Homework 2: Sensitivity and Option Strategies

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Abstract. In this exercise we continue using the program written for the last homework, analyzing now the sensitivity of the option prices to changes in several parameters. In the second part, we analyze the stategy *long call option* and compute the *put-call parity*.

1 Sensitivity of the option price

Firstly we will show how the option price is affected by changes in several variables of the Weiner process. In the following plots we show the mean and standard deviation values for each one, the second one using error bars. Also, the option price for call options will be shown in red, while the one for put options will be shown in blue.

Some comparisons can be done against the Black-Scholes model, which solves analytically the stochastic differential equation of the Weiner process, giving the following solution [1] for the probability density distribution of S:

$$\frac{1}{\sigma S\sqrt{2\pi t}} \exp\left[-\frac{\left(\log\frac{S}{S_0} - (\mu - \frac{1}{2}\sigma^2)t\right)^2}{2\sigma^2 t}\right] \quad , \qquad (1)$$

also known as lognormal distribution.

Just by looking at this equation we can predict the behavior of the option price as parameters change — σ and t, which will clearly widen the distribution of points as they increase, and μ or S_0 which will shift the whole distribution changing the option price but without affecting the width.

1.1 Initial stock price, S_0

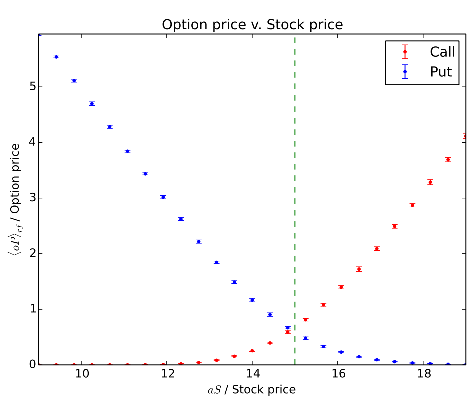

Figure 1: Value and sensitivity of the option price for a range of initial stock prices. The green line represents the exercise price (15€).

As we can see in Figure 1, changing the initial stock price doesn't affect the option price in an unexpected way, and the standard deviation of our results doesn't increase significantly either. This is expected, since what we are doing here is just moving the distribution of share value at exercise date to higher or lower values, so for lower (higher) values smaller (larger) the probability of making profit and therefore the option price will be lower (higher).

1.2 Volatility, σ

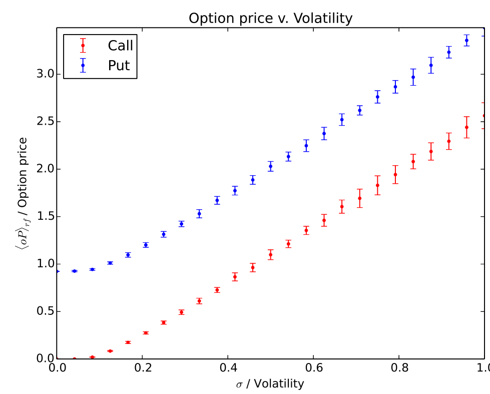

Figure 2: Value and sensitivity of the option price for a range of volatilities.

In this case we are changing the volatility of the action, so its value at exercise date will have a wider range of values. In other words, we expect the standard deviation of our results to be larger as we can predict by looking at equation (1). The denominator of the distribution is given by $2\sigma^2 t$, and it clearly explains this behavior. Also, for $\sigma \to 0$ we find no width at all, since the values will only change due to the drift.

The option price is also increasing, since a higher volatility spreads the share value at expiration date, and since we compute the option price by removing values smaller than the exercise price, it increases the average.

1.3 Duration, aT

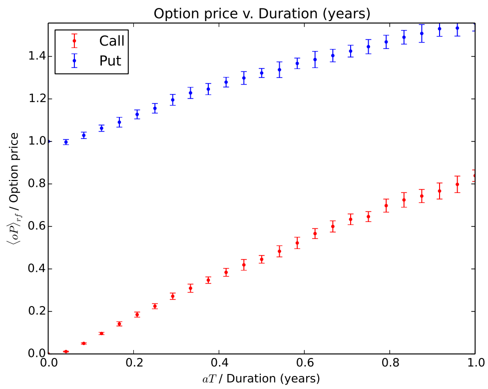

Figure 3: Value and sensitivity of the option price for a range of durations.

Again, the error bars widen as t increases, since the standard deviation depends on $\sqrt{t}$ as we can see from equation 1. Other than that, the option price increases, since the drift has more time to act and thus increase the value of the share. For large t we should include a term of the form e^{-rt}, but we will just assume that r is small enough not to make a significant difference in a year.

As we mentioned before, changing aT is similar to a change in σ in terms of widening of the distribution, and it shows clearly as Figures 2 and 3 both present a similar behavior, the former increasing $\langle oP \rangle_{rf}$ linearly and the latter as $\sqrt{t}$.

1.4 Drift, μ

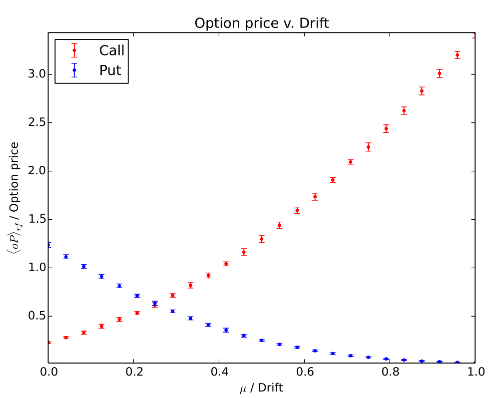

Figure 4: Value and sensitivity of the option price for a range of drifts.

Finally, in Figure 4 we can see how changing the drift produces a very similar result as changing the initial stock price from Figure 1, as we predicted by studying equation (1). We could check that these two plots are equal when $\log S_0$ and μt give the same contribution. Also as we predicted, the standard deviation doesn't significantly change with μ, except for the inherent randomness of the algorithm.

2 Long call option

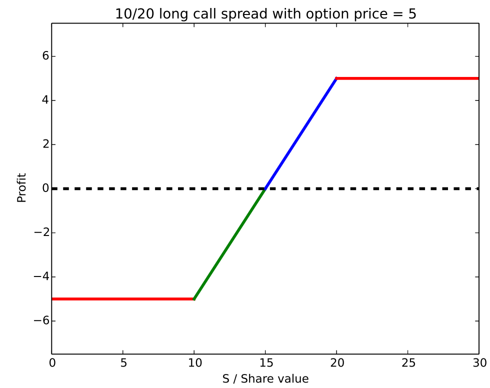

Figure 5: Diagram of the payoff for the strategy *long call option*.

3 Put-call parity

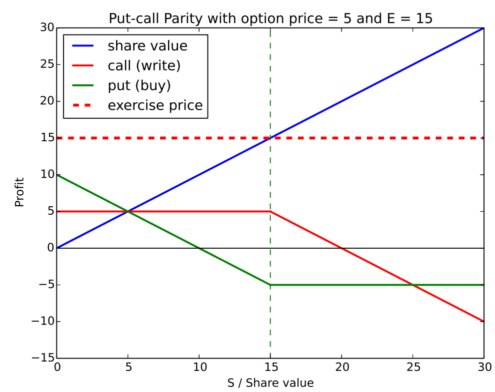

Figure 6: Diagram of the payoff showing the *put call parity*.

References

[1] P. Wilmott et al, *The Mathematics of Financial Derivatives*, 1995.