

1.

$$L = 255$$

$$C_1, C_2 = \frac{1}{\sqrt{L}}$$

Image A



Image B



```
>> HW4Q1  
SSIM = 0.7209
```

3.

$$\exp(j\theta) = c + id, \quad c = \cos\theta, \quad d = \sin\theta$$

$$\begin{bmatrix} c & -d \\ d & c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\Rightarrow \left\{ \begin{array}{l} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \theta = \frac{\pi}{6} \\ \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \theta = \frac{\pi}{4} \\ \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, \theta = \frac{\pi}{3} \\ \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, \theta = \frac{2}{3}\pi \end{array} \right. \Rightarrow \theta = \frac{j\pi}{6}, \quad \theta = \frac{j\pi}{4} \quad (j \in \mathbb{N})$$

#

4.

$$M \times N \times P \text{ DCT} = M \times \text{1D } N\text{-Point DCT} + N \times \text{1D } M\text{-Point DCT} \\ + P \times \text{2D } M \times N \text{ DCT}$$

$$\Rightarrow \text{complexity} = M \times N \log N + N \times M \log M + P \times MN \log MN \\ = MN \log MN + PMN \log MN \\ = (P+1) MN \log MN$$

#

5

$$\begin{bmatrix} X[1] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \begin{matrix} n=1 \\ m=1 \end{matrix} \begin{matrix} 1 & 2 & 3 & 4 \\ \begin{bmatrix} a & b & b & a \\ b & a & -a & -b \\ b & -a & -a & b \\ a & -b & b & -a \end{bmatrix} \end{matrix} \begin{bmatrix} X[1] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} \quad \begin{matrix} a = 0.5878 \\ b = 0.9511 \end{matrix}$$

$$\Rightarrow \begin{cases} X[1] = a(X[1] + X[4]) + b(X[2] + X[3]) \\ X[2] = a(X[2] - X[3]) + b(X[1] - X[4]) \\ X[3] = -a(X[2] + X[3]) + b(X[1] + X[4]) \\ X[4] = a(X[1] - X[4]) + b(X[3] - X[2]) \end{cases}$$

$$\Rightarrow \begin{bmatrix} X[1] \\ X[3] \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} X[1] & X[4] \\ X[2] & X[3] \end{bmatrix}$$

$$\Rightarrow 2 \text{ MVLs} + 2 \text{ MVLs} = 4 \text{ MVLs}$$

$$\begin{bmatrix} X[2] \\ X[4] \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} X[2] - X[3] \\ X[1] - X[4] \end{bmatrix}$$

6.

(a)

$$\begin{aligned} MV_{L193} &= 11MV_{L13} + 13MV_{L11} \\ &= 11 \times 52 + 13 \times 40 \\ &= 572 + 520 \\ &= 1092 \end{aligned}$$

(c)

$$\begin{aligned} MV_{L196} &= 14MV_{L14} + 14MV_{L14} \\ &= 14 \times 32 + 14 \times 32 \\ &= 896 \end{aligned}$$

(b)

$$\begin{aligned} MV_{L195} &= 15MV_{L13} + 13MV_{L15} \\ &= 15 \times 52 + 13 \times 40 \\ &= 780 + 520 \\ &= 1300 \end{aligned}$$