Homework 5 (Due: 28th Dec.)

(1) What are the roles of (a) admissibility criterion and (b) scaling function for continuous wavelet transform design? (10 scores)

(2) What are the vanish moments of (a) the sinc wavelet, (b) the 18-point coiflet wavelet, and (c) $G(f) = (1+\exp(-j2\pi f))^5\cos(\pi f)/32$ where $G(f)$ is the discrete-time Fourier transform of gk defined on page 415? (15 scores)

(3) What is the maximal possible frequency for a discrete sequence h_k where k = …, 0, 1, 2, 3, ….? (5 scores)

(4) Why the complexity of the 1-D discrete wavelet transform is linear with N? (10 scores)

(5) (a) What is the advantage of the symlet? (b) What is the advantage of the curvelet compared to the original wavelet? (10 scores)

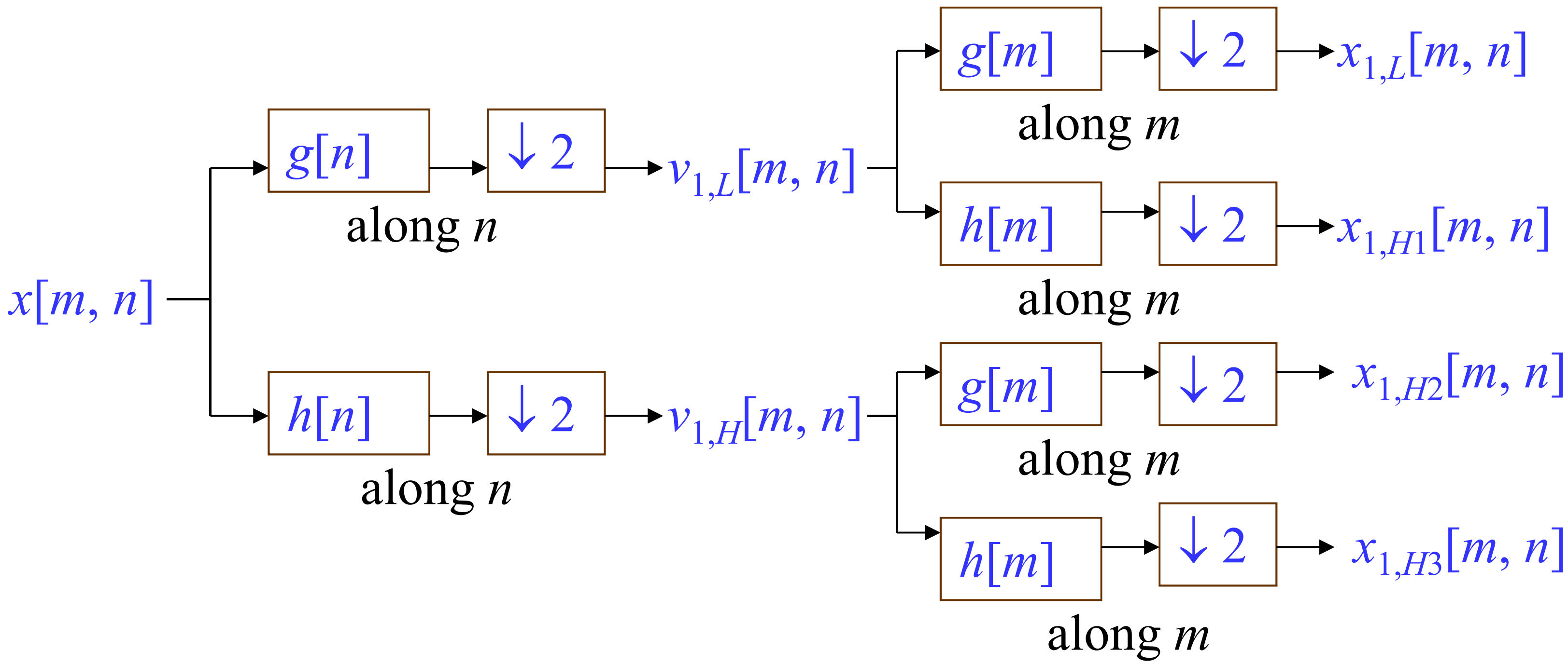
(6) Why the wavelet transform is useful for (a) adaptive filter design and (b) pattern recognition? (10 scores)

(7) For a two-point wavelet filter, if $g[0] = 3/5$, $g[1] = a$, and $g[n] = 0$ otherwise. Determine a if (a) $g[n]$ is a <u>quadratic mirror filter</u> and (b) $g[n]$ is an <u>orthonormal filter</u>. (10 scores)

(8) (a) Write a Matlab or Python code for the following 2-D discrete 10-point Daubechies wavelet.

x = double(imread('filename'))

[x1L, x1H1, x1H2, x1H3] = wavedbc10(x)



(b) Also write the program for the inverse 2-D discrete 10-point Daubechies wavelet transform.

x = iwavedbc10(x1L, x1H1, x1H2, x1H3)

The code should be handed out by NTUCool. (30 scores)

(Extra): Answer the questions according to your student ID number.

(ended with 1, 2, 3, 4, 6, 7, 8, 9)

1.

(a) Admissibility Criterion $\int_{-\infty}^{\infty} |\Psi(t)|^2 \frac{dt}{|t|} < \infty$，wavelet function ($\Psi(t)$) 必須收斂，其條件可用來捕捉 singnal 中的 HF、LF 成份

(b) scaling function $\phi_t = \int_{-\infty}^{\infty} \Phi(f) e^{j2\pi ft} df$ 主要功能為提高 wavelet 頻譜的範圍

2.

(a)

$\psi(t) =$ sinc wavelet

$\int_{-\infty}^{\infty} t^n \psi(t) = 0$

$\Rightarrow$ sinc wavelet 在 -0.25~0.25 值為 0

$\Rightarrow$ 積分後 $= 0$

$\Rightarrow$ vanish moment 無限大

(b)

6p - point Coilfet wavelet vanish moment $= p$

$\Rightarrow$ 18p - point vanish moment $= 3$

(c)

$$G(f) = \frac{(1+e^{-2\pi f})^5 \cos\pi f}{32}$$

$\Rightarrow$ At $f=0$ $(1+e^{-2\pi f})^5$ has 5 vanishing moment

〃 $\cos(\pi f)$ has no vanishing moment

$\Rightarrow$ $G(f)$ vanishing moment $= 5$

3.

$$f_{max} = \frac{f_s}{2}$$

離散序列中的 maximal possible frequency 為樣本數的 $\frac{1}{2}$

4.

1-D discrete wavelet transform 中 $x(n)$, $g(n)$ 的 convolution 為 $O(N)$ 的複雜度

N 為 $x(n)$ 長度，時間複雜度為 $O[(N+L-1)\log(N+L-1)] \Rightarrow O[N\log N]$, $\log N$ 為 constant ⇒ 與 linear with N

5.

(a)

在做 wavelet transform 後圖片偏移幅度小

(b)

Curvelet transform is a multiscale direction transform and higher dimensional of the wavelet transform

6.

(a)

wavelet transform 可過濾 HF signal 也可作為 low pass filter, 能夠保留 edge 的特徵

(b)

因為 wavelet transform 能在保留特徵下壓縮圖片, 所以可以在壓縮後的圖片判斷特徵

7.

$$G(z) = \sum_{n=-\infty}^{\infty} g[n] z^{-n} = g[0] + g[1] z^{-1}$$

(a)

$$G^2(z) - G^2(-z) = 2z^k$$

$$\Rightarrow \left(\frac{3}{5} + az^{-1}\right)^2 - \left(\frac{3}{5} - az^{-1}\right)^2 = 2z^k$$

$$\Rightarrow 4 \times \frac{3}{5} \times a z^{-1} = 2z^k$$

$$\Rightarrow \frac{12a}{5} z^{-1} = 2z^k$$

$$\Rightarrow a = \frac{5}{6}, \quad k = -1$$

(b)

$$\begin{cases} G_2(z) G_2(z^{-1}) + G_2(-z) G_2(-z^{-1}) = 2 \\ G_2(z) = G(z^{-1}) = \frac{3}{5} + az \end{cases}$$

$$\Rightarrow \left(\frac{3}{5} + az\right)\left(\frac{3}{5} + az^{-1}\right) + \left(\frac{3}{5} - az\right)\left(\frac{3}{5} - az^{-1}\right) = 2$$

$$\Rightarrow 2\left[\left(\frac{3}{5}\right)^2 + (b)^2\right] = 2$$

$$\Rightarrow \frac{9}{25} + b^2 = 1$$

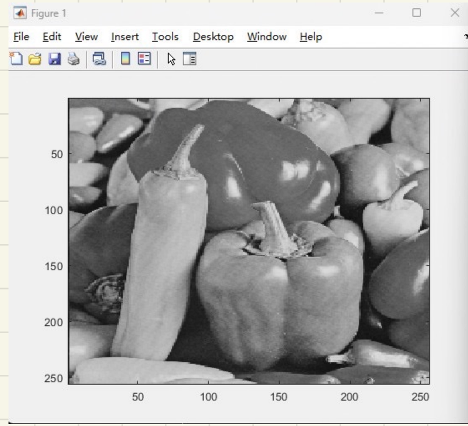
$$\Rightarrow b^2 = \frac{16}{25}$$

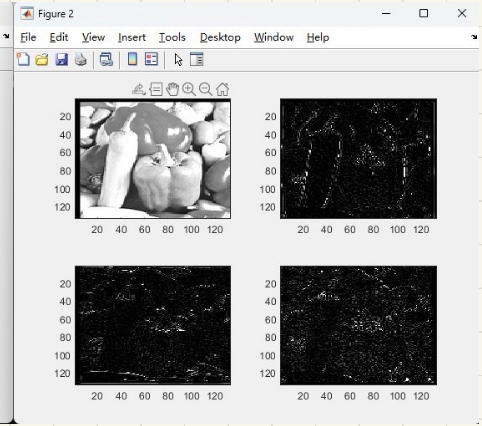
$$\Rightarrow b = \pm \frac{4}{5}$$

8.

(a)

```
clear all
x = double(imread('pepper.jpg'));
tic
y = wavedbc10(x);
toc
L = size(y, 2)/4;
x1L = y(:, 1:L); x1H1 = y(:, L+1:2*L); x1H2 = y(:, 2*L+1:3*L); x1H3 = y(:, 3*L+1:4*L);
figure(1);
colormap(gray(256));
image(x);
hold on;
figure(2);
colormap(gray(256));
subplot(2, 2, 1);
image(x1L./sqrt(2));
hold on;
subplot(2, 2, 3);
image(x1H1.*5);
hold on;
subplot(2, 2, 2);
image(x1H2.*5);
hold on;
subplot(2, 2, 4);
image(x1H3.*10);
hold on;

function y = wavedbc10(x)
%Step1: to set g[n] and h[n]
g = [0.0033 -0.0126 -0.0062 0.0776 -0.0322 -0.2423 0.1384 0.7243 0.6038 0.1601];
gT = transpose(g);
h = [0.1601 -0.6038 0.7243 -0.1384 -0.2423 0.0322 0.0776 0.0062 -0.0126 -0.0033];
hT = transpose(h);
%Step2: to compute v1L[m,n] and v1H[m,n]
xg = conv2(g, x);
xgT = transpose(xg);
v1L = transpose(downsample(xgT, 2));
xh=conv2(h, x);
xhT = transpose(xh);
v1H = transpose(downsample(xhT, 2));

%Step 3: to compute x1L[m,n] and x1H1[m,n]
v1Lg = conv2(gT, v1L);
x1L = downsample(v1Lg, 2);
v1Lh = conv2(hT, v1L);
x1H1 = downsample(v1Lh, 2);

%Step 4: to compute x1H2[m,n] and x1H3[m,n]
v1Hg = conv2(gT, v1H);
x1H2 = downsample(v1Hg, 2);
v1Hh = conv2(hT, v1H);
x1H3 = downsample(v1Hh, 2);
y = [x1L, x1H1, x1H2, x1H3];
end
```



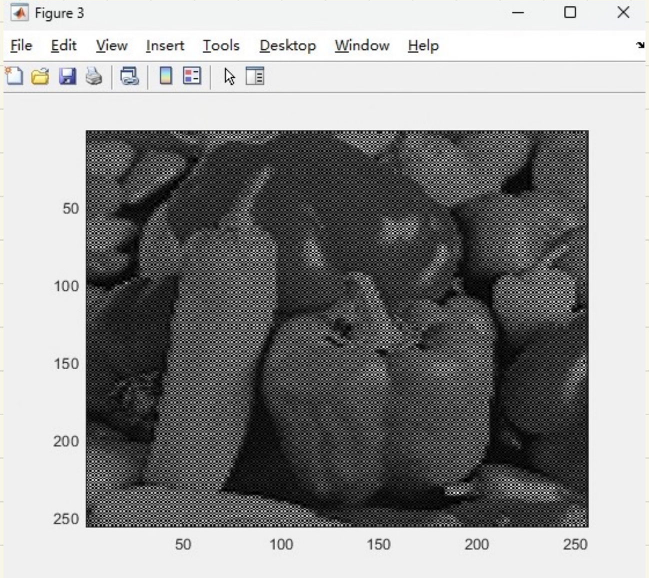



```
>> Q8a
Elapsed time is 0.008277 seconds.
>> Q8a
Elapsed time is 0.009488 seconds.
>> Q8a
Elapsed time is 0.008242 seconds.
>> Q8a
Elapsed time is 0.009892 seconds.
```

8.

b.

```matlab
clear all
x = double(imread('pepper.jpg'));
tic
y = wavedbc10(x);
toc
L = size(y, 2)/4;
x1L = y(:, 1:L); x1H1 = y(:, L+1:2*L); x1H2 = y(:, 2*L+1:3*L); x1H3 = y(:, 3*L+1:4*L);

z = iwavedbc10(x1L, x1H1, x1H2, x1H3);
figure(3);
colormap(gray(256));
image(z);

function y = iwavedbc10(x1L, x1H1, x1H2, x1H3)
g1 = [0.1601 -0.6038 0.7243 -0.1384 -0.2423 0.0322 0.0776 0.0062 -0.0126 -0.0033];
h1 = [-0.0033 -0.0126 0.0062 0.0776 0.0322 -0.2423 -0.1384 0.7243 -0.6038 0.1601];
h1T = transpose(h1); g1T = transpose(g1);

X1L = upsample(x1L, 2);
X1H1 = upsample(x1H1, 2);
x0 = conv2(X1L, g1T)+conv2(X1H1, h1T);

X1H2 = upsample(x1H2, 2);
X1H3 = upsample(x1H3, 2);
x1 = conv2(X1H2, g1T)+conv2(X1H3, h1T);

x0T = transpose(x0);
X0 = transpose(upsample(x0T, 2));
x1T = transpose(x1);
X1 = transpose(upsample(x1T, 2));
newx = conv2(X0, g1)+conv2(X1, h1);
for i=1:10
    newx(1, :)=[]; newx(:, 1)=[];
end
L = size(newx, 1);
for i=1:9
    newx(L-8, :)=[]; newx(:, L-8)=[];
end
y = newx;
end
function y = wavedbc10(x)
%Step1: to set g[n] and h[n]
g = [0.0033 -0.0126 -0.0062 0.0776 -0.0322 -0.2423 0.1384 0.7243 0.6038 0.1601];
gT = transpose(g);
h = [0.1601 -0.6038 0.7243 -0.1384 -0.2423 0.0322 0.0776 0.0062 -0.0126 -0.0033];
hT = transpose(h);
%Step2: to compute v1L[m,n] and v1H[m,n]
xg = conv2(g, x);
xgT = transpose(xg);
v1L = transpose(downsample(xgT, 2));
xh=conv2(h, x);
xhT = transpose(xh);
v1H = transpose(downsample(xhT, 2));

%Step 3: to compute x1L[m,n] and x1H1[m,n]
v1Lg = conv2(gT, v1L);
x1L = downsample(v1Lg, 2);
v1Lh = conv2(hT, v1L);
x1H1 = downsample(v1Lh, 2);

%Step 4: to compute x1H2[m,n] and x1H3[m,n]
v1Hg = conv2(gT, v1H);
x1H2 = downsample(v1Hg, 2);
v1Hh = conv2(hT, v1H);
x1H3 = downsample(v1Hh, 2);
y = [x1L, x1H1, x1H2, x1H3];
end
```




```
>> Q8b
Elapsed time is 0.008608 seconds.
>> Q8b
Elapsed time is 0.010557 seconds.
>> Q8b
Elapsed time is 0.010714 seconds.
>> Q8b
Elapsed time is 0.011352 seconds.
```