Green University of Bangladesh
Department of Computer Science and Engineering (CSE)
Faculty of Sciences and Engineering
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ASSAIGNMENT: KSA TEST 1
Course Title: Differential Equations and Co-ordinate Geometry
Course Code: MAT 201 Section: 232_D6

MATH ASSAIGNMENT NAME: Co-ordinate Geometry

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Assaignment Status
Marks: ………………………………… **Signature:.....................**
Comments:... **Date:..............................**

(1)

Given,

$3x^2 - 16xy + 5y^2 = 0$ — (1)

$\Rightarrow 3x^2 - 16xy + 5y^2 + 2.0.x + 2.0.y + 0 = 0$

General equations of 2nd degree,

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ — (ii)

comparing (1) & (ii),

$a = 3,\ h = -8,\ b = 5,\ g = 0,\ f = 0,\ c = 0$

$\therefore \Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \cancel{= 0}$

$\Rightarrow \Delta \cancel{=0} 0 + 0 - 0 - 0 - 0 \cancel{= 0}$

$\therefore \Delta = 0$

$\therefore$ so (1) represents ~~a pair of~~ the straight lines.

$\therefore \cancel{ab - h^2 = 3 \times (-8)}$ ~~3×5 18 − (49 − 49)~~

Let, $F(x,y) = 3x^2 - 16xy + 5y^2$

$\frac{\partial F}{\partial x} = 6x - 16y \Rightarrow 8x - 8y = 0$

$\frac{\partial F}{\partial y} = -16x + 10y \Rightarrow 8x - 5y = 0$

$3x - 8y + 0 = 0$
$8x - 5y + 0 = 0$

cross product we get,

$$\frac{x}{0 - 0} = \frac{y}{0} = \frac{1}{-15+64}$$

$$\Rightarrow \frac{x}{0} = \frac{y}{0} = \frac{1}{49}$$

$\therefore x = 0, \quad y = 0$

straight line are

$3x - 8y = 0$
$8x - 5y = 0$

Both are perpendicular.

(2)

Given,

$$ax^2+4xy+y^2-4x-2y-3=0 \quad \text{--- (i)}$$

Genarel equation of 2nd degree,

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad \text{--- (ii)}$$

comparing (i) & (ii),

$a=a$, $b=1$, $f=-1$, $g=-2$, $h=2$, $c=-3$

We know,

$$\Delta = abc+2fgh-af^2-bg^2-ch^2=0$$

$$\Rightarrow -3a+8-a-4+6=0$$

$$\Rightarrow -4a+10=0$$

$$\Rightarrow \cancel{4a-10=}$$

$$\Rightarrow 2a-5=0$$

$$\therefore a=5/2$$

(3)

Given,

$(x^2+y^2)(\cos^2\theta\sin^2\theta+\sin^2\theta) = (x\tan\theta-y\sin\theta)^2$

$\Rightarrow x^2\cos^2\theta\sin^2\theta+x^2\sin^2\theta+y^2\cos^2\theta\sin^2\theta+y^2\sin^2\theta = (x^2\tan^2\theta-2xy\tan\theta\sin\theta+y^2\sin^2\theta)$

$\Rightarrow x^2(\cos^2\theta\sin^2\theta+\sin^2\theta)+y^2(\cos^2\theta\sin^2\theta+\sin^2\theta)$
$= x^2\tan^2\theta-2xy\tan\theta\sin\theta+y^2\sin^2\theta$

Genarel equation of 2nd degree,

$ax^2+2hxy+by^2+2gx+2fy+c=0$

$\therefore$ Both equations are simillar with Genar 2nd degree quandratic equation

~~$\tan\theta=\infty$~~

$\therefore \theta=90^\circ$

$\therefore$ The root is very complex that means angle will be $\theta=90^\circ$, $\tan 90^\circ=\infty$

(4)

Given,

$$8x^2+4xy+5y^2-24x+24y=0 \quad -(i)$$

$$\Rightarrow 8x^2+4xy+5y^2-24x+24y+0=0$$

Genaral equation of 2nd degree,

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad -(ii)$$

comparing (i) & (ii),

$a=8, b=5, f=12, g=-12, h=2, c=0$

$\Delta = abc+2fgh-af^2-bg^2-ch^2$

~~$= 0 - 288$~~

$= 0-576-1152-720-0$

$= -2448$

$\therefore \Delta \neq 0$

$\therefore ab-h^2 = (8\times5)-2^2 = 40-4 = 36 > 0$

So, ~~It~~ Equation (i) is a ellipse.

Let,

$F(x, y) = 8x^2 + 4xy + 5y^2 - 24x + 24y$

$\frac{\partial f}{\partial x} = 16x + 4y - 24 = 0 \Rightarrow 4x + y - 6 = 0$

$\frac{\partial f}{\partial y} = 4x + 10y + 24 \Rightarrow 2x + 5y + 12 = 0$

solving,

$\frac{x}{12+30} = \frac{y}{48+12} = \frac{1}{20-2}$

$\Rightarrow \frac{x}{42} = \frac{y}{60} = \frac{1}{18}$

$\Rightarrow \therefore x = 7/3 \; , y = \cancel{10} \; 10/3$

$\therefore$ center is $(7/3, 10/3)$

~~Shifting the origin the reduce form~~

is

$\therefore$ New constant

$c_1 = gx + fy + c = -12 \times 7/3 + 12 \times 10/3 +$

$= -28 + 40$

$= 12$

$\therefore$ Reduce ~~to~~ form of equation (I),

$$\cancel{3x^2}$$

$$8x^2+4xy+5y^2+12=0 \quad \text{—} \quad (\text{II})$$

If we remove the xy term then equation (II),

$$a'x^2+b'y^2+12=0$$

where,

$$\therefore a'+b' = a+b = 8+5 = 13$$

and $a'b' = ab-h^2 = 36$

$$a'-b' = \sqrt{(a'+b')^2 - 4a'b'}$$

$$= \sqrt{13^2-(4\times 36)}$$

$$= 5$$

solving,

$a' = 9$, $b' = 4$

Hence the equation is,

$$9x^2+4y^2 = -12$$

$$\Rightarrow \frac{x^2}{\left(-\frac{4}{3}\right)} + \frac{y^2}{(-3)} = 1$$

$$\therefore \frac{x^2}{\left(\sqrt{\left(-\frac{4}{3}\right)}\right)^2} - \frac{y^2}{(\sqrt{3})^2} = 1$$

(5)

Given the equation of paraboloa,

$$2x^2+\sqrt{3}xy+4x-3y-3y^2=2a^2 \quad -①$$

Here,

$$\theta=60^\circ$$

$$x = x'\cos60^\circ - y'\sin60^\circ$$

$$=\frac{1}{2}x'-\frac{\sqrt{3}}{2}y'$$

$$y=x'\sin60^\circ+y'\cos60^\circ$$

$$=\frac{\sqrt{3}}{2}x'+\frac{1}{2}y'$$

Then the corresponding equation,

$$2\left(\frac{1}{2}x'-\frac{\sqrt{3}}{2}y'\right)^2+\sqrt{3}\left(\frac{1}{2}x'-\frac{\sqrt{3}}{2}y'\right)\left(\frac{\sqrt{3}}{2}x'+\frac{1}{2}y'\right)$$
$$+4\left(\frac{1}{2}x'-\frac{\sqrt{3}}{2}y'\right)-3\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}y'\right)-$$
$$3\left(\frac{\sqrt{3}}{2}x'+\frac{1}{2}y'\right)^2-2a^2=0$$

$$\Rightarrow 2\left(\frac{1}{4}x'^2-2\cdot\frac{1}{2}x'\cdot\frac{\sqrt{3}}{2}y'+\frac{3}{4}y'^2\right)+\frac{\sqrt{3}}{2}x'-\frac{3}{2}y'$$
$$\sqrt{3}\left(\frac{\sqrt{3}}{4}x'^2+\frac{1}{4}x'y'-\frac{3}{2}x'y'-\frac{\sqrt{3}}{4}y'^2\right.$$

$$2x' - 2\sqrt{3}y' - \frac{3\sqrt{3}}{2}x' - \frac{3}{2}y' - 3\left(\frac{3}{4}x'^2 + 2\cdot\frac{\sqrt{3}}{2}x'\cdot\frac{1}{2} \ldots + \frac{1}{4}y'^2\right) - 2a^2 = $$

$$= \frac{1}{2}x'^2 - \sqrt{3}x'y' + \frac{3}{2}y'^2 + \frac{3}{4}x'^2 + \frac{\sqrt{3}}{4}x'y' - \frac{3\sqrt{3}}{2}x'y' - \frac{3}{4}y'^2 + 2x' - 2\sqrt{3}y' - \frac{3\sqrt{3}}{2}x' - \frac{3}{2}y' - \frac{9}{4}x'^2 - \frac{3\sqrt{3}}{2}x'y' - \frac{3}{4}y'^2 - 2a^2 = 0$$

$$\Rightarrow x'^2\left(\frac{1}{2} + \frac{3}{4} + \frac{9}{4}\right) - x'y'\left(\sqrt{3} + \frac{3\sqrt{3}}{2} - \frac{\sqrt{3}}{4} + \frac{3\sqrt{3}}{2}\right) + y'^2\left(\frac{3}{2} - \frac{3}{4} - \frac{3}{4}\right) \cancel{+2x' - 2\sqrt{3}y'} + x'\left(2 - \frac{3\sqrt{3}}{2}\right) - y'\left(2\sqrt{3} + \frac{3}{2}\right) - 2a^2 = 0$$

$$\Rightarrow \frac{7}{2}x'^2 - \frac{9\sqrt{3}}{4}x'y' + y'^2 \times 0 + \frac{4 - 3\sqrt{3}}{2}x' - \frac{4\sqrt{3} + 3}{2}y' - 2a^2 = 0$$

$$\Rightarrow \frac{7}{2}x'^2 - \frac{9\sqrt{3}}{4}x'y' + \frac{4 - 3\sqrt{3}}{2}x' - \frac{4\sqrt{3} + 3}{2}y' - 2a^2 = 0$$