

How Many Bits are Needed For Memory:

Miller (+Tom's) Useless Piece of Sheet

$$2^{-1} \leq \# \text{ of bytes}$$

Extra Note Space:

"Courage Need Not Be Remembered, For It Is Never Forgotten"

Conversions:

Binary: 128 64 32 16 8 4 2 1

Binary (2) → Decimal (10) → Add up the locations of 1s, sum is decimal

→ Hex (16) → Add up locations of 1s, split into 2 groups of 4, first group is first letter/number, second is the second

Decimal To Binary: Make a tree, find largest 2^x in it, keep going until 1 or 0 is left, the x in 2^x is the location of 1 in binary

Decimal To Hex: Convert to binary, then to decimal

In Hex:
10: A
11: B
12: C
13: D
14: E
15: F

How to Convert from 1 Base to Any Base:

I.E. 357_{10} to base 4

$$\begin{array}{r} 4 \overline{) 357} \rightarrow 89 \text{ r } 1 \\ 89 \rightarrow 22 \text{ r } 1 \\ 22 \rightarrow 5 \text{ r } 2 \\ 5 \rightarrow 1 \text{ r } 1 \end{array} \rightarrow 357_{10} = 1121_4$$

Extra Notes:

$$(5743)_8 \rightarrow 10$$

$$5 \times 8 + 7 = 47$$

$$\rightarrow 47 \times 8 + 3 = 380$$

$$\rightarrow 380 \times 8 + 3 = 3043$$

$$1 + 1 = 2$$

$$\rightarrow 2 + 2 = 4$$

$$\rightarrow 4 + 4 = 8$$

CMOS Power:

$$\text{Power} = C \times V_{DD}^2 \times \text{frequency}$$

$$A + \bar{A}B = A + B$$

$$\bar{\bar{A}} = A \checkmark$$

Simplification Methods:

DeMorgan's Law:

$$\overline{(X+Y)} = \bar{X} \cdot \bar{Y}$$

$$\begin{array}{l} + \leftrightarrow \cdot \\ \cdot \leftrightarrow + \end{array}$$

Principle of Duality:

- Switch $+ \leftrightarrow \cdot$
- Switch $0 \leftrightarrow 1$
- Keep variables the same $Y \leftrightarrow \bar{X} \rightarrow \bar{Y}$

Various Rules:

$$(X+Y) = \overline{(\overline{X+Y})} \rightarrow \overline{(\bar{X} \cdot \bar{Y})} \rightarrow (X+Y)$$

Equivalent

Identities	$X+0=X$	$X \cdot 1=X$
Null Elements	$X+1=1$	$X \cdot 0=0$
Idempotency	$X+X=X$	$X \cdot X=X$
Involutions	$\overline{\bar{X}}=X$	
Complements	$X+\bar{X}=1$	$X \cdot \bar{X}=0$

Commutativity

$$X+Y=Y+X$$

$$X \cdot Y=Y \cdot X$$

Associativity

$$(X+Y)+Z=X+(Y+Z)$$

$$(X \cdot Y) \cdot Z=X \cdot (Y \cdot Z)$$

Distributivity

$$X \cdot (Y+Z)=(X \cdot Y)+(X \cdot Z)$$

$$(X+Y) \cdot (X+Z)=X+(Y \cdot Z)$$

Covering

$$X+X \cdot Y=X$$

$$X \cdot (X+Y)=X$$

Combining

$$X \cdot Y+X \cdot \bar{Y}=X$$

$$(X+Y) \cdot (X+\bar{Y})=X$$

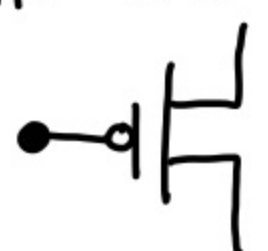
Consensus

$$X \cdot Y+\bar{X} \cdot Z+Y \cdot Z=X \cdot Y+\bar{X} \cdot Z$$

$$(X+Y) \cdot (\bar{X}+Z) \cdot (Y+Z)=(X+Y) \cdot (\bar{X}+Z)$$

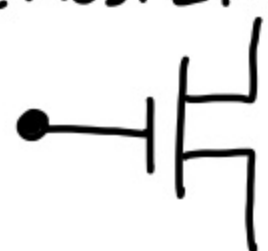
CMOS Gates:

P-Type MOSFET:



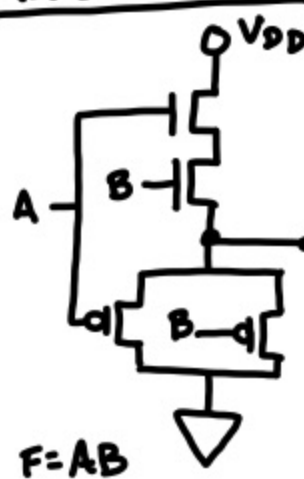
On when low input
Off when high input

N-Type MOSFET:



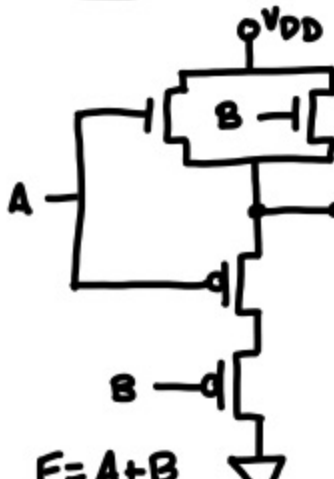
On when high input
Off when low input

AND Gate:



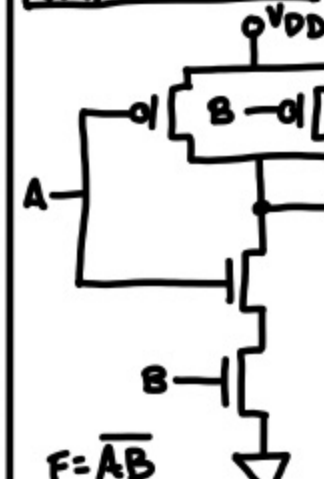
$$F=AB$$

OR Gate:



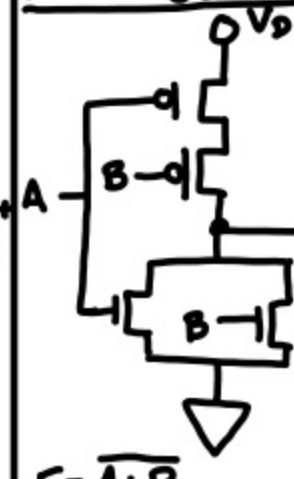
$$F=A+B$$

NAND Gate:



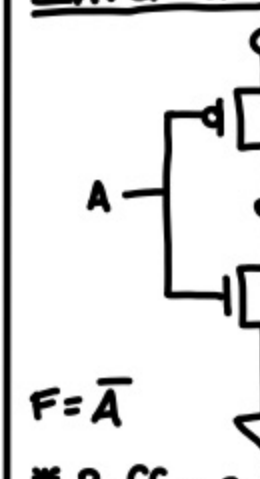
$$F=\bar{A}\bar{B}$$

NOR Gate:



$$F=\bar{A}+\bar{B}$$

Inverter:



$$F=\bar{A}$$

* Buffer = 2 inverters

- Fan in: # of inputs
- Can use inverter to invert \rightarrow NAND \leftrightarrow AND
- \rightarrow NOR \leftrightarrow OR

Extra Notes:

$$V_{out} = V_{DD} e^{-\frac{t}{RC}} \rightarrow H \rightarrow L$$

$$V_{out} = V_{DD} (1 - e^{-\frac{t}{RC}}) \rightarrow L \rightarrow H$$

$$\tau = RC = \frac{1}{2\pi f_c}$$

Noise Margins:

$$NM_L: V_{IL} - V_{OL}$$

$$NM_H: V_{OH} - V_{IH}$$

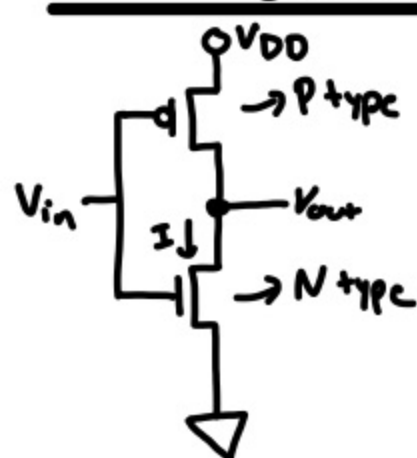
$$\rightarrow \text{i.e. } NM_L(A \rightarrow B): V_{IL, \max}(B) - V_{OL, \max}(A)$$

$$\rightarrow \text{i.e. } NM_H(B \rightarrow A): V_{OH, \min}(B) - V_{IH, \min}(A)$$

Extra Notes:

"You must believe in your own courage, which has led you to triumph over the many hardships you have faced... and you triumph once again!"

"On" and "Off" Resistance:



Channel	On Ω	Off Ω
N	50	0.6 M
P	100	0.6 M

$$V_{DD} = 5V$$

$$V_{in} = 0 \rightarrow \text{Low, } P_{on}, N_{off}$$

$$V_{out} = V_{DD} \times \left(\frac{0.6M}{50 + 0.6M} \right)$$

$$V_{out} \approx 4.99$$

$$I = V/R$$

$$4.99/0.6M = 8.33 \mu A$$

$$P = IV \rightarrow 4.99$$

$$\rightarrow P = 41.66 \mu W$$

For answers to the exam you are currently taking, scan this QR Code:



Propagation Delay:

- 50% of input to 50% of output
- For NAND Gate: Last to turn to high or first to turn to low
- For NOR Gate: First to turn high or last to turn low
- For AND Gate: Last gate to turn to high or first gate to turn to low
- For OR Gate: First gate to turn to high or last gate to turn to low

Verilog Notes:

- End each statement with a Semicolon ;

- Example Verilog Statement:

gate name (output, input1, input2 ...)

and u0 (F, A, B)

or

assign variable = expression \rightarrow assign $F = A \& B$

XNOR of

11010100

1: $1 \oplus 1 = 1$

2: $1 \oplus 0 = 0$

3: $0 \oplus 0 = 0$

4: $0 \oplus 1 = 1$

5: $1 \oplus 1 = 0$

6: $1 \oplus 0 = 1$

7: $0 \oplus 0 = 0$

Output is 1

Gate:

$F = V_{out}$

$F = \overline{AB}$

$$\begin{array}{c}
 143 \\
 \swarrow \quad \searrow \\
 \frac{128}{7} \qquad 15 \\
 \qquad \swarrow \searrow \\
 \qquad \frac{8}{3} \quad 7 \\
 \qquad \quad \swarrow \searrow \\
 \qquad \quad \frac{4}{2} \quad 3 \\
 \qquad \quad \quad \swarrow \searrow \\
 \qquad \quad \quad \frac{2}{1} \quad \frac{1}{0}
 \end{array}$$

$$2^x - 1 \geq 32000 \rightarrow x \geq 15$$
$$F = \overline{M}N + MP$$

$\overline{M}N \rightarrow M \text{ and } N = 0$
 $MP \rightarrow M \text{ and } P = 1$

M	N	P	F
0	0	0	1
0	0	0	1
0	1	0	0
0	1	0	0
1	0	0	0
1	1	0	1

Baba Pooooooooosh!

Convert the following base 16 (hexadecimal) number to base 2

$(F6)_{16}$ $\rightarrow F = 15$

$\frac{1}{8} \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{1}{1}$ $\frac{0}{8} \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{0}{1}$

15 6

$$\frac{1}{7} \frac{0}{6} \frac{1}{5} \frac{0}{4} \frac{1}{3} \frac{1}{2} \frac{0}{1} \frac{0}{0}$$

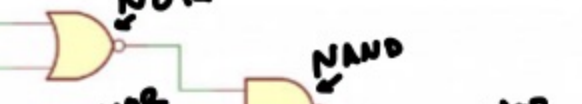
$$2^2 + 2^3 + 2^5 + 2^7 \rightarrow 4 + 8 + 32 + 128 = 172$$

The Buffer Passes input
to output

AND

- OR
1. Smoke detector turns on (condition S) AND fire alarm turns on (condition F).
 2. A person presses an emergency button (condition E)

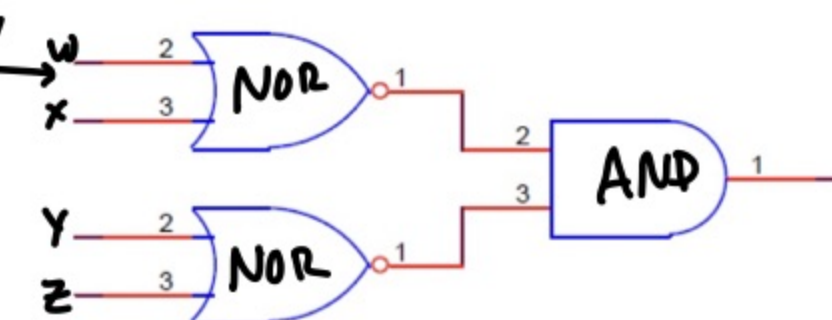
The diagram shows a hierarchical circuit with two sub-circuits, AND2 and OR2, connected in series. The AND2 sub-circuit has two inputs, S and F, and its output is connected to the input of the OR2 sub-circuit. The OR2 sub-circuit has two inputs, the output of AND2 and a constant input T, and its output is D. The sub-circuits are labeled inst1 and inst2.



$$\begin{aligned}
 & \overline{((\overline{W+X}) \cdot (\overline{X+Y})) + ((\overline{Y+Z}) \cdot (\overline{X+Y}))} \\
 & \downarrow \\
 & \overline{((W+X) + (X+Y)) + ((Y+Z) + (X+Y))} \\
 & \downarrow \\
 & \overline{(W+X+Y) + (Y+X+Z)} \\
 & \rightarrow W+X+Y+Z
 \end{aligned}$$

The circuit diagram implements the expression $Y = ((\overline{A+B}) + C) \cdot (((\overline{A+B}) + D) \cdot (((\overline{A+B}) \cdot E)))$. It features several logic gates: OR gates for $\overline{A+B}$ and $\overline{A+B} + D$; a NOR gate for $\overline{A+B} \cdot E$; and a final AND gate for the product. Handwritten labels 'NOR', 'OR', and 'AND' identify the gate types. The final output is labeled 'Y'.

What is this logically equivalent to?


$$\begin{array}{c} \overline{(w+x)} \cdot \overline{(y+z)} \\ \hline \overline{w+x+y+z} \\ \downarrow \\ \boxed{w+x+y+z} \end{array}$$

Extra Notes:	
$2^0 = 1$	$2^5 = 32$
$2^1 = 2$	$2^6 = 64$
$2^2 = 4$	$2^7 = 128$
$2^3 = 8$	$2^8 = 256$
$2^4 = 16$	$2^9 = 512$

X	Y	Z	X NOR
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$(\overline{X \cdot Y}) + (X \cdot Z) = \overline{X} + Y + Z$$

Did you scan the QR Code?

In
Case
of
emergence


$$\overline{(x+y) + z} = \overline{x+y} \cdot \overline{z}$$

A CMOS circuit dissipates 1200 mW when its V_{dd} is 4 V and its load capacitance is 5 nF (nanofarads). How much power will the CMOS circuit dissipate if the load capacitance is changed to 6 nF (same V_{dd} and frequency)?

$$P_{\text{new}} = \frac{6}{5} P_{\text{old}} \rightarrow 1200 \text{ mW}$$

$$W' \cdot (X+Y) \cdot (X'+Y) \cdot W + W \cdot (X+Y+Z) + W' \cdot (X'+Y+Z) + Z'$$

$0 \cdot (x+y) = (0+x+y)$
 opposites
 $\rightarrow 0$

$$\begin{aligned} & wx + wy + wz + \overline{w}x + \overline{w}y + \overline{w}z + z \\ & Y(w + \overline{w}) = Y \\ & Z(w + \overline{w}) = Z \\ & \underline{wx + \overline{w}x + y + z + \overline{z}} \\ & \quad | + y + wx + \overline{w}x \rightarrow 0 \end{aligned}$$

$$0 + 1 = 1$$

The diagram shows three waveforms: A, B, and Z. Waveform A transitions from low to high. Waveform B transitions from low to high. Waveform Z transitions from low to high. A vertical arrow indicates the time delay between the rising edge of A and the rising edge of Z, labeled 4 ns . A scale bar at the bottom indicates 1 ns . An AND gate symbol is shown with inputs A and B, and output Z.

wire K, L, D, E;

not n1(K, A); $\rightarrow K = \bar{A}$

not n2(L, B); $\rightarrow L = \bar{B}$

and a1(D, A, B); $D = A \cdot B$

and a2(E, K, L); $E = \bar{A} \cdot \bar{B}$

nor o1(Y, D, E); $Y = (A \cdot B) + (\bar{A} \cdot \bar{B})$

end module

$\rightarrow A(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B})$

\downarrow

$(\bar{A}\bar{B} + B\bar{A})$

$\hookrightarrow Y = (\bar{A} + \bar{B}) \cdot (A + B)$

```
1 module myCircuit (
2     input wire x,
3     input wire y,
4     input wire z,
5     output wire f
6 );
7     wire w;
8     assign f = w ^ z;
9     assign w = ~(y & x);
10 endmodule
```

$$\begin{aligned} w &= \overline{(y \cdot x)} \\ f &= w \oplus z \\ f &= \overline{(y \cdot x)} \oplus z \end{aligned}$$

$\hookrightarrow x=0$
 $y=1$
 $z=0$

$f = \overline{(0 \cdot 1)} \oplus 0$
 $\quad \quad \quad \overline{0} \oplus 0 \rightarrow f=1$

Logic Family A DC Characteristics				
$V_{CC} = 5.0 \text{ V}$	$V_{OHmin} = 4.60 \text{ V}$	$V_{OLmax} = 0.30 \text{ V}$	$V_{IHmin} = 2.70 \text{ V}$	$V_{ILmax} = 1.30 \text{ V}$
$V_{TH} = (V_{OH} + V_{OL})/2$	$I_{OH} = -11 \text{ mA}$	$I_{OL} = 15 \text{ mA}$	$I_{IH} = 2 \text{ }\mu\text{A}$	$I_{IL} = -1 \text{ }\mu\text{A}$

Logic Family B DC Characteristics				
$V_{CC} = 4.5\text{ V}$	$V_{OHmin} = 4.30\text{ V}$	$V_{OLmax} = 0.20\text{ V}$	$V_{IHmin} = 2.90\text{ V}$	$V_{ILmax} = 1.00\text{ V}$
$V_{TH} = (V_{OH} + V_{OL})/2$	$I_{OH} = -3\text{ mA}$	$I_{OL} = 15\text{ mA}$	$I_{IH} = 8\text{ }\mu\text{A}$	$I_{IL} = -4\text{ }\mu\text{A}$

$$NM_{H(A \rightarrow A)} = V_{OH, \min}^{4.6}(A) - V_{IH, \min}^{2.7}(A) = 1.9$$