MillenniumDB: A Persistent, Open-Source, Graph Database

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ABSTRACT

In this systems paper, we present MillenniumDB: a novel graph database engine that is modular, persistent, and open source. MillenniumDB is based on a novel graph data model, called domain graphs, which provides a simple abstraction upon which a variety of popular graph models can be supported. The engine itself is founded on a combination of tried and tested techniques from relational data management, state-of-the-art algorithms for worst-case-optimal joins, as well as graph-specific algorithms for evaluating path queries. In this paper, we present the main design principles underlying MillenniumDB, describing the abstract graph model and query semantics supported, the concrete data model and query syntax implemented, as well as the storage, indexing, query planning and query evaluation techniques used. We evaluate MillenniumDB over real-world data and queries from the Wikidata knowledge graph, where we find that it outperforms other popular persistent graph database engines (including both enterprise and open source alternatives) that support similar query features.

1 INTRODUCTION

Recent years have seen growing interest in graph databases [3], wherein nodes represent entities of interest, and edges represent relations between those entities. In comparison with alternative data models, graphs offer a flexible and often more intuitive representation of particular domains [2]. Graphs forgo the need to define a fixed (e.g., relational) schema for the domain upfront, and allow for modelling and querying cyclical relations between entities that are not well-supported in other data models (e.g., tree-based models, such as XML, JSON, etc.). Graphs have long been used as an intuitive way to model data in domains such as social networks, transport networks, genealogy, biological networks, etc. Graph databases further enable specific forms of querying, such as path queries that find entities related by arbitrary-length paths in the graph. Graph databases have become popular in the context of NoSQL [8], where alternatives to relational databases are sought for specialised scenarios; Linked Data [16], where graph-structured data are published and interlinked on the Web; and more recently Knowledge Graphs [19], where diverse data are integrated at large scale under a common graph abstraction.

Alongside this growing interest, recent years have seen a growing number of models, languages, techniques and systems for managing and querying graph databases [2, 3]. In the context of NoSQL systems, Neo4j [41], which uses the query language Cypher [12], is a leading graph database system in practice.[1] Other popular graph database systems include ArangoDB [28], JanusGraph [30], OrientDB [35], TigerGraph [37], etc., which support Gremlin [25] and other custom graph query languages. We also find graph database systems supporting the RDF data model and SPARQL query language [1], including Allegrograph [27], Amazon Neptune [29], Blazegraph [38], GraphDB [6], Jena TDB [31], Stardog [36], Virtuoso [10], and (many) more besides [1]. In summary, there now exist many graph database systems to choose from. Within this graph database landscape, however, we foresee the need for yet another alternative: an open source graph database that implements state-of-the-art techniques; offers performance, scale and reliability competitive with (or ideally better than) enterprise systems; supports a variety of graph data models and query languages; and is extensible. To the best of our knowledge, no existing open source graph database system combines these features.

For this reason, we have designed and implemented *MillenniumDB*: a persistent graph database system that aims to better align the theory and practice for graph database systems, being based on a solid theoretical foundation and state-of-the-art techniques, while being applicable in practical settings. The overall design and implementation of MillenniumDB is based on the following goals:

- *Generality*: MillenniumDB can support various graph database models and query languages by generalising them and implementing techniques for the more general setting.
- *State-of-the-art*: With the goal of reaching state-of-the-art performance, MillenniumDB incorporates and combines techniques from the recent research literature.
- *Theoretically well-founded*: MillenniumDB provides clear semantics for its underlying interfaces, and prioritises techniques that provide theoretical guarantees.
- *Modularity*: In order to enable extensibility, MillenniumDB follows a modular design that decouples a graph database system into components with clearly defined interfaces.
- *Open source*: The code for MillenniumDB is available under an open source license [32], with the goal that it may be extended and reused by other researchers and practitioners.

These research goals imply significant engineering and research challenges that form a road-map for the development of MillenniumDB. In this systems paper, we present a first milestone in this development: the first release of MillenniumDB, which establishes the foundations and general architecture of a graph database system that will be extended in the coming years. These foundations include the proposal of a novel data model – called *domain graphs* – that provides a suitable abstraction for supporting a range of popular graph models and query languages, a novel query syntax adapted for this model, and a query engine that includes state-of-the-art join and path algorithms. A variety of benchmarks over the Wikidata knowledge graph [40] show that this first release of MillenniumDB generally outperforms prominent graph database systems – specifically Blazegraph, Neo4j, Jena and Virtuoso – in terms of query performance.

[1] See, e.g., https://db-engines.com/en/ranking/graph+dbms

Paper structure. The rest of this paper is structured as follows:

- In Section 2, we discuss graph data models that are popular in current practice, and propose *domain graphs* as an abstraction of these models that we implement in MillenniumDB.
- In Section 3, we describe the query language of MillenniumDB, and how it takes advantage of domain graphs.
- In Section 4 we explain how MillenniumDB stores the data and evaluates its queries.
- Section 5 is devoted to empirical testing of the proposed methods on a large body of queries over the WikiData knowledge graph.
- We conclude in Section 6.

2 DATA MODEL

In this section, we present the graph data model upon which MillenniumDB is based, and discuss how it generalizes existing graph data models such as RDF and property graphs. We also show its utility in concisely modeling real-world knowledge graphs that contain higher-arity relations, such as Wikidata [40].

2.1 Domain Graphs

The structure of knowledge graphs is captured in MillenniumDB via *domain graphs*. Formally, assume a universe Obj of objects (ids, strings, numbers, IRIs, etc.). We define domain graphs as follows:

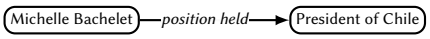
Definition 2.1. A *domain graph* $G = (O, \gamma)$ consists of a finite set of objects $O \subseteq \text{Obj}$ and a partial mapping $\gamma : O \to O \times O \times O$.

Intuitively, O is the set of objects that appear in our graph database, and γ models edges between objects. If $\gamma(e) = (n_1, t, n_2)$, this states that the edge (n_1, t, n_2) has id e, type t, and links the source node n_1 to the target node n_2.[2] We can analogously define our model as a relation:

$$\textsc{DomainGraph}(\text{source}, \text{type}, \text{target}, \underline{\text{eid}})$$

where eid (edge id) is a primary key of the relation.

The domain graph model of MillenniumDB already subsumes the RDF graph model [9]. Recall that an RDF graph is a set of triples of the form (a, b, c). An RDF graph can be visualized as a *directed labelled graph* [2, 22], which is a set of edges of the form $(a) \text{–} b \rightarrow (c)$, where a is the source node (aka *subject*), b the edge type (aka *predicate*), and c the target node (aka *object*). Alternatively, an RDF graph can be seen as a relation RDFGraph(source, type, target). To show how RDF is modelled in domain graphs, consider the following edge, claiming that Michelle Bachelet was the president of Chile.



We can encode this triple in a domain graph by storing the tuple (Michelle Bachelet, position held, President of Chile, e) in the DomainGraph relation, where e denotes a unique (potentially autogenerated) edge id, or equivalently stating that:

γ(e) = (Michelle Bachelet, position held, President of Chile).

The id of the edge itself is not accessible in the RDF data model. However, the edge id can be useful when modelling RDF* graphs or named graphs based on RDF, as we will discuss in Section 2.2.

[2]Herein, we say "*edge type*" rather than "*edge label*" to highlight that the type forms part of the edge, rather than being an annotation on the edge, as in property graphs.

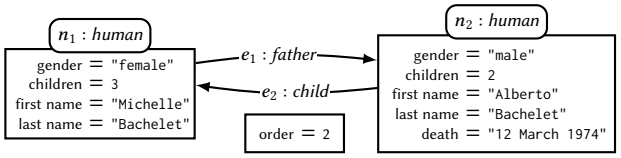


Figure 1: A property graph with two nodes and two edges. Notice that order is a property of edge e_2 that is used to indicate that Michelle Bachelet is the second child of Alberto Bachelet

Property graphs [2] further allow nodes and edges to be annotated with labels and property–value pairs, as shown in Figure 1. Domain graphs can directly capture property graphs, where, for example, the property–value pair (gender, "male") on node n_1 can be represented by an edge $\gamma(e_3) = (n_1, \text{gender}, \text{"male"})$, the property–value pair (order, 2) on an edge e_2 becomes $\gamma(e_4) = (e_2, \text{order}, 2)$, the label on n_1 becomes $\gamma(e_5) = (n_1, \text{label}, \text{human})$, the label on e_1 becomes the type of the edge $\gamma(e_1) = (n_1, \text{father}, n_2)$, etc.[3] However, given a legacy property graph, there are some potential "incompatibilities" with the resulting domain graph; for example, strings like "male", labels like human, etc., now become nodes in the graph, generating new paths through them that may affect query results.

For stricter backwards compatibility with legacy property graphs (where desired), MillenniumDB implements a simple extension of the domain graph model, called *property domain graphs*, which allows for *external annotation*, i.e., adding labels and property–value pairs to nodes and edges without creating new nodes and edges. Formally, if $\mathcal{L}$ is a set of labels, $\mathcal{P}$ a set of properties, and $\mathcal{V}$ a set of values, we define a property domain graph as follows:

Definition 2.2. A *property domain graph* is defined as a tuple $G = (O, \gamma, \text{lab}, \text{prop})$, where:

- (O, γ) is a domain graph;
- $\text{lab} : O \to 2^{\mathcal{L}}$ is a function assigning a finite set of labels to an object; and
- $\text{prop} : O \times \mathcal{P} \to \mathcal{V}$ is a partial function assigning a value to a certain property of an object. Besides, we assume that for each object $o \in O$, there exists a finite number of properties $p \in \mathcal{P}$ such that $\text{prop}(o, p)$ is defined.

In summary, domain graphs capture the graph structure of our model, while property domain graphs additionally permit annotating that graph structure by adding labels and property–value pairs to objects, as ordinary property graphs do; such annotations are then considered external from the graph structure. The property graph in Figure 1 can be represented with the following property domain graph $G = (O, \gamma, \text{lab}, \text{prop})$, where the graph structure is as follows:

$$O = \{n_1, n_2, e_1, e_2\},$$
$$\gamma(e_1) = (n_1, \text{father}, n_2),$$
$$\gamma(e_2) = (n_2, \text{child}, n_1),$$

and the annotations of this structure are as follows:

[3]To represent edges in property graphs that permit multiple labels, multiple edges with different types can be added (or the labels can be added on the edge ids).

lab(n_1) = human	prop(n_1, last name) = "Bachelet"
lab(n_2) = human	prop(n_2, gender) = "male"
prop(e_2, order) = 2	prop(n_2, children) = 2
prop(n_1, gender) = "female"	prop(n_2, first name) = "Alberto"
prop(n_1, children) = 3	prop(n_2, last name) = "Bachelet"
prop(n_1, first name) = "Michelle"	prop(n_2, death) = "12 March 1974"

The relational representation of property domain graph then adds two new annotation relations alongside DomainGraph:

$$\textsc{Labels}(\text{object}, \text{label}),$$
$$\textsc{Properties}(\underline{\text{object}, \text{property}}, \text{value}),$$

where object, property is a primary key of the second relation, with the first relation allowing multiple labels per object.

2.2 Why domain graphs?

Why did we choose domain graphs as the model of MillenniumDB? As discussed in the previous section, it can be used to model both directed labeled graphs (like RDF) as well as property graphs. It also has a natural relational expression, which facilitates its implementation in a query engine. But it is also heavily inspired by the needs of real-world knowledge graphs like Wikidata [40].

Take the two Wikidata statements shown in Figure 2. Both statements claim that Michelle Bachelet was a president of Chile, and both are associated with nested *qualifiers* that provide additional information: in this case a start date, an end date, who replaced her, and whom she was replaced by. There are two statements, indicating two distinct periods when she held the position. Also the ids for objects (for example, Q320 and P39) are shown; any positional element can have an id and be viewed as a node in the knowledge graph.

Michelle Bachelet [Q320]

position held [P39] President of Chile [Q466956]
- start date [P580] 2014-03-11
- end date [P582] 2018-03-11
- replaces [P155] Sebastián Piñera [Q306]
- replaced by [P156] Sebastián Piñera [Q306]

position held [P39] President of Chile [Q466956]
- start date [P580] 2006-03-11
- end date [P582] 2010-03-11
- replaces [P155] Ricardo Lagos [Q331]
- replaced by [P156] Sebastián Piñera [Q306]

Figure 2: Wikidata statement group for Michelle Bachelet

Representing statements like this in a directed labeled graph requires some form of *reification* to decompose n-ary relations into binary relations [17]. For example, Figure 3 shows a graph where e_1 and e_2 are nodes representing n-ary relationships. The reification is given by the use of the edges typed as source, type and target. As before, we use human-readable nodes and labels, where in practice, a node (Sebastián Piñera) will rather be given as the identifier (Q306), and an edge type replaces will rather be given as P155.

A number of graph models have been proposed to capture higher-arity relations more concisely, including property graphs [12] and

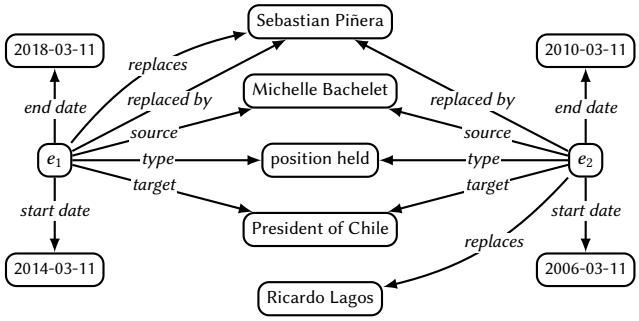


Figure 3: Directed edge-labelled graph reifying the statements of Figure 2

RDF* [15]. However, both have limitations that render them incapable of modeling the statements shown in Figure 2 without resorting to reification [20].

On the one hand, Property graphs allow labels and property–value pairs to be associated with both nodes and edges, as shown in Figure 4. Though more concise than reification, labels, properties and values are considered to be simple strings, which are disjoint with nodes; for example, "Ricardo Lagos" is neither a node nor a pointer to a node, but a string, which would complicate, for example, querying for the parties of presidents that Michelle Bachelet replaced.

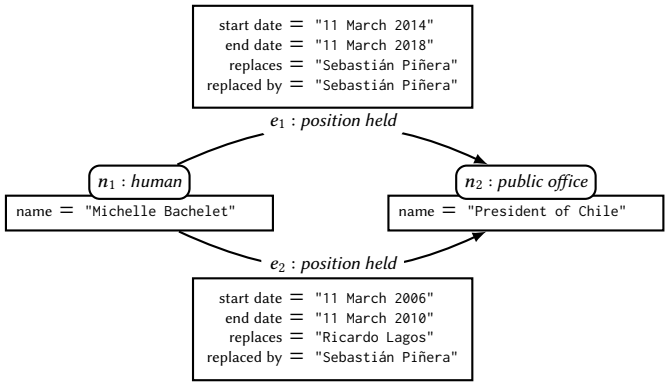
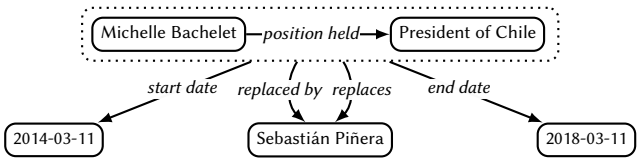


Figure 4: Property graph for the statements of Figure 2

On the other hand, RDF* allows an edge to be a node. For example, the first statement of Figure 2 can be represented as follows in RDF*:



However, we can only represent one of the statements (without reification), as we can only have one distinct node per edge; if we add the qualifiers for both statements, then we would not know which start date pairs with which end date, for example.

The domain graph model allows us to capture higher-arity relations more directly. In Figure 5 we have one possible representation

of the statements from Figure 2. We only show edge ids as needed (all edges have ids). We do not use the "property part" of our data model for external annotation, considering that the elements of Wikidata statements shown can form nodes in the graph itself.

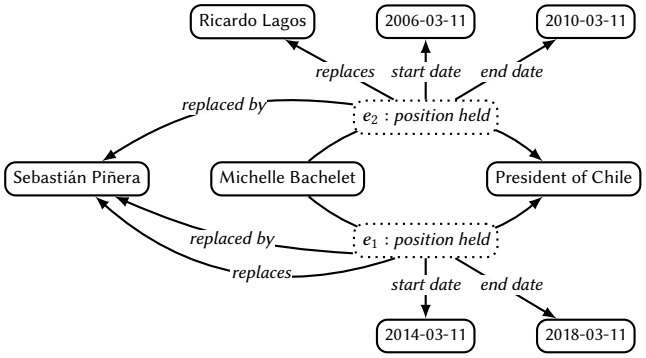


Figure 5: Domain graph for Figure 2

Domain graphs are inspired by *named graphs* in RDF/SPARQL, and can be viewed as *singleton named graphs* (where named graphs have one triple/edge each). Both domain graphs and named graphs can be represented as quads. However, the edge ids of domain graphs identify each quad, which, as we will discuss in Section **??**, simplifies indexing. Named graphs were proposed to represent multiple RDF graphs for publishing and querying. SPARQL thus uses syntax – such as `FROM`, `FROM NAMED`, `GRAPH` – that is unintuitive when querying singleton named graphs representing higher-arity relations. Moreover, SPARQL does not support querying paths that span named graphs; in order to support path queries over singleton named graphs, all edges would need to be duplicated (virtually or physically) into a single graph [17]. Named graphs (with multiple edges) could be supported in domain graphs using a reserved term graph, and edges of the form $\gamma(e_3) = (e_1, \text{graph}, g_1)$, $\gamma(e_4) = (e_2, \text{graph}, g_1)$; optionally, *named domain graphs* could be considered in the future to support multiple domain graphs with quins.

Table 1 summarizes the features that are directly supported by the respective graph models themselves without requiring *reserved terms*, which would include, for example, source, label and target in Figure 3 (all features except *External annotation* can be supported in all models with reserved vocabulary). Reserved terms can add indirection to modeling (e.g., reification [17]), and can clutter the data, necessitating more tuples or higher-arity tuples to store, leading to more joins and/or index permutations. The features are then defined as follows, considering directed (labeled) edges:

- *Edge type/label*: assign a type or label to an edge.
- *Node label*: assign labels to nodes.
- *Edge annotation*: assign property–value pairs to an edge.
- *Node annotation*: assign property–value pairs to a node.
- *External annotation*: nodes/edges can be annotated without adding new nodes or edges.
- *Edge as node*: an edge can be referenced as a node (this allows edges to be connected to nodes of the graph).
- *Edge as nodes*: a single unique edge can be referenced as multiple nodes.

Table 1: The features supported by graph models without reserved terms (NG = Named Graphs, PG = Property Graphs, DG = Domain Graphs, PDG = Property Domain Graphs)

	RDF	RDF*	NG	PG	DG	PDG
Edge type/label	✓	✓	✓	✓	✓	✓
Node label	✗	✗	✗	✓	✗	✓
Edge annotation	✗	✓	✓	✓	✓	✓
Node annotation	✓	✓	✓	✓	✓	✓
External annotation	✗	✗	✗	✓	✗	✓
Edge as node	✗	✓	✓	✗	✓	✓
Edge as nodes	✗	✗	✓	✗	✓	✓
Nested edge nodes	✗	✓	✓	✗	✓	✓
Graph as node	✗	✗	✓	✗	✗	✗

- *Nested edge nodes*: an edge involving an edge node can itself be referenced as a node, and so on, recursively.
- *Graph as node*: a graph can be referenced as a node.

Some ✗'s in Table 1 are more benign than others; for example, *Node label* requires a reserved term (e.g., `rdf:type`), but no extra tuples; on the other hand, *Edge as node* requires reification, using at least one extra tuple, and at least one reserved term.

Wikidata requires *Edge as nodes* for cases as shown in Figure 2 (since values such as Ricardo Lagos are themselves nodes). Only named graphs, domain graphs and property domain graphs can model such examples without reserved terms. Comparing named graphs and domain graphs, the latter sacrifices the "*Graph as node*" feature without reserved vocabulary to reduce indexing permutations (discussed in Section **??**). Such a feature is not needed by Wikidata. Property domain graphs further support external annotation, and thus better compatibility with legacy property graphs.

3 QUERY LANGUAGE

We now discuss the query language that MillenniumDB currently supports. We first introduce the common primitives supported by graph query languages, and then discuss the syntax and semantics of a concrete query language currently supported by our system.

3.1 Features of Graph Query Languages

A variety of query languages for graphs have been proposed down through the years [3], including AQL [28] (used by ArangoDB), Cypher [12] (used by Neo4j [41]), Gremlin [25] (used by Amazon Neptune [29], JanusGraph [30], Neo4j [41], OrientDB [35]), GSQL [37] (used by TigerGraph [37]) SPARQL [14] (used by various systems [1], including Allegrograph [27], Amazon Neptune [29], Blazegraph [38], GraphDB [6], Jena TDB [31], Stardog [36] and Virtuoso [10]), and SQL-Match [35] (used by OrientDB [35]).

While the syntax of these query languages vary widely, they share a large common core in terms of querying primitives. Specifically, all such query languages support different types of *graph patterns* that transform a graph into a relation (aka. a table) of results [2]. The simplest form of graph pattern is a *basic graph pattern*, which in its most general form, is a graph pattern structured like the data, but that also allows variables to replace constants. Next we have *navigational graph patterns*, that allow for specifying path

expressions that can match arbitrary-length paths in the graphs. We also have *relational graph patterns* which, noting that a graph pattern converts a graph into a relation, allows the use of relational operators to transform and/or combine the results of one or more graph patterns into a single relation. Finally, query languages may support other features, such as *solution modifiers* that order or limit results, convert results back into a graph, etc.

3.2 Domain Graph Queries

Per our goal of supporting multiple graph models, MillenniumDB aims to support a number of graph query languages, such as those that we previously discussed. However, no existing query language would take full advantage of the property domain graph model defined in the previous section. We have thus implemented a base query language, called DGQL, which closely resembles Cypher [12], but is adapted for the property domain graph model, and adds features of other query languages, such as SPARQL, that are commonly used for querying knowledge graphs like Wikidata [7]. Herein we provide a guided tour of the syntax of DGQL; a syntactic grammar for DGQL is presented in Appendix A, while its abstract syntax and formal semantics is given in Appendix B.

A DGQL query takes the following high-level form:

```
SELECT Variables
MATCH Pattern
WHERE Filters
```

When evaluated over a property domain graph, such a query will return a multiset of mappings binding `Variables` to database objects (or values) that satisfy the `Pattern` specified in the **MATCH** clause and the `Filters` specified in the **WHERE** clause.

Querying objects. The most basic query will return all the objects (or more precisely, their ids) in our property domain graph. In MillenniumDB we can achieve this via the following query:

```
SELECT ?x
MATCH (?x)
```

Of course, one usually wants to select objects with a certain label, or a certain value in a specific key. For instance, if we want to select all people in the property (domain) graph from Figure 1 with two children, we could do this as follows:

```
SELECT ?x, ?x.gender
MATCH (?x :human { "children":2 })
```

This would return the ids of nodes with label `human` and value `2` for the key `children`, along with their associated value for the key `gender`, i.e., a single result $\{\{?x \mapsto n_2, ?x.gender \mapsto "male"\}\}$. If n_2 did not have a gender, we would still want to return the node that makes the match, but signal that the gender is not specified. To do so, we would return $\{\{?x \mapsto n_2, ?x.gender \mapsto null\}\}$.

If we wish to specify a range, we can rather use the **WHERE** clause:

```
SELECT ?x, ?x.gender
MATCH (?x :human)
WHERE ?x.children >= 2
```

which returns two solutions: $\{\{?x \mapsto n_2, ?x.gender \mapsto "male"\}, \{?x \mapsto n_1, ?x.gender \mapsto "female"\}\}$. If we replaced `>=` with `==`, the results would be the same as for the previous query.

Querying edges. In order to query over edges, we can write the following query, which returns γ, i.e., the relation DomainGraph:

```
SELECT *
MATCH (?x)-[?e TYPE(?t)]->(?y)
```

The **SELECT** * operation projects all variables specified in the **MATCH** pattern, while the construct `(?x)-[?e TYPE(?t)]->(?y)` specifies that we want to connect the object in `?x` with an object in `?y`, via an edge with type `?t` and id `?e`. This is akin to a query DomainGraph(?x,?t,?y,?e) over the domain graph relation.

We can also restrict which edges are matched. The following query in DGQL will return ids for edges of type child, where both nodes have the same last name, and the child is not the oldest; it also returns the order of the child:

```
SELECT ?e, ?e.order
MATCH (?x)-[?e child]->(?y)
WHERE (?x.lastname == ?y.lastname) AND (?e.order > 1)
```

This returns $\{\{?e \mapsto e_2, ?e.order \mapsto 2\}\}$ over Figure 1. If we wanted to query for the second oldest (an equality condition), we could use the syntax `(?x)-[?e child {"order"=2}]->(?y)` on the edge (or replace `(?e.order > 1)` with `(?e.order == 1)`). As shown here, the **WHERE** clause may use boolean combinations. Notice that edge types are written "as is". This is in contrast to object labels, which are prefixed with the `:` symbol.

Querying known objects. Knowing the id of an object, we might want to query more about it. For instance, in the domain graph of Figure 4 representing statements from Wikidata, we may query for the positions held by Michelle Bachelet as follows:

```
SELECT ?x
MATCH (Michelle Bachelet)-[position held]->(?x)
```

This would return duplicate results $\{\{?x \mapsto$ `President of Chile`$\}, \{?x \mapsto$ `President of Chile`$\}\}$ due to the two edges e_1 and e_2. Recall that for clarity we use human-readable ids, where for Wikidata, the node id `(Michelle Bachelet)` may rather be given as `(Q320)`, and `position held` by `P39`.

Path queries. A key feature of graph databases is their ability to explore paths of arbitrary length. Like SPARQL [14] (but not Cypher[4]), DGQL supports two-way regular path queries (2RPQs), which specify regular expressions over edge types, including concatenation (`/`), disjunction (`|`), inverses (`^`), optional (`?`), Kleene star (`*`) and Kleene plus (`+`). For instance, if we wished to find all descendants of people named Alberto, we could use the regular expression `child+` in the following way:

```
SELECT ?y
MATCH (?x { "first name":"Alberto" })=[child+]=>(?y)
```

This returns $\{\{?y \mapsto n_1\}\}$ over Figure 1, and would include children of n_1, and their children, recursively, if present. We use `=[]=>` (rather than `-[]->`) to signal a path query in DGQL. If we wanted to capture more results, we could replace `[child+]` with an extended expression like `[(child|^father|^mother)+]` which also traverses backwards along edges of type `father` and `mother`, and

[4] Of these 2RPQ features, Cypher only supports Kleene star. However, 2RPQs are widely used for querying Wikidata [7], and thus we have prioritized adding this feature. We do not support SPARQL's negated property sets, which are used infrequently [7].

may thus return more results. We can also concatenate paths with /, where the expression [(father|mother)*/sister] could find sisters, aunts, grand-aunts, and so forth.

Like Cypher (but not SPARQL), DGQL can return a single shortest path witnessing the query result via the following construct:

```
SELECT ?y, ?p
MATCH (?x { "first name":Alberto })=[?p child+]=>(?y)
```

The variable ?p stores a shortest path connecting each Alberto with each descendant ?y. No manipulation of path variables, apart from outputting the result, is currently supported in MillenniumDB, but a full path algebra will be supported in future versions.

Basic and navigational graph patterns. Basic graph patterns [2] lie at th core of many graph query languages, including DGQL. These are graphs following the same structure as the data model, but where variables are allowed in any position. They can also be seen as expressing natural (multi)joins over sets of atomic edge patterns. In DGQL, they are given in the **MATCH** clause. To illustrate, the following query finds pairs of nodes who share the same father:

```
SELECT ?x, ?y
MATCH (?x)-[father]->(?z),
      (?y)-[father]->(?z)
```

We evaluate basic graph patterns under *homomorphism-based semantics* [2], which allows multiple variables in a single result to map to the same element of the data. If we evaluate this query over Figure 1, we would thus get one solution: $\{\{?x \mapsto n_1, ?y \mapsto n_1, \}\}$.

If we further allow path queries within basic graph patterns, we arrive at *navigational graph patterns* [2]. Suppose we wished to find, for example, pairs of nodes that shared a common ancestor along the paternal line and held the same position; we could write this as:

```
SELECT ?x, ?y
MATCH (?x)=[father+]=>(?z),
      (?y)=[father+]=>(?z),
      (?x)-[position held]->(?w)
      (?y)-[position held]->(?w)
WHERE ?x != ?y
```

This time we filter results that map ?x and ?y to the same node.

Taking advantage of the domain graph model. The DGQL query language allows us take full advantage of the domain graph, by allowing joins between edges, types, etc. To illustrate this, consider the following query, evaluated over the domain graph of Figure 5:

```
SELECT ?x, ?d
MATCH (Michelle Bachelet)-[?e position held]->
                          (President of Chile),
      (?e)-[replaces]->(?x),
      (?e)-[start date]->(?d)
```

The variable ?e invokes a join between an edge and a node, returning two solutions: $\{\{?x \mapsto$ Ricardo Lagos, ?d $\mapsto$ 2006-03-11$\}$, $\{?x \mapsto$ Sebastián Piñera, ?d $\mapsto$ 2014-03-11$\}\}$. Illustrating a join on an edge type, consider the following query:

```
SELECT ?x, ?y
MATCH (?x)-[?e TYPE(?t)]->(?y),
      (?t)=[subproperty of*]=>(parent)
```

This query returns all pairs of nodes with an edge whose type is parent, or a transitive sub-property of parent; for example, Wikidata defines father and mother to be sub-properties of parents.

Optional matches. DGQL also supports optional graph patterns, which behave akin to left outer joins. For instance, we may wish to query for presidents of Chile, and if available, the president they replaced, and if further available, the president that the latter president replaced. The following DGQL query accomplishes this:

```
SELECT ?x, ?y, ?z
MATCH (?x)-[?e1 position held]->(President of Chile),
      OPTIONAL {
        (?e1)-[replaces]->(?y)
        OPTIONAL {
          (?y)-[?e2 position held]->(President of Chile),
          (?e2)-[replaces]->(?z)
        }
      }
```

If evaluated on the domain graph of Figure 5, we would get two solutions: $\{\{?x \mapsto$ Michelle Bachelet, ?y $\mapsto$ Ricardo Lagos$\}$, $\{?x \mapsto$ Michelle Bachelet, ?y $\mapsto$ Sebastián Piñera$\}\}$, where in both cases, the variable ?z is left *unbound* as the data are not available for the inner optional graph pattern. As shown, nested optionals are supported, but with the restriction that they form *well-designed patterns* as defined by Pérez et al. [23].

Limits and ordering. Some additional operators that MillenniumDB supports are **LIMIT** and **ORDER BY**. These allow us to limit the number of output mappings, and sort the obtained results. For instance, if we wished to obtain the ten most recent presidents of Chile, we could accomplish this with the following query:

```
SELECT ?x
MATCH (?x)-[?e position held]->(President of Chile),
      (?e)-[start date]->(?d)
LIMIT 10
ORDER BY DESC (?d)
```

Ordering is always applied before limiting results.

Formal definitions. At [33], we include an extended version of this paper, which contains a full specification of DGQL, as well as a formal definition of its semantics. A grammar for generating DGQL queries is presented in Appendix A of [33]. The abstract syntax and formal semantics of the language is given in Appendix B of [33].

4 SYSTEM ARCHITECTURE

In this section we describe the architecture underlying MillennumDB. We begin by explaining how one can store domain graphs and access them from disk. We then outline the process of query evaluation, and also provide some details on less conventional algorithms deployed in this process.

Storage. Internally, all object are represented as 8 byte identifiers. To optimize query execution, identifiers are divided into classes and the first byte of the identifier specifies a class it belongs to. The main classes in a property domain graph $G = (O, \gamma, \mathtt{lab}, \mathtt{prop})$ are:

- *Nodes*, which are objects in the range of γ. They are divided in two subclasses: *named nodes*, which are objects in the domain

graph for which an explicit name is available (e.g. Q320 in WikiData), and *anonymous nodes*, which are internally generated objects without an explicit name available to the user (similar as blank nodes in RDF [18]).
- *Edges*, which are objects in the domain and range of γ, and are always anonymous and internally generated objects.
- *Values*, which are data objects like strings, integers, etc. These values are classified in two subclasses: *inlined values*, which are values that fit into 7 bytes of the identifier after the mask (e.g. 7 byte strings, integers, etc.), and *external values*, which are values longer than 7 bytes (e.g. long strings).

All records guarded in MilleniumDB are composed of these identifiers. Therefore, when we write $o \in O$, we are referring to the identifier of the object o, with some of the classes above. As we mentioned, the first byte of the 8 byte identifier define the class, and the rest 7 bytes are used to store an id (e.g., edges), a name (e.g., named nodes), or data like an integer or a short string (e.g., inlined values). We will later explain how long strings for external values are handled.

To store property domain graphs, MillenniumDB deploys B+ trees [24]. For this purpose, we built a B+ tree template for fixed sized records, which allows to guard all classes of identifiers. To store a property domain graph $G = (O, \gamma, \texttt{lab}, \texttt{prop})$, we simply index the four components defining it:

- Objects(id) which contains the identifiers of all the objects in the database (i.e., O), and store it as a B+tree with records of size 1.
- DomainGraph(source,type,target,eid), which contains all the information on connections in the graph (i.e., γ), where eid is an edge identifier, and source, type, and target can be ids of any class (i.e., node, edge, or value). By default four permutation of the attributes are indexed in order to aid query evaluation. These are: *SourceTargetTypeEdge*, *TargetTypeSourceEdge*, *TypeSourceTargetEdge* and *TypeTargetSourceEdge*;
- Labels(id,label), which guard labels of objects (i.e., lab). The id can be any identifier, and label are stored as values identifiers. Both possible attribute permutations are indexed;
- Properties(id,key,value), containing key-value pairs associated with each object (i.e., prop). The id can be any identifier, and key and value are value identifiers. An additional attribute permutation allowing to search objects by their keys is also created.

All the B+ trees are created through a bulk-import phase, which loads multiple tuples of sorted data, instead of creating the trees by inserting records one by one. In order to allow fast lookups by edge identifier, we use the fact that this attribute is the key for the relation. Therefore, we also store a table called EdgeTable, which contains triples of the form $(source, type, target)$, such that the position in the table equals to the identifier of the object e such that $\gamma(e) = (source, type, target)$. This implies that edge identifiers must be assigned consecutive ids starting from zero, and are thus generated by MillenniumDB, and not specified by the user. In total, we use ten B+-trees for storing the data.

Finally, we need to explain how external-values identifiers are converted to actual objects. To transform these identifiers to real values, we have a single binary file called ObjectFile, which contains all of these long values concatenated together. In particular, the identifier of such an explicit value will be equal to the position where it is written in the ObjectFile, thus allowing efficient lookups of a value via its id. The identifiers are generated upon loading, and an additional hash table is kept to map (hash) of a string to its identifier, in order to ensure that no value is inserted twice, and to transform explicit values given in a query to their internal ids upon query execution. At this moment, only external value supported is a long string; however, the implementation interface allows to add different value types in a simple manner.

Access methods. All of the stored relations are accessed through linear iterators which allow accessing the information one tuple at a time. All of the data is stored on pages of fixed (but parametrizable) size (4kB at this point). The data from disc is loaded into a shared main memory buffer, whose size can be specified upon initializing the MillenniumDB server. The buffer uses the standard clock page replacement policy [24]. Additionally, for improved performance, upon initializing the server, it can be specified that the *ObjectFile* be loaded into main memory in order to quickly convert internal identifiers to string and integer values that do not fit into 7 bytes.

Evaluating a query. The query execution pipeline follows the standard database template:

(1) The string of the query is parsed into an Abstract Syntax Tree (AST), and checked to be syntactically correct.
(2) A logical plan is generated using a depth-first traversal of the AST, instantiating nodes of the tree to logical operators.
(3) Using a visitor-pattern, the logical tree is parsed in order to apply some basic simplifications (e.g. pushing constant from filters into the search pattern)[5].
(4) Finally, another pass of the logical tree using the visitor-pattern is made in order to create a physical plan.

An illustration of what a logical plan looks like for a generic MillenniumDB query is given in Figure 6. In the plan tree, there will be nodes corresponding to the `MATCH`, `WHERE`, or any other clause in the query. We prefix each logical operator with an "`Op`" (so the names will be `OpMatch`, `OpSelect`, etc.). In Figure 6 each `OpMatch` is also associated with the node, edge, or property path patterns appearing in it, but we omit those for brevity. As we can notice, some parts of the query are mandatory (corresponding to the `SELECT` and `MATCH` clauses), while the other need not be present. Finally, a physical plan is generated by assigning a linear iterator returning solution mappings to each `Op` node in the logical plan. This allows each `Op` node to pass its results to the node above following the red arrows in Figure 6, thus resulting in a pipelined evaluation of queries.

The interesting part is evaluating the `OpMatch` pattern, which is a list of relations that can be edges (i.e., connections), labels, properties, or property paths. In essence, evaluating `OpMatch` is analogous to selecting an appropriate join plan for the relations representing the different elements. This also goes in hand with selecting the appropriate join algorithm for each of the joins. Given that edges, labels, and properties are all indexed, this will most

[5] At this point MillenniumDB does not have a fully developed query rewriting module which would naturally fit here. This is left as future work.

```
SELECT   SelOptions
MATCH    MatchPattern
WHERE    Condition
ORDER BY VarList
```

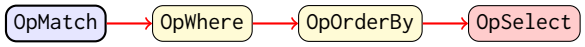
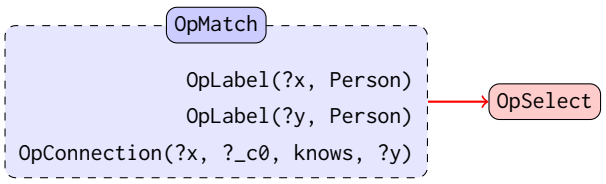


Figure 6: A logical plan for a generic MillenniumDB query. Red and blue nodes must always be present in the query plan, while the yellow ones can be omitted. OpMatch is either a single conjunction of atomic patterns (node, edge, or property path patterns), or an OPTIONAL pattern.

```
SELECT ?x, ?y
MATCH (?x :Person)-[:knows]->(?y :Person)
```




```
IndexNestedLoopJoin(
    IndexNestedLoopJoin(
      Label(node: ?x, label: "Person"),
      Connection(from: ?x, to: ?y, type: knows, edge: ?_c0)
    ),
    Label(node: ?y, label: "Person")
)
```

Figure 7: Generating a physical plan based on a logical plan. Notice that the `OpMatch` now lists the relations that are to be joined in the query, and an implicit edge variable `?_c0` is introduced. The physical plan in this case simply joins the used relations.

commonly be index nested loops join. Property paths on the other hand are not indexed and require to be evaluated before they are joined with other relations. For this reason, they are currently pushed to the very end of the join plan and joined via a nested-loop join with the remaining part of `OpMatch` [6]. We illustrate a sample query, and a possible plan generated for this query in Figure 7.

Currently, MillenniumDB supports two different evaluation mechanisms for evaluating `OpMatch`:

- The classical relational optimizer which is based on cost estimation, and tries to order base relations in such a way as to minimize the amount of (intermediate) results. We currently support two modes of execution here:
  (i) Selinger-style join plans [26] which use dynamic programming to determine the optimal order of relations.
  (ii) In the presence of a large number of relations, a greedy planner [13] is used which simply determines the cheapest relation to use in each step.
- A worst-case optimal query plan as described in [20] is used whenever possible. This approach implements a modified leapfrog algorithm of [39] in order to minimize the number of intermediate results that are generated.

Two points of interest in the way that MillenniumDB handles query execution are the worst-case optimal query planner for `OpMatch`, and the way that property paths are evaluated. Both of these deploy state-of-the-art research ideas, and are usually not, to the best of our knowledge, implemented in graph database systems. We provide some additional details on these next.

Worst-case optimal query plan. Evaluating `OpMatch` in a worst-case optimal way is done using a modified leapfrog algorithm [39], similarly as in [20]. Roughly speaking, a classical join plan does a nested for-loop for each relation, iterating through each relation one tuple at a time. Instead, the worst-case optimal join plan of [39] does a nested for-loop for each variable `?x`, intersecting all relations where `?x` appears. We refer the reader to [39] and [20] for a detailed explanation. Two critical aspects for supporting this approach are indices and variable ordering, which we will explain next.

To support the leapfrog algorithm, we should index all relations in all possible orders of their attributes, which greatly increases disk storage [20]. In MilleniumDB, we include four orders for DomainGraph, and all orders for Labels and Properties. By considering these orders, we can cover most of the queries that appear in practice by a worst-case optimal query plan. We use the classical relational optimizer if the plan needs an unsupported order or one of the relations is a property path.

The leapfrog algorithm requires choosing a variable ordering for the iteration, which is crucial for its performance. The heuristic we deploy for selecting the variable ordering mixes a greedy approach, and the ideas of the GYO reduction [42]. More precisely, we first order the variables based on the minimal cost of the relations they appear in and resolve ties by selecting the variable that appears in more distinct relations participating in the join. The variables "connected" to the first one chosen are then processed in the same manner (where connected meant appearing in the same relation) until the process can not continue. The isolated variables are then treated last.

Evaluating property paths. For evaluating a property path, the path pattern is compiled into an automaton, and a "virtual" cross-product of this automaton and the graph is constructed on-the-fly, and navigated in a breadth-first manner[7], as commonly suggested in the theoretical literature [4, 5, 22]. Our assumption is that each property path pattern will have at least one of the endpoints assigned before evaluation. This can be done either explicitly in the pattern, or via the remainder of the query. For instance, a property paths pattern `(Q1)=[:P31*]=>(?x)` has the starting point of our search assigned to `Q1`. On the other hand, `(?x)=[:P31*]=>(?y:Person)` does not have any of the endpoints assigned, however, the `(?y :Person)` allows us to instantiate `?y` with any node with the label `:Person`.

[6] We admit that this is not always the best option. However, based on extensive empirical evidence (see Section 5), this solution seems to be adequate in practice.

[7] BFS is the default evaluation method for property paths. The source code also includes four other algorithms, each with their merits and cons. Detailed exploration of those is outside of the scope of this paper.

Intuitively, from a starting node (tagged with the initial state of the query automaton), all connections with the type specified by the outgoing transitions from this state are followed. The process is repeated until reaching an end state of the automaton, upon which, a result can be returned. This allows a fully pipelined evaluation of property paths, and additionally requires to have at most one page pinned in the buffer on execution (the neighbors of the node on the top of the BFS queue).

Additionally, the BFS algorithm also allows us to return a single shortest path between each pair of endpoints. We allow specifying this by adding a variable in the property path pattern as follows:

```
SELECT *
MATCH (Kevin_Bacon)=[?p (:actedIn/^:actedIn)*]=>
                                        (?actor)
```

This query anchors the search at the node corresponding to `Kevin_Bacon`, and looks for all the actors who acted with him in the same movie, or in a movie with someone who acted with him, etc. The variable `?p` will return a single shortest path between Kevin Bacon and any other actor who can be reached via a path satisfying the query. We note that returning path comes basically for free, given that they can be reconstructed using the set of visited nodes which is used for bookkeeping in the BFS algorithm.

5 BENCHMARKING

In this section we provide a detailed experimental evaluation of the core features of MillenniumDB. We base our experiments on the Wikidata knowledge graph [40], as it is one of the largest real-world knowledge graphs that is publicly available, and provides an extensive query log of real-world queries [7, 21]. The experiments focus on two main query features: (i) basic graph patterns (BGPs); and (ii) path queries. We compare the performance of MillenniumDB (MillDB) with several popular persistent graph database engines that support BGPs and at least the Kleene star feature for paths. For the purposes of reproducibility, we publish online the data, queries, scripts, and configuration files for each engine, together with the scripts used to load the data and run the experiments [34].

The engines. We compare the performance of MillenniumDB [32] with five persistent graph query engines. First, we include three popular RDF engines: Jena TDB version 4.1.0 [31], Blazegraph (BlazeG for short) version 2.1.6 [38], and Virtuoso version 7.2.6 [10]. We further include a property graph engine: Neo4J community edition 4.3.5 [41]. Finally, we also tested with Jena Leapfrog (Jena LF, for short) – a version of Jena TDB implementing a worst-case optimal leapfrog algorithm [20] – in order to compare the performance of our worst case optimal algorithm with a persistent engine implementing a similar algorithm for evaluating BGPs.

The data. To uniformly test all of the engines, we base ourselves on the Wikidata dataset. In particular, we used the truthy dump version 20210623-truthy-BETA [11], and kept only triples in which the subject position is a Wikidata entity, and in which the predicate is a direct property. The size of the dataset after this was 1,257,169,959 triples. The simplification of the dataset is done for two reasons: (i) to keep the nodes and edges which will allow to test the performance of BGPs and property paths; and (ii) to make data loading times manageable. We made this clean version of the dataset available at [34]. The size of this dataset, when loaded into the respective systems, is summarized in Table 2.

	MillDB	BlazeG	Jena	Jena LF	Virtuoso	Neo4J
size	203GB	70GB	110GB	195GB	70GB	112GB

Table 2: The dataset size when loaded into each engine. The original RDF dataset consists of roughly 1.25 billion triples.

To store the data, default indices were used on Jena TDB, Blazegraph and Virtuoso. Jena LF stores three additional permutations of the stored triples to efficiently support the leapfrog algorithm for any join query. Neo4j by default creates an index for edge types (as of version 4.3.5). To support faster searches for particular entities and properties, we also created an index linking a Wikidata identifier (such as, e.g., Q510) to its internal id in Neo4j. We also tried to index literals in Neo4j, but the process failed.

Aidan: What are "literals" here? They were removed from data? Or are these the literals that sometimes appear as values for direct properties?

As we can see, MillenniumDB uses extra disc space, because of the additional indices needed to support the domain graph model. Similarly, Jena LF indexes more permutations than Jena TDB, hence using more space. It is important to remark that the truthy dump of Wikidata is an RDF dataset, and is thus very poorly suited for MillenniumDB, which will generate additional items (e.g., edge identifiers) which are never used. In this section we show that despite of this limitation, MillenniumDB performs at the same level, or better, than the other engines.

Aidan: Shooting ourselves in the foot here. :) Saying that Wikidata truthy is poorly suited for MillenniumDB kind of calls into question our selection of dataset, and the design of our engine.

At this point it is natural to ask why Wikidata was not loaded from the JSON dump, which could exploit two key features of MillenniumDB's data model: (i) the fact that an object can have properties and labels associated to them; and (ii) the fact that qualifiers, which are absent from the truthy version of the dataset, are naturally modeled as edged on edges, a feature intrinsic to MillenniumDB. There are several reasons for this. First, this would result in a dataset different from the RDF dump provided by the Wikimedia foundation for the same version of data, due to some intrinsic restriction of the RDF data model, and the particular choice of reification [17]. This would mean that the query results would be different for equivalent queries, thus making it difficult to draw an exact comparison. Second, we were not able to load the entire dataset into several of the engines, even when letting them run for an entire week uninterrupted.

Aidan: It was loaded, so the previous paragraph is contradicted by the next one. Minor issue, but this could be discussed more cleanly.

Nevertheless, in order to show scalability we loaded the entire knowledge graph of Wikidata into MillenniumDB using the aforementioned features. The version JSON dump 0201102-all.json was

used and preprocessed as outlined at [34]. The total size of this dataset is 827GB on disc. In the final subsection below, we illustrate that this data bloat does not significantly affect query evaluation, and show comparable times to the ones MillenniumDB achieves on the smaller dataset, with the caveat that query evaluation might slightly change based on the data modelling we deploy.

How we ran the queries. We detail the query sets used for the experiments in their respective subsections. To simulate a realistic database load, we do not split queries into cold/hot run segments. Rather, we run them in succession, one after another after a cold start of each system (and after cleaning the OS cache). This simulates the fact that query performance can vary significantly based on the state of the system buffer, or even on the state of the hard drive, or the state of OS's virtual memory. For each system, queries were run in the same order. We set a limit of 100,000 distinct results for each query. Additionally, 64GB of RAM were assigned to Jena and Blazegraph, and Virtuoso was set up with 64GB or more of RAM as is recommended. Neo4J was run with default settings, while MillenniumDB had access to 32GB for main-memory buffer, and it uses an additional 10GB for in-memory dictionaries. Queries were run on a server with Intel(R) Xeon(R) Silver 4110 CPU, and 128GB of DDR4/2666MHz RAM, running Linux Debian 10 operating system and kernel version 5.10. The hard disc used to store the data was SEAGATE ST14000NM001G with 14TB of storage.

Handling timeouts. We defined a timeout of 10 minutes per query for each system. As shown below, MillenniumDB was the only system not to timeout on any query. Apart from that, we note that most systems had to be restarted upon a timeout. This was done without cleaning the OS cache in order to preserve some of the virtual memory mapping that the OS built up to that point. An interesting observation is also that, when provided a very small timeout window, MillenniumDB managed to return a non-trivial amount of query results on each query, and did not need to be restarted, thus allowing us to handle timeouts gracefully. We also speculate that there is an issue in the Virtuoso version we used in this paper, since timeouts on some queries resulted in a significant bloat to the database (in order of 10 of GB), without resulting in a bloat to the log file.

Riveros: I would remove the last statement. It is ambiguous and does not add to the story.

5.1 Basic Graph Patterns

We focus first on basic graph pattern queries. To test different query execution strategies of MillenniumDB, we use two benchmarks: *Troublesome BGPs* and *Complex BGPs*, which are described next.

Troublesome BGPs. This is a benchmark generated from a log of queries that timed-out on the Wikidata public endpoint [21]. From these queries we extracted their BGPs. The original list of queries is linked from [34]. We removed queries that give zero results when working with the reduced version of the original dataset. We distinguish queries consisting of a single triple pattern (*Single*) from those containing more than one triple (*Multiple*). The former set tests the triple matching capabilities of the systems, whereas queries in the latter set require joins to be computed. *Single* contains 399 queries, whereas *Multiple* has 436 queries.

Single							
	MillDB LF	MillDB NL	BlazeG	Jena	Jena LF	Virtuoso	Neo4J
Supported	399	399	399	399	395	399	394
Error	0	0	0	0	4	0	5
Timeouts	0	0	0	0	0	0	0
Average	0.07	0.07	2.21	14.10	10.08	2.22	28.00
Median	0.05	0.05	0.09	0.34	0.44	0.32	1.33

Multiple							
	MillDB LF	MillDB NL	BlazeG	Jena	Jena LF	Virtuoso	Neo4J
Supported	436	436	436	426	418	436	405
Error	0	0	0	10	18	0	31
Timeouts	0	1	3	0	0	0	0
Average	4.77	11.01	31.79	35.43	16.78	7.87	75.55
Median	0.27	0.30	2.42	4.90	3.39	5.11	6.84

Table 3: Summary of query times, in seconds, for BGPs. Average and median are for all the queries, including timeouts.

Table 3 summarizes the query times on this set, whereas Figure 8 (left and middle) shows boxplots with more detailed statistics. MillDB LF and MillDB NL stand for MillenniumDB using the worst-case-optimal algorithm LeapFrog TrieJoin and using the basic Nested Loop strategy, respectively. MillDB LF is the default execution strategy of MillenniumDB, while the nested loops version was included to measure the performance of classical approach to join evaluation. MillenniumDB sharply outperforms the alternatives on the Single queries. The next best performers on average, Blazegraph and Virtuoso, are more than 30 times slower. In terms of medians, Blazegraph is almost twice as slow as MillenniumDB. It is also much less stable: The third quartile of Blazegraph is an order of magnitude higher than the most costly queries in MillenniumDB.[8] The other systems are even slower, with their averages being two orders of magnitude higher than those of MillenniumDB and their medians being higher than all the times of MillenniumDB.

MillenniumDB also outperforms the other systems on queries of the *Multiple* set. Its medians are an order of magnitude faster than those of Blazegraph, the next best contender. Indeed, the median of Blazegraph is an order of magnitude higher than all the queries in MillenniumDB (excluding outliers). Even the first quartile of Blazegraph is close to the most costly queries in MillDB LF. The difference is less sharp on averages, but still MillDB LF takes 60% of the time of Virtuoso, the next best contender. The comparison between both query strategies of MillenniumDB shows that, though both medians are close, the worst-case-optimal MillDB LF offers more stable times than the nested-loop strategy of MillDB NL.

Complex BGPs. This is a benchmark used to test the performance of worst-case optimal algorithms [20]. Here, 17 different complex join patterns were selected, and 50 different queries generated for each pattern, resulting in a total of 850 queries. Figure 8 (right) shows the resulting query times. In this case, the difference between the two evaluation strategies of MillenniumDB is more clear. The worst-case-optimal version (MillDB LF) is not only considerably more stable than the nested-loop version (MillDB NL), but also twice as fast in the median. After MillDB LF, the next-best competitor is

[8] Excluding the outliers, which are not included between whiskers.

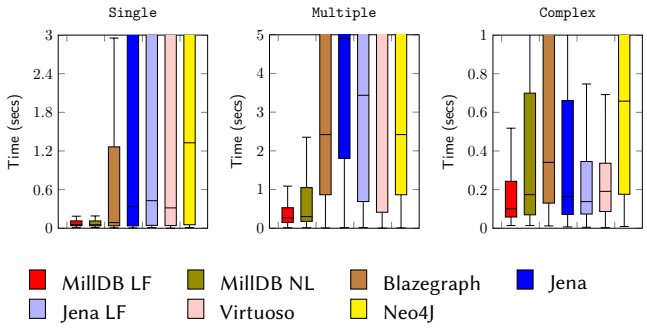


Figure 8: Boxplots for the query time distribution on the set of troublesome Single BPG queries (left), troublesome Multiple BPG queries (middle), and Complex BGP queries (right).

	MillDB	BlazeG	Jena	Virtuoso	Neo4J
Supported	1683	1683	1683	1683	1550
Error	0	46	60	59	33
Average	1.1	27.6	22.8	5.8	23.3
Median	0.094	0.396	0.207	0.325	0.328
Avg. succ.	1.1	11.2	6.3	1.8	8.0
Median succ.	0.094	0.382	0.175	0.294	0.277

Table 4: Summary of query times, in seconds, for property paths, when limiting the queries to 100,000 results. Average and median are for all the queries, including timeouts. Avg. succ. and median succ. excludes the queries that gave a timeout for that particular engine.

Jena LF, showing the benefits of worse-case optimal joins. Virtuoso follows not far behind, while MillDB NL, Jena, Blazegraph and Neo4j are considerably slower. Overall, MillDB LF offers the best performance for every statistic shown in the plot.

5.2 Property Paths

To test the performance of path queries, we extracted 2RPQ expressions from a log of queries that timed out on the Wikidata endpoint [21]. The original log has 2110 queries. After removing queries that yield no results (due to missing data), we ended up with 1683 queries. These were run in succession, each restricted to return at most 100,000 results. Each system was started after cleaning the system cache, and with a timeout of 10 minutes. Since these are originally SPARQL queries, not all of them were supported by Neo4J given the restricted regular-expression syntax it supports. We remark that only MillenniumDB and Neo4J were the only systems able to handle timeouts without being restarted.[9] In this comparison we did not include MillDB NL, nor Jena LF, since these deploy precisely the same execution strategy for property paths as do MillDB LF, and Jena, respectively. The experimental results are summarized in Table 4 and Figure 9.

[9]In fact, MillenniumDB did not give any timeouts. However, we re-ran the experiments with a lower timeout, and observed that the system could recover from interrupting the query gracefully and was able to return the results found before being interrupted.

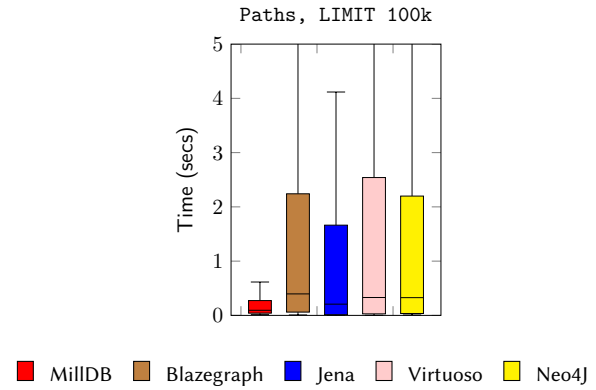


Figure 9: Boxplots of query times on property paths, limiting the results to 100,000.

We can observe that, again, MillenniumDB is the fastest and has the most stable performance, being able to run all the queries. Its average is near a second, 5 times faster than the next best contender (Virtuoso). Its median, below 0.1 seconds, is half the next one (Jena's). Even after removing the queries that timed-out on the other systems, they are considerably slower than MillenniumDB. In particular, if we only consider the queries that run successfully on Virtuoso (i.e., excluding the 59 queries that timed-out or gave an error), we get an average time of 0.85 seconds and a median time of 0.086 seconds on MillenniumDB, less than half the times of Virtuoso. The boxplots further illustrate the stability of MillenniumDB: the medians of the alternatives are over the third quartile of MillenniumDB. Their third quartile is 5–10 times higher than that of MillenniumDB, and higher than all its query times (again, excluding outliers).

To further test robustness, we also ran all of the queries *without limiting the output size* on MillenniumDB. In this test, the engine timed out in only 15 queries, each returning between 800 thousand and 44 million results before timing out. When running queries to completion, MillenniumDB averaged 13.4 seconds per query (8 seconds if timeouts were excluded), with a mean of 0.1 seconds (both with and without timeouts).

5.3 The Full Monty

To show the scalability of MillenniumDB, we took the full Wikidata JSON dump version 0201102-all.json, and converted it into the format readable by MillenniumDB. This allowed us to model qualifiers (i.e. edges on edges), put labels on objects, and assign them properties with values. We use properties to guard the language value of each string in Wikidata, and also to model elements of complex data values (e.g., for coordinates we would have objects with properties latitude and longitude, and similarly for amounts, date/time, limits, etc.). Each object representing a complex data value also has a label specifying its data type (e.g. `coord` for geographical coordinates). Qualifiers were loaded in their totality (i.e., not only the preferred qualifiers, but all specified ones). The only elements ignored from the data dump were sitelinks and references.

	Single	Multiple	Synthetic	Paths
Average	0.08	4.04	0.35	1.04
Median	0.07	0.28	0.095	0.10

Table 5: Average and median for MillenniumDB on the 827GB version of Wikidata with the same query load as in the previous subsections.

This full version of Wikidata resulted in a knowledge graph with roughly 300 million objects, participating in 4.3×10^9 connections. The total size on disc of this data was 827GB in MillenniumDB. It is important to note that JSON and RDF dumps of Wikidata do not result in precisely the same knowledge graph due to some restrictions of the particular reification used in RDF, however, they do result in very similar knowledge graphs. We then ran again the same queries from the four benchmarks above (Single, Multiple, Complex, and property paths). The number of outputs on the two versions of the data, while not the same, was within the same order of magnitude averaged over all the queries. The results are presented in Table 5. As we can observe, MillenniumDB shows no deterioration in performance when a lager database is considered for similar queries. This is mostly due to the fact that the buffer only loads the necessary pages into the main memory, and will probably require a rather similar effort in both cases. We also note that no queries resulted in a timeout.

6 CONCLUSIONS AND LOOKING AHEAD

This paper presents MillenniumDB: a persistent, open-source, graph database engine, which implements the domain graph data model. Domain graphs can naturally represent popular existing graph database formats such as RDF and property graphs, and are at the same time powerful enough to express higher-arity relations such as WikiData qualifiers, which usually require some sort of reification in other models. We created MilleniumDB as a proof-of-concept that state-of-the-art techniques are enough to support efficient evaluation of realistic graph DB query workloads.

In essence, there is no magic in MillenniumDB. It implements the well established research findings (both new, such as the leapfrog algorithm [39], and old, such as evaluating path queries via an automaton [4, 22]), and incorporates them into a classical database pipeline [24]. It creates indices via B+trees in the same way as a relational database would (or as many RDF engines do), and it loads the data from the disc into a main memory buffer, thus being able to handle big datasets without incurring a major cost in main memory. As for how well this works, we believe that the experimental evaluation speaks for itself.

Riveros: I would remove this paragraph. It goes against the presentation strategy. Furthermore, a reviewer could ask to explain why, if there is no magic, it perform better than other systems. Even worst, this will make reviewers starts taking their own conclusions, inventing reasons why the comparison is not fair (e.g., MilleniumDB does not support ACID, etc). Personally, it is not a good idea.

Looking forward, we see many exciting extensions for MillenniumDB: returning entire graphs, supporting more complex path constraints (e.g. by comparing data points along the paths), returning a set of paths connecting a pair of nodes (e.g. all shortest paths instead of a single path), full transactional support, just to name a few. More importantly, we hope that the research community can view the MillenniumDB source code as a sort of a sandbox for trying new research algorithms without the need to remake storage, indexing, access methods, or query parsing. In the end, MillenniumDB is just a proposal of how a graph database can be; and is definitely not a final word on the matter.

REFERENCES

[1] Waqas Ali, Muhammad Saleem, Bin Yao, Aidan Hogan, and Axel-Cyrille Ngonga Ngomo. 2021. A Survey of RDF Stores & SPARQL Engines for Querying Knowledge Graphs. *CoRR* abs/2102.13027 (2021). arXiv:2102.13027 https://arxiv.org/abs/2102.13027

[2] Renzo Angles, Marcelo Arenas, Pablo Barceló, Aidan Hogan, Juan L. Reutter, and Domagoj Vrgoc. 2017. Foundations of Modern Query Languages for Graph Databases. *ACM Comput. Surv.* 50, 5 (2017), 68:1–68:40. https://doi.org/10.1145/3104031

[3] Renzo Angles and Claudio Gutiérrez. 2008. Survey of graph database models. *ACM Comput. Surv.* 40, 1 (2008), 1:1–1:39. https://doi.org/10.1145/1322432.1322433

[4] Pablo Barceló Baeza. 2013. Querying graph databases. In *Proceedings of the 32nd ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems, PODS 2013, New York, NY, USA - June 22 - 27, 2013*. 175–188. https://doi.org/10.1145/2463664.2465216

[5] Jorge A. Baier, Dietrich Daroch, Juan L. Reutter, and Domagoj Vrgoc. 2017. Evaluating Navigational RDF Queries over the Web. In *Proceedings of the 28th ACM Conference on Hypertext and Social Media, HT 2017, Prague, Czech Republic, July 4-7, 2017*. 165–174. https://doi.org/10.1145/3078714.3078731

[6] Barry Bishop, Atanas Kiryakov, Damyan Ognyanoff, Ivan Peikov, Zdravko Tashev, and Ruslan Velkov. 2011. OWLIM: A family of scalable semantic repositories. *Semantic Web* 2, 1 (2011), 33–42. https://doi.org/10.3233/SW-2011-0026

[7] Angela Bonifati, Wim Martens, and Thomas Timm. 2020. An analytical study of large SPARQL query logs. *VLDB J.* 29, 2-3 (2020), 655–679. https://doi.org/10.1007/s00778-019-00558-9

[8] Rick Cattell. 2010. Scalable SQL and NoSQL data stores. *SIGMOD Rec.* 39, 4 (2010), 12–27. https://doi.org/10.1145/1978915.1978919

[9] Richard Cyganiak, David Wood, and Markus Lanthaler. 2014. RDF 1.1 Concepts and Abstract Syntax. W3C Recommendation. https://www.w3.org/TR/rdf11-concepts/

[10] Orri Erling. 2012. Virtuoso, a Hybrid RDBMS/Graph Column Store. *IEEE Data Eng. Bull.* 35, 1 (2012), 3–8. http://sites.computer.org/debull/A12mar/vicol.pdf

[11] The Wikimedia Foundation. 2021. *Wikidata:Database download.* https://www.wikidata.org/wiki/Wikidata:Database_download

[12] Nadime Francis, Alastair Green, Paolo Guagliardo, Leonid Libkin, Tobias Lindaaker, Victor Marsault, Stefan Plantikow, Mats Rydberg, Petra Selmer, and Andrés Taylor. 2018. Cypher: An Evolving Query Language for Property Graphs. In *Proceedings of the 2018 International Conference on Management of Data, SIGMOD Conference 2018, Houston, TX, USA, June 10-15, 2018*, Gautam Das, Christopher M. Jermaine, and Philip A. Bernstein (Eds.). ACM, 1433–1445. https://doi.org/10.1145/3183713.3190657

[13] Hector Garcia-Molina, Jeffrey D. Ullman, and Jennifer Widom. 2009. *Database systems - the complete book (2. ed.).* Pearson Education.

[14] Steve Harris, Andy Seaborne, and Eric Prud'hommeaux. 2013. SPARQL 1.1 Query Language. W3C Recommendation. https://www.w3.org/TR/sparql11-query/

[15] Olaf Hartig. 2017. Foundations of RDF★ and SPARQL★ (An Alternative Approach to Statement-Level Metadata in RDF). In *Proceedings of the 11th Alberto Mendelzon International Workshop on Foundations of Data Management and the Web, Montevideo, Uruguay, June 7-9, 2017 (CEUR Workshop Proceedings)*, Juan L. Reutter and Divesh Srivastava (Eds.), Vol. 1912. CEUR-WS.org. http://ceur-ws.org/Vol-1912/paper12.pdf

[16] Tom Heath and Christian Bizer. 2011. *Linked Data: Evolving the Web into a Global Data Space.* Morgan & Claypool Publishers. https://doi.org/10.2200/S00334ED1V01Y201102WBE001

[17] Daniel Hernández, Aidan Hogan, and Markus Krötzsch. 2015. Reifying RDF: What Works Well With Wikidata?. In *Proceedings of the 11th International Workshop on Scalable Semantic Web Knowledge Base Systems co-located with 14th International Semantic Web Conference (ISWC 2015), Bethlehem, PA, USA, October 11, 2015 (CEUR Workshop Proceedings)*, Thorsten Liebig and Achille Fokoue (Eds.), Vol. 1457. CEUR-WS.org, 32–47. http://ceur-ws.org/Vol-1457/SSWS2015_paper3.pdf

[18] Aidan Hogan, Marcelo Arenas, Alejandro Mallea, and Axel Polleres. 2014. Everything you always wanted to know about blank nodes. *J. Web Semant.* 27-28 (2014), 42–69. https://doi.org/10.1016/j.websem.2014.06.004

[19] Aidan Hogan, Eva Blomqvist, Michael Cochez, Claudia d'Amato, Gerard de Melo, Claudio Gutiérrez, José Emilio Labra Gayo, Sabrina Kirrane, Sebastian Neumaier, Axel Polleres, Roberto Navigli, Axel-Cyrille Ngonga Ngomo, Sabbir M. Rashid, Anisa Rula, Lukas Schmelzeisen, Juan F. Sequeda, Steffen Staab, and Antoine Zimmermann. 2020. Knowledge Graphs. *CoRR* abs/2003.02320 (2020). arXiv:2003.02320 https://arxiv.org/abs/2003.02320

[20] Aidan Hogan, Cristian Riveros, Carlos Rojas, and Adrián Soto. 2019. A Worst-Case Optimal Join Algorithm for SPARQL. In *The Semantic Web - ISWC 2019 - 18th International Semantic Web Conference, Auckland, New Zealand, October 26-30, 2019, Proceedings, Part I (Lecture Notes in Computer Science)*, Chiara Ghidini, Olaf Hartig, Maria Maleshkova, Vojtech Svátek, Isabel F. Cruz, Aidan Hogan, Jie Song, Maxime Lefrançois, and Fabien Gandon (Eds.), Vol. 11778. Springer, 258–275. https://doi.org/10.1007/978-3-030-30793-6_15

[21] Stanislav Malyshev, Markus Krötzsch, Larry González, Julius Gonsior, and Adrian Bielefeldt. 2018. Getting the Most Out of Wikidata: Semantic Technology Usage in Wikipedia's Knowledge Graph. In *International Semantic Web Conference (ISWC)*. Springer, 376–394.

[22] Alberto O. Mendelzon and Peter T. Wood. 1989. Finding Regular Simple Paths in Graph Databases. In *Proceedings of the Fifteenth International Conference on Very Large Data Bases, August 22-25, 1989, Amsterdam, The Netherlands*. 185–193.

[23] Jorge Pérez, Marcelo Arenas, and Claudio Gutiérrez. 2009. Semantics and complexity of SPARQL. *ACM Trans. Database Syst.* 34, 3 (2009), 16:1–16:45.

[24] Raghu Ramakrishnan and Johannes Gehrke. 2000. *Database management systems.* McGraw-Hill.

[25] Marko A. Rodriguez. 2015. The Gremlin graph traversal machine and language (invited talk). In *Proceedings of the 15th Symposium on Database Programming Languages, Pittsburgh, PA, USA, October 25-30, 2015*, James Cheney and Thomas Neumann (Eds.). ACM, 1–10. https://doi.org/10.1145/2815072.2815073

[26] Edward Sciore. 2020. *Database Design and Implementation - Second Edition.* Springer. https://doi.org/10.1007/978-3-030-33836-7

[27] AllegroGraph Team. 2021. AllegroGraph 7.1.0 Documentation. https://franz.com/agraph/support/documentation/current/

[28] ArangoDB Team. 2021. ArangoDB v3.7.11 Documentation. https://www.arangodb.com/docs/stable/

[29] Amazon Neptune Team. 2021. What Is Amazon Neptune? https://docs.aws.amazon.com/neptune/latest/userguide/intro.html

[30] JanusGraph Team. 2021. JanusGraph Documentation, v.0.5. https://docs.janusgraph.org/

[31] Jena Team. 2021. TDB Documentation. https://jena.apache.org/documentation/tdb/

[32] MillenniumDB Team. 2021. MillenniumDB 0.1. https://github.com/MillenniumDB/MillenniumDB

[33] MillenniumDB Team. 2021. MillenniumDB: A persistent, Open-Source, Graph Database [Extended Version]. https://github.com/MillenniumDB/MillenniumDB/whitepaper/whitepaper.pdf

[34] MillenniumDB Team. 2021. Wikidata Benchmark. https://github.com/MillenniumDB/benchmark

[35] OrientDB Team. 2021. OrientDB Manual – version 3.0.34. https://orientdb.org/docs/3.0.x/

[36] Stardog Team. 2021. Stardog 7.6.3 Documentation. https://docs.stardog.com/

[37] TigerGraph Team. 2021. TigerGraph Documentation – version 3.1. https://docs.tigergraph.com/

[38] Bryan B. Thompson, Mike Personick, and Martyn Cutcher. 2014. The Bigdata® RDF Graph Database. In *Linked Data Management*, Andreas Harth, Katja Hose, and Ralf Schenkel (Eds.). Chapman and Hall/CRC, 193–237. http://www.crcnetbase.com/doi/abs/10.1201/b16859-12

[39] Todd L. Veldhuizen. 2014. Triejoin: A Simple, Worst-Case Optimal Join Algorithm. In *Proc. 17th International Conference on Database Theory (ICDT), Athens, Greece, March 24-28, 2014*. 96–106. https://doi.org/10.5441/002/icdt.2014.13

[40] Denny Vrandecic and Markus Krötzsch. 2014. Wikidata: a free collaborative knowledgebase. *Commun. ACM* 57, 10 (2014), 78–85. https://doi.org/10.1145/2629489

[41] Jim Webber. 2012. A programmatic introduction to Neo4j. In *Conference on Systems, Programming, and Applications: Software for Humanity, SPLASH '12, Tucson, AZ, USA, October 21-25, 2012*, Gary T. Leavens (Ed.). ACM, 217–218. https://doi.org/10.1145/2384716.2384777

[42] C. T. Yu and M. Z. Ozsoyoglu. 1979. An algorithm for tree-query membership of a distributed query. In *The IEEE Computer Society's Third International Computer Software and Applications Conference, COMPSAC 1979, 6-8 November, 1979, Chicago, Illinois, USA*. 306–312.

APPENDIX

A SYNTAX OF MILLENNIUMDB QUERIES

The language supported by MillenniumDB borrows syntax from popular graph query languages SPARQL [14] and Cypher [12]. General structure of queries in MillenniumDB can be visualized as follows:

```
SELECT Selectors
MATCH  MatchPattern
WHERE  Condition
ORDER BY OrderSelectors
LIMIT Number
```

Figure 10: General structure of queries in MillenniumDB

Intuitively, the SELECT clause specifies which of the matched variables will be returned, while the MATCH clause specifies the basic or navigational graph pattern which we will look for in our graph. The WHERE clause is used to filter result based on a selection, usually by restricting the values of some of the attributes of a matched object. The ORDER BY allows us to reorder the results based on the values of some output variables, while LIMIT cuts off the evaluation after a specific number of results had been found. The syntax of different patters that appear in Figure 10 is defined in Figure 11.

As an example of a query occupying many of these elements, consider the following:

```
SELECT ?x.age, ?y.age, ?z
MATCH (?x :Person)-[?e :knows+]->(?y :Person {country:"Chile"})
      OPTIONAL {(?x)-[:studiedAt]->(?z)}
WHERE ?x.age >= 35 AND ?e.since = 1/1/2020
ORDER BY ?x.age ASC, ?y.age DESC
LIMIT 1000
```

When evaluated on a social network graph, similar to the one from Figure **??** in Section 2, the query is tying to find the ages of all pairs of people who know each other. Additionally, it is stating that the second person lives in Chile, that the first one has age greater than or equal 35, and that the edge connecting them was created on 1/1/2020. If the university where the first person studied is known, this is also returned (and is otherwise `null`). The results are ordered such that the age of the first person is ascending, and the age of the second person is descending. Finally, only 1000 results are solicited. The syntax here can be build rater easily using the specification of Figure 11. The only interesting portion is the `MatchPattern`, which is composed as a join (comma) of an edge pattern `(?x :Person)-[?e :knows+]->(?y :Person country:"Chile")` and an optional pattern `(?x)-[:studiedAt]->(?z)`, which is itself a `EdgePattern`.

B FORMAL DEFINITION OF DOMAIN GRAPH QUERIES

Queries in MillenniumDB are based on an abstract notion of *domain graph query*, which generalise the types of graph patterns used by modern graph query languages [2]. This query abstraction provides modularity in terms of how the database is constructed, flexibility in terms of what concrete query syntax is supported, and allows for defining its semantics and studying its theoretical properties in a clean way.

This section provides the formal definition of the MillenniumDB query language From now on, assume a countably infinite set Var of variables disjoint with the set of objects Obj.

B.1 Basic graph patterns

At the core of domain queries are basic graph patterns.[10] A *basic graph pattern* is defined as a pair (V, ϕ) such that $\phi : (\text{Obj} \cup \text{Var}) \to (\text{Obj} \cup \text{Var}) \times (\text{Obj} \cup \text{Var}) \times (\text{Obj} \cup \text{Var})$ is a partial mapping with a finite domain and $V \subseteq \text{var}(\phi)$, where $\text{var}(\phi)$ is the set of variables occurring in the domain or in the range of ϕ. Thus, ϕ can be thought of as a domain graph that allows a variable in any position, together with a set V of output variables (hence the restriction that each variable in V occurs in ϕ).

The evaluation of a basic graph pattern returns a set of solution mappings. A *solution mapping* (or simply *mapping*) is a partial function $\mu : \text{Var} \to \text{Obj}$. The *domain* of a mapping μ, denoted by $\text{dom}(\mu)$, is the set of variables on which μ is defined. Given $v \in \text{Var}$ and $o \in \text{Obj}$, we use $\mu(v) = o$ to denote that μ maps variable v to object o. Besides, given a set V of variables, the term $\mu_{|_V}$ is used to denote the mapping obtained by restricting μ to V, that is, $\mu_{|_V} : (V \cap \text{dom}(\mu)) \to \text{Obj}$ such that $\mu_{|_V}(v) = \mu(v)$ for every $v \in (V \cap \text{dom}(\mu))$ (notice that V is not assumed to be subset of $\text{dom}(\mu)$). Finally, for the sake of presentation, we assume that $\mu(o) = o$, for all $o \in \text{Obj}$.

[10] Basic graph patterns correspond to conjunctive queries (CQs) over graphs.

Selectors

Selectors : – * | Variables

Variables : – v | v.k | Variables, Variables , $v \in \text{Var}, k \in \mathcal{K}$

MatchPattern

MatchPattern : – GraphPattern | (MatchPattern) OPTIONAL { MatchPattern }

GraphPattern : – GraphElement | GraphElement, (GraphPattern)

GraphElement : – NodePattern | EdgePattern | PropertyPath

NodePattern : – (Node? Labels? Properties?)

Node : – $v \in \text{Var}$ | $o \in \text{Obj}$

Labels : – :l | :l Labels , $l \in \mathcal{L}$

Properties : – { PropertyList }

PropertyList : – k:val | k:val, PropertyList , $k \in \mathcal{K}, val \in \mathcal{V}$

EdgePattern : – NodePattern -Edge?> NodePattern | NodePattern <-Edge? NodePattern

Edge : – [v? Type? Properties?]- , $v \in \text{Var}$

Type : – :o | =TYPE(v) , $o \in \text{Obj}$, $v \in \text{Var}$

PropertyPath : – NodePattern =[pathExp]=> NodePattern | NodePattern <=[pathExp]= NodePattern

pathExp : – :o | ^pathExp | (pathExp/pathExp) | pathExp* |
pathExp+ | pathExp{n,m} | (pathExp | pathExp) | pathExp? , $o \in \text{Obj}, \mathtt{n}, \mathtt{m} \in \mathbf{N}, \mathtt{n} \leq \mathtt{m}$

Conditions

Condition : – Comparison | (Condition AND Condition) | (Condition OR Condition)

Comparison : – $v.k \sim val$ | $v.k \sim v'.k'$, $v, v' \in \text{Var}, k, k' \in \mathcal{K}, val \in \mathcal{V}, \sim \in \{==, <=, >=, <, >\}$

OrderSelectors

OrderSelectors : – v | v ASC | v DESC | $v.k$ | $v.k$ ASC | $v.k$ DESC | OrderSelectors, OrderSelectors , $v \in \text{Var}, k \in \mathcal{K}$

Figure 11: Specification of query patterns in MilleniumDB. Blue symbols are taken literally. Red brackets in `MatchPattern` and `GraphPattern` are grouping specification for unambiguous parsing, and are not specified when writing the query. The `Number` going in the `LIMIT` command is any integer.

The evaluation of a basic graph pattern $B = (V, \phi)$ over a property-domain graph $G = (O, \gamma)$, denoted by $[\![B]\!]_G$, is defined as $[\![B]\!]_G = \{\mu_{|_V} \mid \mu \in [\![\phi]\!]_G\}$, where

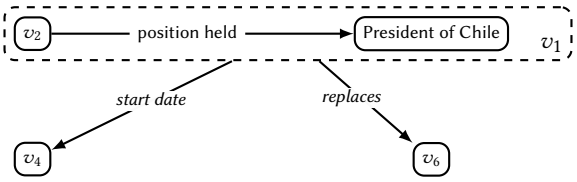
$$[\![\phi]\!]_G = \{\mu \mid \text{dom}(\mu) = \text{var}(\phi) \text{ and if } \phi(a) = (a_1, a_2, a_3), \text{then } \gamma(\mu(a)) = (\mu(a_1), \mu(a_2), \mu(a_3))\}.$$

For example, consider the basic graph pattern (V, ϕ) where $V = \{v_2, v_4, v_6\}$ and ϕ is given by the assignments:

$\phi(v_1) = (v_2, \texttt{position held}, \texttt{President of Chile})$,
$\phi(v_3) = (v_1, \texttt{start date}, v_4)$, and
$\phi(v_5) = (v_1, \texttt{replaces}, v_6)$.

In Figure 12, we provide a graphical representation of the above graph pattern, and the solution mappings obtained by evaluating the graph pattern over the property domain graph shown in Figure **??**. The solution mappings are presented as a table with columns v_2, v_4, v_6 (i.e. the variables in V), and each row represents an individual mapping. Per our definitions, different variables may map to the same object in a single solution. Thus, our notion of evaluation follows a homomorphism-based semantics, similar to query languages such as SPARQL [2].[11]

[11] Isomorphism-based semantics [2] – such as Cypher's no-repeated-edge semantics, which disallows the same solution to use the same edge twice – can be emulated by filtering solutions after they are generated.



v_2	v_4	v_6
Michelle bachelet	2006-03-11	Ricardo Lagos
Michelle bachelet	2018-03-11	Sebastián Piñera

Figure 12: Graphical representation of a basic graph pattern (top), and the tabular representation of the solution mappings (bottom) obtained by evaluating the basic graph pattern over the property domain graph shown in Figure ?? .

B.2 Navigational graph patterns

A characteristic feature of graph query languages is the ability to match paths of arbitrary length that satisfy certain criteria. We call basic graph patterns enhanced with this feature *navigational graph patterns*, and we define them next.

A popular way to express criteria that paths should match is through regular expressions on their labels, aka. *regular path queries* (*rpqs*). More precisely, an *rpq expression* r is defined by the following grammar:

$$r \quad ::= \quad \varepsilon \mid o \in \mathsf{Obj} \mid (r/r) \mid (r + r) \mid r^{-} \mid r^{*}.$$

The semantics of an rpq expression r is defined in terms of its evaluation on a property-domain graph G, denoted by $[\![r]\!]_G$, which returns a set of pair of nodes in the graph that are connected by paths satisfying r. More precisely, assuming that $G = (O, \gamma)$, $o \in \mathsf{Obj}$ and r, r_1, r_2 are rpq expressions, we have that:

$$\begin{aligned}
[\![\varepsilon]\!]_G &= \{(o, o) \mid o \in O\},\\
[\![o]\!]_G &= \{(o_1, o_2) \mid \exists o' \in \mathsf{Obj} : \gamma(o') = (o_1, o, o_2)\},\\
[\![(r_1/r_2)]\!]_G &= \{(o_1, o_2) \mid \exists o' \in \mathsf{Obj} : (o_1, o') \in [\![r_1]\!]_G \text{ and } (o', o_2) \in [\![r_2]\!]_G\},\\
[\![(r_1 + r_2)]\!]_G &= [\![r_1]\!]_G \cup [\![r_2]\!]_G,\\
[\![r^{-}]\!]_G &= \{(o_1, o_2) \mid (o_2, o_1) \in [\![r]\!]_G\}.
\end{aligned}$$

Moreover, assuming that $r^1 = r$ and $r^{n+1} = r/r^n$ for every $n \geq 1$, we have that:

$$[\![r^{*}]\!]_G \quad = \quad [\![\varepsilon]\!]_G \cup \bigcup_{k \geq 1} [\![r^k]\!]_G.$$

Other rpq expressions widely used in practice can be defined by combining the previous operators. In particular, we have that $r? = \varepsilon + r$ and $r^{+} = r/r^{*}$.

A *path pattern* is a tuple (a_1, r, a_2) such that $a_1, a_2 \in \mathsf{Obj} \cup \mathsf{Var}$ and r is an rpq expression. As for the case of basic graph patterns, given a path pattern p, we use the term $\mathsf{var}(p)$ to denote the set of variables occurring in p. Moreover, the evaluation of $p = (a_1, r, a_2)$ over a property-domain graph G, denoted by $[\![p]\!]_G$, is defined as:

$$[\![p]\!]_G \quad = \quad \{\mu \mid \mathsf{dom}(\mu) = \mathsf{var}(p) \text{ and } (\mu(a_1), \mu(a_2)) \in [\![r]\!]_G\}.$$

For example, the expression (`Michelle Bachelet`, `replaced by`$^{+}$, v) is a path pattern that returns all the Presidents of Chile after Michelle Bachelet. Given a set ψ of path patterns, $\mathsf{var}(\psi)$ also denotes the set of variables occurring in ψ, and the evaluation of ψ over a property-domain graph G is defined as:

$$[\![\psi]\!]_G \quad = \quad \{\mu \mid \mathsf{dom}(\mu) = \mathsf{var}(\psi) \text{ and } \mu_{|_{\mathsf{var}(p)}} \in [\![p]\!]_G \text{ for each } p \in \psi\}.$$

Definition B.1. A *navigational graph pattern* is a triple (V, ϕ, ψ) where $(V' \subseteq V, \phi)$ is a basic graph pattern, ψ is a set of path patterns and $V \subseteq \mathsf{var}(\phi) \cup \mathsf{var}(\psi)$. The semantics of a navigational graph pattern $N = (V, \phi, \psi)$ is defined as:

$$[\![N]\!]_G \quad = \quad \{\mu_{|_V} \mid \mathsf{var}(\mu) = \mathsf{var}(\phi) \cup \mathsf{var}(\psi), \mu_{|_{\mathsf{var}(\phi)}} \in [\![\phi]\!]_G \text{ and } \mu_{|_{\mathsf{var}(\psi)}} \in [\![\psi]\!]_G\}. \quad \square$$

Hence, the result of a navigational graph pattern $N = (V, \phi, \psi)$ is a set of mappings μ projected onto the set V of output variables, where μ satisfies the structural restrictions imposed by ϕ and the path constrainst imposed by ψ. Notice that multiple rpq expressions can link the same pair of nodes. This is similar to the existential semantics of path queries, as specified in the SPARQL standard [14].

Given a domain graph $G = (O, \gamma)$, currently we define paths over the directed edge-labelled graph that forms the range of γ; in other words, we do not allow for matching paths that emanate from an edge object. Such a feature could be considered in the future. We may also consider adding semantics for shortest paths, additional criteria on node or edges in the path, etc.

B.3 Relational graph patterns

As previously discussed (and seen in the example of Figure 12), graph patterns return relations (tables) as solutions. Thus we can – and many practical graph query languages do – use a relational-style algebra to transform and/or combine one or more sets of solution mappings into a final result.

Towards defining this algebra, we need the following terminology. Two mappings μ_1 and μ_2 are *compatible*, denoted by $\mu_1 \sim \mu_2$, if $\mu_1(v) = \mu_2(v)$ for all variables v which are in both $\mathsf{dom}(\mu_1)$ and $\mathsf{dom}(\mu_2)$. If $\mu_1 \sim \mu_2$, then we write $\mu_1 \cup \mu_2$ for the mapping obtained by extending μ_1 according to μ_2 on all the variables in $\mathsf{dom}(\mu_2) \setminus \mathsf{dom}(\mu_1)$. Given two sets of mappings Ω_1 and Ω_2, the *join*, *union*, *difference* and *left outer join* between Ω_1 and Ω_2 are defined respectively as follows:

$$\begin{aligned}
\Omega_1 \bowtie \Omega_2 &= \{\mu_1 \cup \mu_2 \mid \mu_1 \in \Omega_1,\ \mu_2 \in \Omega_2 \text{ and } \mu_1 \sim \mu_2\},\\
\Omega_1 \cup \Omega_2 &= \{\mu \mid \mu \in \Omega_1 \text{ or } \mu \in \Omega_2\},\\
\Omega_1 \setminus \Omega_2 &= \{\mu_1 \in \Omega_1 \mid \forall \mu_2 \in \Omega_2 : \mu_1 \sim \mu_2 \text{ does not hold}\},\\
\Omega_1 \text{⟕} \Omega_2 &= (\Omega_1 \bowtie \Omega_2) \cup (\Omega_1 \setminus \Omega_2).
\end{aligned}$$

With this terminology, a *relational graph pattern* is recursively defined as follows:

- If N is a navigational graph pattern, then N is also relational graph pattern;
- If R_1 and R_2 are relational graph patterns, then $(R_1 \text{ AND } R_2)$ and $(R_1 \text{ OPT } R_2)$ are relational graph patterns.

The evaluation of a relational graph pattern R over a property-domain graph G, denoted by $[\![R]\!]_G$, is recursively defined as follows:

- if R is a navigational graph pattern N, then $[\![R]\!]_G = [\![N]\!]_G$;
- if R is $(R_1 \text{ AND } R_2)$ then $[\![R]\!]_G = [\![R_1]\!]_G \bowtie [\![R_2]\!]_G$;
- if R is $(R_1 \text{ OPT } R_2)$ then $[\![R]\!]_G = [\![R_1]\!]_G \text{⟕} [\![R_2]\!]_G$.

B.4 Selection conditions

In addition to match a graph pattern against a property-domain graph, we would like to filter the solutions by imposing selection conditions over the resulting objects (i.e. nodes and edges). More precisely, a *selection condition* is defined recursively as follows: (a) if $v_1, v_2 \in \mathsf{Var}$ and $o \in \mathsf{Obj}$, $k_1, k_2 \in \mathcal{K}$, $v \in \mathcal{V}$ then $(v_1 = v_2)$, $(v_1 = o)$, $v_1.k_1 = v_2.k_2$ and $v_1.k_1 = v$ are selection conditions; and (b) if C_1, C_2 are selection conditions, then $(\neg\ C_1)$, $(C_1 \wedge C_2)$, $(C_1 \vee C_2)$ are selection conditions.

Given a property-domain graph $G = (O, \gamma, \mathtt{lab}, \mathtt{prop})$, a mapping μ, and a selection condition C, we say that μ satisfies C under G, denoted by $\mu \models_G C$, if

- C is $(v_1 = v_2)$, $v_1, v_2 \in \mathsf{dom}(\mu)$ and $\mu(v_1) = \mu(v_2)$;
- C is $(v_1 = o_1)$, $v_1 \in \mathsf{dom}(\mu)$ and $\mu(v_1) = o_1$;
- C is $(v_1.k_1 = v_2.k_2)$, $v_1, v_2 \in \mathsf{dom}(\mu)$, $\mathtt{prop}(\mu(v_1), k_1) = \mathtt{prop}(\mu(v_2), k_2)$;
- C is $(v_1.k_1 = v)$, $v_1 \in \mathsf{dom}(\mu)$ and $\mathtt{prop}(\mu(v_1), k_1) = v$;
- C is $(\neg C_1)$, and it is not the case that $\mu \models_G C_1$;
- C is $(C_1 \wedge C_2)$, $\mu \models_G C_1$ and $\mu \models_G C_2$;
- C is $(C_1 \vee C_2)$, $\mu \models_G C_1$ or $\mu \models_G C_2$.

B.5 Solution modifiers

We consider an initial set of solution modifiers that allow for applying a final transformation on the solutions generated by a graph pattern. These include: selection, which defines a set of elements (variables and properties) to be returned; order by, which orders the solutions according to a sort criteria; and limit, which returns the first n mappings in a sequence of solutions.

Let $\mathbf{S}$ be the set of strings, $v \in var$ and $k \in \mathcal{K}$. A *selection mapping* is a function $\tau : \mathbf{S} \to \mathsf{Obj} \cup \mathcal{V}$. A *selection element* is either a variable v or an expression $v.k$. Assume that there is a simple way to transform a selection element into a string in $\mathbf{S}$. Given a sequence of selection mappings S and an integer n, the function $\mathtt{limit}(S, n)$ returns the first n elements of S when $n > 0$, and returns S otherwise.

Given a property-domain graph $G = (O, \gamma, \mathtt{lab}, \mathtt{prop})$, a mapping μ, and a sequence of selection elements E, the function $\mathtt{sel}(\mu, E)_G$ returns a selection mapping τ defined as follows: if $v \in E$ then "v" $\in \mathsf{dom}(\tau)$ and τ("v") $= \mu(v)$; if $v.k \in E$ then "$v.k$" $\in \mathsf{dom}(\tau)$ and τ("$v.k$") $= \mathtt{prop}(\mu(v), k)$. Moreover, given a set of solution mappings Ω, the function `select` returns a set of selection mappings defined as $\mathtt{select}(\Omega, E)_G = \{\mathtt{sel}(\mu, E)_G \mid \mu \in \Omega\}$.

An *order modifier* is a tuple (e, β) where e is a selection element and β is either `asc` or `desc`. Given a sequence of selection mappings S and an order modifier $o = (e, \beta)$, we say that S satisfies o, denoted $S \models o$, if it applies that: (i) β is `asc` and S satisfies an ascending order with respect to e; or (ii) β is `desc` and S satisfies a descending order with respect to e. Moreover, given a sequence of order modifiers $O = (o_1, \ldots, o_n)$, we say that S satisfies O, denoted $S \models O$, if it applies that: (i) $S \models o_1$ when $n = 1$; or (ii) $S \models o_1$ and, for every sub-sequence of selection mappings $S' \subseteq S$ it applies that $S' \models (o_2, \ldots, o_n)$ such that $\tau_i(e_1) = \tau_j(e_1)$ for any pair of selection mappings $\tau_i, \tau_j \in S'$, with $o_1 = (e_1, \beta_1)$.

B.6 Graph Queries

A *graph query* Q is defined as a tuple (R, C, E, O, n), where R is a relational graph pattern, C is a selection condition, E is a sequence of selection elements, $O = \{o_1, \ldots, o_n\}$ is a sequence of order modifiers, and n is a positive integer. We assume that R is the unique mandatory component. Given a variable $v \in \text{dom}(R)$, the rest of components have the following expressions by default: C is $v = v$, E is v, O is $(v, \texttt{asc})$ and $n = 0$.

The evaluation of Q over G is defined as $\texttt{limit}(S, n)$ where $S = \texttt{Select}(\Omega, E)_G$, $S \models O$, and $\Omega = \{\mu_{|_V} \mid \mu \in [\![R]\!]_G \wedge \mu \models^G C\}$.

We will assume that every graph query $Q = (R, C, E, O, n)$ satisfies the following two conditions: (i) For every sub-pattern $R' = (R_1 \text{ OPT } R_2)$ of R and for every variable v occurring in R, it applies that, if v occurs both inside R_2 and outside R', then it also occurs in R_1; (ii) It applies that $\textbf{Var}(C) \subseteq \textbf{Var}(R)$. Then, we say that Q is a *well-designed graph query*.

It is easy to see that a declarative query expression **SELECT** E **MATCH** R **WHERE** C **LIMIT** n **ORDER BY** O, can be transformed into a graph query (R, C, E, O, n).