Assignment 4

KDSMIL001 MAM2000W 2IA

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7. (a) True. Given some $\sigma = (k_1\, k_2)(k_3\, k_4)\dots(k_{r-1}\, k_r)$ which is a product of disjoint transpositions, we know that the inverse of a transposition is the transposition itself, so we can write

$$\begin{aligned}\sigma^{-1} &= ((k_1\, k_2)(k_3\, k_4)\dots(k_{r-1}\, k_r))^{-1}\\ &= (k_1\, k_2)^{-1}(k_3\, k_4)^{-1}\dots(k_{r-1}\, k_r)^{-1}\\ &= (k_1\, k_2)(k_3\, k_4)\dots(k_{r-1}\, k_r)\\ &= \sigma\end{aligned}$$

$\square$

(b) True. Any permutation in S_n, aside from the identity permutation ϵ, is not disjoint with its inverse. Firstly $\epsilon^{-1} = \epsilon$ and ϵ fixes every value so its inverse fixes every element, making them disjoint as neither move the same element. Then any other element in S_n will be a product of one or more disjoint cycles of length at least 2 (by the Cycle Decomposition Theorem) and any cycle is not disjoint with its inverse since the inverse *must* move the same elements as the original cycle in order to reverse the moves.

$\square$