Homework 1 (Due: 5th Oct.)

(1) (a) What are the main advantage and the main disadvantage of the wavelet transform when compared with the STFT? (b) Why the wavelet transform is suitable for image compression and directional edge detection? (20 scores)

(2) Write at least three conditions where the chirp signal may be generated. (10 scores)

(3) Suppose that $f(t) = \cos(60\pi t^3 - 540\pi t^2 + 3020\pi t)$), $0 \le t \le 3$. (a) What is the instantaneous frequency of $f(t)$? (b) What is the lower bound of the number of samples of $f(t)$ if we do not change the sampling interval? (c) What is the lower bound of the number of samples of $f(t)$ if we change the sampling interval every second? (15 scores)

(4) (a) How does the window width B affect the resolution of the rec-STFT? (b) Determine the rec-STFT of $\sin(4\pi t)$. (10 scores)

(5) (a) What is the main advantage of the STFT with an asymmetric window? (b) What are the two advantages of the STFT with a Gaussian window? (10 scores)

(6) Write a program for the rectangular short time Fourier transform .

y = recSTFT(x, t, f, B) (35 scores)

x: input, *t*: samples on *t*-axis, *f*: samples on *f*-axis,

[-B, B]: interval of integration, y: output

$w(t)=1$ for $|t|<B$

$w(t)=0$ for $|t|>B$

(i) 要交本題的程式碼 (*.m 檔或 *.py檔，可用 Matlab 或 Python寫)，

(ii) 自己選一個 input x, 用你們的程式將 output y 算出來並畫出來

(iiii) 計算程式的 computation time

(iv) 不可以用 direct implementation 的方法

例子：

$t=n\Delta t$, $\Delta t=0.05$

$n=0\sim 601$

$Q=\frac{B}{\Delta t}=20B$ ($Q=20$ if $B=1$

$2Q+1=41$

$N=\frac{1}{\Delta t\Delta f}=400$

```
dt=0.05;
df=0.05;
t1=[0:dt:10-dt]; t2=[10:dt:20-dt]; t3=[20:dt:30];
t=[0:dt:30];
f=[-5:df:5];
x=[cos(2*pi*t1),cos(6*pi*t2),cos(4*pi*t3)];
B=1;
tic
y=recSTFT(x,t,f,B);
toc
```

$f=m\Delta f$, $m=-100\sim 100$

(Extra): Answer the questions according to your student ID number.

(ended with 0, 1, 2, 3, 5, 6, 7, 8)

Note: Do not forget the extra question.

1.

(a)

Advantage: Better locality in time and frequency domain

Disadvantage: Not significant as the solutions in specturm

(b)

Wavelet transform can separates lower frequency and higher frequency

2.

Radar system, Optics, wireless communication

3.

$f(t) = \cos(60\pi t^3 - 540\pi t^2 + 3030\pi t)$, $0 \le t \le 3$

(a)

$f(t) = 0.5\exp[j(60\pi t^3 - 540\pi t^2 + 3030\pi t)] + 0.5\exp[-j(60\pi t^3 - 540\pi t^2 + 3030\pi t)]$

$$\frac{f'(t)}{2\pi} = \pm(90t^2 - 540t + 1510) = \pm[90t(t-6) + 1510]$$

(b)

$0 \le t \le 3 \Rightarrow \Delta t = 3$

max at $t = 0, 6 \Rightarrow \frac{f'(t)}{2\pi} = \pm(1510)$

$3 \times (1510 + 1510) = 9060$ (points)

(c)

$0 < t < 1$, $\frac{f'(t)}{2\pi} = \pm(1510) \Rightarrow \Delta t = \frac{1}{3020}$

$1 < t < 2$, $\frac{f(t)}{2\pi} = \pm(1060) \Rightarrow \Delta t = \frac{1}{2120}$

$2 < t < 3$, $\frac{f'(t)}{2\pi} = \pm(790) \Rightarrow \Delta t = \frac{1}{1580}$

$\Rightarrow 3020 \times 1 + 2120 \times 1 + 1580 \times 1 = 6720$ (point)

4.

(a)

	t resolution	f resolution
B↑	↓	↑
B↓	↑	↓

(b)

$\sin 4\pi t$

$\Rightarrow W(t) = 1, |t| < \frac{1}{8}$

$W(t) = 0, |t| \ge \frac{1}{8}$

$\Rightarrow$ window width $= \frac{1}{4}$ (s)

5.

(a)

Real-time processing

(b)

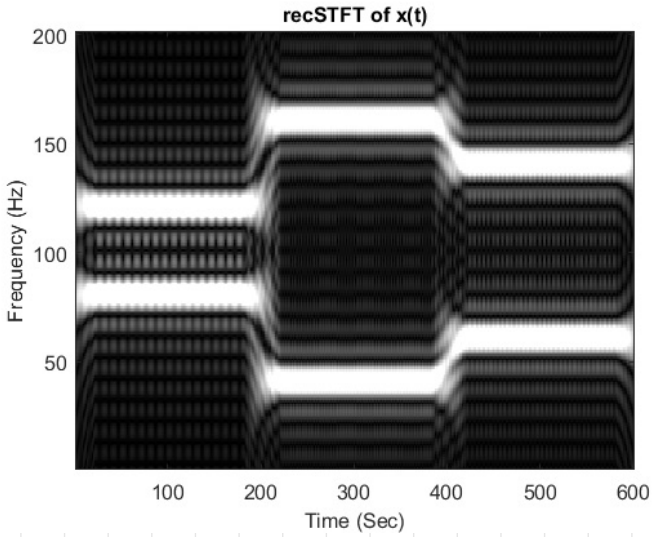
Best result, No sidelobe effect

6.

MATLAB Code

```
clear all
dt = 0.05;
df = 0.05;
t1 = [0:dt:10-dt]; t2=[10:dt:20-dt]; t3=[20:dt:30];
t = [0:dt:30];
f = [-5:df:5];
x = [cos(2 * pi * t1), cos(6 * pi * t2), cos(4 * pi * t3)];
B = 1;
tic
y=recSTFT(x,t,f,B);
toc

function y = recSTFT(x, t, f, B)
%% Step 1 Calculate n0, m0, T, F, N, Q
C = 400;

% Calculate delta t and f
dt = t(2) - t(1);
df = f(2) - f(1);

% Calculate numbers of N points
N = round(1 / (dt * df));

% Calculate numbers of t points
nt = t ./ dt;
nt = round(nt);
nt0 = nt(1);
T = length(nt);

% Calculate numbers of f points
nf = f ./ df;
nf = round(nf);
nf0 = nf(1);
F = length(nf);

% Calculate points in interval[-B , B]
Q = round(B/dt);

% Define parameter name
X = zeros(T,F);
t_value = zeros(1 , T);
f_value = zeros(1 , F);
x=[x,0];

%% Step 2 n = nt0
% time domain calculation
for n = 1 : T
%% Step 3 determine x1(q)
    % q tatol range
    q_range= zeros(1 , N);
    % q = 0 for 2Q < q <N
    q=[0 : 2 * Q];
    p = round(nt(n) - Q + q);
    p(p<1) = T + 1;
    p(p>T) = T + 1;
    % Set x1 = 0 if x is out of range
    x1=[x(p), q_range];

%% Step 4 : X1(m) = FFT[x1(q)]
    X1=fft(x1 , N);

%% Step 5:Convert X1(m) into X(ndt, mdf)
% frequency domain calculation
    for m = 1 : F
        t_value(1, n) = nt(n) * dt;
        f_value(1, m) = nf(m) * df;
        X(n,m) = X1(1, mod(nf(m), N) + 1) * exp(1i * 2 * pi * (Q - nt(n)) * nf(m) / N) * dt;
    end

%% Step 6 : Set n = n + 1 and return to Step 3 until n = n0 + T - 1
    n = n + 1;
end
%% Step 7 : Draw plot
% Transpose X to let time on x-axis
y = transpose(X);
image(abs(y) / max(max(abs(y))) * C);
% Gray-level figure
colormap(gray(256));
% Let y-axis from small to large
set(gca, 'Ydir', 'normal');
% Set figure description
set(gca, 'Fontsize', 12)
xlabel('Time (Sec)','FontSize', 12)
ylabel('Frequency (Hz)','FontSize', 12)
title('recSTFT of x(t)', 'FontSize', 12)
end
```

Result




```
>> HW1
Elapsed time is 0.185768 seconds.
```