线性代数第三次作业

1.Find the inverse matrix of

$$A=\begin{bmatrix}1&1&2\\0&0&1\\1&0&1\end{bmatrix}$$

if it exists. If you think there is no inverse matrix of A, then give a reason.

2.Let A,B,C be the following 3×3 matrices.

$$A=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix},B=\begin{bmatrix}1&0&1\\0&3&0\\1&0&5\end{bmatrix},C=\begin{bmatrix}-1&0&1\\0&5&6\\3&0&1\end{bmatrix}$$

Then compute and simplify the following expression.

$$(A^T-B)^T+C(B^{-1}C)^{-1}$$

3.Let A and B are $n\times n$ matrices with real entries.Assume that $A+B$ is invertible.Then show that

$$A(A+B)^{-1}B=B(A+B)^{-1}A.$$

4.Let A be an $n\times n$ invertible matrix.Prove that the inverse matrix of A is uniques.

5.Let

$$A=\begin{bmatrix}a&0\\0&b\end{bmatrix}$$

Show that

(1)

$$A^n=\begin{bmatrix}a^n&0\\0&b^n\end{bmatrix} for\quad any\quad n\in N$$

(2)Let $B=S^{-1}AS$,where S be an invertible 2×2 matrix.Show that $B^n=S^{-1}A^nS$ for any $n\in N$.

Hint :Use mathematical induction.

6.Let A be a real symmetric matrix whose diagonal entries are all positive real numbers. Is it true that the all of the diagonal entries of the inverse matrix A^{-1} are also positive? If so, prove it. Otherwise, give a counterexample.

Hint :$A^{-1} = \frac{A^*}{|A|}$

7.Consider the system of linear equations

$$\begin{cases} x_1 = 2 \\ -2x_1 + x_2 = 3 \\ 5x_1 - 4x_2 + x_3 = 2 \end{cases}$$

(a). Find the coefficient matrix A for this system.

(b). Find the inverse matrix of the coefficient matrix found in (a)

(c). Solve the system using the inverse matrix A^{-1}.

8.Determine the values of x so that the matrix

$$A = \begin{bmatrix} 1 & 1 & x \\ 1 & x & x \\ x & x & x \end{bmatrix}$$

is invertible. For those values of x, find the inverse matrix A^{-1}.

9.Let

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Then find the value of

det$(A^2 B^{-1} A^{-2} B^2)$

(Hint：Without a proof, you may assume that A and B are invertible matrices.)

10. Find the determinant of the matix

$$A=\begin{bmatrix}100 & 101 & 102\\ 101 & 102 & 103\\ 102 & 103 & 104\end{bmatrix}$$

11.

设

$$A=\left[\begin{array}{ccc|cc}2 & -3 & 1 & 0 & -4\\ 1 & 5 & -2 & 3 & -1\\ \hline 0 & -4 & -2 & 7 & -1\end{array}\right]=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}, B=\left[\begin{array}{cc}6 & 4\\ -2 & 1\\ -3 & 7\\ \hline -1 & 3\\ 5 & 2\end{array}\right]=\begin{bmatrix}B_1\\ B_2\end{bmatrix}$$

请验证：用**分块矩阵乘法**和**矩阵乘法（按行列计算）**这两种方法计算出来的矩阵乘积是一样的。并说明，使用分块矩阵来计算矩阵的乘积需要注意什么？（就是计算 AB 的时候，从**对 A 的列的分法**和对 **B 行的分法** 的角度来说明）