UNIVERSITY OF HULL

Session 3: Linear solver - Forward Euler

- Leonhard Euler was a prolific mathematician of his time, at his peak, producing one mathematical paper per week.
- This led to the development of the **Forward Euler Method** in his book Institutionum calculi integralis.
- This method is simple, but powerful. During this morning you will find out how to implement this method to solve solutions for keplerian orbits.

UNIVERSITY OF HULL

What is the Euler method?

- The Euler method is a numerical first order, ordinary differential equation solver.

- This means that it is a method of numerically integrating an equation.

- The Forward Euler can be defined as:

$$y_{n+1} = y_n + hf(t_n, y_n).$$

Deriving the Euler method with Taylor Series

- We can derive the Euler method with the Taylor Expansion:

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots,$$

Or.. $$\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n,$$

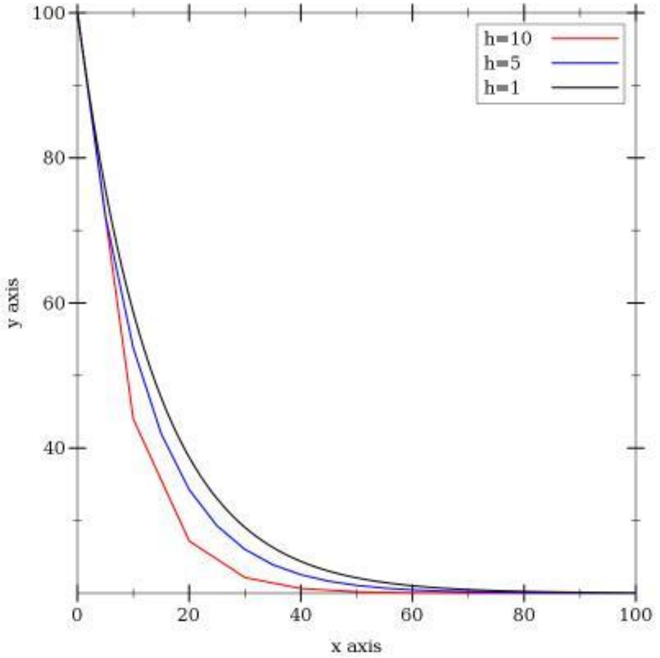
- This eventually gives:

$$y(t_0+h)=y(t_0)+hy'(t_0)$$

UNIVERSITY OF HULL

How does it work?

- It achieves this by taking a known differential equation, and treating each point as the slope of a tangent line to the curve.
- By taking small steps along the curve and solving the next step we can compute an approximation to the curve.
- The size of the step impacts the accuracy of the approximated curve



Numerical derivative

$$f'(x) = \frac{f(x+\delta) - f(x)}{\delta}$$

derivative

A key part of the equation for the Euler method is calculating the derivative

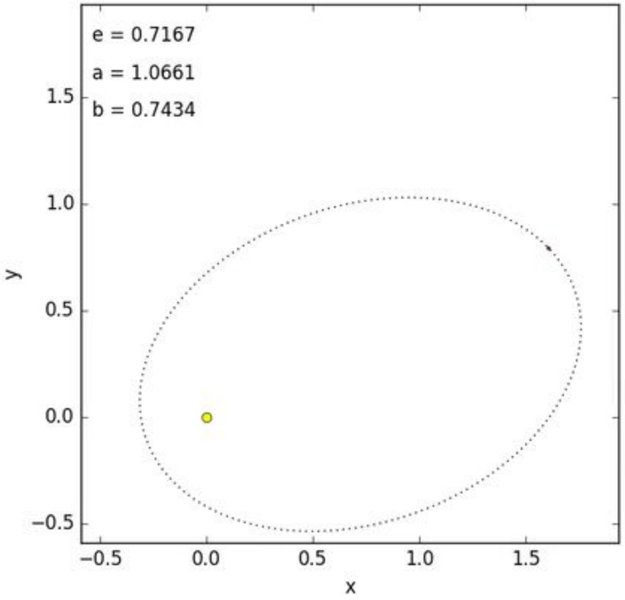
Calculating the derivative computationally can be estimated using the following simple equation:

This works by calculating the difference between your function at the value you input, and at a step forward, delta.

This method of estimating the derivative is only true if the delta value is small enough.

Where would we use this method?

- The primary reason an astronomer codes is to perform numerical calculations.
- Think back to the Kepler mini project…
- We can continue to expand on this mini project by implementing the Euler method to model the orbits themselves.



UNIVERSITY OF HULL

Creating a solver

$$x(t_0 + h) = x(t_0) + hx'(t_0)$$

$$y(t_0 + h) = y(t_0) + hy'(t_0)$$

$$\mathbf{v}_x(t_0 + h) = \mathbf{v}_x(t_0) + h\mathbf{v}'_x(t_0)$$

$$\mathbf{v}_y(t_0 + h) = \mathbf{v}_y(t_0) + h\mathbf{v}'_y(t_0)$$

You will be using the Euler method to calculate the orbital path of an object.

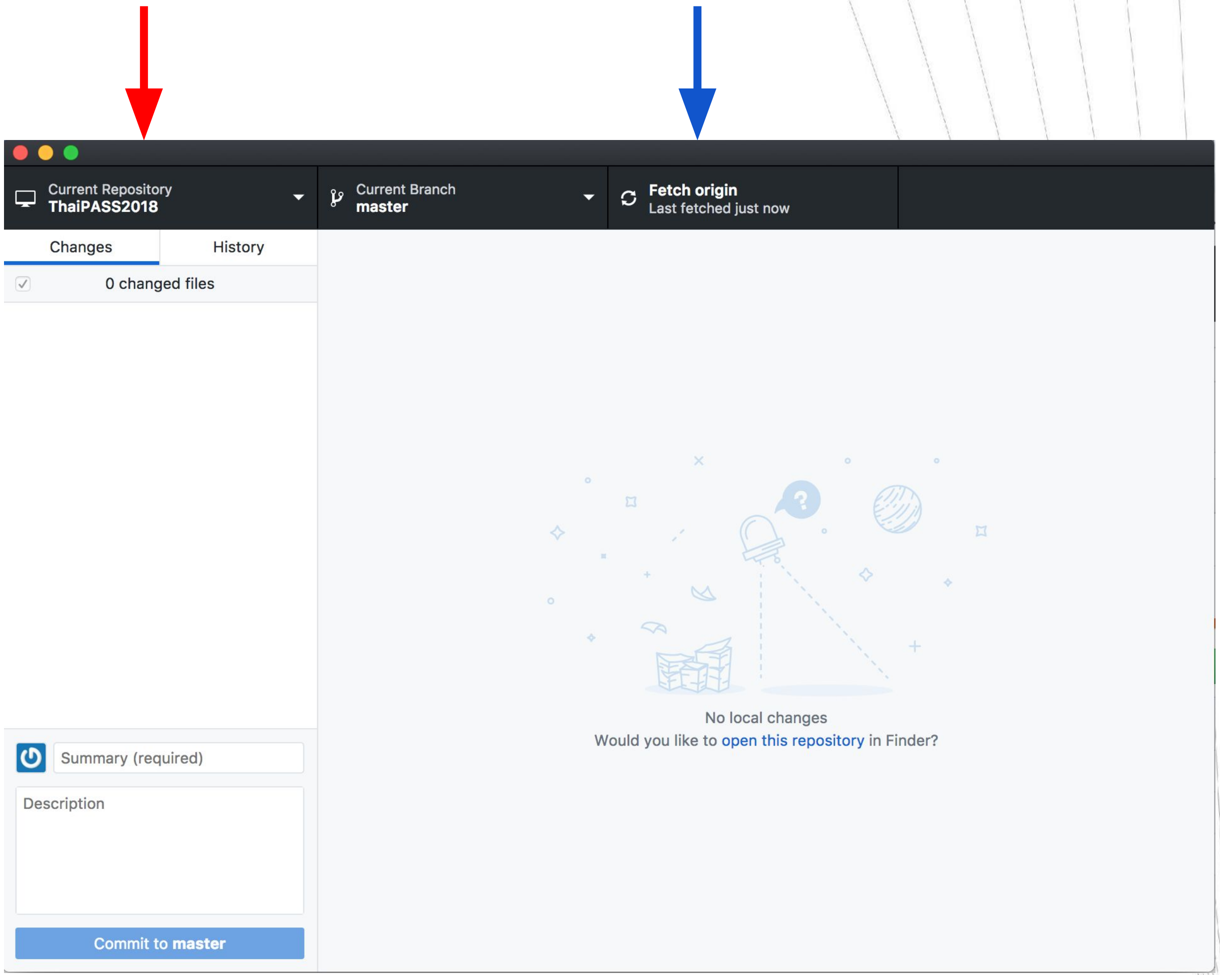
This will require four key parameters: x position, y position, x velocity contribution, y velocity contribution.

We can calculate the next step in the orbit using the derived velocity and acceleration, and putting them into the Euler equation.

UNIVERSITY OF HULL

Let’s update your ThaiPASS GitHub Directory.

- Go to GitHub Desktop, the programme used to download the ‘StarterPack’ files.
- Make sure you have the “ThaiPASS2018” repository selected (red arrow).
- Then, click on the “Fetch origin” button (blue arrow).
- This will now update and download new tasks! :-)



UNIVERSITY OF HULL

If you finish everything...

Check with us for extra tasks!