Meccanica classica

Marco Militello

Indice

$(q_1, \ldots, q_d) \to (\tilde{q}_1, \ldots, \tilde{q}_d)$

$$\begin{cases} q_j = q_j(\tilde{q}_1, \ldots, \tilde{q}_d) \to \dot{q}_j = \dot{q}_j(\underline{\tilde{q}}, \underline{\dot{\tilde{q}}}) \qquad J_{kj} = \frac{\delta q_k}{\delta \tilde{q}_j} \quad \det J \neq 0 \\ j = 1, \ldots, d \end{cases}$$

$\begin{cases} \tilde{q}_j(q_1, \ldots, q_d) \\ j = 1, \ldots, d \end{cases}$

$\mathcal{L}(q_1, \ldots, q_d, \dot{q}_1, \ldots, \dot{q}_d) \qquad \tilde{\mathcal{L}}(\underline{\tilde{q}}, \underline{\dot{\tilde{q}}}) = \mathcal{L}(\underline{q}(\underline{\tilde{q}}), \dot{q}(\underline{\tilde{q}}, \underline{\dot{\tilde{q}}}))$

Prop. q(t) è soluzione E-L per $\mathcal{L}(\underline{q}, \underline{\dot{q}}) \iff \tilde{q}(t)$ è soluzione per $\mathcal{L}(\underline{\tilde{q}}, \underline{\dot{\tilde{q}}})$

Dimostrazione. $(\Rightarrow)$ $q(t)$ è soluzione E-L per $\mathcal{L}$

$$\frac{d}{dt}\frac{\delta \tilde{\mathcal{L}}}{\delta \dot{\tilde{q}}_j} = \frac{d}{dt}\left(\sum_{i=1}^{d} \frac{\delta \mathcal{L}}{\delta \dot{q}_i}\frac{\delta \dot{q}_i}{\delta \dot{\tilde{q}}_j}\right) = \frac{d}{dt}\left(\sum_{i=1}^{d} \frac{\delta \mathcal{L}}{\delta \dot{q}_i} \overbrace{\frac{\delta q_i}{\delta \tilde{q}_j}}^{lemma1}\right) = \sum_{i=1}^{d}\left[\left(\frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{q}_i}\right)\frac{\delta q_i}{\delta \dot{\tilde{q}}_j} + \frac{\delta \mathcal{L}}{\delta \dot{q}_i} \underbrace{\frac{d}{dt}\frac{\delta q_i}{\delta \tilde{q}_j}}_{lemma2}\right] =$$

$$= \sum_{i=1}^{d}\left[\left(\frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{q}_i}\right)\frac{\delta q_i}{\delta \dot{q}_j} + \frac{\delta \mathcal{L}}{\delta \dot{q}_i}\frac{\delta \dot{q}_i}{\delta \tilde{q}_j}\right]$$

Uso ipotesi

$$= \sum_{i=1}^{d}\left(\frac{\delta \mathcal{L}}{\delta q_i}\frac{\delta q_i}{\delta \dot{\tilde{q}}_j} + \frac{\delta \mathcal{L}}{\delta \dot{q}_i}\frac{\delta \dot{q}_i}{\delta \tilde{q}_j}\right) = \frac{\delta \tilde{\mathcal{L}}}{\delta \tilde{q}_j}$$

□

Prop. $\forall$ scelta di F($\underline{q}$,t) e $\alpha \neq 0 \in \mathbb{R}$ allora

$$\mathcal{L}(\underline{q}, \underline{\dot{q}}, t) \quad \mathcal{L}'(\underline{q}, \underline{\dot{q}}, t) = \alpha(\underline{q}, \underline{\dot{q}}, t) + \frac{dF}{dt}$$

conducono alla stessa soluzion per E-L

Dimostrazione.
$L_0 = \frac{dF}{dt} = \dot{F} \qquad \mathcal{L}' = \alpha\mathcal{L} + L_0$

$$\frac{d}{dt}\frac{\delta \mathcal{L}'}{\delta \dot{q}_j} = \alpha\frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{q}_j} + \frac{d}{dt}\frac{\delta L_0}{\delta \dot{q}_j}$$

$\frac{d}{dt}\frac{\delta L_0}{\delta \dot{q}_j} = \frac{d}{dt}\frac{\delta \dot{F}}{\delta \dot{q}_j} \underbrace{=}_{Lemma1} \frac{d}{dt}\frac{\delta F}{\delta q_j} \underbrace{=}_{Lemma2} \frac{\delta \dot{F}}{\delta \dot{q}_j} = \alpha\frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{q}_j} + \frac{\delta L_0}{\delta q_j}$ □