PHY2004W: PHYLAB 2
Experimental Physics Lab Session
The Hall Effect

1 Introduction

Since its discovery in 1879, the Hall effect has been observed in a number of materials and has helped in clarifying the mechanism of conduction of electric charge in solids, liquids and gases, as well as being used in a number of technological applications such as "Hall effect probes" for measuring magnetic fields.
The Hall effect is most easily observed in semiconductors, and we will measure it in a sample of germanium. From these measurements you will calculate certain properties of the sample. In addition you are required to look for any deviation of your data from the ideal case, in order to identify certain subsidiary effects.

2 References

Purcell, *Electricity and Magnetism*, section 6.9. Note non-SI units.
Young and Freedman, *University Physics (11th Ed)*, Addison-Wesley (2004): 27.9.

3 Purpose

New examples and concepts, measurement techniques, data and error analysis.

4 Theory

4.1 Conduction of current in the presence of a static magnetic field.

Current in conductors is carried by mobile charge carriers which move under the influence of electric and magnetic fields. The fields accelerate the charges, transferring momentum to them, and if a steady state is to be maintained the charges must transfer momentum to some other component of the system. In solid conductors this component is the fixed atomic lattice through which the charges move, the transfer of momentum being effected by means of collisions of the charges with the lattice atoms.

Suppose there are n charges per unit volume, each with mass m and charge q, and that the average time between collisions with the lattice is τ. Let $\mathbf{v}$ be the average velocity acquired by the charges due to the action of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$. Since the average value of momentum lost by a charge on colliding with a lattice atom is $m\mathbf{v}$, the equation of motion of the charges may be written:

$$m\dot{\mathbf{v}} \equiv m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{m\mathbf{v}}{\tau}$$

In the steady state $\dot{\mathbf{v}} = 0$, so

$$\mathbf{v} = \frac{\tau q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \tag{1}$$

Consider initially the case with $\mathbf{B} = 0$. Then

$$\mathbf{v} = \frac{\tau q}{m}\mathbf{E} = \mu\,\mathbf{E}$$

where μ is called the mobility of the charges. The current density $\mathbf{J}$ is given by

$$\mathbf{J} = nq\,\mathbf{v} = nq\mu\,\mathbf{E} = \frac{nq^2\tau}{m}\,\mathbf{E}$$

The electrical conductivity σ is defined by $\mathbf{J} = \sigma\mathbf{E}$, so it is clear that

$$\sigma = nq\mu = \frac{nq^2\tau}{m} \tag{2}$$

Returning to the general case, $\mathbf{B} \neq 0$, we see that equation (1) may be written in the form

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

so that

$$\mathbf{E} = \frac{\mathbf{J}}{\sigma} - \mathbf{v} \times \mathbf{B}.$$

But since

$$\mathbf{v} = \frac{\mathbf{J}}{nq},$$

it follows that

$$\mathbf{E} = \frac{\mathbf{J}}{\sigma} - R_H\left(\mathbf{J} \times \mathbf{B}\right) \tag{3}$$

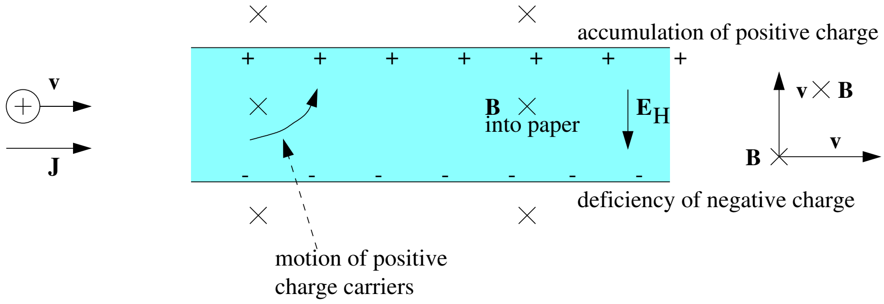
where

$$R_H \equiv 1/nq \tag{4}$$

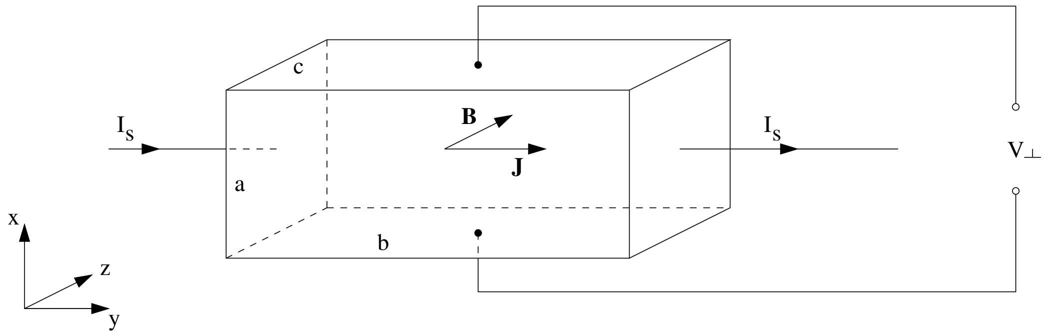
is known as the Hall coefficient.

4.2 The Hall effect.

From equation (3) we see that if a current density $\mathbf{J}$ exists in a conductor in a magnetic field $\mathbf{B}$, there will be the usual Ohmic electric field $\mathbf{E}_{\parallel} = \mathbf{J}/\sigma$ parallel to the current density, and in addition a Hall electric field $\mathbf{E}_{\perp} = -R_H\,\mathbf{J} \times \mathbf{B}$ perpendicular to both the current density and the magnetic field. The physical origin of the Hall field can be explained in terms of the build up of charge as sketched below



If a current is sent through a block of material similar to the one below, the Hall effect can be investigated by measuring the Hall voltage $V_\perp = aE_\perp = aR_H JB$



Since the current density $J = I_s/ac$ where I_s is the current through the sample, this relation can be written

$$V_\perp = R_H I_s B/c \tag{5}$$

For the analysis above to hold, the Hall voltage must be measured without drawing any current. (If a current is drawn from the Hall electrodes, there will be a component J_x of current density which complicates the analysis). In this experiment, a high input resistance (10 MΩ) voltmeter is used so that current drawn is negligible.
The voltage $V_\shortparallel$ across the length of the sample can also be measured

$$\begin{aligned} V_\shortparallel &= E_\shortparallel b = Jb/\sigma \\ V_\shortparallel &= \frac{I_s b}{\sigma ac} \end{aligned} \tag{6}$$

From equations (5) and (6) you can see that if the magnetic field B and the sample dimensions a, b and c are known, measurements of $I_s, V_\shortparallel$ and $V_\perp$ can be used to obtain values of R_H and σ

4.3 The Hall effect in semiconductors.

So far we have considered conductors with charge carriers of one sign only. In conductors in which the current is carried by both negative and positive charges the expression for the Hall coefficient is more complicated, but reduces to simple expressions (2) in the case of strongly "doped" semiconductors. In p-type semiconductors in which the current is predominantly carried by positive "holes"

$$R_H = \frac{1.4}{n_+ e} \tag{4a}$$

and in n-type semiconductors where negative electrons are the main charge carriers

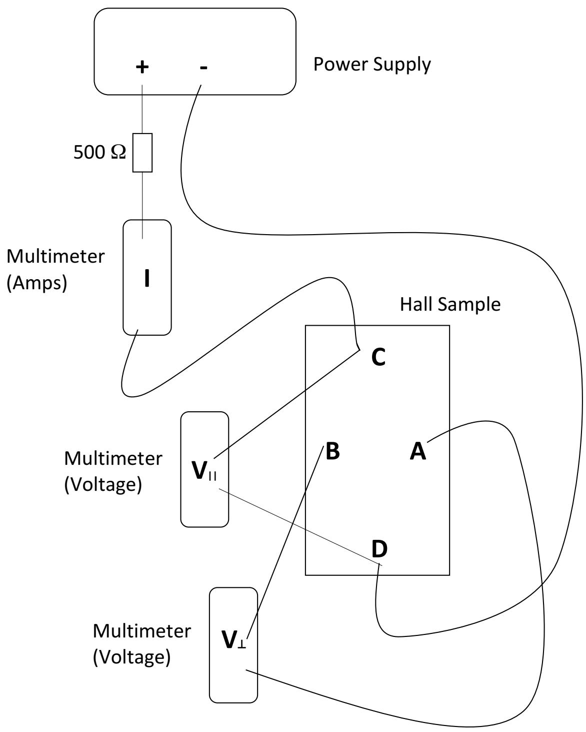
$$R_H = \frac{0.93}{n_- e} \tag{4b}$$

In these expressions e is the magnitude of the charge on an electron, and n_+ and n_- are respectively the number densities of mobile positive and negative charges.

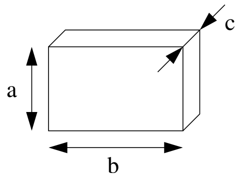
The theory developed above has implicitly assumed that the temperature is constant. In a semiconductor both the number density of charges and their mobility are temperature dependent. Thus we must expect the conductivity and the Hall coefficient to be temperature dependent. As the current through a sample is increased there will be increased Joule heating so the sample temperature will be raised. You will be measuring $V_{\shortparallel}$ and $V_{\perp}$ as functions of the current I_s. According to equations (5) and (6) these should give linear relations, but in view of the variation of the "constants" R_H and σ with temperature, you can expect some deviation from linearity, especially at the higher values of current (remember that the heating varies as I^2).

5 Apparatus

A germanium sample (p-type) is mounted on a board with the necessary terminals and an offset potentiometer R_p. It is to be connected up as shown below.



The dimensions of the sample are as follows.



a = 5 mm

b = 10 mm

c = 1 mm

The Hall voltage $V_{\perp}$ is measured directly by means of a digital voltmeter.
The offset potentiometer R_p is included because the Hall electrode on the sample may not be attached exactly opposite each other, resulting in an Ohmic voltage drop between them even in the absence of a magnetic field. To eliminate the effect of this, $V_{\perp}$ is observed when B is zero and the potentiometer R_p is varied to give zero voltage $V_{\perp}$.
The magnet is mounted on a wooden frame to which the sample board can be attached. Strong magnets like this should have their "magnetic circuit" completed with an iron bar when not in use.

6 Instructions

1. The circuit is first connected as shown in the figure.

   The current through the germanium sample must not exceed 50 mA, so before switching on power supply we need to ensure that the 500 Ω resistor is in place.

2. We start with the power supply OFF.

3. We make sure that all the Current Controls on the power supply (fine and course) are set to maximum.

4. We make sure that all the Voltage Controls on the power supply (fine and course) are set to minimum.

5. Only then is the power supply turned ON.

6. Using the voltage course control we set the voltage on the power supply voltage display to 22 V. We. make sure that the current as measured on the multimeter does not go above 39 mA.

7. We set the current in the sample to about 38 mA, and with the magnet far away from the sample, vary R_p to obtain zero $V_{\perp}$.

8. We are now ready to carry out the measurements. We measure $V_{\perp}$ and $V_{\parallel}$ as a function of I with the sample in the magnetic field B. We take at least 10 values of I over the range 0 to 40 mA. We can control the current by turning the course knob on the voltage supply. We do not go above 22 V as indicated on the power supply display.

   We repeat the measurements for the 39 mA case after leaving the current flowing for 5 to 10 minutes.

   Under normal circumstances, you would be expected to carryout the experimental setup yourself as well as collect all the relevant data. Instead watch the following video of the setup and data taking procedure: **Hall Setup** HALL01

   Click on the data folder **Hall** DATA01 to download the Excel data for this experiment.

9. You will need to plot graphs of $V_{\parallel}$ vs I_s and $V_{\perp}$ vs I_s.

7 Results

Are the graphs linear? Remember that we expect the deviations from linearity to be more pronounced at high currents.

As a check on the linearity, calculate V_H/I_s, $V_\perp/V_{\shortparallel}$ for each of the measured values and see if there is any systematic variation in these quantities.

Decide on the best values of $V_{\shortparallel}/I_s$ and $V_\perp/I_s$ for room temperature germanium, and hence use equations (5) and (6) to find values for R_H and σ.

Use either equation (4a) or (4b) to find n from your value of R_H.

Since $\mu = \sigma/ne$ [equation (2)] you can now calculate μ, the mobility. Note that $\mu \propto V_\perp/V_{\shortparallel}$.

Exercises - Now consider the deviations of the graphs from linearity and answer the following questions. Include full answers in your final submitted lab report.

1. Does the conductivity of the sample increase or decrease with temperature? Does your answer to this question agree with what you know about semiconductors?
2. Does the number density of charges increase or decrease with temperature? Does your answer to this question agree with what you know about semiconductors?
3. Does the mobility of the charges increase or decrease with temperature? Compare your answer to this question with the information on charge mobilities in pure germanium (at 20° C) given below.

Electron mobility μ_-	0.38 $\mathrm{m^2V^{-1}s^{-1}}$
Temperature dependence $\mu_-(T)$	$\propto T^{-1.66}$
Hole mobility μ_+	0.18 $\mathrm{m^2V^{-1}s^{-1}}$
Temperature dependence $\mu_+(T)$	$\propto T^{-2.33}$

(where T is absolute temperature)