**Fractional Share Selling**

Bryce Miller

**The Study**: to test the historical performance of selling fractional shares when positive price movements result in gains greater than 0.50%. For example if a share has a price appreciation going from $100 to $101 then the price rise is 1% (greater than 0.50% thus triggering our event) and the investor sells 0.99% of the share to get $1 in revenue while their investment is then back to being worth $100 and is 99% of a whole share. This will be compared to the buy-and-hold strategy where an asset is simply purchased and sold in whole at a later date.

Prices used in our data are end-of-day prices. Asset is the SPY S&P500 ETF, adjusted prices. Holding period is about two years, or more exactly 253 trading days per year times two (the average number of active trading days). Historical data consists of 5,283 daily SPY prices from 11/20/2000 to 11/18/2021.

In total there are 4,777 samples where the two-year investments were started on each day, e.g. from 11/20/200 going two years, then another sample as 11/21/2000 going two years, etc. The asset has fractional shares sold on positive gains, then at the end of the period the remaining fractional share is sold at its fraction of the market price. All returns are annualized on a compounded-basis.

We will also look at the effects of leveraged products, in that we will lever the SPY to make its movements more volatile to measure the results.

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The overall average return from the 4,777 samples at 1X:

Average holding return: 9.18%

Average trading return: 7.96%

Therefore when the trading strategy is applied the average returns were lower by 1.22%.

Comparison, how many times (as a percentage of total) trading returns beat holding returns:

Comparison when the holding returns are negative, how many times trading is better:

Therefore, when the holding strategy is negative, the trading strategy had better returns 89% of the time.

When trading return is better, the average rate is 0.85% better than the holding return.

Comparison when the holding returns are negative, how many times trading return is positive:

Therefore while the trading strategy was better when the alternative was negative, it was only positive 2% of the time.

Probability (percentage of total samples) that returns were positive between holding and trading:

Therefore it seems that the trading strategy was slightly more likely to be positive compared to the alternative.

Next we will look at the distribution of returns experienced. The below chart details the classifications used in the distribution:

A picture containing table

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The distribution of returns between trading and holding shows that the trading strategy “normalizes” the distribution, limiting the high and low extremes:

Now we will look at including leverage. Leveraged investing may have effects because it increases the volatility which may increase the number of opportunities to sell fractional shares. We will compare a buy-and-hold and fractional trading strategy on the same leveraged asset. Leveraged at 3X, therefore 3X the daily volatility of the underlying asset.

Looking at a triple leveraged asset, using the same comparisons we have above:

|  |  |
| --- | --- |
| 1X Trading better than holding | 21% |
| 3X Trading better than holding | 27% |
| 1X Better when negative | 89% |
| 3X Better when negative | 91% |
| 1X Positive when negative | 2% |
| 3X Positive when negative | 10% |

In summary:

* In the comparison between better returns, using a triple leveraged asset had better returns 27% of the time compared to the 1x levered above as 21% better.
* When the holding return is negative, 91% of the time the leveraged trading was better, compared with 89% of the time with the 1x leverage.
* Moreover, the triple leveraged was positive 10% of the time when the holding was negative, as opposed to 2% with the 1x leverage.

When the trading return is better than the holding return, on average it is 7.98% better, compared to 0.85% better for the 1x leverage.

However, the probability (percentage of total samples) that returns were positive between holding and trading, with the triple leverage asset, falls. Although like above the trading strategy is slightly more likely to be positive than holding.

Like the 1x levered asset, the leveraged trading strategy reduced the frequency of the extreme returns in the distribution However, the leveraged returns still remain more volatile and disbursed.

See the 3X distribution:

Overall the 3X product still remains volatile, the distribution is more “wide” than the 1X, yet still we show some degree of “normalization” with the fractional trading strategy. With normalization we expect to see smaller distributions on the extreme ends, and returns that gravitate more around “Bad” and “Good”. Above we see that “Premium” is higher from trading, but we should recognize that this comes from a sharp fall in the “Gold” category.

Below is the results from 2X leverage:

Moving on, we will look closer at the impacts of adding leverage. The distributions for different leverage is posted below.

Below we have a table comparing the changes in “Very Bad” returns from the different holding and trading strategies. The left column strategy is compared to the column strategies, e.g. the 1X trading strategy had less returns in the “Very Bad” category, amounting to 1.25% better difference. While the 3X Holding had 12.12% greater returns in the “Very Bad” category compared to the 1X hold.

An interesting observation is that while the trading strategy always has reduced the risk of Very Bad returns the amount diminishes, where the 1X trade is 1.26 better than 1X hold, the 2X trade is 1.07 better than the 2X hold, and lastly the 3X trade is only 0.46 better than the 3X hold. (Although, if we look closely at another category in the distribution table of 3X returns, (posted below) the 3X trade reduced the “Extreme Bad” from 0.31 to 0 which is not counted as reducing the “Very Bad” category, and actually may increase the category as the returns become less-risky overall. However the trend of diminishing results still seems clear regardless.)

Table

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Below we graph the decline:

Lastly, we will see the effect of dollar-cost-average (DCA) investing. We investigate this phenomena because as an asset’s price fluctuates different moments of investing can lead to different volatility opportunities to exit and sell fractional shares. For example, an investor can start an investment at high prices and over the two year period there may be little to no exit opportunities to sell fractional shares. While the opposite example is starting a period on a low where plenty of opportunities exist to exit fractional shares. Therefore a DCA approach can diversify the starting investments by splitting up the period where investments enter at different times to try to minimize the two extremes of timing: investing at highs and investing at lows. We will introduce DCA investing combined with the fractional share strategy, meaning we invest with DCA while also selling fractional share on positive price movements.

To facilitate the DCA approach we expand our investment horizon from two years to around two and a half years, composing of three 2-year investments that start 90 days apart, i.e. one investment starts then 90 days later the next starts. The final investment ends in around two years and 180 days. This will be compared to a holding strategy where an asset is bought and held for about two years and 180 days. Fractional shares are still sold at movements above a positive 0.50%.

We must note that the compounded DCA trading return will be slightly inaccurate. Due to the structure of the data, it was most convenient to treat each of the three investments individually where each is compounded for two-years. Because the whole investment period is two years and 180 days, the first investment should be compounded at (about) 2.5 years, the second should be (about) 2.25 years, and the third investment just at two years. The results here are simply the three investments compounded individually at two years then averaged together. The real DCA trading returns would be slightly larger. Lastly, we will mention that when we compare DCA trading returns directly to holding returns we compare holding over the (about) 2.5 year period and compounding the holding at 2.5 years. Unlike the other comparisons between trading and holding which are at two years.

Below we present the percentage the trading is better than holding, and summarize past results:

|  |  |
| --- | --- |
| 1X trading better % | 21% |
| 3x trading better % | 27% |
| 1x DCA trading better% | 25% |
| 3X DCA trading better % | 29% |

We will consolidate the results that indicate when the trading was better than holding when the holding was negative. Below we notice that the 1X DCA trading was the most likely to be negative when the holding alternative is also negative. It does not seem that anything theoretical about the trading strategy would explain this, and that perhaps it is simply a statistical phenomena coming from our data:

|  |  |
| --- | --- |
| 1X trading | 89% |
| 3x trading | 91% |
| 1x DCA trading | 46.35% |
| 3X DCA trading | 76.53% |

When the holding returns were negative and the trading were positive:

|  |  |
| --- | --- |
| 1X trading | 2% |
| 3x trading | 10% |
| 1x DCA trading | 12% |
| 3X DCA trading | 23% |

Probability of positive returns when trading:

|  |  |
| --- | --- |
| 1X trading | 83.63% |
| 3x trading | 81.12% |
| 1x DCA trading | 84.16% |
| 3X DCA trading | 81.99% |

On the frequency distribution, the dollar-cost-average trading strategy has also proven to reduce the extremes of the holding returns.

Below is the 1x DCA histogram:

And the 3x DCA distribution:

From the above it would seem that the DCA approach did well in all categories except for being more likely to be negative if the holding is negative. The overall average returns:

|  |  |
| --- | --- |
| 1X holding | 9.18% |
| 1X trading | 7.96% |
| 3x holding | 22.04% |
| 3x trading | 15.74% |
| 1X DCA holding comparison | 9.70% |
| 1X DCA trading | 7.94% |
| 3X DCA holding comparison | 21.93% |
| 3X DCA trading | 16.04% |

Below are three distribution graphs that show holding returns going from 1X, 2X, to 3X leverage:

Now the same but with the trading strategy:

And lastly the DCA trading (only 1X and 3X):

Conclusion:

The trading strategy historically produced lower risk and lower returns consistently the majority of the time. This means that while the trading returns were normally lower they also reduced the frequency and severity of negative and low returns. This is mainly because the trading strategy seizes the opportunities of high price fluctuations. However the downside is that it eliminates the compound effect of the holding strategy and hence the lower returns. Adding leveraged had mixed results, the increased risk of leveraged assets could not be completely eliminated. The leveraged assets still represent higher risk and higher reward as they would with a buy-and-hold strategy. While the trading strategy did consistently reduce the risk and reward of leveraged assets, still the higher-risk component remained. Additionally, we noticed a diminishing return where increased leverage increasingly diminished the results of the trading strategy.

The dollar cost-average (DCA) trading approach (DCA investing and fractional share selling) created better results for the most part. On its own DCA investing produces lower returns according to a 2012 study from Vanguard where they reported that DCA investing was worse than holding 66% of the time. Therefore we should expect lower returns from the DCA approach, but we noticed that combined with the trading strategy these results are mixed.

See below where we compare 1x strategies, holding, trading, and DCA trading. Although we must note that the 1X trading is the aggregate data for two years while the holding and DCA trading are two and a half years.

As we noticed above comparing the DCA strategy to the holding strategy over the same period we see the same hedging effect, although including the 1x trading, despite the fact that the time period is slightly inaccurate as a comparison, we see a continued pattern that most likely reflects reality. The DCA trading strategy had the lowest “Premium” returns, the highest “Good” returns, and, despite it having only the second best “Bad” performance, overall for negative returns it performs the best if we add the “Bad” and “Very Bad” categories together where its total is 15.81%, and then 16.35% for trading and 17.61% for holding. Below we will make a similar comparison with triple leverage:

The results from adding leverage here become mixed. Regarding negative returns we have Holding: 23.22%, Trading: 18.88%, DCA Trading: 17.99%. Thus the DCA trading has a slight hedge advantage against negative returns, in line with what we have seen. However for positive returns the image is not so clear. This is likely a result from the same phenomena we now understand about leverage: higher risk and high reward. Therefore buying and holding the triple leveraged asset produces extreme highs and lows, moving into the “Gold” range. Still however we can see the extra hedging effects of the DCA approach in that it is the only strategy that has no existence above premium returns, the lowest (by a small amount) in the negative category.

Again we re-iterate the comparison time period between the trading and the other two strategies is slightly different and thus is slightly inaccurate. Also, as mentioned above the DCA compounding is slightly inaccurate.