 'serif',
    'text.usetex': True,
    'pgf.rcfonts': False,
})

file = open('PHY2004W Practicals and Reports\Induction\Report\Data\
    FieldAxisData.txt', 'r')
header = file.readline()
lines = file.readlines()
N = len(lines)
i=0
data = np.zeros((2,N))
# Reading the file, getting the data into the data array
for line in lines:
    line = line.strip()
    columns = line.split()
    data[0][i] = float(columns[0])
    data[1][i] = float(columns[1])
    i += 1
file.close()
# Converting the data to SI
data[0]*=1e-2
data[1]*=1e-3*0.5
# Defining some useful constants
mu0=pi*4e-7
omega=1000*2*pi
omegaUn=0.02*omega
primaryCoilWinds=120
secondaryCoilWinds=175
# Radius measured in m
primaryCoilRadius=3.4e-2
primaryCoilRadiusUn=0.05e-2
secondaryCoilRadius=6.5e-3
SecondaryCoilRadiusUn=0.05e-2
# Area measured in m^2
primaryCoilArea=pi*primaryCoilRadius**2
primaryCoilAreaUn=2*pi*primaryCoilRadius*primaryCoilRadiusUn
secondaryCoilArea=pi*secondaryCoilRadius**2
```

```python
secondaryCoilAreaUn=2*pi*secondaryCoilRadius*SecondaryCoilRadiusUn
# Current measured in amps
measuredCurrent=0.494975
measuredCurrentUn=0.02857738033
# An array used for plotting the theoretical values
distances=np.linspace(-0.1,0.05,1000)
# The Functions used to calculate B in different ways
def experimentalMagField(emf):
    return emf/(secondaryCoilWinds*secondaryCoilArea*omega)
def theoreticalMagField(z):
    return (mu0*primaryCoilWinds*measuredCurrent/2)*(((
        primaryCoilRadius)**2)/((primaryCoilRadius**2 + z**2)**(3/2)))

experimentalMax=max(experimentalMagField(data[1]))
theoreticalMax=max(theoreticalMagField(data[0]))
scaleFactor=experimentalMax/theoreticalMax
print(experimentalMax,theoreticalMax)
# Uncertainty calculations, my favourite
calibrationFactor=1/(secondaryCoilWinds*secondaryCoilArea*omega)
# This is 2% of the scale that the oscilloscope was at, 1 V
experimentalEmfUn=sqrt((1e-5/(2*sqrt(3)))**2 + (0.02)**2)*0.5
experimentalBUn=[]
for emf in data[1]:
    experimentalBUn.append((emf*calibrationFactor)*sqrt((
        experimentalEmfUn/emf)**2 + (secondaryCoilAreaUn/
        secondaryCoilArea)**2 + (omegaUn/omega)))

plt.errorbar(data[0],experimentalMagField(data[1]),yerr=
    experimentalBUn,fmt='bs',ms=3,elinewidth=1,capsize=4,label='
    Experimental Data')
plt.plot(distances,theoreticalMagField(distances),'r',label='
    Theoretical Model')
plt.xlabel('Distance Along the Primary Axis (m)')
plt.ylabel('Amplitude of the Magnetic Field (T)')
plt.legend()
plt.grid(color='#CCCCCC', linestyle=':')
plt.show()
# plt.savefig(r'PHY2004W Practicals and Reports\Induction\Report\Data
    \FieldAxisData.pgf')
```

Appendix 2: Code for subsection 3.2

```python
from matplotlib import pyplot as plt
import numpy as np
from scipy.optimize import curve_fit
from numpy import cos, pi, sin, sqrt, exp, random
import matplotlib
# matplotlib.use('pgf')
matplotlib.rcParams.update({
    'pgf.texsystem': 'pdflatex',
```

```python
    'font.family': 'serif',
    'text.usetex': True,
    'pgf.rcfonts': False,
})
# The usual stripping of data from the files
file = open('PHY2004W Practicals and Reports\Induction\Report\Data\
    InductanceData.txt', 'r')
header = file.readline()
lines = file.readlines()
N = len(lines)
i=0
# data[0] is frequency (not omega), data[1] is primary coil emf. data
    [2] is axial coil emf
data = np.zeros((3,N))
# Reading the file, getting the data into the data array
for line in lines:
    line = line.strip()
    columns = line.split()
    data[0][i] = float(columns[0])
    data[1][i] = float(columns[1])
    data[2][i] = float(columns[2])
    i += 1
file.close()
# Converting to Volts and rad/s, now data[0] is omega, not f
data[2]*=1e-3*0.5
data[0]*=2*pi
# Defining some useful constants
mu0=pi*4e-7
primaryCoilWinds=120
secondaryCoilWinds=175
# Radius measured in m
primaryCoilRadius=3.4e-2
primaryCoilRadiusUn=0.05e-2
secondaryCoilRadius=6.5e-3
SecondaryCoilRadiusUn=0.05e-2
# Area measured in m^2
secondaryCoilArea=pi*secondaryCoilRadius**2
secondaryCoilAreaUn=2*pi*secondaryCoilRadius*SecondaryCoilRadiusUn
# Current measured in amps
measuredCurrent=0.806101
measuredCurrentUn=0.02857738033
# Linear least squares fit to find slope of correlation
x=data[0]
y=data[2]
m = ((N*sum(x*y)) - sum(x)*sum(y))/((N*sum(x**2))-(sum(x))**2)
c = ((sum(x**2)*sum(y))-(sum(x*y)*sum(x)))/((N*sum(x**2))-(sum(x)**2)
    )
di=y-((m*x)+c)
um = sqrt(((sum(di**2)/((N*sum(x**2))-(sum(x)**2)))*(N/(N-2))))
uc = sqrt((((sum(di**2)*sum(x**2))/(N*((N*sum(x**2))-(sum(x)**2))))*(
    N/(N-2))))
```

```python
# Calculating B from the induced voltages (expression 1) and its uncertainty (easier here)
Bs=np.zeros(N)
experimentalBUn=np.zeros(N)
experimentalEmfUn=sqrt((1e-5/(2*sqrt(3)))**2 + (0.02)**2)*0.5
for index,freq in enumerate(data[0]):
    calibrationFactor=1/(secondaryCoilWinds*secondaryCoilArea*freq)
    Bs[index]=(data[2][index]*calibrationFactor)
    experimentalBUn[index]=((data[2][index]*calibrationFactor)*sqrt((experimentalEmfUn/data[2][index])**2 + (secondaryCoilAreaUn/secondaryCoilArea)**2 + (1000*2*pi*0.02/freq)))
BAverage=np.mean(Bs)
# Calculating B from expression 2
B2=(mu0*primaryCoilWinds*measuredCurrent)/(2*primaryCoilRadius)
# Finding "expected" gradient value
expectedM1=secondaryCoilArea*secondaryCoilWinds*BAverage
expectedM2=secondaryCoilArea*secondaryCoilWinds*B2
# Uncertainties
BAverageUn=(1/N)*sqrt(np.sum(experimentalBUn**2))
B2Un=B2*sqrt((measuredCurrentUn/measuredCurrent)**2 + (primaryCoilRadiusUn/primaryCoilRadius)**2)
expectedM1Un=expectedM1*sqrt((BAverageUn/BAverage)**2 + (secondaryCoilAreaUn/secondaryCoilArea)**2)
expectedM2Un=expectedM2*sqrt((B2Un/B2)**2 + (secondaryCoilAreaUn/secondaryCoilArea)**2)
axialEmfUn=0.02*np.mean(data[2])
# Printing results
print('Gradient from Linear fit:',m,'+/-',um)
print('Average B amplitude from axial emf:',BAverage,'+/-',BAverageUn)
print('B determined from expression 2:',B2,'+/-',B2Un)
print('Gradient expected, from expression 1:',expectedM1,'+/-',expectedM1Un)
print('Gradient expected, from expression 2:',expectedM2,'+/-',expectedM2Un)
# Plotting
freqs=np.linspace(200*pi,4000*pi,10000,endpoint=True)
plt.errorbar(data[0],data[2],yerr=axialEmfUn,fmt='bs',ms=2,elinewidth=0.7,capsize=3,label='Axial (induced) emf')
plt.plot(freqs,(freqs*m)+c,'r',label='Linear Least Squares Fit')
plt.xlabel('Angular Frequency $\omega$ (rad/s)')
plt.ylabel('Amplitude of the Induced emf (V)')
plt.legend()
plt.show()
# plt.savefig(r'PHY2004W Practicals and Reports\Induction\Report\Data\EmfPropToFreq.pgf')
```

Appendix 3: Code for subsection 3.3

```python
from matplotlib import pyplot as plt
```

```python
import numpy as np
from scipy.optimize import curve_fit
from numpy import cos, pi, sin, sqrt, exp, random
import matplotlib
matplotlib.use('pgf')
matplotlib.rcParams.update({
    'pgf.texsystem': 'pdflatex',
    'font.family': 'serif',
    'text.usetex': True,
    'pgf.rcfonts': False,
})
# The usual stripping of data from the files
file = open('PHY2004W Practicals and Reports\Induction\Report\Data\InductanceData.txt', 'r')
header = file.readline()
lines = file.readlines()
N = len(lines)
i=0
# data[0] is frequency (not omega), data[1] is primary coil emf. data[2] is axial coil emf
data = np.zeros((3,N))
# Reading the file, getting the data into the data array
for line in lines:
    line = line.strip()
    columns = line.split()
    data[0][i] = float(columns[0])
    data[1][i] = float(columns[1])
    data[2][i] = float(columns[2])
    i += 1
file.close()
# Converting to Volts and rad/s, now data[0] is omega, not f
data[1]*=0.5
data[0]*=2*pi
# Current measured in amps
theoreticalCurrent=0.8
measuredCurrent=0.806101
measuredCurrentUn=0.02857738033
# The function for curve_fit
def emfFromFreq(omega,R,L):
    return theoreticalCurrent*sqrt(R**2 + (L**2)*(omega**2))
# Things needed for curve_fit
freqs=np.linspace(200*pi,4000*pi,N,endpoint=True)
u=[sqrt((1e-2/(2*sqrt(3)))**2 + (2)**2)*0.5]*N
p0=[10,0.01]
# Jackknife curve_fitting
jackknifeData = np.zeros((4, N, N-1))
for c in range(N):
    r = random.randint(0, N)
    jackknifeData[0, c] = np.delete(data[0], r)
    jackknifeData[1, c] = np.delete(data[1], r)
    jackknifeData[2, c] = np.delete(u, r)
```

```python
    jackknifeData[3, c] = np.delete(freqs, r)
# Fitting to the jackknifed datasets
jackknifeFits = np.zeros((N, N-1))
popts = []
for k in range(N):
    popt, pcov = curve_fit(emfFromFreq, jackknifeData[0, k], jackknifeData[1, k], p0, sigma=jackknifeData[2, k], absolute_sigma=True)
    jackknifeFits[k] = emfFromFreq(jackknifeData[3, k], *popt)
    popts.append(popt)
# Isolating arrays of each optimal fitting parameter
poptNp = np.zeros((2, N))
for d, ds in enumerate(popts):
    poptNp[0, d] = ds[0]
    poptNp[1, d] = ds[1]
# Calculating means and standard uncertainties
pOptimals = np.zeros((2, 2))
for j, js in enumerate(poptNp):
    mean = np.mean(js)
    pOptimals[0][j] = mean
    sumI = 0
    for i in js:
        sumI += (i-mean)**2
    pOptimals[1][j] = sqrt(float(((N-1)/N)*sumI))/sqrt(N-1)
# Plotting the function with the optimal parameters
yfit=emfFromFreq(freqs,*pOptimals[0])
print('Resistance of the Large Coil:',pOptimals[0][0],'+/-',pOptimals[1][0])
print('Inductance of the Large Coil:',pOptimals[0][1],'+/-',pOptimals[1][1])
# Plotting but more
plt.errorbar(data[0],data[1],yerr=u,fmt='bs',ms=3,elinewidth=0.7,capsize=3,label='Experimental Data')
plt.plot(freqs,yfit,'r',label='curve\_fit Approximation')
plt.xlabel('Angular Frequency $\omega$ (rad/s)')
plt.ylabel('Amplitude of Voltage Across the Large Coil (V)')
plt.legend()
# plt.show()
plt.savefig(r'PHY2004W Practicals and Reports\Induction\Report\Data\ResistanceInductance.pgf')
```