Homework 4 (Due: 7[h] Dec.)

(1) Illustrate the following terms. (a) color noise; (b) vanish moment (about the wavelet transform); (c) the Fresnel transform . (15 scores)

(2) Suppose that $x(t)$ is a white noise. Which of the following function is also a white noise? Why? (i) $x(2t+3)$); (ii) $x(t)\exp(-\pi t^2)$; (iii) the FT of $x(t)$; (iv) the LCT of $x(t)$ (10 scores)

(3) What are the disadvantages when using the Hilbert transform to determine the instantaneous frequency? Write at least <u>three disadvantages</u>. (10 scores)

(4) Among the <u>STFT</u>, the <u>WDF</u>, the <u>Hilbert-Huang transform</u>, and the <u>wavelet transform</u>, which one is better for the applications of (a) video compression, (b) random process analysis, (d) analyzing the variation of temperature, and (d) modulation?
<u>Also illustrate the reasons</u>. (15 scores)

(5) (a) What is the most important advantage of the Haar transform nowadays?

(b) How many entries of the 2^k-point Haar transform are equal to 0, 1, and -1? Express the solutions in term of k. (10 scores)

(6) What are the vanish moments of

(a) $\dfrac{d^5}{dt^5} e^{-\pi t^2}$

(b) $x(t) = 1 - |t|$ for $-2 < t < 2$, $x(t) = 0$ otherwise. (10 scores)

(*Continued*)

(7) Write a Matlab or Python program of the Hilbert-Huang transform.

y = hht(x, t, thr)

x: input, y: output (each row of y is one of the IMFs of x), t: samples on the *t*-axis, thr : the threshold used in Step 7.

In Step 8, the number of non-boundary extremes can be no more than 3.

Just write Steps 1~8 and Step 9 is unnecessary.

<u>The Matlab or Python code should be handed out by NTUCool</u>. (30 scores)

Example:
```
t = [0: 0.01: 10];
x = 0.2*t + cos(2*pi*t) + 0.4*cos(10*pi*t);
thr = 0.2;
y = hht(x, t, thr);
```

(Extra): Answer the questions according to your student ID number.

(ended with 0, 2, 3, 4, 5, 7, 8, 9)

1.

(a)

color noise 為當在單位頻域內的譜密度 $1/f^{\beta}$，$\beta \neq 0$ 時為 color noise，$\beta = 0$ 為 white noise

(b)

Vanish moment $m_k = \int t^k \psi(t)\,dt$，$k$ = kth moment

若 $m_0 = m_1 = m_2 = \cdots\cdots = m_{x-1} = 0 \Rightarrow \psi(t)$ 有 x 個 vanish moment

當 vanish moment 值越大 ⇒ 被濾掉的 low-f 越多

也可用來檢視 Mother Wavelet 是否為 High-f 函數

(c)

Fresnel transform $V_0(x,y) = e^{ikz}\sqrt{\frac{1}{j\lambda z}}\int_{-\infty}^{\infty} e^{j\frac{k}{2z}(y-y_i)^2}\sqrt{\frac{1}{j\lambda z}}\int_{-\infty}^{\infty} e^{j\frac{k}{2z}(x-x_i)^2}dx_i dy_i$

描述了電磁波在空氣中傳播隨著距離增加而產生波形的相位和振幅改變

2、

if $x(t)$ is a white noise $\Rightarrow$ $\begin{cases} \text{mean}[x(t)] = 0 \\ x(t) \text{ 自相關} = 0 \\ \text{Power spectral density of } x(t) = \text{constant} \end{cases}$

(i)

$x(2t+3)$ 為訊號平移、縮放不會改變其特性

(ii)

$x(t)\exp(-\pi t^2)$ 為訊號 × 高斯函數，結果會使其特性改變

(iii)

FT of $x(t)$，FT 會將 $x(t)$ 的頻譜變為在所有頻率上皆為平坦

(iv)

LCT of $x(t)$ 與 FT 類似會將 $x(t)$ 的頻譜變為所有頻率上的平坦譜

$\Rightarrow$ (i) is also a with noise

#

3.

(a)

Sensitivity to noise

(b)

Edge Effect

(c)

Assumption of stationarity

4、

(a) video compression：Wavelet transform

在 vide compression 中 HHT的計算過於複雜且對於 noise較敏感

STFT 則有時間-頻率的 trade-off 現象

WDF則有 cross term 使分析複雜

Wavelet transform的計算複雜度最小且可以使用多解析度分析改善 trade-off

(b) random process analysis：STFT

STFT 能夠透過短 window 更好的分析 signal 在瞬時時間上的特性，也能根據 random process 進行多個時間尺度分析

(c) analyzing Variation of Temperature：CWT

溫度為非平穩 signal 而 CWT具有可變的 window，能更好分析溫度在某些情況下的劇烈變化

(d) Modulation：HHT

HHT 將 signal 分解為 IMFs，每個 IMF 代表不同的頻率成分，能夠有效的進行 Modulation

5.

(a)

計算速度快、簡單、只需使用 + − 法

(b)

Haar transform $= 0$, $2^k \Rightarrow 2^{k-1}(2^{k+1} - 2k - 2)$

$= 1$, $2^k \Rightarrow 2^{k-1}(k+2)$

$= -1$, $2^k \Rightarrow 2^{k-1} \times k$

6.

(a)

$$m_k = \int_{-\infty}^{\infty} t^k \psi(t)\, dt = \mathcal{F}\{t^k \psi(t)\}$$

$$\Rightarrow m_k = \frac{1}{-2\pi j} \frac{d^k}{df^k} \left(\mathcal{F}\{t^k \psi(t)\} \right)$$

$$\Rightarrow m_k = \frac{1}{-2\pi j} \frac{d^k}{df^k} (-2\pi j f)^5 \left(\mathcal{F}\{e^{-\pi t^2}\} \right)$$

$$\Rightarrow m_k = \frac{(2\pi j)^5}{(2\pi j)^k} \frac{d^k}{df^k} f^5 e^{-\pi t^2}$$

$$\Rightarrow m_k = 0 \text{ for } k < 5$$

$\Rightarrow$ Vanishing moment $= 5$

(b)

$$m_0 = \int_{-2}^{2} 1 - |t|\, dt$$

$$\Rightarrow m_0 = \left[t - \frac{t|t|}{2} \right]_{-2}^{2} = \left(2 - \frac{4}{2}\right) - \left(-2 + \frac{4}{2}\right) = 0$$

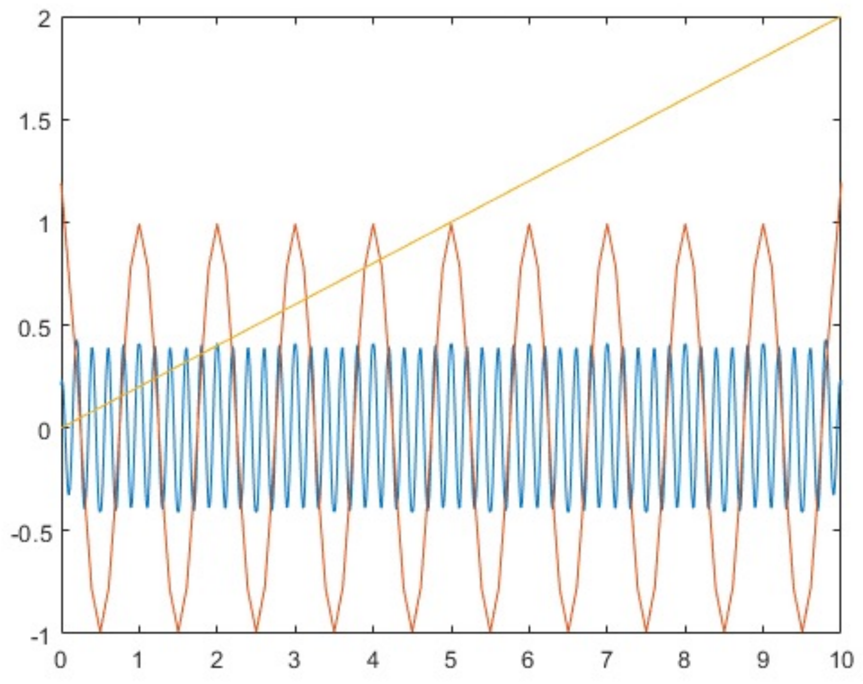
$$m_1 = \left[\frac{4 - \left(\frac{4\times 4}{2}\right)}{2} \right] - \left[\frac{-4 + \left(\frac{4\times 4}{2}\right)}{2} \right] = -4$$

$\Rightarrow$ Vanishing moment $= 1$

7.

```
clear all

t = [0: 0.01: 10];
x = 0.2*t + cos(2*pi*t) + 0.4*cos(10*pi*t);
thr = 0.2;
tic                                              % Timer Start
y = hht(x, t, thr);
toc                                              % Timer End



function y = hht(x, t, thr)
n = 1;
dt = diff(t(1:2));
max_iterations = 100;  % Set a maximum number of iterations to avoid infinite loo
iteration_count = 0;   % Initialize iteration counter
while (iteration_count < max_iterations)
    if length(findpeaks(x)) <= 3
        y(n,:) = x;
        break
    end

    temp = x;
    test = 1;
    k = 1;
    max_iterations_k = 3;  % Set a maximum number of iterations for k
    iteration_count_k = 0;   % Initialize iteration counter

    while (iteration_count_k < max_iterations_k)
        iteration_count_k = iteration_count_k + 1;

        [max, maxloc] = findpeaks(temp);            % Step02
        peaks = interp1((maxloc-1)*dt, max, t, 'linear', 'extrap');


        [neg_min, minloc] = findpeaks(temp*(-1));   % Step04
        min = -1 * neg_min;

        dips = interp1((minloc-1)*dt, min, t, 'linear', 'extrap');      % Step05
        z = (peaks + dips) / 2;                      % Step06_1
        h = (temp - z);                              % Step06_2
```

```
            % Step07
            hpeaks = findpeaks(h);
            hdips = -findpeaks(-h);

            test = 0;
            for i = 1:(length(hpeaks)-1)
                if ((hpeaks(i) <= 0) || (hdips(i) >= 0) || (abs((hpeaks(i) + hdips(i))/2) >= thr))
                    temp = h;
                    test = 1;
                    break
                end
            end
        end
        y (n,:) = h;
        % Step08
        x = x - h;
        n = n + 1;
    end
    plot(t, y)
end
```




```
>> Q7
Elapsed time is 0.235298 seconds.
>> Q7
Elapsed time is 0.151803 seconds.
>> Q7
Elapsed time is 0.216724 seconds.
>> Q7
Elapsed time is 0.230771 seconds.
```