Tarea 5 — Philip Mateo Millán Patiño
20202005096

Video 1

- sistema de control por realimentación de estados

$G(s) = \frac{20(s+5)}{s(s+1)(s+4)}$ $\begin{cases} Os \;/\; 4,5\,\% \\ ts = 9+4sy \end{cases}$

$U(s) \longrightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X(s)} \boxed{0s^2+20s+100} \longrightarrow Y(s)$

$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s} \longrightarrow (s^3+5s^2+4s)\,X_1(s) = U(s)$

- $\dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u$

Variables de Estado:
$x_1 = x_1$
$x_2 = \dot{x}_1$
$x_3 = \dot{x}_2 = \ddot{x}_1$
$\dot{x}_3 = \dddot{x}_1$

reemplazando:

- $\dot{x}_3 + 5x_3 + 4x_2 = u \longrightarrow \boxed{\dot{x}_3 = -5x_3 - 4x_2 + u}$

$Y(s) = (b_2 s^2 + b_1 s + b_0)\,X_1(s)$
$= (0s^2 + 20s + 100)\,X_1(s) \longrightarrow (20s+100)\,X_1(s)$
$= 20\dot{x}_1 + 100x_1 \longrightarrow$ Remplazando variables de estado

$\boxed{y = 20x_2 + 100x_1}$

Representación en espacio de estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\%0{,}05 = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100$$

$$0{,}095 = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100 \rightarrow \ln(0{,}045) \cdot \ln\left(e^{-(\zeta\pi/\sqrt{1-\zeta^2})}\right)$$

$$-3{,}3539 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \longrightarrow 2{,}3539\left(\sqrt{1-\zeta^2}\right) = \cancel{-}\,\zeta\pi$$

- Elevando al cuadrado ambos lados

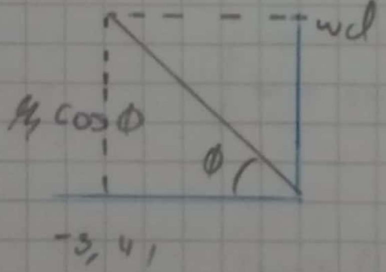
$$5{,}5407(1-\zeta^2) = \zeta^2\pi^2$$

- $5{,}5407 - 5{,}5407\,\zeta^2 - \zeta^2\pi^2 \longrightarrow 5{,}5407 = \zeta^2\pi^2 + 5{,}54\,\zeta^2$

$$5{,}5407 = \zeta^2(\pi^2 + 5{,}5407) \longrightarrow \zeta^2 - \frac{5{,}5407}{\pi^2 + 5{,}5407}.$$

$$\zeta = \sqrt{\frac{5{,}5407}{\pi^2 + 5{,}5407}} \longrightarrow \boxed{\zeta = 0{,}5946}$$

plano s



$$s = \sigma + j\omega d \longrightarrow \sigma = \zeta\omega_n$$

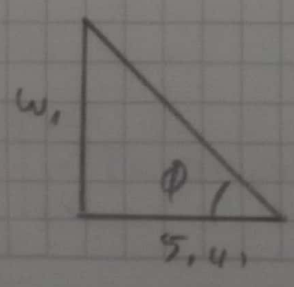
$$\phi = \cos^{-1}(0{,}5946) = 53{,}16°$$

$$T_s = \frac{4}{\sigma} \longrightarrow 0{,}74 = \frac{4}{\sigma}$$

$$\sigma = \frac{4}{0{,}74} = 5{,}405$$

$$\sigma = \zeta\omega_n \rightarrow 5{,}405 = 0{,}5946\,\omega_n$$

$$\omega_n = 9{,}02 \text{ rad/seg}$$



$$\tan\phi = \frac{\omega d}{3{,}41}$$

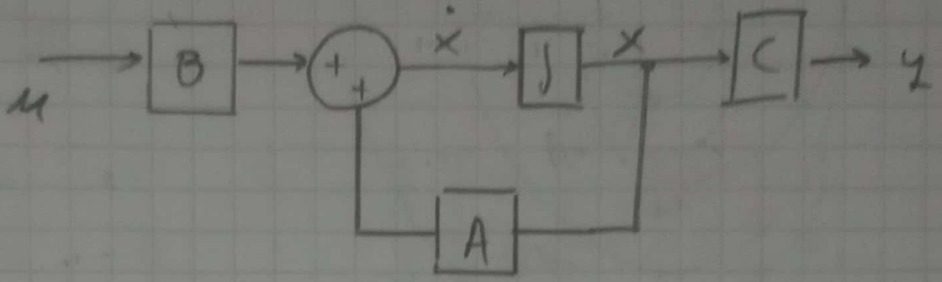
$$\tan^{-1}(53{,}16)(5{,}41) = \omega d$$

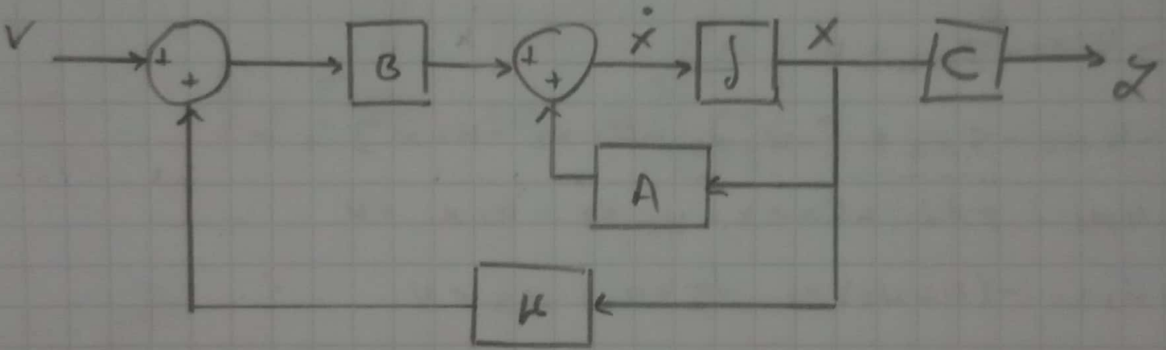
$$\boxed{\omega d = 7{,}2146}$$

realimentación en espacio de estados:

$\dot{x} = Ax + Bx$

$y = Cx$





$\dot{x} = Ax + Bu$

$= Ax + B(-Kx + r)$

$= Ax - BKx + Bv \longrightarrow \boxed{\dot{x} = (A - B)x + v}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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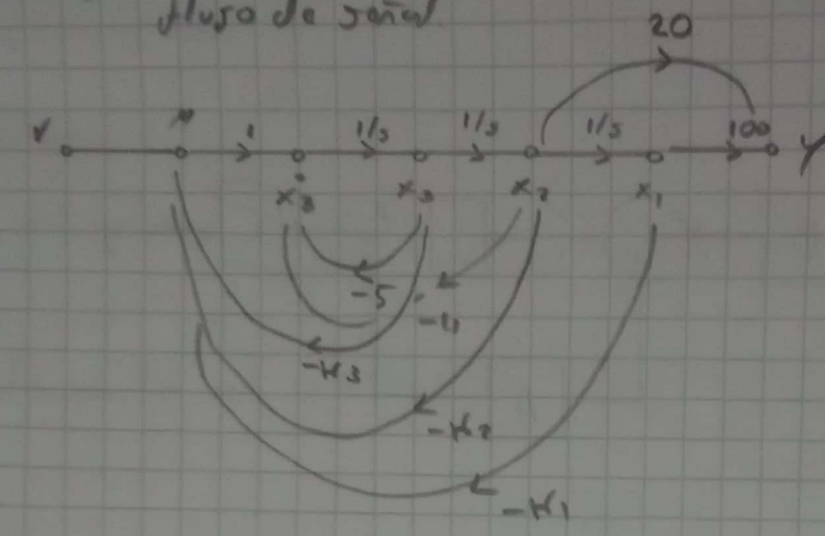
Diagrama de flujo de señal



$$\dot{x}_3 = -4x_2 - 5x_3 + u$$

$$= -4x_2 - 5x_3 + [-k_3x_3 - k_2x_2 - k_1x_1] + r$$

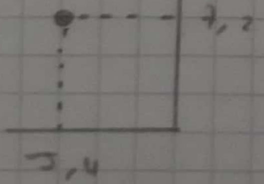
$$= -4x_2 - 5x_3 - k_3x_3 - k_2x_2 - k_1x_1 + v$$

$$= -k_1x_1 - (4+k_2)x_2 - (5+k_3)x_3 + v$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -(4+k_2) & -5+k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$\det(sI - A(34)) = \boxed{s^3 + (5+k_3)s^2 + (4+k_2)s + k_1 = 0}$$

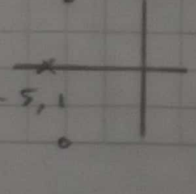
→ función característica del sistema



$$(s+5,4-j7,2)(s+5,4+j7,2)(s+5,1) = 0$$

$$T. \rightarrow s^3 + 15,9s^2 + 136,21s + 413,83 = 0$$

$$s^3 + (5+k_3)s^2 + (4+k_2)s + k_1 = s^3 + 15,9s^2 + 136,22s + 413,83$$



$(s+k_3)s^2 = 15{,}9s\cancel{s^2}$

$s + k_3 = 15{,}9$

$k_3 = 10{,}9$

$(4+k_2)s + 136{,}2s$

$4 + k_2 = 136{,}1$

$k_2 = 132{,}22$

$k_1 = 413{,}83$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -413{,}9 & -136{,}2 & -15{,}9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$