PHY2004W: PHYLAB 2
Experimental Physics Lab Session
Wave Guides

1 Introduction

Electromagnetic waves can propagate on a transmission line as well as in free space. One example of such a transmission line is the *coaxial cable*, familiar as the cable connecting a TV set to an aerial.

2 References

Pollack and Stump, *Electromagnetism*, Addison-Wesley (2002): 10.4.
Young and Freedman, *University Physics (11th Ed)*, Addison-Wesley (2004): 32.5.
J M Serra, et al. Eur. J. Phys, **25** (2004) 581-591.

3 Emphasis

New concepts (impedance matching, transmission line), measurements.

4 Transmission lines

One can consider the properties of the transmission line in terms of its capacitance and inductance per unit length. For the coaxial cable these are

$$C = \frac{2\pi\epsilon}{\ln r_{\text{out}}/r_{\text{in}}} \qquad \text{and} \qquad L = \frac{\mu}{2\pi}\ln\frac{r_{\text{out}}}{r_{\text{in}}}$$

where ϵ is the permittivity of the dielectric in the cable, $\mu = \mu_0$ is the permeability of the dielectric, which can be taken to be that of free space ($\mu_0 = 4\pi \times 10^{-7}$ H/m) and r_{out} and r_{in} is the outer radius and inner radius of the cable respectively.

For an electromagnetic wave in the cable, the electric and magnetic energy densities are equal; in terms of the above these can be written

$$E_E = \frac{1}{2}CV^2 \qquad \text{and} \qquad E_B = \frac{1}{2}LI^2$$

where V and I represent the potential and current at a point.

Exercise 1 - Since $E_E = E_B$, the ratio V/I depends only on ϵ, μ and the geometry of the cable. This ratio has the dimensions of an impedance, and is a basic characterisation of the cable. Show that the *characteristic impedance* Z_0 is given by:

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\frac{r_{\text{out}}}{r_{\text{in}}}$$

Note that electromagnetic waves will propagate in the cable at a speed

$$v = \frac{c}{\sqrt{\epsilon\mu}} = \frac{c}{n}$$

where c is the speed of light in vacuum, and n is the *refractive index* of the dielectric (at the frequency of propagation).

If the characteristic impedance of the cable changes at some point, the propagation of the wave will be affected; the wave might be partially reflected. The reflection coefficient for a purely resistive load R is given by

$$\mathcal{R} = \frac{R - Z_0}{R + Z_0}$$

Note that there are no reflections if $R = Z_0$; the load is said to be *matched* to the cable. If there is a short circuit, $R = 0$, then $\mathcal{R} = -1$ so that the reflected wave will be inverted.

Exercise 2 - Show if $R = \infty$, in other words an open circuit at the end of the cable, then $\mathcal{R} = 1$.

A short pulse will travel along the cable and be reflected from the end. Thus if we monitor the voltage at the transmitting end with an oscilloscope we should see two pulses, the original and the reflected. By measuring the time between them we can determine the speed of propagation, if we know the length of the line. The cable length in this experiment is $L = (55.845 \pm 0.005)$ m long.

In the same way, a more extended wave, for example a sine wave, will be reflected.
At appropriate frequencies we should expect to see the formation of standing electromagnetic waves. The nature of the standing waves in the cable depend on the boundary conditions in the cable. At the driven end (i.e. the signal generator end) the boundary condition can be taken to be a short circuit, since the signal generator has a low output impedance. At the other end of the cable, a switch setting permits the termination to be set to a short circuit, an open circuit or to a variable resistance which can be measured with a multimeter after first disconnecting the signal generator from the circuit.

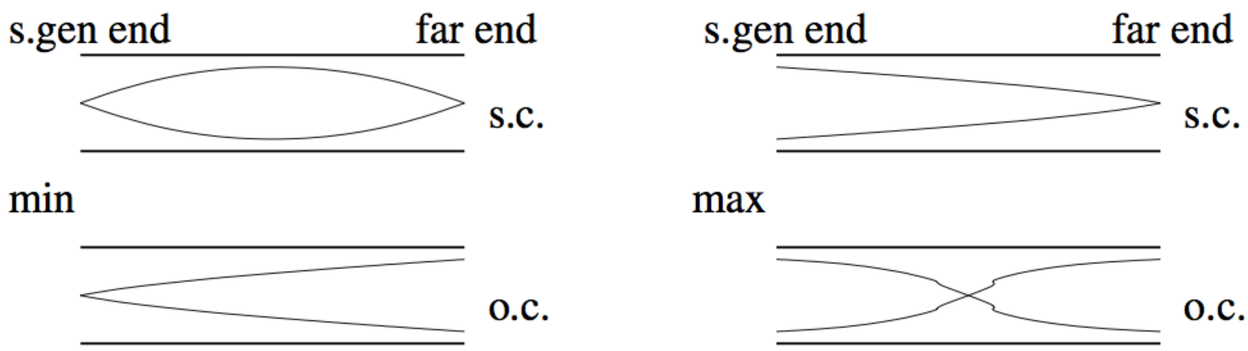
For the open circuit termination the standing wave condition is given by

$$f_n = n\frac{v}{4L}, \qquad n = 1, 3, 5\ldots$$

where the potential at the end of the cable can take any value. For a short circuit, where the potential at the end is zero, the standing wave condition is given by,

$$f_n = n\frac{v}{2L}. \qquad n = 1, 2, 3\ldots$$

The various cases are illustrated below:



5 Measurement

The signal generator and oscilloscope is first connected to the long coaxial cable. We adjust the output of the signal generator to give a square pulse with a low duty cycle (set to a minimum). We set the frequency to 1 MHz. Watch the following video of the setup of the experiment **Setup** Wave01. For an explanation of what the duty cycle is see the following video **Duty** Wave02.

5.1 Propagation Speed of the Wave

For the first part of this prac, we observe the waveform on the scope as we select the open circuit (oc) or short circuit (sc) condition by means of the toggle switch, paying careful attention to the time difference between the input signal and the reflected signal as seen on the osciloscope. Watch the following video showing how the experiment is carried out and what the signals produced look like **Time** Wave03.

To measure the time between the initial and reflected pulses in both the oc and sc, photos of the oscilloscope were taken. Download the following pictures **Photo Open** Wave04, **Photo Open Zoom** Wave05, **Photo Short** Wave06 and **Photo Short Zoom** Wave07, to measure the time difference between the input signal and the reflected signal directly from the photos. The photos labelled with Zoom are the same signals with a change in time base on the oscilloscope. From the time difference between the input and reflected signals determine the propagation speed of the wave.

The propagation speed can also be determined using standing waves. We select a sine wave output on the signal generator and set the termination on the cable to first oc and then sc. For both termination settings we measure the frequency values for which a standing wave occurs, as determined by a maximum amplitude in the signal on the scope. Watch the following video showing how the experiment is carried out **Standing** Wave08.
The frequencies can be used determine the propagation speed. The first two frequencies corresponding to standing waves in both oc and sc termination are given in the following link **Data Standing** Wave09. Compare the values you obtain here for the propagation speed of the wave with that obtained using the time difference between the input and reflected wave.

5.2 Characteristic Impedance Z_0

We now attempt to determine the characteristic impedance Z_0 of the cable.

We select the variable resistor termination. By turning the knob from one end of its travel to the other, we move from a termination of sc to oc or vice versa. Somewhere in the middle (careful adjustment required!) we find that the reflected pulse vanishes. Watch the following video showing how the experiment is carried out and what the signal produced looks like **Imped** Wave10.

Obviously $\mathcal{R} = 0$ and you can determine Z_0 by measuring the value of the resistance. A photo of the resistance value when the reflected pulse vanishes is given here: **Photo Resistance** Wave11.