Example – Continuous-Time Optimal Control with Infinite Horizon

Same problem setting/dynamic as in the Examples 5.1, 5.2, and 5.5, i.e. we have a unit mass that moves on a line under the influence of a force u. The corresponding system is

$$\dot{x}_1(t) = x_2(t),$$
$$\dot{x}_2(t) = u(t).$$

We will not assume any constraint on the control u, and we will assume that the horizon is infinite, with the objective

$$\text{minimize} \int_{t=0}^{\infty} x_1(t)^2 + x_2(t)^2 + u(t)^2 dt$$

with the initial conditions:

$$x_1(0) = -2, \quad x_2(0) = 10.$$

For this infinite time problem, it can be shown that J^* is not a function of time, but only a function of x, i.e. that the following equation holds.

$$0 = \min_u [g(x,u) + \nabla_x J^*(x)' f(x,u)]$$

Assignment: Find the optimal state/control trajectories for the problem above either

- analytically - deriving the solution by "guessing" that $J^*(x) = x'Kx$, with a positive definite K
- numerically - solving the Hamiltonian formulation using the two-point boundary problem method (for this, you can use a finite horizon T, for which $p(T) = 2Ix$)
- numerically - discretizing the space and using Value Iteration algorithm (this seems simple but is in fact rather tricky w.r.t the used discretization)