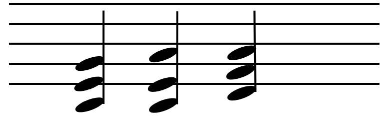
Homework 2 (Due: 26th Oct.)

(1) (a) Compute the Fourier transform of $g_1(t) = 2\exp(-\pi(2t^2+4t+2))$. (b) Calculate the Gabor transform of $g_1(t)$. (c) Does $g_1(t)$ satisfy the lower bound of the uncertainty principle? Why? (15 scores)

(2) Compare the 4 methods to implement the STFT in terms of (a) complexity and (b) constraints. (c) Which methods can also be used for implementing the WDF? (15 scores)

(3) Which of the following signals are suitable to be analyzed by the Wigner distribution function (WDF)? Why? (a) $\exp(j2t^3)$, (b) Music signal, (c) $\sin(|t|^{0.5}+1)$, (d) Gaussian functions?



(10 scores)

(4) (a) Prove that the WDF of any signal is a real function. (b) How do we make the windowed WDF for any function always be real (show the constraint for the window $w(\tau)$) ? (c) How do we make Cohen's class distribution for any function always be real (show the constraint for the mask function $\Phi(\tau, \eta)$) ?

(15 scores)

(5) Why (a) Cohen's class distribution and (b) the polynomial WDF can avoid the cross term in some case? (10 scores)

(6) Write a Matlab or Python code for the scaled Gabor transform (unbalanced form). (page 102)

y = Gabor(x, tau, t, f, sgm) (35 scores)

x: input, *tau*: samples on *t*-axis for the input, *t*: samples on *t*-axis for the output
f: samples on *f*-axis, *sgm*: scaling parameter, *y*: output

(i) The Matlab or Python code should be handed out by NTUCool, (ii) Choose an input x (Use *.wav), plot the output y. (iii) Also show the running time, (iv) Determine tau of the following example, (v) The running time should be as short as possible (for the following example, within 1.5 seconds)

```
[a1, fs] = audioread('Chord.wav');
x=a1(:,1).';    % only extract the first channel
tau = (?  Please think how to determine tau);
dt = 0.01;              df= 1;
t= 0:dt:max(tau);       f= 20:df:1000;
sgm= 200;
tic
y= Gabor (x, tau, t, f, sgm);
toc
```

Add scores for the top 20.

(Extra): Answer the questions according to your student ID number.

(ended with 0, 1, 2, 4, 5, 6, 7, 9)

1.

$$g(t) = 2\exp[-\pi(2t^2+4t+2)]$$

$$\Rightarrow g(t) = 2\exp\{-\pi[2(t+1)^2+1]\}$$

(a)

$$F\{g(t)\} = 2\int_{-\infty}^{\infty} \exp[-\pi[2(t+1)^2+1]]\, e^{-j\omega t}\, dt$$

$$= 2e^{-\pi}\int_{-\infty}^{\infty} \exp[-2\pi(t+1)^2 - j\omega t]\, dt$$

$$= 2e^{-\pi} \times \frac{1}{\sqrt{2\pi}} \times \exp\left[\frac{-w^2}{8\pi}\right]$$

$$= \frac{2\exp\left[\frac{-w^2}{8\pi} - \pi\right]}{\sqrt{2\pi}}$$

(b)

$$G(a,w) = \int_{-\infty}^{\infty} g(t) \times h(t-a) \times e^{-j\omega t}\, dt$$

$$h(t) = e^{-\pi t^2}$$

$$\Rightarrow G(a,w) = \int_{-\infty}^{\infty} 2e^{-\pi(2t^2+4t+2)} \times e^{-\pi(t-a)^2} \times e^{-j\omega t}\, dt$$

$$\Rightarrow G(a,w) = 2\int_{-\infty}^{\infty} e^{-\pi(t^2+4t+2+2ta-a^2+j\omega t)}\, dt$$

$$\Rightarrow G(a,w) = 2e^{-2\pi a^2}\int_{-\infty}^{\infty} e^{-\pi[t^2+(4+a-jw)t+4-\frac{a^2}{2}-\frac{1}{2}]}\, dt$$

$$\Rightarrow G(a,w) = 2e^{-2\pi a^2} \times e^{\pi(2+\frac{a-jw}{2})^2-\frac{a^2}{2}-\frac{1}{2}} \times 1$$

$$\Rightarrow G(a,w) = 2\sqrt{\pi}\, e^{-2\pi a^2} \times \exp\left[\pi\left(2+\frac{a-jw}{2}\right)^2 - \frac{a^2}{2} - \frac{1}{2}\right]$$

(c)

No, $g_1(t)$ is Gaussian

$\Rightarrow \sigma_t \cdot \sigma_f \simeq 0$

2.

(a)

Direct > Chirp - Z transform > FFT - based > FFT - based with recursive

(b)

Driect : no constraints

Chirp-Z : $\Delta t < \dfrac{1}{2(\Omega_x + \Omega_w)}$, $N > 2Q+1$

FFT : $\Delta t < \dfrac{1}{2(\Omega_x + \Omega_w)}$, $\Delta f \Delta N = \dfrac{1}{N}$, $N \geq 2Q+1$

FFT with recursive : Same as FFT-based but only for rectangular window.

(c)

FFT - based

3.

(a) $\exp(j2t^3)$: t^3 具有時頻變化特性

(b) music signal : 多樂器(頻率)在時間上皆有變化

(c) $\sin(|t|^{0.5}+1)$: 正弦波，不與時間作變化

(d) Gaussian : stable with time

$\Rightarrow$ (a)、(b) suitable to be analyzeb by Wigner distribution function

4.

(a)

$$W(t,f) = \int_{-\infty}^{\infty} x(t+\frac{\tau}{2}) \times x^*(t-\frac{\tau}{2}) e^{-j2\pi f} dt$$

for application $x(t)$ usually a real function $\Rightarrow x(t) = x^*(t)$

$$\Rightarrow W(t,f) = \int_{-\infty}^{\infty} x(t+\frac{\tau}{2}) x(t-\frac{\tau}{2}) e^{-j2\pi f} dt$$

$x(t+\frac{\tau}{2})$ 、 $x(t-\frac{\pi}{2})$ 、 $e^{-j2\pi f}$ are real $\Rightarrow W(t,f)$ is real

(b)

$$W(t) = W(-t)$$

(c)

$$\Phi(t,f) = \Phi^*(-t,-f)$$

5.

(a)

Cohen's Class distribution : Window × signal

By choosing the suitable window (Gaussian) to avoid cross term

(b)

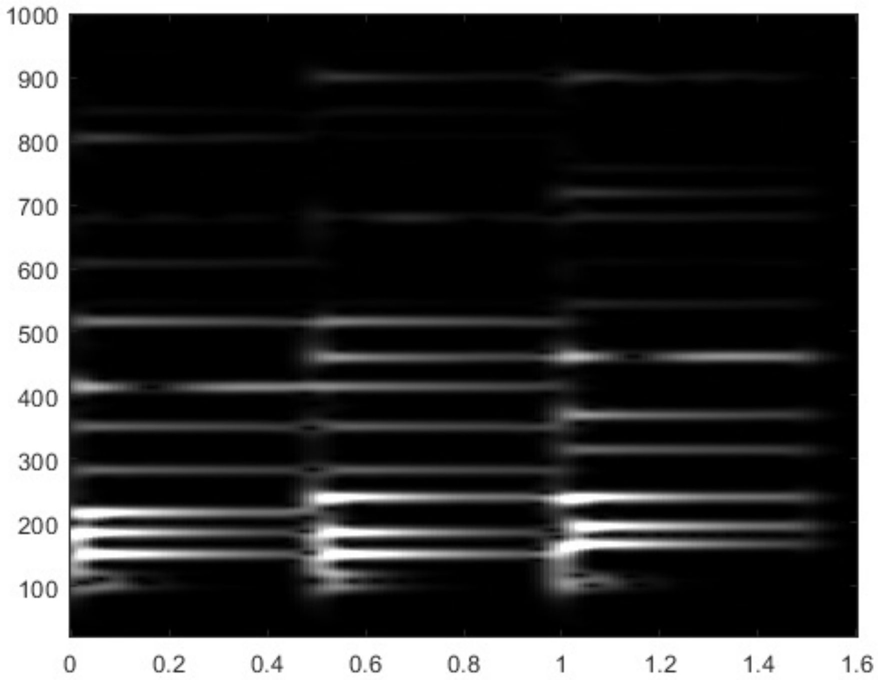
polynomial window 在時間上有局限性可用來限制分析的時頻區域來避免 cross term

6. (Code)

```
clear all

[a1, fs]=audioread('Chord.wav');

x=a1(:,1);

dtau=1/44100;
dt=0.01;
df=1;
sgm=200;

tau = 0 : dtau : 1.6;
t = 0: dt : max(tau);
f = 20 : df : 1000;

tic
y=Gabor(x,tau,t,f,sgm);
toc

function y = Gabor(x, tau, t, f, sgm)
%% Step 1:Calculate n0, f0, tauo, T, F, Tau, N, Q
dtau = diff(tau(1:2));
dt = diff(t(1:2));
df = diff(f(1:2));

Tau = numel(tau);
F = numel(f);
T = numel(t);

N = fix(1 / (dtau * df));
Q = fix(1.9143 / (sqrt(sgm) * dtau));
S = fix(dt / dtau);
```

```
%% Step 2:n=c0
for time = 1 : T
    C = 400;
    %% Step3:Determine x1(q)

    q = (0 : 1 : N - 1)';
    q(q >= 2 * Q) = 0;

    x1 = zeros(N, 1);

    for i = 1:N
        q_val = time * S - Q + q(i); % 算q值
        if q_val < 1
            q_val = 1;  % 限制下界
        elseif q_val > Tau
            q_val = Tau; % 限制上界
        end
        x1(i) = x(q_val) * exp(-sgm * pi * ((Q - q(i)) * dtau) ^ 2);
    end

%% Step4:X1(m)=FFT(x1(q))
X1 = fft(x1, N);

%% Step5:ConvertX1(m) into X(ndt, mdf)
frequencies = 1:F;  % 頻率範圍
X(frequencies, time) = dtau * exp(j * 2 * pi * (Q - time * S) * frequencies / N) .* X1(frequencies)';

end
y=X;
image(t, f, abs(y) / max(max(abs(y))) * C);
colormap(gray(256));
set(gca, 'Ydir', 'normal');
end
```

6. (result)




```
>> HW2
Elapsed time is 0.443270 seconds.
>> HW2
Elapsed time is 0.408431 seconds.
>> HW2
Elapsed time is 0.431078 seconds.
>> HW2
Elapsed time is 0.399280 seconds.
>> HW2
Elapsed time is 0.453245 seconds.
```

Extra

Chirp - Z transform 無法使用 unbalance form 降低複雜度