

In MATLAB.

DFT(y) ... what happens to the spectrum of the output?

$$\text{DFT}(x * h) = \sum_{n=1}^N (x * h) e^{\frac{-j2\pi nk}{N}} \dots$$

$$\left[ \text{Recall: } x * h = \sum_{m=1}^M h[m] \cdot x[n-m+1] \right]$$

$$\dots = \sum_{n=1}^N \left[ \sum_{m=1}^M h[m] \cdot x[n-m+1] \right] e^{\frac{-j2\pi nk}{N}}$$

$$= \sum_{m=1}^M h[m] e^{\frac{-j2\pi nk}{N}} \cdot \sum_{n=1}^N x[n-m+1] e^{\frac{-j2\pi nk}{N}} \quad (\text{useless step!})$$

Note: Let  $d = n - m + 1 \rightarrow n = d + m - 1$

$$= \sum_{m=1}^M h[m] \cdot \sum_{n=1}^N x[d] \cdot e^{\frac{-j2\pi (d+m-1)k}{N}}$$

$$= \underbrace{\sum_{m=1}^M h[m] \cdot e^{\frac{-j2\pi (m-1)k}{N}}}_{H[f]} \cdot \underbrace{\sum_{n=1}^N x[d] \cdot e^{\frac{-j2\pi dk}{N}}}_{X[f]}$$

$$\dots Y[k] = H[f] \cdot X[f]$$

The output spectrum is equal to the spectrum of "h" multiplied by the spectrum of "x".