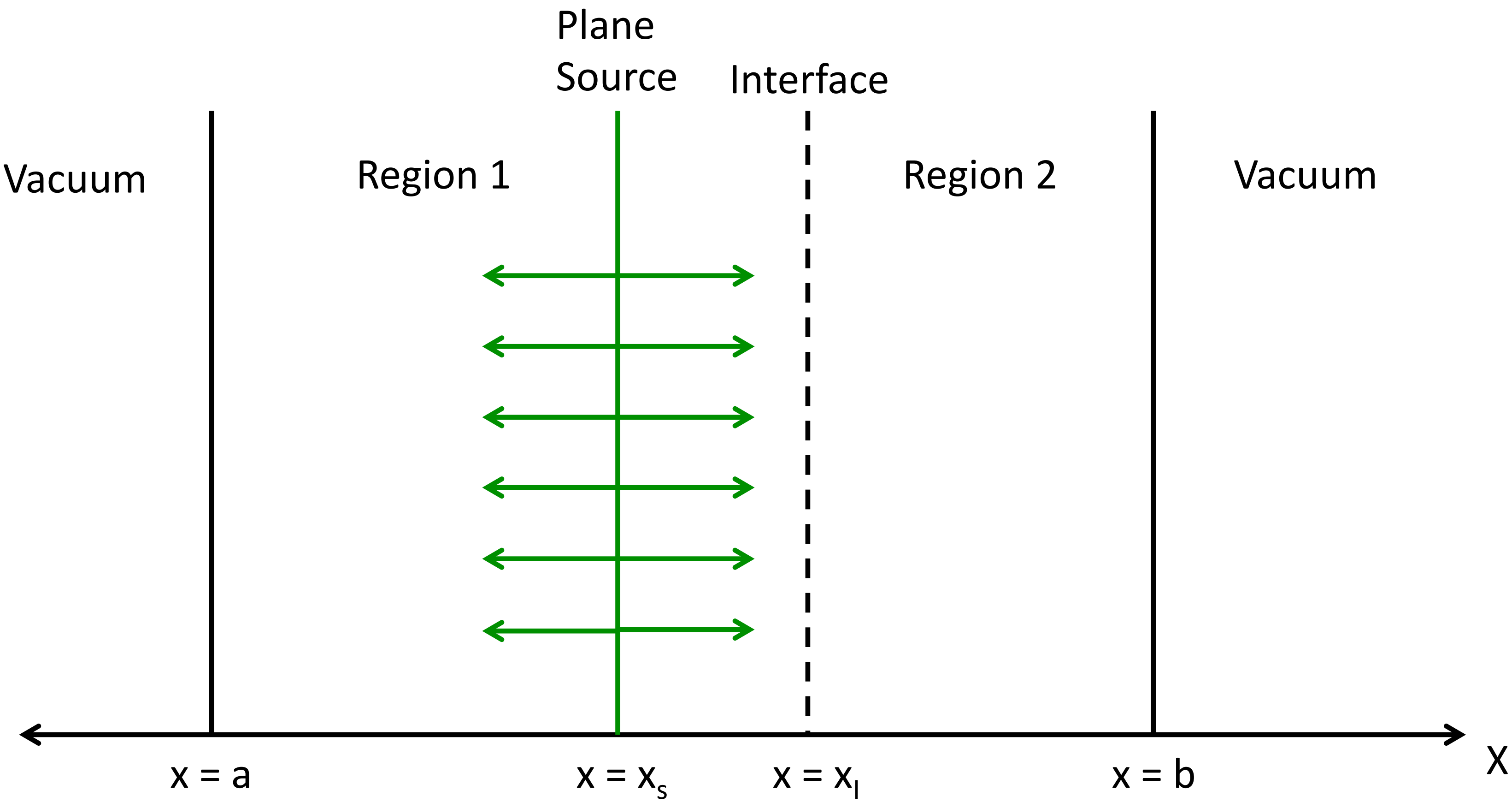
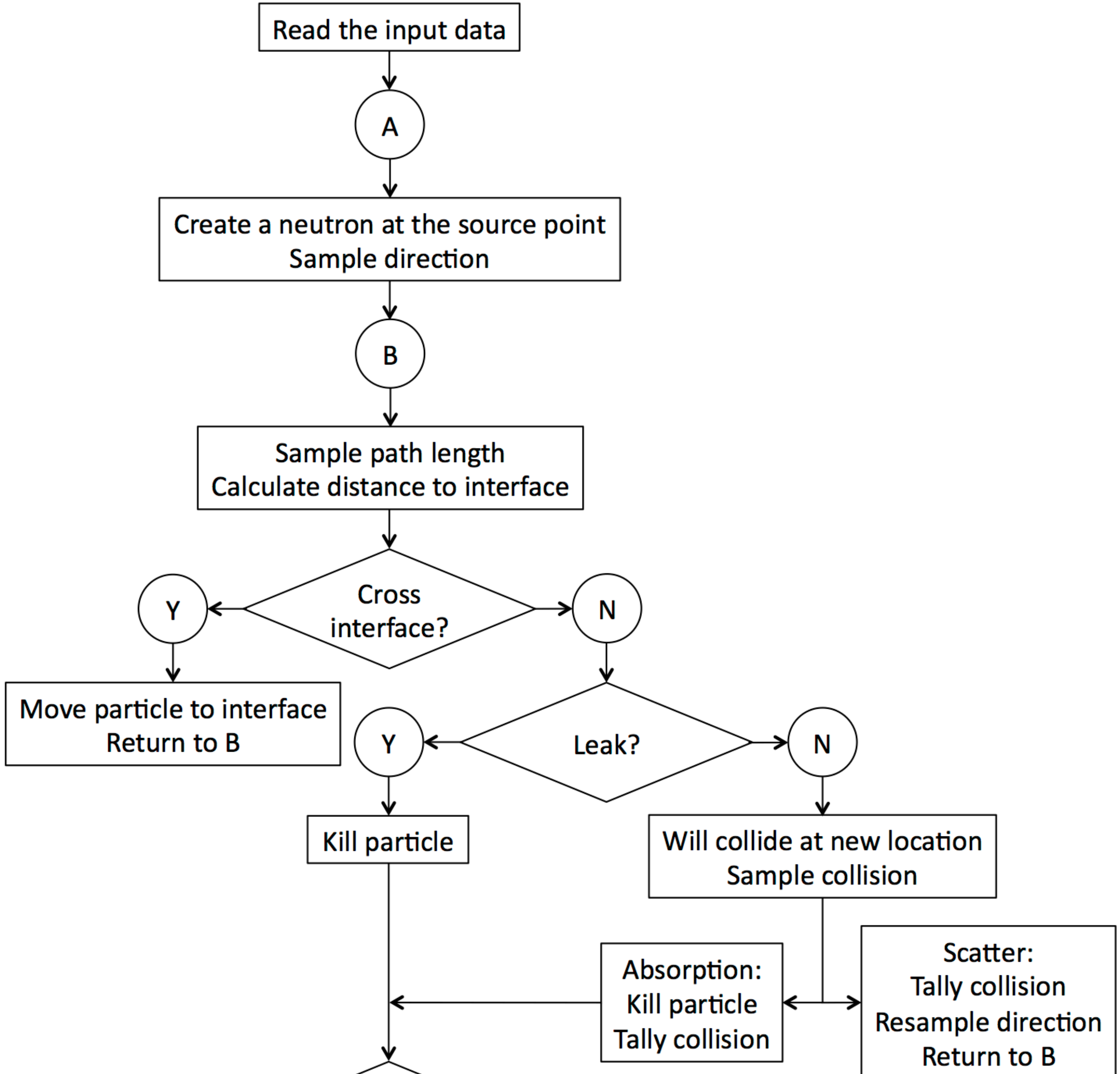
Problem Description and Geometry




Read the input data
A
Create a neutron at the source point
Sample direction
B
Sample path length
Calculate distance to interface
Cross interface?
Y
N
Move particle to interface
Return to B
Leak?
Y
N
Kill particle
Will collide at new location
Sample collision
Absorption:
Kill particle
Tally collision
Scatter:
Tally collision
Resample direction
Return to B

Sampling Routines

Sampling direction

- Generate a random number ξ on the uniform distribution [0, 1).
- If $\xi < 0.5$, direction = left (-)
- else, direction = right (+)

Sampling path length

- Generate a random number ξ on the uniform distribution [0, 1).
- Plug into inverted CDF for path length:

$$s = -\log\left(\frac{\xi}{\Sigma_t}\right)$$

Sampling angle

Isotropic source and scattering; angle can range from [0, 2π].
Interested in cosine of angle [-1, 1].
Direction covers half of angle space; reduce sampling of cosine of angle to [0, 1].

- Generate a random number ξ on the uniform distribution [0, 1).
- ξ multiplied by the direction is the cosine of angle

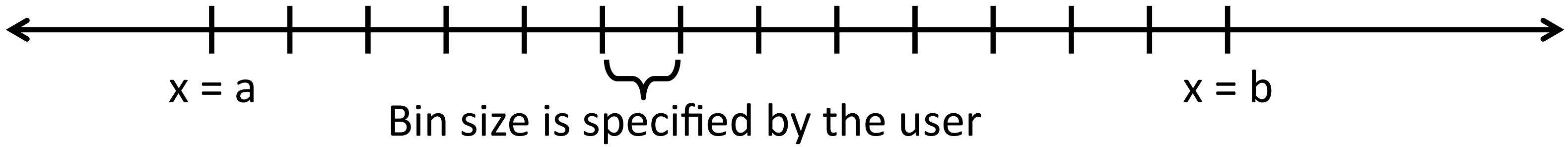
Sampling reaction

Two reactions possible: scattering (s) and absorption (a).
Probability of reaction *i*:

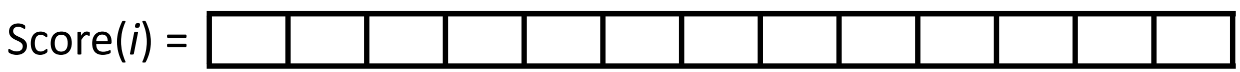
$$p_i = \frac{\Sigma_i}{\Sigma_t}$$

- Generate a random number ξ on the uniform distribution [0, 1).
- If $\xi \geq P_s$, neutron is absorbed.
- Else, it scatters

To resolve a collision tally over space, a spatial tally mesh is used for scoring and statistics.



For each neutron history *i*, a score vector is initialized corresponding to this spatial binning. Each score corresponds to a collision of particle *i*.



Once the history is complete (the particle is terminated):

- The score is added to a cumulative score (a vector of equal size representing the sum of all previous histories). Each entry is given by:

$$T^{\in 1xJ} = \sum_i x_{ij}$$

- The score in each bin of the score vector is squared and added to a vector containing the cumulative sum of the squares of the scores.

$$Q^{\in 1xJ} = \sum_i x_{ij}^2$$

After N histories, sum of the scores and squares of scores in each bin are used to calculate the flux and statistics in each bin.

Flux

$$\bar{\phi}_j \approx \frac{1}{V}\bar{x}_j = \frac{1}{L_j}\frac{1}{N}\sum_{i=1}^{N} x_{ij}$$

Statistics:

Sample Variance

$$S_{x,j}^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_{ij} - \bar{x}_j\right)^2 = \frac{1}{N-1}\left(\sum_{i=1}^{N} x_{ij}^2 - 2\bar{x}_j\sum_{i=1}^{N} x_{ij} + \bar{x}_j^2\sum_{i=1}^{N} 1\right)$$

Estimated Variance of Mean

$$S_{\bar{x},j}^2 = \frac{S_{xj}^2}{N}$$

Relative Error

$$R_j = \frac{S_{\bar{x},j}}{\bar{x}_j}$$

The method can be validated by solving the diffusion equation for a simplified system (finite slab, plane source).

$$\frac{1}{v}\frac{\partial \phi}{dt} - D(x)\phi(x,t) - \Sigma_a \phi(x,t) = S(x,t)$$

Two solution are required for flux on each side of the plane source.
Assume steady state, homogeneous material properties
Boundary conditions for each region are:
(1) Vacuum at slab boundaries (a and b)

$$\phi(\tilde{a}) = \phi(\tilde{b}) = 0$$

(2) Constant current at the source

$$\lim_{x\to 0} J(x) = \lim_{x\to 0} \frac{d\phi}{dx} = \frac{S}{2}$$

$$\phi(x) = \frac{SL}{2D}\left(1 + e^{\frac{-2\tilde{a}}{L}}\right)^{-1}\left(e^{\frac{-x}{L}} - e^{\frac{-2\tilde{a}}{L}} e^{\frac{x}{L}}\right) \qquad \tilde{a} \le x < 0$$

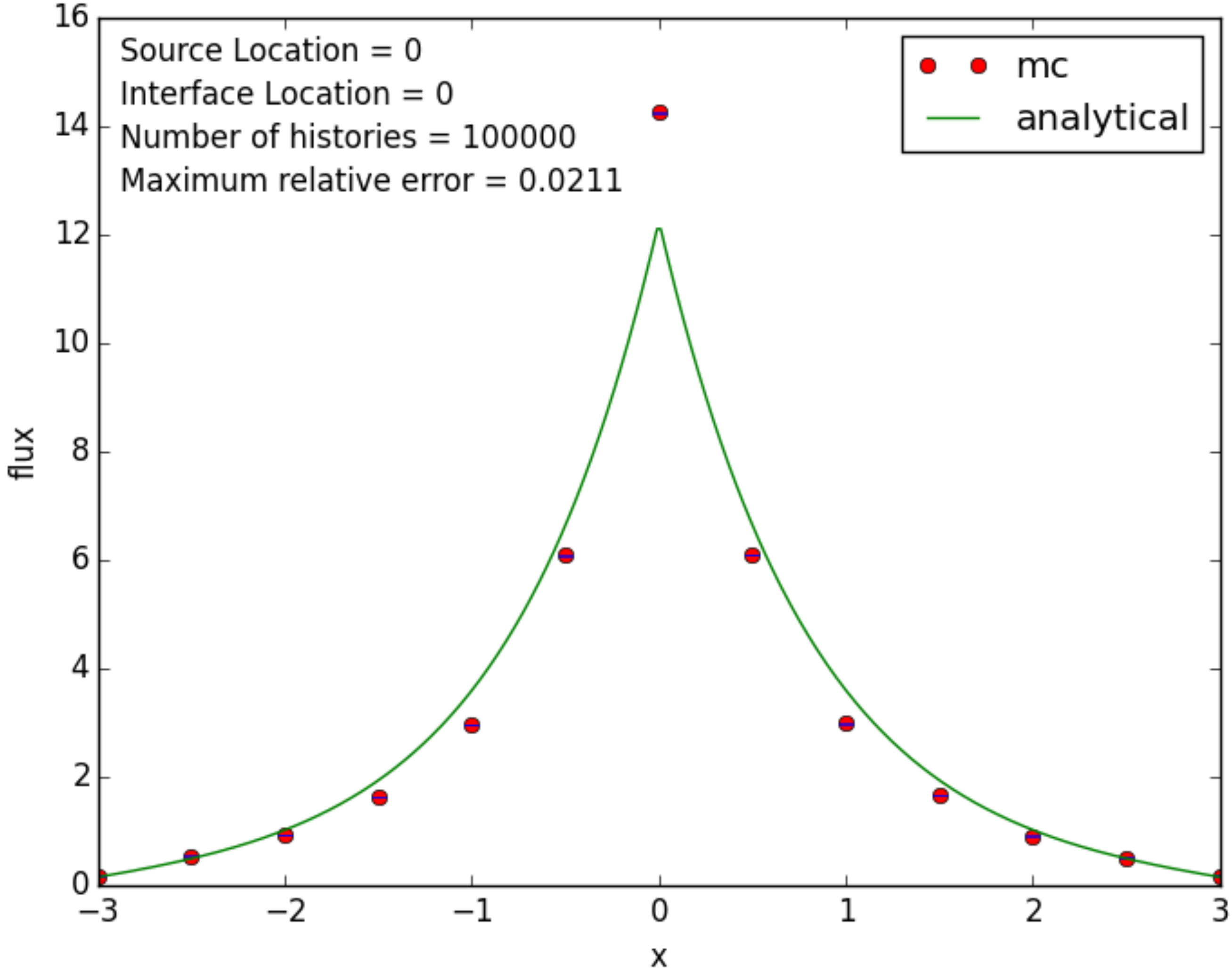
This solution can be adjusted to allow for the source to be anywhere in(a,b)

$$\phi(x) = \frac{SL}{2D}\left(1 + e^{\frac{-2\tilde{b}}{L}}\right)^{-1}\left(e^{\frac{-x}{L}} - e^{\frac{-2\tilde{b}}{L}} e^{\frac{x}{L}}\right) \qquad 0 < x \le \tilde{b}$$

Comparison of the analytical diffusion and MC solutions for the flux

$x_{source} = 0$
$x_{interface} = 0$
$a = 3$, $b = -3$
$S = 10$

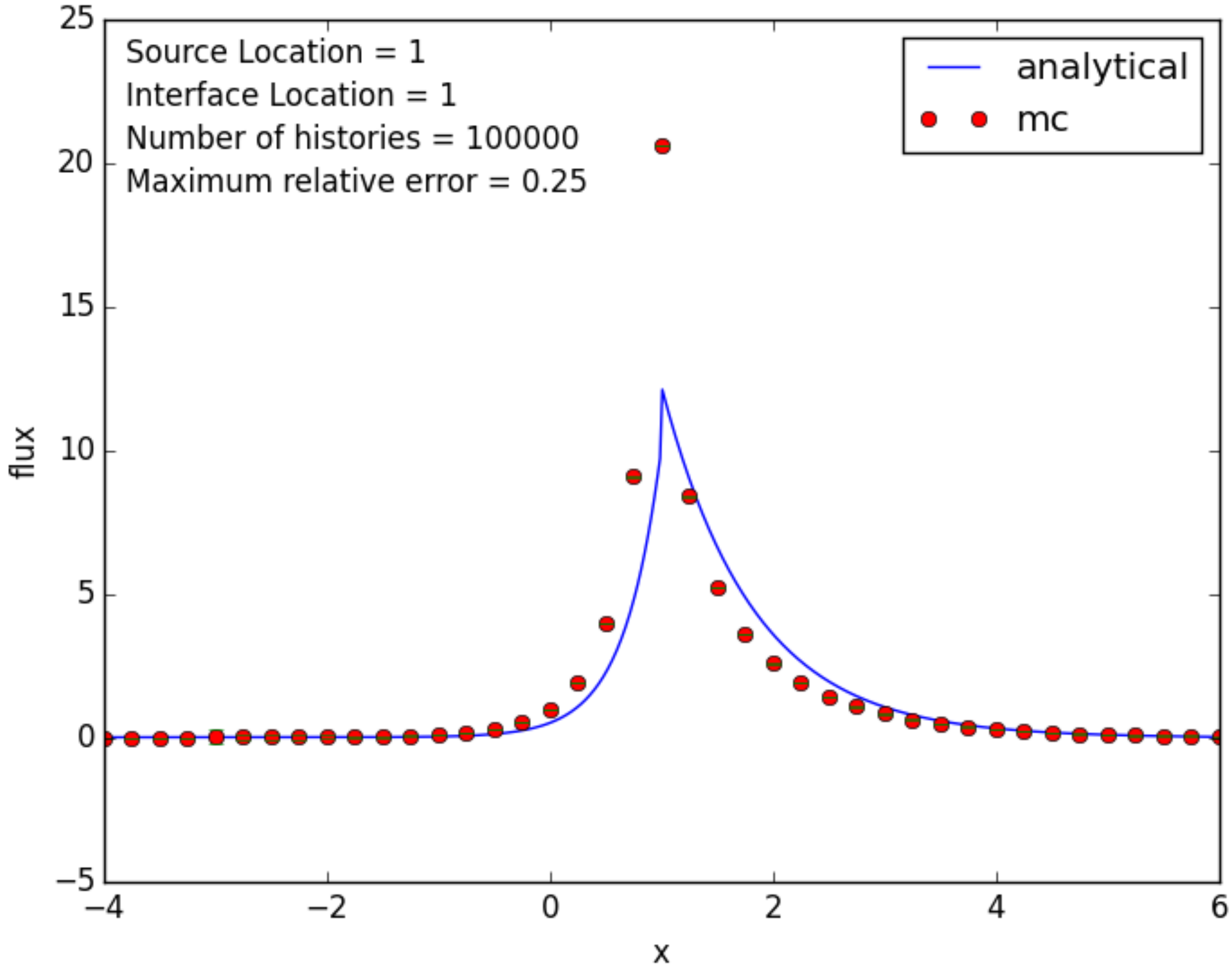
Cross-sections equal in each region:
$\Sigma_t(1) = \Sigma_t(2)$
$\Sigma_s(1) = \Sigma_s(2)$



Comparison of the analytical diffusion and MC solutions for the flux

$x_{source} = 1$
$x_{interface} = 1$
$a = -4$, $b = 6$
$S = 10$

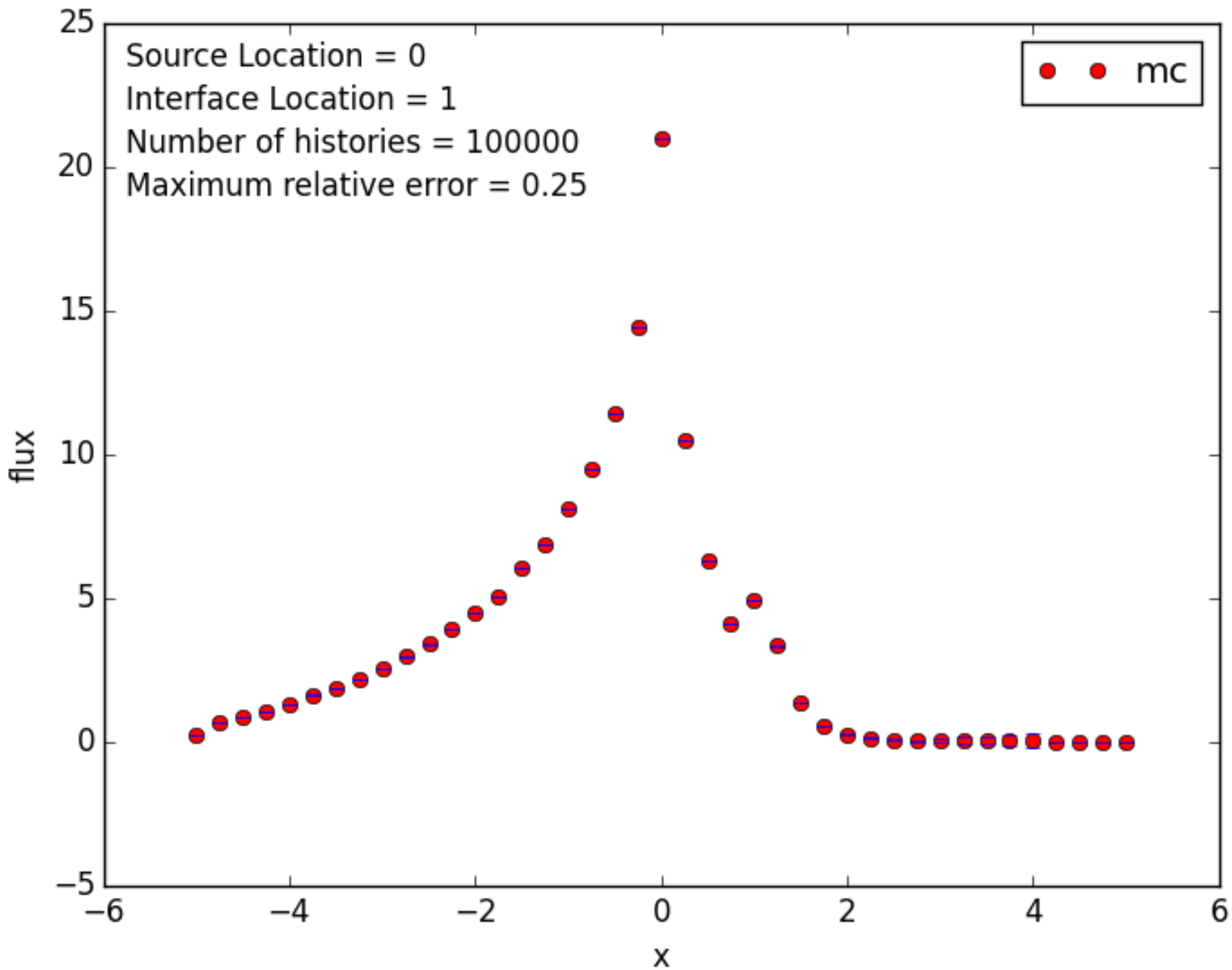
Cross-sections equal in each region:
$\Sigma_t(1) \neq \Sigma_t(2)$
$\Sigma_s(1) \neq \Sigma_s(2)$



Comparison of the analytical diffusion and MC solutions for the flux

$x_{source} = 0$
$x_{interface} = 1$
$a = -6$, $b = 6$
$S = 10$

Cross-sections equal in each region:
$\Sigma_t(1) < \Sigma_t(2)$
$\Sigma_s(1) = \Sigma_s(2)$



References

[1] MCNP5 - A General Monte Carlo N-Particle Transport Code, Version 5. Volume 1: Overview and Theory. X-5 Monte Carlo Team, Los Alamos National Laboratory (rev. 2008).

[2] J. J. Duderstadt, L. J. Hamilton. *Nuclear Reactor Analysis*. John Wiley & Sons, Inc. 1976.