

timeseries_q4

December 16, 2018

1 Time Series Problem Set: Question 4

```
In [1]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

1.1 Part (a)

```
In [2]: N = 250
v = np.random.multivariate_normal(mean=np.zeros(2), cov=np.identity(2), size=N)
eta1 = v[:-1, 0] - 0.7*v[1:, 0] - 0.6*v[1:, 1]
eta2 = -0.5*v[1:, 0] + v[:-1, 1] - 0.7*v[1:, 1]

corr = np.corrcoef(eta1, eta2)[0, 1]
print(f'Correlation coefficient: {corr}')
```

Correlation coefficient: 0.31834882275290255

1.2 Part (b)

```
In [3]: N = 250
T = np.array([[1, -2], [0.5, 1]])

fig, axarr = plt.subplots(ncols=1, nrows=5, sharex=True, figsize=[15, 18])

for i, ax in enumerate(axarr.flatten()):
    v = np.random.multivariate_normal(mean=np.zeros(2), cov=np.identity(2), size=N)
    eta1 = v[:-1, 0] - 0.7*v[1:, 0] - 0.6*v[1:, 1]
    eta2 = -0.5*v[1:, 0] + v[:-1, 1] - 0.7*v[1:, 1]

    # Simple cumulative sum, since x_0 = 0
    x1 = eta1.cumsum()
    x2 = eta2.cumsum()
    x = np.vstack([x1, x2]).T

    y = (T @ x.T).T # Multiplying each row of x by T
```

```

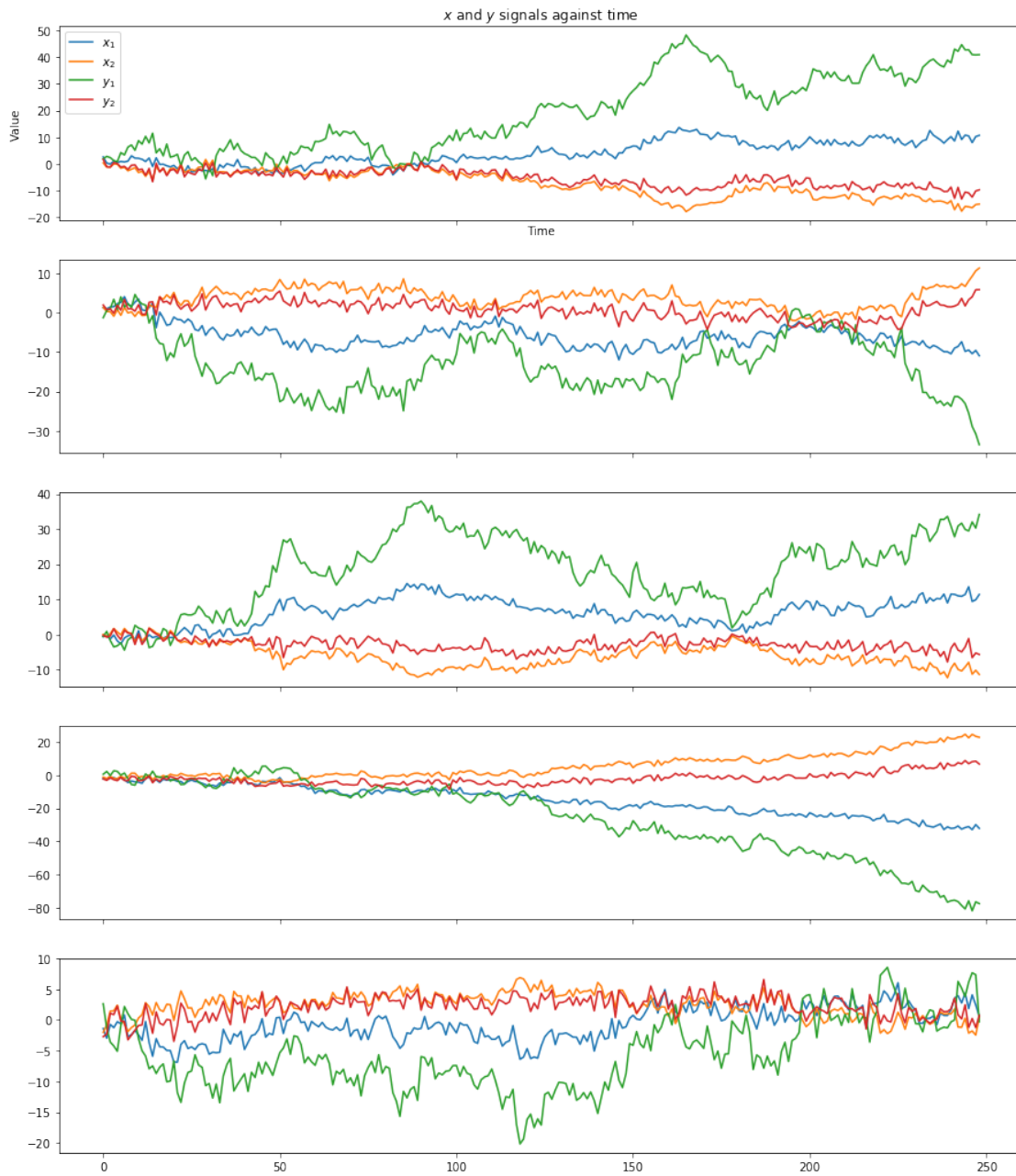
ax.plot(x[:, 0], label='$x_1$')
ax.plot(x[:, 1], label='$x_2$')
ax.plot(y[:, 0], label='$y_1$')
ax.plot(y[:, 1], label='$y_2$')

```

```

axarr[0].set_title('$x$ and $y$ signals against time')
axarr[0].set_xlabel('Time')
axarr[0].set_ylabel('Value')
axarr[0].legend();

```



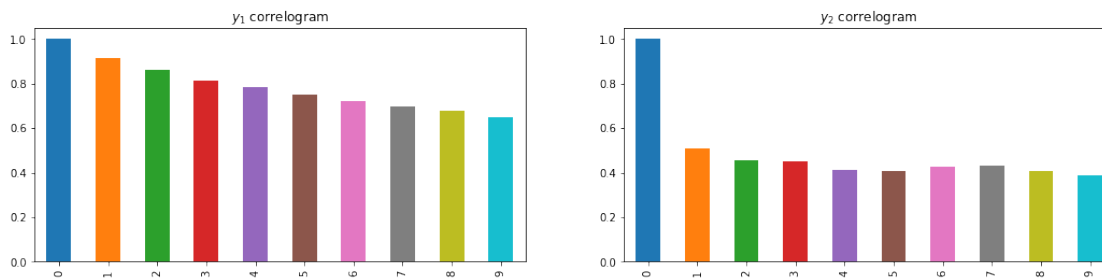
1.3 Part (c)

y_2 has a correlogram that dies out much more sharply than y_1 . Qualitatively, this indicates that y_2 is "more stationary" than y_1 .

```
In [4]: fig, axarr = plt.subplots(ncols=2, nrows=1, figsize=[18, 4])
```

```
y1_series = pd.Series(y[:, 0])  
y2_series = pd.Series(y[:, 1])
```

```
pd.Series([y1_series.autocorr(lag=1) for l in range(10)]).plot(kind='bar', title='$y_1$ autocorrelation')  
pd.Series([y2_series.autocorr(lag=1) for l in range(10)]).plot(kind='bar', title='$y_2$ autocorrelation')
```



1.4 Part (d)

The y_1 time series generally has a non-zero "slope": that is, it looks like it has some long-term linear trend. On the other hand, the y_2 time series more or less "hugs" the x-axis (i.e. zero mean), and is stationary.