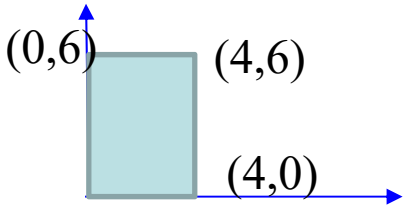
Homework 3 (Due: Nov. 16th)

(1) (a) Compared to the original STFT, what is the advantage of the S transform? (b) Compared to the Fourier series, what is the advantage of the 3-parameter atom? (c) Compared to the original FT, what is the advantage of the FrFT in filter design? (15 scores)

(2) Why using (a) time-frequency reassignment and (b) the Gabor-Wigner transform can obtain even clearer time-frequency distribution than using the original Gabor transform? (10 scores)

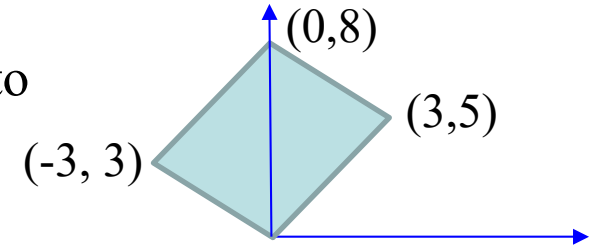
(3) Suppose that the WDF of $x(t)$ is



(10 scores)

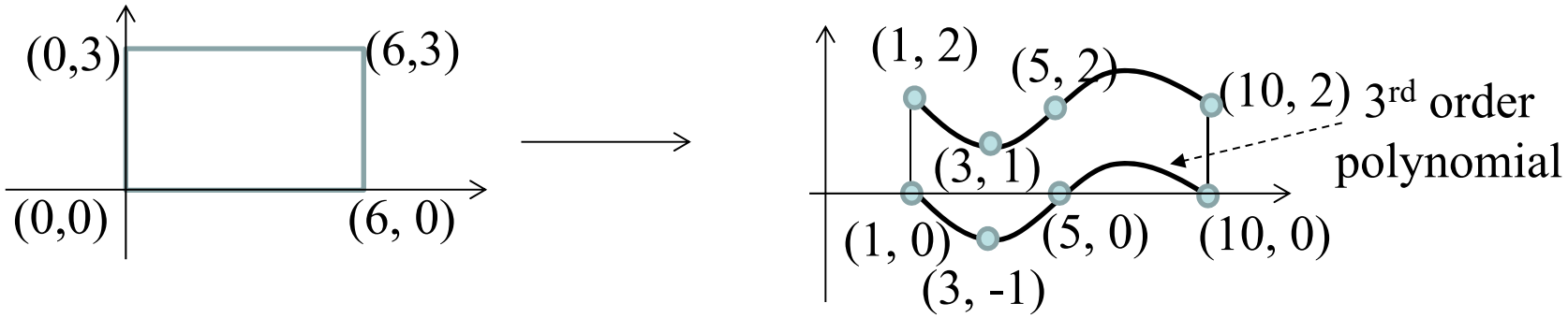
(a) What is the WDF of $FT[\exp(j\pi t^2)x(t/2-1)]$?

(b) How do we convert the WDF into by performing the LCT on $x(t)$?



(10 scores)

(4) Suppose that the WDF of a signal is as the left figure. How do we change its WDF into the right figure? (10 scores)



(5) Suppose that $X(t, f)$ is the STFT of $x(t)$ and $X(t, f)$ > threshold for

$$2t + f < 8, -t + 2f < 6, \text{ and } f > 1.$$

If $x(t)$ is interfered by white noise, how do we use two filters designed by the FrFTs together with one filter design by the FT to reduce the effect of noise? (10 scores)

(6) Write a Matlab or a Python program for the Wigner distribution function when the input function has a finite duration.

y = wdf(x, t, f) (35 scores)

x: input, *t*: samples on *t*-axis, *f*: samples on *f*-axis

(i) The code should be handed out by NTUCool， (ii) 用 function 的指令寫成函式，(iii) 自己選一個 input *x*， 用你們的程式將 output *y* 算出來並畫出來，(iv) 用 tic 和 toc 的指令來計算程式的 running time，(v) 程式執行的時間，越短越好 (使用 unbalanced form 有額外加分)

Ex:

```
del_t = 0.0125;    del_f = 0.025;
t = -9:del_t:9;      f = -4:del_f:4;
x = exp(j*t.^2/10-j*3*t).*((t>=-9)&(t<=1))+exp(j*t.^2/2+j*6*t).*exp(-(t-4).^2/10);
tic
y=wdf (x,t,f);
toc
```

(Extra): Answer the questions according to your student ID number.

(ended with 0, 1, 3, 4, 5, 6, 8, 9)

1.

(a) S transform 可以隨時間調整不同的 window width

(b)

3-parameter atom 可以有更好的七、f 局部性，能多分析瞬時的訊號

Fourier series 適合表示週期性平穩訊號

(c)

FrFT 可以對 signal 做非整数次的FT來調整filter的 frequency response 以達到特定要求

2、

(a)

time-frequency reasignment 能讓能量向中心集中以呈現更清析的結果

(b)

將 Gabor transform × wigner transform

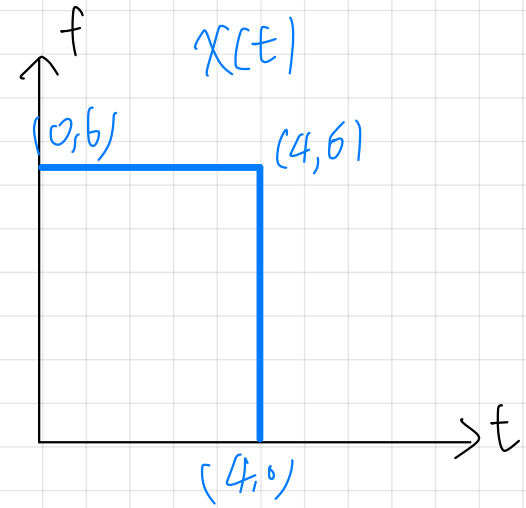
Gabor transform 有 cross term 能量集中

wigner cross term = 0 能量不集中

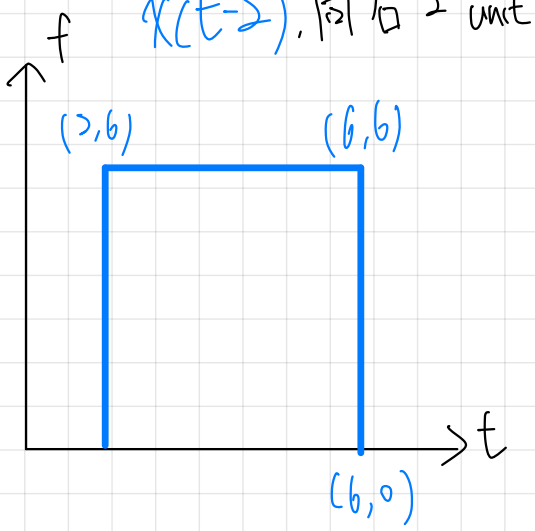
⇒ 結果能使 cross term 消失並得到較好的時頻分析結果

3.

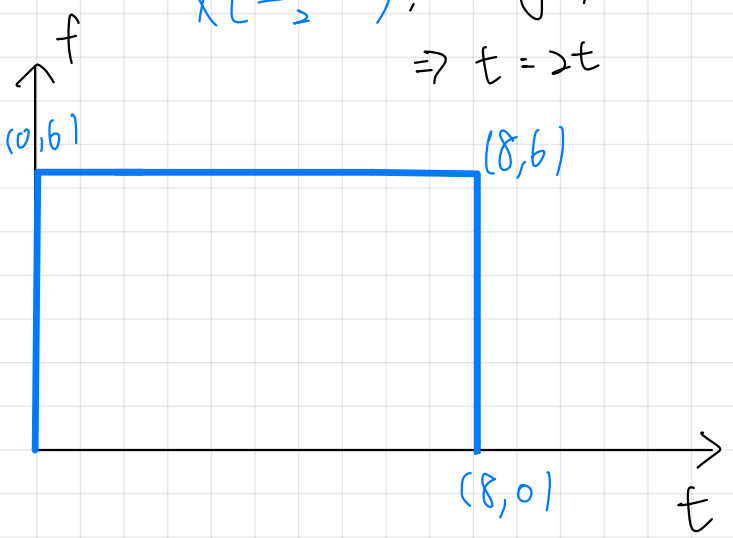
(a)


f
x(t)
(0,6)
(4,6)
t
(4,0)

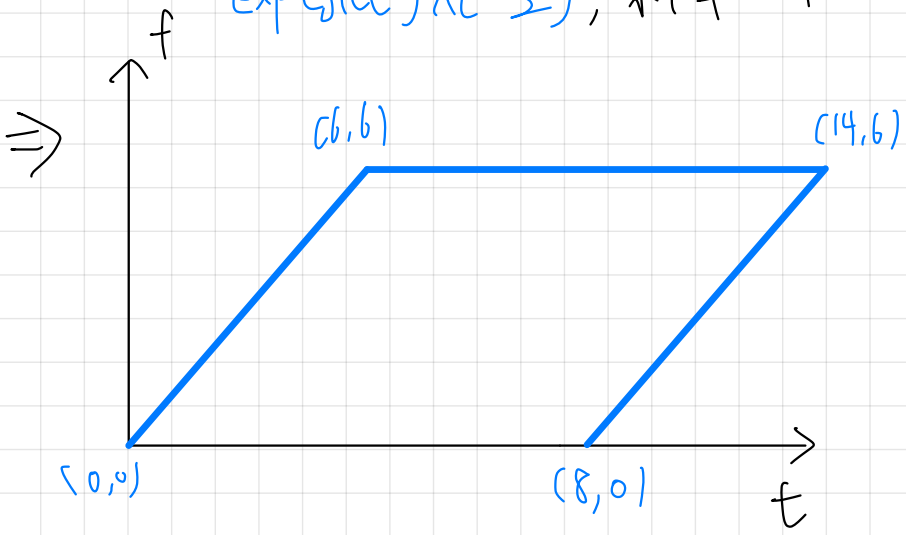
⇒


f
x(t-2), 向右 2 unit
(2,6)
(6,6)
t
(6,0)

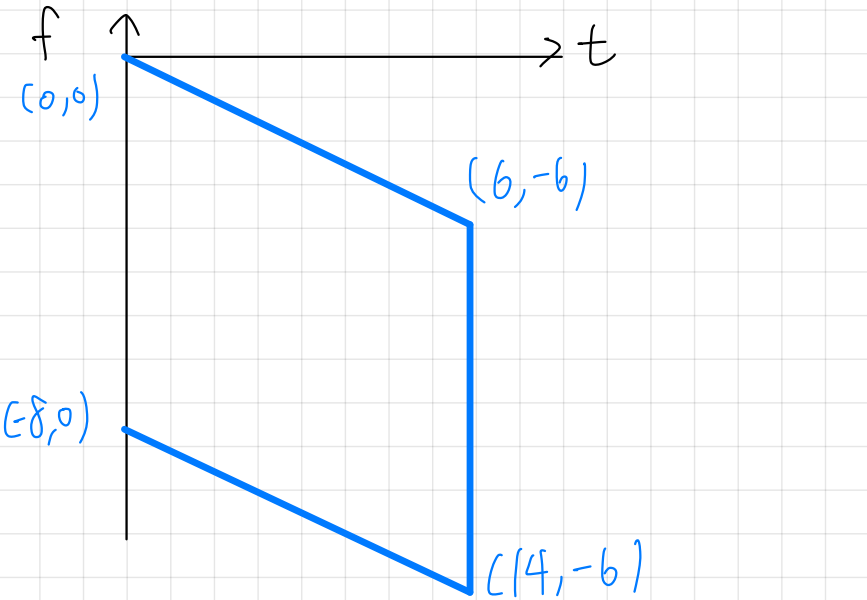
⇒


x((t-2)/2), scaling by 1/2
⇒ t = 2t
f
(0,6)
(8,6)
t
(8,0)

⇒


exp(jπt²)x((t-2)/2), 斜率 = 1
f
(6,6)
(14,6)
(0,0)
(8,0)
t

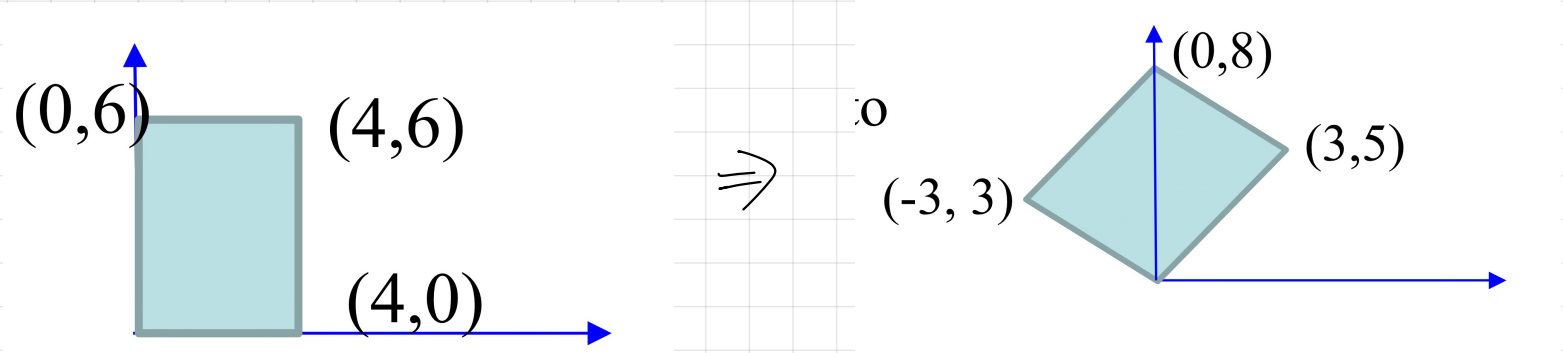
⇒


FT[exp(jπt²)x((t-2)/2)], 順時針 90°
f
t
(0,0)
(6,-6)
(-8,0)
(14,-6)

3.

(b)

Twisting - Linear Canonical transform $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $ad-bc=1$

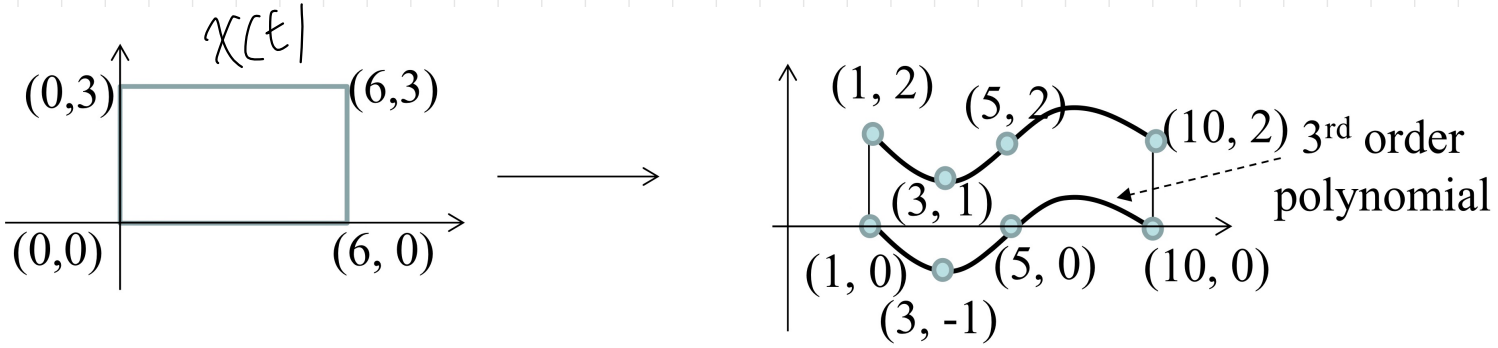


$$\begin{cases} (0,6) \Rightarrow (-3,3) \\ (4,6) \Rightarrow (0,8) \end{cases} \Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 4 \\ 6 & 6 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 3 & 8 \end{pmatrix}$$

$$\Rightarrow A=\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \end{pmatrix}$$

$$\Rightarrow \chi_{\{a,b,c,d\}}(u) = \sqrt{-\frac{2}{j}}\, e^{-j\pi u^2} \int_{-\infty}^{\infty} e^{j4\pi ut}\, e^{-j\pi\frac{3}{2}t^2}\, x(t)\, dt, \quad b\neq 0$$

4、



$x(t)$ 變寬 $\frac{9}{6}=\frac{3}{2}$ $\Rightarrow x(\frac{2t}{3})$

$x(\frac{2}{3}t)$ 變矮 $\frac{2}{3}$ $\Rightarrow$ $\frac{2}{3}x(\frac{2}{3}t)$

$\frac{2}{3}x(\frac{2}{3}t)$ 向右位移 2.5 單位 $\Rightarrow x(\frac{2t-2.5}{3})$

$C\times(t-1)(t-5)(t-10)=-1,\ t=3$

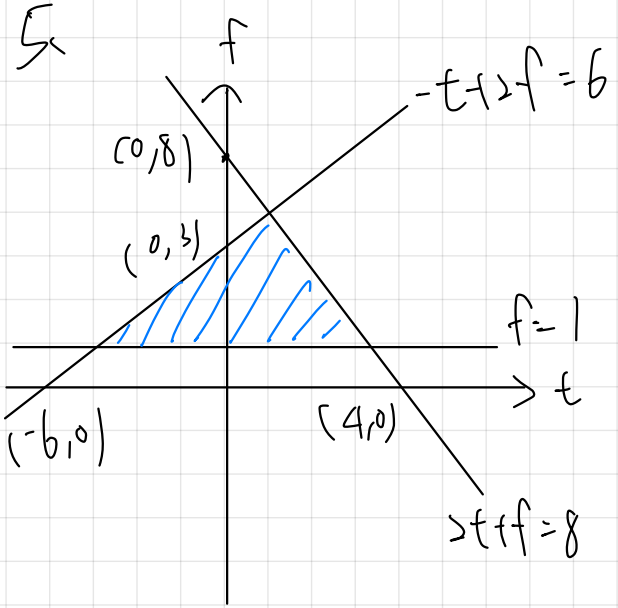
$\Rightarrow C=-\frac{1}{28}$

$\Rightarrow -\frac{1}{28}(t^3-16t^2+65t-50)$

$\Rightarrow \phi(t)=-\frac{1}{28}(\frac{t^4}{4}-\frac{16}{3}t^3+\frac{65t^2}{2}-50t)\times 2\pi$

$\frac{2}{3}\exp\left[-j\frac{1}{14}(\frac{t^4}{4}-\frac{16}{3}t^3+\frac{65}{2}t^2-50t)\right]x(\frac{2t-2.5}{3})$

5.



$$\chi_\phi(u) = O_F^{\phi}[x(t)], \quad \phi = 0.5a\pi$$

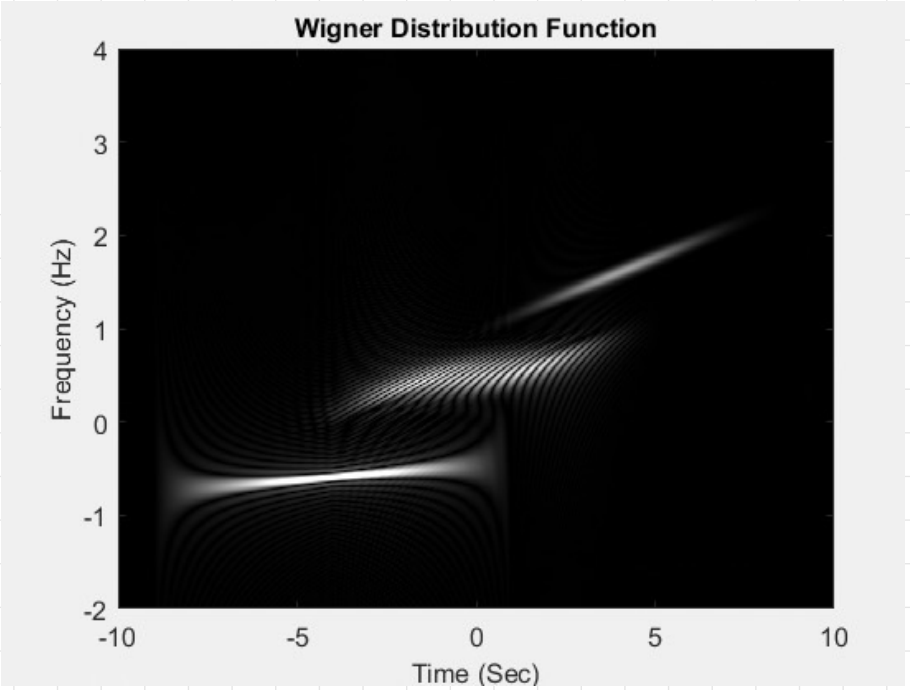
for $f=1$, $a=1 \Rightarrow$ FT

for $-t+2f=6$, $a=2.214 \Rightarrow \chi_\phi(u) = O_F^{1.107\pi}[x(t)]$

for $2t+f=8$, $a=0.926 \Rightarrow \chi_\phi(u) = O_F^{0.463\pi}[x(t)]$

6.

```m
clear all

% Define the time axis with increased resolution
t = -10:0.01:10;
% Define the frequency axis with increased resolution
f = -2:0.01:4;
% Generate the signal x
x = exp(1i*t.^2/10 - 1i*3*t).*((t>=-9)&(t<=1)) + exp(1i*t.^2/2 + 1i*6*t).*exp(-(t-4).^2/10);

tic                 % Timer Start
y = wdf(x, t, f);   % Call the wdf function to compute and plot WDF
toc                 % Timer End

function y = wdf(x, t, f)
    dt = mean(diff(t));
    df = mean(diff(f));

    % Calculate the number of samples in time and frequency
    N = floor(1 / (2 * dt * df));
    % Define indices for time and frequency
    n1 = floor(t(1) / dt);
    n2 = ceil(t(end) / dt);
    m1 = floor(f(1) / df);
    m2 = ceil(f(end) / df);
    m = mod((m1:m2) - 1, N) + 1;

    % Compute the dimensions of the WDF matrix
    Lt = ceil(t(end) / dt) - floor(t(1) / dt) + 1;
    Lf = ceil(f(end) / df) - floor(f(1) / df) + 1;
    y = zeros(Lf, Lt);

    % Loop over time indices
    for n = n1:n2
        % Calculate the lag parameter U
        U = max(0, min(n2 - n, n - n1));

        % Calculate the Q parameter
        Q = 2 * U;

        % Compute the auto-correlation function A
        A = x(1 - n1 + [n - U:n + U]) .* (x(1 - n1 + [n + U:-1:n - U])').';
        % Compute the FFT of A
        A1 = fft(A, N) * 2 * dt;
        % Calculate a1 parameter
        a1 = ceil(Q / N + 0.5) - 1;

        % Loop over a2 parameter
        for a2 = 2:a1
            % Perform inverse FFT and update A1
            A1 = ifft(A((a2 - 1) * N + 1:min(a2 * N, Q)), N) * sqrt(N / (2 * dt)) + A1;
        end
        % Compute the final WDF matrix entry
        y(:, n - n1 + 1) = (A1(m) .* exp(1i * 2 * pi / N * U * (m - 1))).';
    end

    % Plot the WDF
    image(t, f, abs(y) / max(max(abs(y))) * 400)
    colormap(gray(256))
    set(gca, 'Ydir', 'normal')
    set(gca, 'Fontsize', 12)
    xlabel('Time (Sec)', 'Fontsize', 12)
    ylabel('Frequency (Hz)', 'Fontsize', 12)
    title('Wigner Distribution Function', 'Fontsize', 12)
end
```




```
>> HW3
Elapsed time is 0.438824 seconds.
>> HW3
Elapsed time is 0.408156 seconds.
>> HW3
Elapsed time is 0.397075 seconds.
>> HW3
Elapsed time is 0.422282 seconds.
>> HW3
Elapsed time is 0.426411 seconds.
```

Extra