TAREAS 2do Corte.

1ra Entrega.

Sistemas Dinamicos Grp 005 - 1

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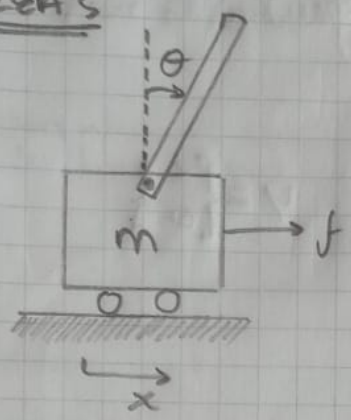
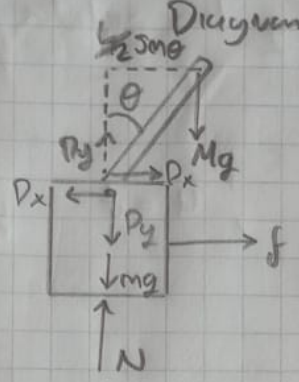
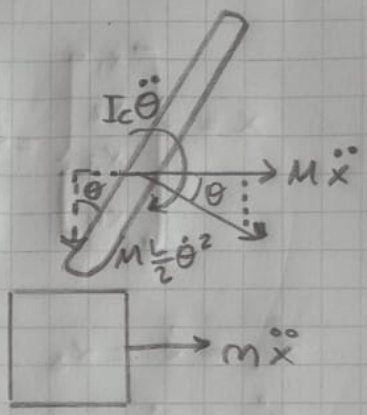
TAREA 5







$$+\rightarrow x: \sum F_x = \sum_{i=1}^{2} m_i (a_{ci})_x$$

$$f = m\ddot{x} + M\ddot{x} - M\frac{L}{2}\dot{\theta}^2 \operatorname{sen}\theta + M\frac{L}{2}\ddot{\theta}\cos\theta$$

$$+\circlearrowleft: \sum M_P = I_c\alpha + M_{efd\_mas}$$

$$Mg\cdot\frac{L}{2}\operatorname{sen}\theta = \frac{1}{12}ML^2\ddot{\theta} + M\ddot{x}\frac{L}{2}\cos\theta + M\frac{L}{2}\ddot{\theta}\cdot\frac{L}{2}$$

reordenando las ecuaciones:

$$(m+M)\ddot{x} + \frac{1}{2}ML\ddot{\theta}\cos\theta - \frac{1}{2}ML\dot{\theta}^2\operatorname{sen}\theta = f$$

$$\frac{1}{3}ML^2\ddot{\theta} + \frac{1}{2}ML\ddot{x}\cos\theta - \frac{1}{2}MgL\operatorname{sen}\theta = 0$$

- Para pequeños movimientos angulares, $\cos\theta \approx 1$, $\operatorname{sen}\theta \approx \theta$, $\dot{\theta}^2\theta = 0$

$$(m+M)\ddot{x} + \frac{1}{2}ML\ddot{\theta} = f$$

$$\frac{1}{3}ML^2\ddot{\theta} + \frac{1}{2}ML\ddot{x} - \frac{1}{2}MgL\theta = 0$$

los estados, la entrada y las salidas especificadas son:

$$x = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} x \\ \theta \\ \dot{x} \\ \theta \end{Bmatrix}, \quad u = f \quad y \quad y = \begin{Bmatrix} x \\ \theta \end{Bmatrix}$$

$$\dot{x}_1 = \dot{x} = x_3$$
$$\dot{x}_2 = \dot{\theta} = x_4$$

tomando: • resolviendo por "Cramer"

$$\ddot{x} = \frac{(1/3)\, ML^2 f - (1/4)\, M^2 L^2 g \theta}{(1/12)\, ML^2 (M + 4m)}$$

$$\ddot{\theta} = \frac{-(1/2)\, ML f + (M+m)(1/2)\, Mg L \theta}{(1/12)\, ML^2 (M+4m)}$$

reemplazando:

$$\dot{x}_3 = \frac{4}{M+4m} u - \frac{3Mg}{M+4m} x_2$$

$$\dot{x}_4 = \frac{-6}{L(M+4m)} u + \frac{6(M+m)g}{L(M+4m)} x_2$$

• Espacio de estados:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{3Mg}{M+4m} & 0 & 0 \\ 0 & \frac{6(M+m)g}{L(M+4m)} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{4}{M+4m} \\ -\frac{6}{L(M+4m)} \end{bmatrix} u$$

$$y = \begin{bmatrix} x \\ \theta \end{bmatrix} = \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$