Homework 1 (Due: March 20th)

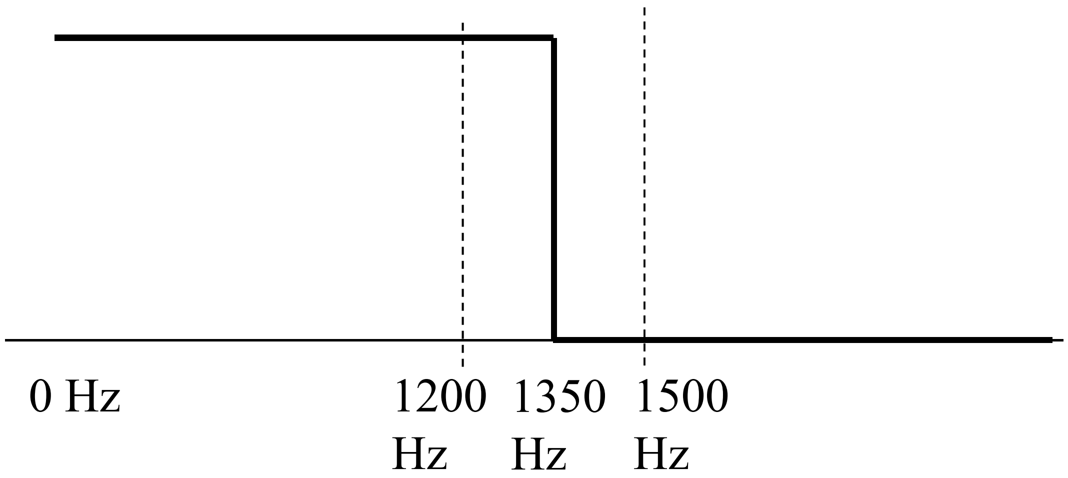
(1) Design a Mini-max **lowpass** FIR filter such that (40 scores)

① Filter length = 17, ② Sampling frequency f_s = 6000Hz,

③ Pass Band 0~1200Hz ④ Transition band: 1200~1500 Hz,

⑤ Weighting function: $W(F)$ = 1 for passband, $W(F)$ = 0.6 for stop band .

⑥ Set Δ = 0.0001 in Step 5.



※ **The code should be handed out by NTUCool, too.**

Show (a) the frequency response, (b) the impulse response $h[n]$, and (c) the maximal error for each iteration.

(2) How do we implement $y[n] = x[n] * (0.8^n u[n] + 0.5^n u[n])$ efficiently where * means convolution and $u[n]$ is the unit step function? (10 scores)

(3) (a) What are the two main advantages of the Fourier transform (FT)? (b) What are the two main problems to implement the FT? (10 scores)

(4) Suppose that $x[n] = y(0.002n)$ and the length of $x[n]$ is 2000. If $X[m]$ is the FFT of $x[n]$, which frequencies do (a) $X[200]$ and (b) $X[1600]$ correspond to? (10 scores)

(5) Why (a) the step invariance method and (b) the bilinear transform can reduce or avoid the aliasing effect in IIR filter design? (10 scores)

(6) (a) Which of the following filters are usually even? (b) Which of the following filters are usually odd? (i) Notch filter; (ii) highpass filter; (iii) edge detector; (iv) integral; (v) differentiation 4 times; (vi) particle filter; (vii) matched filter. (10 scores)

(7) Use the MSE method to design the 7-point FIR filter that approximates the lowpass filter of $H_d(\mathrm{F}) = 1$ for $|F| < 0.25$ and $H_d(\mathrm{F}) = 0$ for $0.25 < |F| < 0.5$.

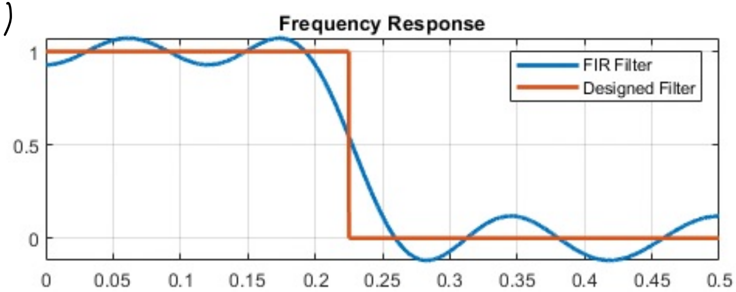
(15 scores)

(Extra): Answer the questions according to your student ID number.

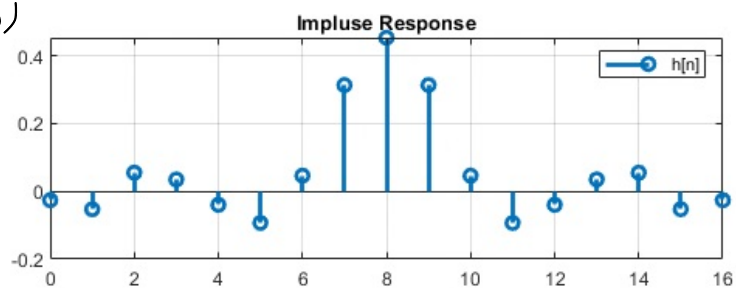
(ended with 0, 1, 2, 3, 5, 6, 7, 8)

1.

(a)



(b)



(c)

```
max_err = 1.236298
max_err = 0.770414
max_err = 0.712761
max_err = 0.712073
max_err = 0.712073
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2.

$$\begin{cases} u[n] = 1, & n \geq 0 \\ u[n] = 0, & n < 0 \end{cases}$$

$$y[n] = x[n] * (0.8^n u[n] + 0.5^n u[n])$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[z] z^{-n} = \sum_{n=-\infty}^{\infty} (0.8^n u[n] + 0.5^n u[n]) z^{-n}$$

$$= \sum_{n=0}^{\infty} (0.8^n z^{-n}) + \sum_{n=0}^{\infty} (0.5^n z^{-n})$$

$$= \frac{1}{1-0.8z^{-1}} + \frac{1}{1-0.5z^{-1}}$$

$$= \frac{(1-0.5z^{-1}) + (1-0.8z^{-1})}{(1-0.8z^{-1})(1-0.5z^{-1})}$$

$$= \frac{2-1.3z^{-1}}{1-1.3z^{-1}+0.4z^{-2}}$$

$$Y[z] = X[z] H[z]$$

$$\Rightarrow Y[z] = X[z] \frac{2-1.3z^{-1}}{1-1.3z^{-1}+0.4z^{-2}}$$

$$\Rightarrow Y[z] - 1.3z^{-1} Y[z] + 0.4z^{-2} Y[z] = 2X[z] - 1.3z^{-1} X[z]$$

$$\Rightarrow Y[z] = 2X[z] - 1.3z^{-1} X[z] + 1.3z^{-1} Y[z] + 0.4z^{-2} Y[z]$$

$$\Rightarrow y[n] = 2x[n] - 1.3x[n-1] + 1.3y[n-1] + 0.4y[n-2]$$

#

3.

(a)

Spectum analysis

Convolution $\rightarrow$ multiplication

(b)

Not real operation

Irrational number multiplication

4、

$$x[n] = y(0.002n) \Rightarrow X[n] = y\left(\frac{n}{500}\right),\quad n = 2000$$

(a)

$$X[200] = 200 \times \frac{500}{2000} = 50\,\text{Hz} \quad \#$$

(b)

$$1600 > \frac{n}{2000} \Rightarrow f = m \times \frac{f_s}{N} - m$$

$$\Rightarrow X[1600] = 1600 \times \frac{500}{2000} - 1600 = -1200\,\text{Hz} \quad \#$$

5.

(a)

Step invarience 利用積用處理高頻的能量，故能降低 HF 中的 aliasing effect

(b)

Bilinear transform 將 $-\infty \sim \infty$ 的 frequency domain mapping 至 $\pm\frac{f_s}{2}$ 間讓 aliasing effect 消失

6.

(a)

(i)、(iv)

(b)

(ii)、(iii)、(v)、(vii)

7.

$$H_d(F) = \begin{cases} 1, & |F| < 0.3 \\ 0, & 0.3 < |F| < 0.5 \end{cases}$$

$$S[n] = 2\int_{-0.5}^{0.5} \cos(2\pi nF) H_d(F)\, dF$$

$$S[0] = \int_{-0.5}^{0.5} H_d(F)\, dF = 0.6$$

$$S[1] = 2\int_{-0.3}^{0.3} \cos(2\pi F)\, dF$$

$$= 2\left[\sin(2F\pi) \times \frac{1}{2\pi}\right]_{-0.3}^{0.3}$$

$$= \frac{1}{\pi}\left[\sin 0.6\pi + \sin 0.6\pi\right]$$

$$= 0.6055$$

$$S[2] = 2\left[\sin(4F\pi) \times \frac{1}{4\pi}\right]_{-0.3}^{0.3}$$

$$= \frac{1}{2\pi}\left[2\sin 1.2\pi\right]$$

$$= -0.1871$$

$$S[3] = 2\left[\sin(6\pi F) \times \frac{1}{6\pi}\right]_{-0.3}^{0.3}$$

$$= \frac{1}{3\pi}\left[2\sin(1.8\pi)\right]$$

$$= -0.125$$

$$k = \frac{(7-1)}{2} = 3$$

$$\Rightarrow h[3] = S[0]$$

$$\Rightarrow h[0] = S[3]/2 = -0.0625$$

$$h[1] = S[2]/2 = -0.09355$$

$$h[2] = S[1]/2 = 0.30275$$

$$h[3] = S[0] = 0.6$$

$$h[4] = S[1]/2 = 0.30275$$

$$h[5] = S[2]/2 = -0.09355$$

$$h[6] = S[3]/2 = -0.0625$$