NE 255 | Fall 2016
Homework #2 | Due October 4th, 2016

On homework:

- If you work with anyone else, document what you worked on together.
- Show your work.
- Always clearly label plots (axis labels, a title, and a legend if applicable).
- Homework should be done "by hand" (i.e. not with a numerical program such as MATLAB, Python, or Wolfram Alpha) unless otherwise specified. You may use a numerical program to check your work.
- If you use a numerical program to solve a problem, submit the associated code, input, and output (email submission is fine).

1. (10 points) Determine the macroscopic scattering cross section of UO_2 as a function of a generic enrichment factor $\gamma = N_{U-235}/N_{U-238}$ where N is the atom density. Find its value assuming a density of $10\,\mathrm{g/cm^3}$, $\sigma_s^U \backsimeq 8.9\,\mathrm{b}$ and $\sigma_s^O \backsimeq 3.75\,\mathrm{b}$, and 5% weight enrichment.

2. A major challenge in scientific computing is navigating the disparate architectures that comprise new supercomputers.
   (a) (5 points) Look up the Top 10 supercomputing list and briefly describe the architecture of the top three machines. List the number of machines of each type on the top 10 (e.g. X are GPU-accelerated, Y contain MICs, etc.)
   (b) (10 points) Describe the main characteristics of GPUs, MICs (multi integrated cores), and CPUs–memory, clock speed, structure, etc.
   (c) (5 points) Based on what we've talked about so far, postulate challenges of solving the neutron transport equation in a way that would work on all of these architectures.

3. (20 points) We often measure convergence by comparing one iteration to the previous iteration (rather than the solution, since we presumably don't know what it is). Imagine that you have software that gives the following solution vectors

$$x_{n-1} = \begin{pmatrix} 0.45 \\ 0.95 \\ 0.2 \\ -0.05 \\ 0.6 \end{pmatrix} \qquad x_n = \begin{pmatrix} 0.5 \\ 0.9 \\ 0.3 \\ -0.1 \\ 0.5 \end{pmatrix}$$

Calculate

a. The absolute and relative error using the 1 norm
b. The absolute and relative error using the 2 norm
c. The absolute and relative error using the infinity norm

What is most restrictive (that is, what would cause the code to converge *first*)?

Image that now $x_{n-1} = (0.49, 0.92, 0.4, -0.09, 0.51)^T$. Recalculate the convergence values.

What do you observe? What does that mean about how you might select convergence criteria?

4. (10 points) You have a piece of software that takes mesh spacing as an input variable. Imagine that you have the following relative error values for each mesh spacing (h) / number of mesh cells (N cells):

h	N cells	rel err
1.0	8	8.44660179e-03
0.5	16	2.30286448e-03
0.1	80	9.84273963e-05
0.05	160	2.48043656e-05
0.01	800	9.98488163e-07

One of the ways we characterize method performance is the order of convergence. We'd like to know how the error changes as we change the resolution of our discretization, in this case, mesh spacing.

Using a log-log plot, plot relative error as a function of

a. mesh spacing and
b. cell count.

What is the functional relationship between error and mesh spacing / number of cells?

5. (10 points) What are *six* underlying assumptions in the neutron transport equation? Write at least one sentence explaining what each assumption means and why we need or want to make it.

6. Consider the transport equation:

$$\underbrace{\frac{1}{v}\frac{\partial\psi}{\partial t}}_{A} + \underbrace{\hat{\Omega}\cdot\nabla\psi(\vec{r},\hat{\Omega},E,t)}_{B} + \underbrace{\Sigma(\vec{r},E)\psi(\vec{r},\hat{\Omega},E,t)}_{C} = \underbrace{S(\vec{r},\hat{\Omega},E,t)}_{D}$$
$$+ \underbrace{\int_0^\infty dE' \int_{4\pi} d\hat{\Omega}'\, \Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\cdot\hat{\Omega})\psi(\vec{r},\hat{\Omega}',E',t)}_{E}$$
$$+ \underbrace{\frac{\chi(E)}{4\pi}\int_0^\infty dE'\, \nu\Sigma_f(\vec{r},E') \int_{4\pi} d\hat{\Omega}'\, \psi(\vec{r},\hat{\Omega}',E',t)}_{F}$$

   (a) (10 points) Briefly describe what each term in the Transport Equation physically represents.

   (b) (10 points) Rewrite the time independent form of the equation to include azimuthal symmetry. Show the steps needed to get there.

7. (10 points) Solve the following differential equation by hand:

$$\frac{d^2y}{dx^2} + 3y(x) = \sin(x) \qquad x \in [0,1]$$

   where $y(0) = 1, y(1) = 3$.

   What simplifications to the transport equation would have been required to get an equation form that looks like this?

8. (10 points) At what energy is the lowest isolated resonance of ^{235}U, ^{238}U, ^{239}Pu, ^{240}Pu, ^{241}Pu, and ^{242}Pu? Why do we care about that?