NE 255 Fall 2016
Homework #3 Due October 18th, 2016

On homework:

- If you work with anyone else, document what you worked on together.
- Show your work.
- Always clearly label plots (axis labels, a title, and a legend if applicable).
- Homework should be done "by hand" (i.e. not with a numerical program such as MATLAB, Python, or Wolfram Alpha) unless otherwise specified. You may use a numerical program to check your work.
- If you use a numerical program to solve a problem, submit the associated code, input, and output (email submission is fine).

1. (20 points) Derive the 1st order form of SP_5 with isotropic source and vacuum boundary conditions.

2. Consider the integral
$$\int_{4\pi} d\hat{\Omega}\,\hat{\Omega}$$
The LQ$_N$ quadrature set is given in Figure 1. Recall that $\mu_i = \eta_i = \xi_i$ for a given level, i.
   (a) (5 points) Use the S_4 LQ$_N$ quadrature set to execute this integral
   (b) (10 points) Repeat it with S_6. What do you observe?
   (c) (10 points) Write a short code to execute this integration (and higher orders if you'd like). Try a few different functions. Turn in the code and the evaluation of these functions. Include comments on what you observe.

3. (a) (5 points) Briefly compare the diffusion equation, deterministic methods, and monte carlo methods in terms of complexity, accuracy, run time, and range of applicability.
   (b) (5 points) Given what you've learned about deterministic methods so far, discuss strengths and weaknesses.

4. Write a function that generates the associated Legendre Polynomials:
$$P_\ell^m(x) = \frac{(-1)^m}{2^\ell \ell!}(1-x^2)^{m/2}\,\frac{d^{\ell+m}}{dx^{\ell+m}}(x^2-1)^\ell.$$

Multidimensional Discrete Ordinates

Table 4-1 Level Symmetric S_N Quadrature Sets LQ_n[a]

Level	n	μ_n	w_n[b]
S_4	1	0.3500212	0.3333333
	2	0.8688903	
S_6	1	0.2666355	0.1761263
	2	0.6815076	0.1572071
	3	0.9261808	
S_8	1	0.2182179	0.1209877
	2	0.5773503	0.0907407
	3	0.7867958	0.0925926
	4	0.9511897	
S_{12}	1	0.1672126	0.0707626
	2	0.4595476	0.0558811
	3	0.6280191	0.0373377
	4	0.7600210	0.0502819
	5	0.8722706	0.0258513
	6	0.9716377	
S_{16}	1	0.1389568	0.0489872
	2	0.3922893	0.0413296
	3	0.5370966	0.0212326
	4	0.6504264	0.0256207
	5	0.7467506	0.0360486
	6	0.8319966	0.0144589
	7	0.9092855	0.0344958
	8	0.9805009	0.0085179

[a]Data from Ref. 5.
[b]See Fig. 4-3 for ordinate directions corresponding to weight

Figure 1: LQ_n quadrature

Use this function in a function that generates spherical harmonics

$$Y_{lm}(\theta,\varphi) = (-1)^m \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}\, P_{lm}(\cos\theta)e^{im\varphi}$$

(a) (30 points) Generate and plot the following $l = 0, 1, 2$ for $-l \le m \le l$ (recall we can relate the negate m to positive m values). You will need to discretize θ and μ fairly finely (I suggest 30 increments in each to start so you get a real sense of the shape of the harmonics).

(b) (20 points) Now, we will approximate the external source. Using the S_4 quadrature to do the integrations and $q_e = 1$ for all angles: use the equations for external source we developed in class (eqns. 19-21), calculate the external source for $l = 0, 1, 2$.

5. (5 points) What are the major nuclear data libraries and which countries manage them?