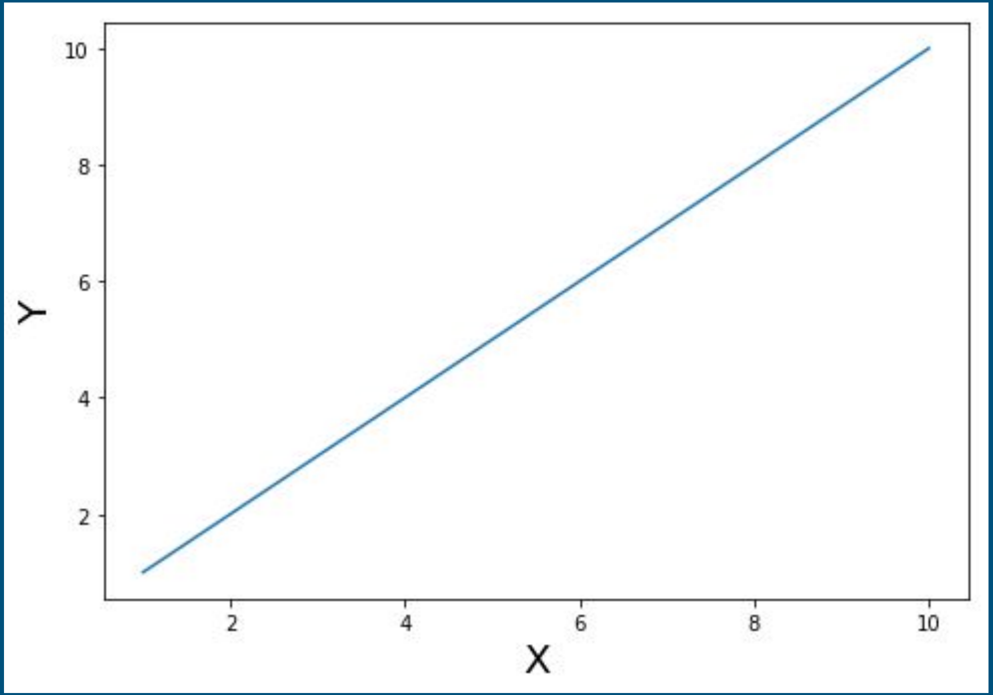
Forward Euler

- Numerical integration
- Time evolution
- Solutions of ODE

How do we find the slope of this line?

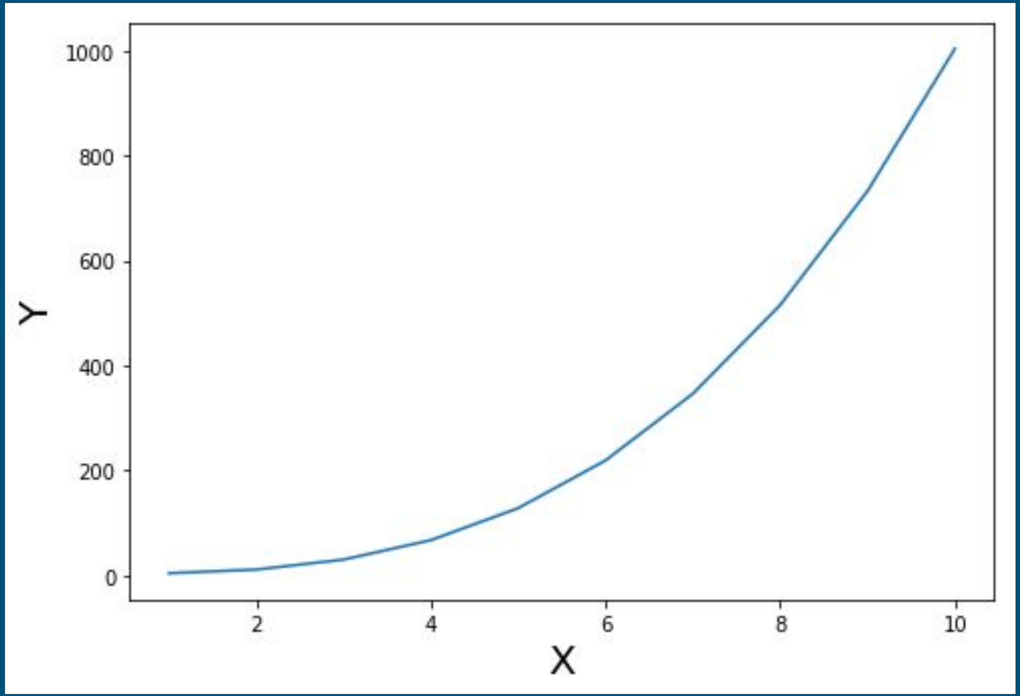
30 s



$$\frac{\Delta y}{\Delta x}$$

How do we find the slope of this line?

1 min



Derivative of the slope changes from point to point

Still valid but need to re-evaluate at new x and have small Δx

$$\frac{\Delta y}{\Delta x}$$

How do we find the slope of this line?

1 min

$$y = x^3 + 7x^2 + 4x - 19$$

For simple equations like these, an analytic solution is available (calculus)

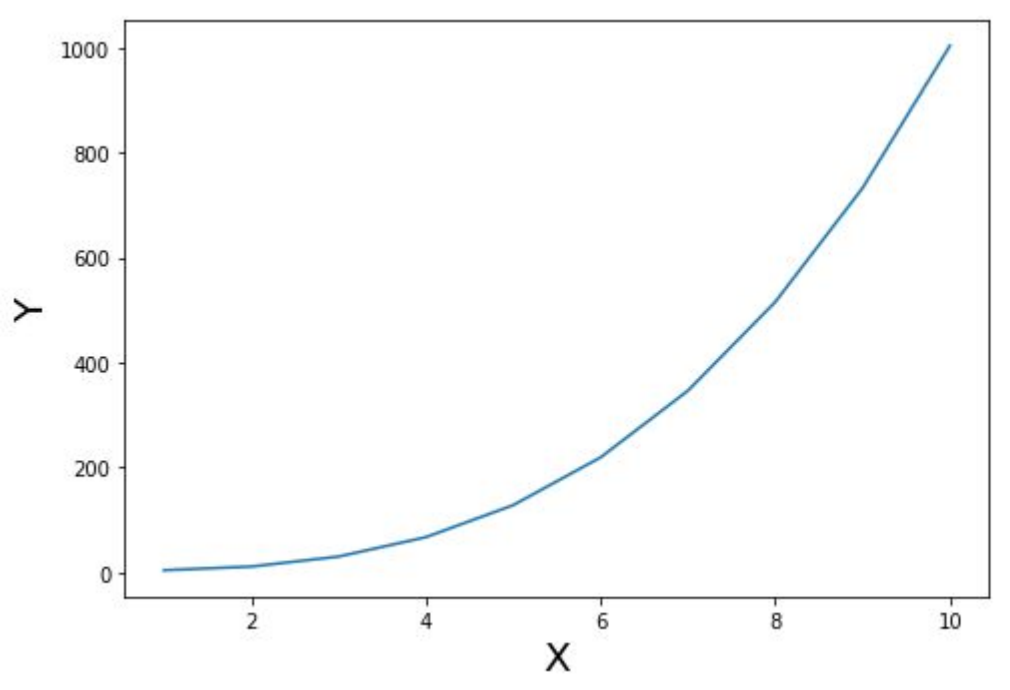
We can also compute the value of y at any point along the curve

However, sometimes there is no easy analytic solution, we cannot compute the exact solution quickly enough, or the problem is more complex...

Assumptions	Number of atoms (N)
$\lambda_1 = \lambda_2 = \lambda_3$ $\lambda_1 = \lambda$	$N_{t(3)} = N_{0(1)} \frac{1}{2} f$
$\lambda_1 = \lambda_2 = \lambda_{12}$ $\lambda_{12} \neq \lambda_3$	$N_{t(3)} = N_{0(1)} f_{(1\to2)} f$
$\lambda_2 = \lambda_3 = \lambda_{23}$ $\lambda_{23} \neq \lambda_1$	$N_{t(3)} = N_{0(1)} f_{(1\to2)} f$
$\lambda_1 = \lambda_3 = \lambda_{13}$ $\lambda_{13} \neq \lambda_2$	$N_{t(3)} = N_{0(1)} f_{(1\to2)} f$
$\lambda_1 \neq \lambda_2$ $\lambda_1 \neq \lambda_3$ $\lambda_2 \neq \lambda_3$	$N_{t(3)} = N_{0(1)} f_{(1\to2)} f_{(2-}$

10	T	1	BE	7	+	1	HE	4	->	1	C	11	+	0	00000		0.000E+00	VITAL	(a,g)	9	1.000E+00	7.280E+18
11	T	1	LI	7	+	1	HE	4	->	1	B	11	+	0	00000		0.000E+00	VITAL	(a,g)	9	1.000E+00	8.360E+18
12	T	1	B	11	+	1	HE	4	->	1	N	14	+	1	NEUT		0.000E+00	VITAL	(a,n)	11	1.000E+00	1.531E+17
13	T	1	B	11	+	1	PROT		->	3	HE	4	+	0	00000		0.000E+00	VITAL	(p,g)	5	1.000E+00	8.377E+18
14	F	1	C	12	+	1	PROT		->	1	N	13	+	0	00000		5.666E-22	VITAL	(p,g)	5	1.000E+00	0.000E+00
15	F	1	C	13	+	1	PROT		->	1	N	14	+	0	00000		2.254E-21	VITAL	(p,g)	5	1.000E+00	0.000E+00
16	T	1	N	13	+	1	PROT		->	1	O	14	+	0	00000		5.162E-25	VITAL	(p,g)	5	1.000E+00	4.464E+18
17	T	1	N	14	+	1	PROT		->	1	O	15	+	0	00000		3.280E-25	VITAL	(p,g)	5	1.000E+00	7.040E+18
18	F	1	N	15	+	1	PROT		->	1	C	12	+	1	HE	4	1.180E-20	VITAL	(p,a)	8	1.000E+00	0.000E+00
19	F	1	N	15	+	1	PROT		->	1	O	16	+	0	00000		1.135E-23	VITAL	(p,g)	5	1.000E+00	0.000E+00
20	F	1	O	16	+	1	PROT		->	1	F	17	+	0	00000		1.081E-27	VITAL	(p,g)	5	1.000E+00	0.000E+00
21	F	1	O	17	+	1	PROT		->	1	N	14	+	1	HE	4	3.186E-27	VITAL	(p,a)	8	1.000E+00	0.000E+00
22	F	1	O	17	+	1	PROT		->	1	F	18	+	0	00000		1.254E-27	VITAL	(p,g)	5	1.000E+00	0.000E+00
23	F	1	O	18	+	1	PROT		->	1	N	15	+	1	HE	4	2.209E-23	VITAL	(p,a)	8	1.000E+00	0.000E+00
24	F	1	O	18	+	1	PROT		->	1	F	19	+	0	00000		2.518E-25	VITAL	(p,g)	5	1.000E+00	0.000E+00
25	F	1	F	19	+	1	PROT		->	1	O	16	+	1	HE	4	2.124E-27	VITAL	(p,a)	8	1.000E+00	0.000E+00
26	F	1	F	19	+	1	PROT		->	1	NE	20	+	0	00000		3.629E-31	VITAL	(p,g)	5	1.000E+00	0.000E+00
27	F	1	NE	20	+	1	PROT		->	1	NA	21	+	0	00000		2.787E-32	VITAL	(p,g)	5	1.000E+00	0.000E+00
28	F	1	NE	21	+	1	PROT		->	1	NA	22	+	0	00000		3.409E-33	VITAL	(p,g)	5	1.000E+00	0.000E+00
29	F	1	NE	22	+	1	PROT		->	1	NA	23	+	0	00000		1.601E-29	VITAL	(p,g)	5	1.000E+00	0.000E+00
30	F	1	NA	22	+	1	PROT		->	1	NA	23	+	0	00000		1.086E-36	VITAL	(p,g)	5	1.000E+00	0.000E+00
31	F	1	NA	23	+	1	PROT		->	1	NE	20	+	1	HE	4	1.694E-34	VITAL	(p,a)	8	1.000E+00	0.000E+00
32	F	1	NA	23	+	1	PROT		->	1	MG	24	+	0	00000		2.190E-35	VITAL	(p,g)	5	1.000E+00	0.000E+00
33	F	1	MG	24	+	1	PROT		->	1	AL	25	+	0	00000		8.574E-39	VITAL	(p,g)	5	1.000E+00	0.000E+00
34	T	1	MG	25	+	1	PROT		->	1	AL	26	+	0	00000		4.112E-42	VITAL	(p,g)	5	1.000E+00	6.084E+18
35	F	1	MG	25	+	1	PROT		->	1	MG	26	+	0	00000		9.641E-43	VITAL	(p,g)	5	1.000E+00	0.000E+00
36	F	1	MG	26	+	1	PROT		->	1	AL	27	+	0	00000		2.557E-42	VITAL	(p,g)	5	1.000E+00	0.000E+00
37	F	1	AL	26	+	1	PROT		->	1	SI	27	+	0	00000		7.955E-51	VITAL	(p,g)	5	1.000E+00	0.000E+00
38	T	1	AL	27	+	1	PROT		->	1	MG	24	+	1	HE	4	1.219E-44	VITAL	(p,a)	8	1.000E+00	1.545E+18
39	T	1	AL	27	+	1	PROT		->	1	SI	28	+	0	00000		3.885E-41	VITAL	(p,g)	5	1.000E+00	1.118E+19
40	F	1	SI	28	+	1	PROT		->	1	P	29	+	0	00000		1.017E-44	VITAL	(p,g)	5	1.000E+00	0.000E+00
41	F	1	SI	29	+	1	PROT		->	1	P	30	+	0	00000		2.725E-43	VITAL	(p,g)	5	1.000E+00	0.000E+00
42	F	1	SI	30	+	1	PROT		->	1	P	31	+	0	00000		2.882E-44	VITAL	(p,g)	5	1.000E+00	0.000E+00
43	T	3	HE	4	+	0	00000		->	1	C	12	+	0	00000		0.000E+00	VITAL	(v,v)	99	1.000E+00	7.019E+18
44	T	1	C	12	+	1	HE	4	->	1	O	16	+	0	00000		0.000E+00	VITAL	(a,g)	9	1.000E+00	6.910E+18
45	T	1	C	13	+	1	HE	4	->	1	O	16	+	1	NEUT		0.000E+00	VITAL	(a,n)	11	1.000E+00	2.138E+18
46	F	1	N	14	+	1	HE	4	->	1	F	18	+	0	00000		0.000E+00	VITAL	(a,g)	9	1.000E+00	0.000E+00
47	F	1	O	16	+	1	HE	4	->	1	NE	20	+	0	00000		0.000E+00	VITAL	(a,g)	9	1.000E+00	0.000E+00
48	F	1	O	18	+	1	HE	4	->	1	NE	22	+	0	00000		0.000E+00	VITAL	(a,g)	9	1.000E+00	0.000E+00
49	F	1	NE	20	+	1	HE	4	->	1	MG	24	+	0	00000		0.000E+00	VITAL	(a,g)	9	1.000E+00	0.000E+00
50	F	1	NE	21	+	1	HE	4	->	1	MG	24	+	1	NEUT		4.764E-68	VITAL	(a,n)	11	1.000E+00	0.000E+00

Why do we want the slope?



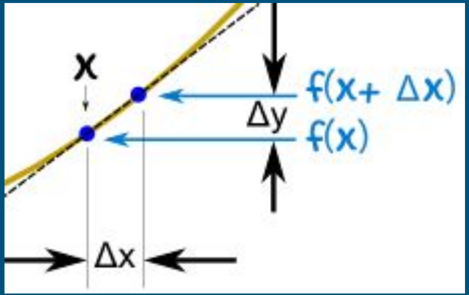
Calculating the value of a function at a later time

Requires gradient and initial conditions (Euler Method)

Fast & Easy(ish)

General formula for determining the slope:

$$f'(x) = \frac{f(x+\delta) - f(x)}{\delta}$$



- Finds the derivative at point x
- Evaluate f(x+δ) - f(x) (Δy) and divide by δ (Δx)

This allows us to use the Euler method to determine our final value for a function

The Euler Method

http://phys23p.sl.psu.edu/~mrg3/mathanim/diff_equ/eulera.html

The Euler Method

$$y_1 = y_0 + hf'(x)$$

1. Start with known initial y value
2. Calculate the gradient (δ)
3. Move along the gradient by an amount h along x
4. Recalculate your gradient
5. Repeat until your simulation is complete

Easy?

(x,y)	$\frac{dy}{dx}$	Δx	$\Delta y = \Delta x\left(\frac{dy}{dx}\right)$	$(x + \Delta x, y + \Delta y)$
(0.0, 2.0)	0.000	0.2	0.0	(0.2, 2.0)
(0.2, 2.0)	0.400	0.2	0.08	(0.4, 2.08)
(0.4, 2.08)	0.832	0.2	0.166	(0.6, 2.246)
(0.6, 2.246)	1.348	0.2	0.270	(0.8, 2.516)
(0.8, 2.516)	2.013	0.2	0.403	(1.0, **2.919**)

Try your own: $f(x) = x^2$; $x = 0$ to $x = 10$

$$f'(x) = \frac{f(x+\delta) - f(x)}{\delta}$$

$$y_{n+1} = y_n + hf'(x)$$

For Kepler's Law:

$$x(t_0 + h) = x(t_0) + hx'(t_0)$$

$$y(t_0 + h) = y(t_0) + hy'(t_0)$$

$$\mathbf{v}_x(t_0 + h) = \mathbf{v}_x(t_0) + h\mathbf{v}'_x(t_0)$$

$$\mathbf{v}_y(t_0 + h) = \mathbf{v}_y(t_0) + h\mathbf{v}'_y(t_0)$$

Note: This Task is VERY Hard

Give it your best! But don't be discouraged if you find it challenging!

If you do get stuck, you can even google the problem for ideas.

Good luck!

For Kepler's Law:

$$x(t_0 + h) = x(t_0) + hx'(t_0)$$

$$y(t_0 + h) = y(t_0) + hy'(t_0)$$

$$\mathbf{v}_x(t_0 + h) = \mathbf{v}_x(t_0) + h\mathbf{v}_x'(t_0)$$

$$\mathbf{v}_y(t_0 + h) = \mathbf{v}_y(t_0) + h\mathbf{v}_y'(t_0)$$