

## Lehmann representation for a non-interacting system

Consider a generic non-interacting system of fermions, whose Hamiltonian is quadratic, i.e.,

$$\hat{H} = \sum_{ij} [\mathbf{h}]_{ij} \hat{c}_i^\dagger \hat{c}_j, \quad (1)$$

where  $\hat{c}_i^\dagger$  creates a fermionic particle in the  $i$ -th spin-orbital, and the spin-orbitals form the orthonormal basis. The single-particle Hamiltonian  $\mathbf{h}$ , which is Hermitian, can be diagonalized as

$$\mathbf{h} = \mathbf{V} \boldsymbol{\epsilon} \mathbf{V}^\dagger, \quad (2)$$

where  $[\boldsymbol{\epsilon}]_{ij} = \epsilon_i \delta_{ij}$  is the diagonal matrix containing the single-particle energy eigenvalues  $\epsilon_i$ , and  $\mathbf{V} = (\vec{v}_1 \vec{v}_2 \cdots)$  is the unitary matrix whose column vectors are the eigenvectors of  $\mathbf{h}$ .

Since the (many-body) Hamiltonian  $\hat{H}$  is quadratic, the retarded Green's function,

$$G[\hat{d}_i, \hat{d}_j^\dagger](t) = -i\theta(t) \text{Tr} \left( \hat{\rho} [\hat{d}_i(t), \hat{d}_j^\dagger]_{\pm} \right), \quad (3)$$

and its spectral function,

$$A[\hat{d}_i, \hat{d}_j^\dagger](\omega) = \frac{-1}{\pi} \text{Im} \int_{-\infty}^{+\infty} dt e^{i\omega t} G[\hat{d}_i, \hat{d}_j^\dagger](t), \quad (4)$$

can be computed exactly.

- (a) Evaluate the spectral function  $A[\hat{d}_i, \hat{d}_j^\dagger](\omega)$  by using the Lehmann representation.
- (b) The local spectral function is the case of  $i = j$  in which the two defining operators  $\hat{d}_i$  and  $\hat{d}_i^\dagger$  act on the same spin-orbital. Explain why the local spectral function can be interpreted as the local density of states.