

Comparative Modeling of USDEUR FX Swaptions

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Abstract

This project investigates the valuation of a USDEUR Cross-Currency FX Swaption by comparing the widely used closed-form pricing solution, the Black '76 model (Benchmark), against the advanced Merton Jump-Diffusion (JD) model (Alternative). The goal was to quantify the pricing impact of assuming fat-tailed (leptokurtic) distributions over the Black model's standard log-normal assumption. Key financial engineering procedures, including yield curve bootstrapping and Vanna-Volga surface interpolation, were performed. The primary finding is that the Merton JD model yields a price that is 8.06% higher than the Black model, accurately capturing a Jump Risk Premium necessary for a more robust and prudent valuation in volatility-sensitive markets.

1 Introduction and Project Objectives

The project addresses the core requirement of the Quantitative Methods in Finance program by modeling an alternative derivative instrument—the FX Swaption—and rigorously comparing valuation methodologies. The specific instrument is a 2-Year into 3-Year Payer FX Swaption on the USDEUR currency pair.

1.1 Model Selection Rationale

The comparison focuses on how each model handles volatility and tail risk:

- Black '76 (Benchmark): Assumes constant volatility (σ) and a log-normal distribution.
- Merton Jump-Diffusion (Alternative): Explicitly models jumps in the FX rate, resulting in a leptokurtic distribution.

2 Methodology and Implementation

The valuation was executed through a five-step pipeline combining market data processing and model computation.

2.1 Data and Curve Construction

1. Data Processing: Par yields were bootstrapped into a continuous-compounding zero-coupon curve for the domestic ($D(t)$) and foreign ($D_f(t)$) currencies.
2. Volatility Surface: FX Option quotes (ATM, Risk Reversal, Butterfly) were used to define the full implied volatility smile via the Vanna-Volga method.

2.2 Valuation Models and Formulas

Black '76 Model (Benchmark)

The FX Swaption price (V_{Black}) is given by the Black-76 formula, adjusted for the swap annuity (P_{fixed}):

$$V_{\text{Black}} = P_{\text{fixed}} \cdot [F \cdot N(d_1) - K \cdot N(d_2)]$$

where the forward swap rate is $F = S_0 \cdot \frac{D_f(T_{\text{exp}})}{D(T_{\text{exp}})}$ and d_1 and d_2 are:

$$d_1 = \frac{\ln(F/K) + \frac{1}{2}\sigma^2 T_{\text{exp}}}{\sigma\sqrt{T_{\text{exp}}}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T_{\text{exp}}}$$

Merton Jump-Diffusion Model (Alternative)

The spot FX process under the risk-neutral measure (\mathbb{Q}) is defined by the SDE:

$$\frac{dS_t}{S_t} = (\mu_{\mathbb{Q}} - \lambda j)dt + \sigma_{\text{JD}}dW_t + dJ_t$$

where j is the average size of the jump, σ_{JD} is the diffusion volatility, and dJ_t is the compound Poisson process (λ is the jump intensity).

The Swaption price (V_{JD}) is calculated using a Monte Carlo Simulation (MC), averaging the discounted payoffs:

$$V_{\text{JD}} = D(T_{\text{exp}}) \cdot P_{\text{fixed}} \cdot \mathbb{E}^{\mathbb{Q}}[\max(S_{T_{\text{exp}}} - K, 0)]$$

3 Results and Key Findings

Table 1: Comparative Valuation Results (2Y \rightarrow 3Y Payer Swaption)

Model	Calculated Price (USD)	Difference to Black
Black '76 (Benchmark)	0.094153	—
Merton Jump-Diffusion (MC)	0.101742	+0.007589

3.1 Inference: Superiority of the JD Model

The numerical results highlight two central findings regarding model performance:

Finding 1: Quantifying the Jump Risk Premium

The Merton JD model yielded a price that was 8.06% higher than the Black model. This difference (\$0.0076) represents the Jump Risk Premium. The JD price is a more prudent, risk-adjusted valuation that reflects true market volatility expectations.

Finding 2: Theoretical Consistency

The Merton JD model is theoretically superior because its leptokurtic distribution better reflects the empirical nature of FX returns, allowing it to naturally reproduce the observed Volatility Smile that the Black model cannot. By moving beyond constant volatility, the JD model provides a valuation that is more robust and consistent with market microstructure and risk management principles.

4 Conclusion

The project successfully demonstrated the implementation and comparative analysis of two critical volatility models for FX Swaption valuation. The reliance on the simplistic Black model results in an artificially low price. For accurate risk management, the inclusion of fat-tail characteristics, as implemented via the Merton Jump-Diffusion model and Monte Carlo simulation, is essential, providing a 8.06% higher, more prudent valuation that explicitly accounts for jump risk.