Mechanical Engineering for Robotics Assignment 1: Statics and Mechanics of Materials

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1 AMR with the reaction forces

The most critical scenario is show in Figure 1. It is identified to be four forces acting directly over the corners of the frame. The reactions on the bogie beam with its components can be determined by looking at each one of the sides of the AMR.

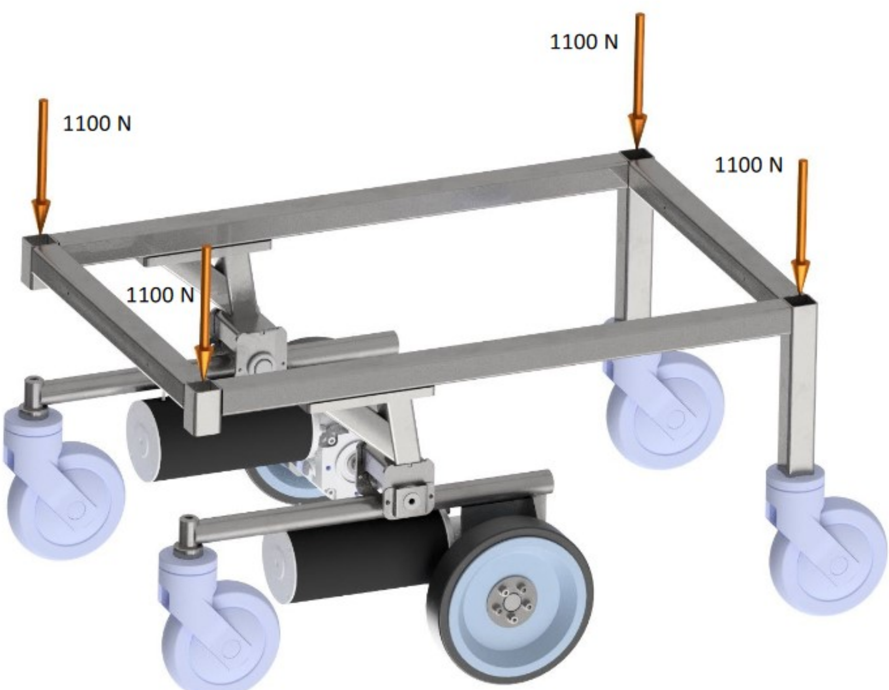

Figure 1: AMR in forward drive with the four applied forces

1.1 Load on the bogie beam

For this assignment we consider two different conditions: the AMR driving forwards and the AMR driving backwards. Those situations will change the position of the caster-wheels, which will result in a different load on the bogie beam from the frame. In order to determine those loads for the two situations, free body diagrams are utilized. The free body diagrams for the two conditions are shown in Figure 2 (forwards) and Figure 3 (backwards).

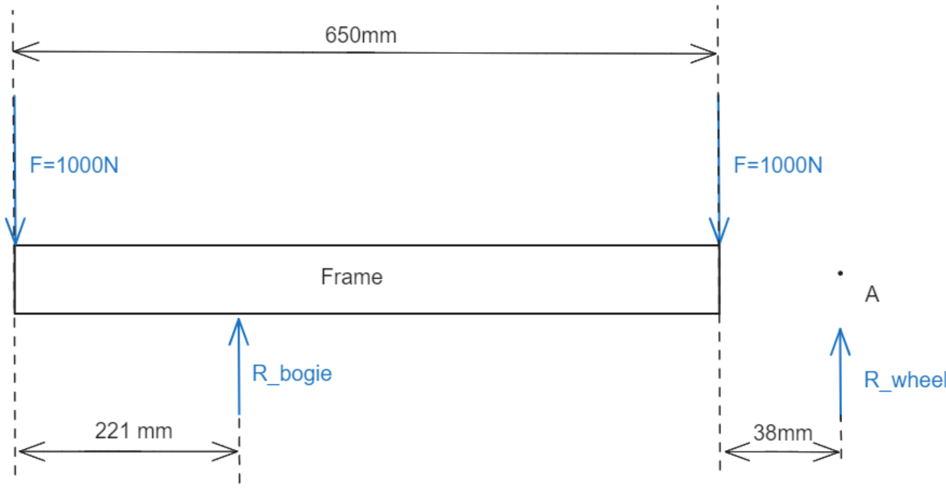

Figure 2: Free body diagram of the robot frame, AMR driving forwards

For the first case, the AMR driving forwards, the load on the bogie beam can be calculated by summing moments around point A (the point where the unknown force R_{wheel} is acting on the frame). It can be calculated using this equation:

$$+\overset{\curvearrowleft}{\sum} M_A = F \cdot (650\,\text{mm} + 38\,\text{mm}) + F \cdot (38\,\text{mm}) - R_{\text{bogie}} \cdot (650\,\text{mm} - 221\,\text{mm} + 38\,\text{mm}) \tag{1}$$

$$R_{\text{bogie}} = \frac{F \cdot (650\,\text{mm} + 38\,\text{mm}) + F \cdot (38\,\text{mm})}{650\,\text{mm} - 221\,\text{mm} + 38\,\text{mm}} = 1554.604\,\text{N} \tag{2}$$

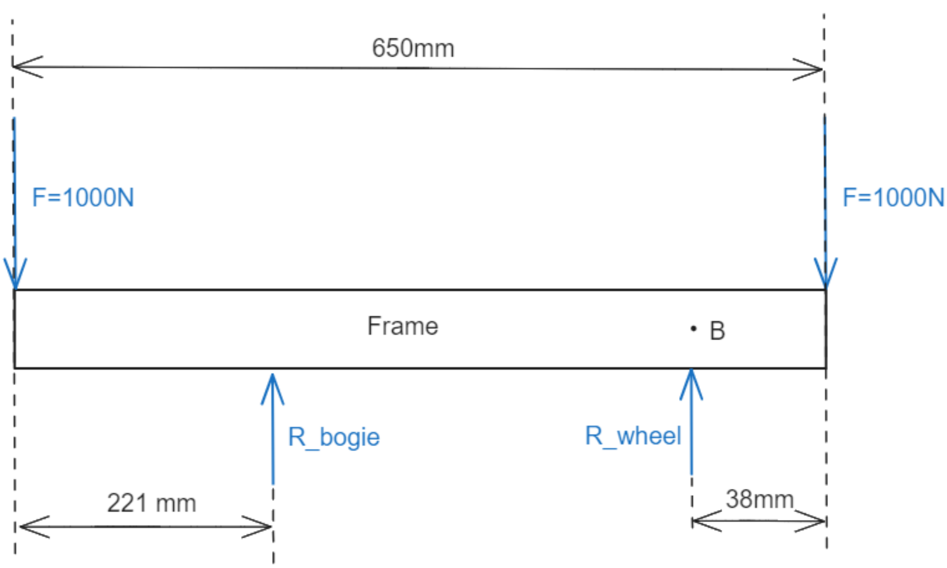

Figure 3: Free body diagram of the robot frame, AMR driving backwards

For the second case, the AMR driving backwards, the load on the bogie beam can be calculated by summing moments around point B (the point where the unknown force R_{wheel} is acting on the frame). This load can be calculated using this equation:

$$\overset{\curvearrowleft}{+}\sum M_B = F \cdot (650\,\text{mm} - 38\,\text{mm}) - F \cdot (38\,\text{mm}) - R_{\text{bogie}} \cdot (650\,\text{mm} - 221\,\text{mm} - 38\,\text{mm}) \tag{3}$$

$$R_{\text{bogie}} = \frac{F \cdot (650\,\text{mm} - 38\,\text{mm}) - F \cdot (38\,\text{mm})}{650\,\text{mm} - 221\,\text{mm} - 38\,\text{mm}} = 1468.031\,\text{N} \tag{4}$$

The two calculated reactions, R_{bogie}, acting on the frame from the bogie beam are equal but opposite to the reaction acting on the bogie beam from the frame, meaning the load applied to the bogie beam from the frame.

1.2 Bogie beam with reaction forces, case 1: AMR driving forwards

In order to calculate the reaction forces for the bogie beam, we can apply similar approach to the one in the previous section. The free body diagram for the case of AMR driving forwards can be seen in Figure 4.

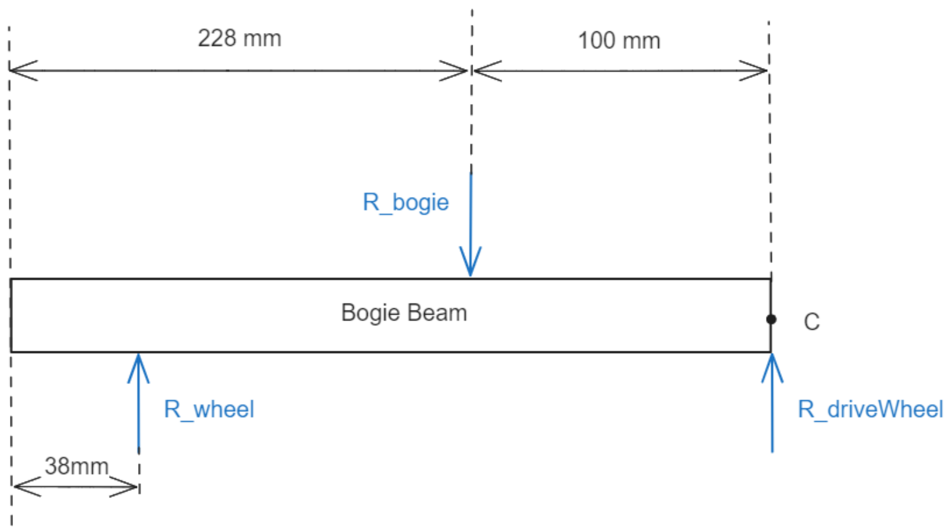

Figure 4: Free body diagram of the bogie beam, AMR driving forwards

The reaction from the caster-wheel R_{wheel} can be calculated by summing moments around point C:

$$\overset{\curvearrowleft}{+}\sum M_C = R_{\text{bogie}} \cdot (100\,\text{mm}) - R_{\text{wheel}} \cdot (228\,\text{mm} + 100\,\text{mm} - 38\,\text{mm}) \tag{5}$$

$$R_{\text{wheel}} = 536.07\,\text{N} \tag{6}$$

The reaction from the drive wheel, $R_{\text{driveWheel}}$, can be calculated by summing forces in the y-direction, because the bogie beam is in equilibrium:

$$\sum F_y = R_{\text{wheel}} + R_{\text{driveWheel}} - R_{\text{bogie}} = 0 \tag{7}$$

$$R_{\text{driveWheel}} = R_{\text{bogie}} - R_{\text{wheel}} = 1018.534\,\text{N} \tag{8}$$

The drive wheel is not placed directly under the bogie beam, as seen in Figure 5, because of that the beam is still not in equilibrium. To keep the beam in equilibrium there has to be torque applied at the bearing bushing.

The magnitude and direction of the torque in the bearing bushing, T_{bushing}, are equal but opposite to the torque generated by the force from the drive wheel. This torque can be calculated by using the force from the drive wheel and the horizontal distance between the center of the wheel and the center of the bearing bushing:

$$T_{\text{bushing}} = R_{\text{driveWheel}} \cdot (76.7\,\text{mm} - \frac{34.5\,\text{mm}}{2}) = 60551.85\,\text{Nmm} \tag{9}$$

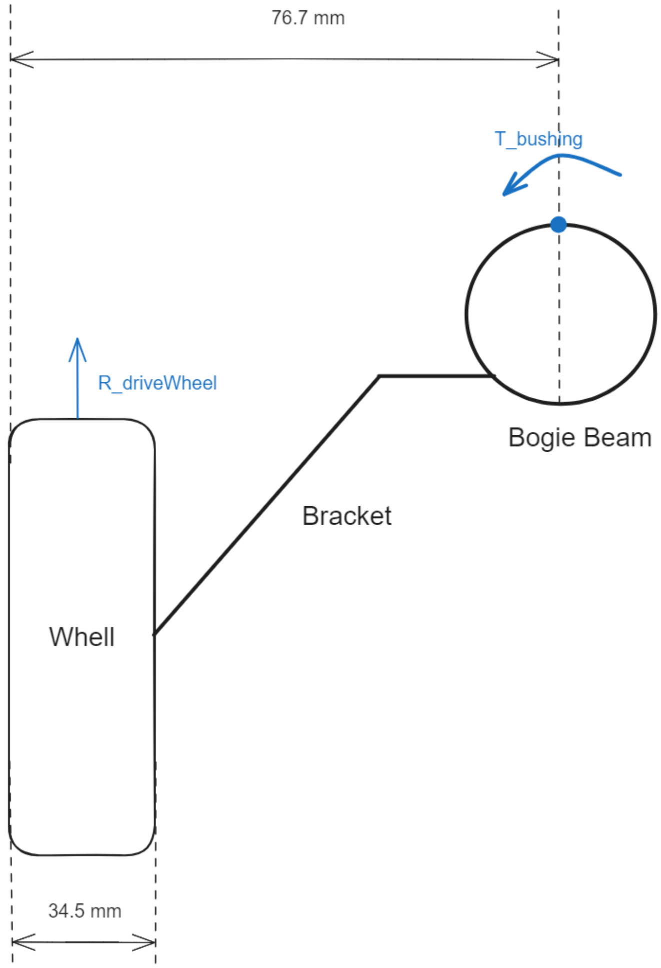

Figure 5: Diagram of the bogie beam and the drive wheel

1.3 Bogie beam with reaction forces, case 2: AMR driving backwards

The same approach can be applied to calculate the reaction forces on the bogie beam with the AMR driving backwards. The free body diagram for the case of AMR driving backwards can be seen in Figure 6. The reaction from the caster-wheel R_{wheel} can be calculated by summing moments around point D:

$$+\overset{\curvearrowleft}{\sum} M_D = R_{\text{bogie}} \cdot (100\,\text{mm}) - R_{\text{wheel}} \cdot (228\,\text{mm} + 100\,\text{mm} + 38\,\text{mm}) \tag{10}$$

$$R_{\text{wheel}} = 401.101\,\text{N} \tag{11}$$

The reaction from the drive wheel, $R_{\text{driveWheel}}$, can be calculated by summing forces in the y-direction, because the bogie beam is in equilibrium:

$$\sum F_y = R_{\text{wheel}} + R_{\text{driveWheel}} - R_{\text{bogie}} = 0 \tag{12}$$

$$R_{\text{driveWheel}} = R_{\text{bogie}} - R_{\text{wheel}} = 1066.93\,\text{N} \tag{13}$$

The torque, T_{bushing}, while the AMR is driving backwards, is calculated using the same way as in the previous section, with a different value of $R_{\text{driveWheel}}$.

$$T_{\text{bushing}} = R_{\text{driveWheel}} \cdot (76.7\,\text{mm} - \frac{34.5\,\text{mm}}{2}) = 63428.989\,\text{Nmm} \tag{14}$$

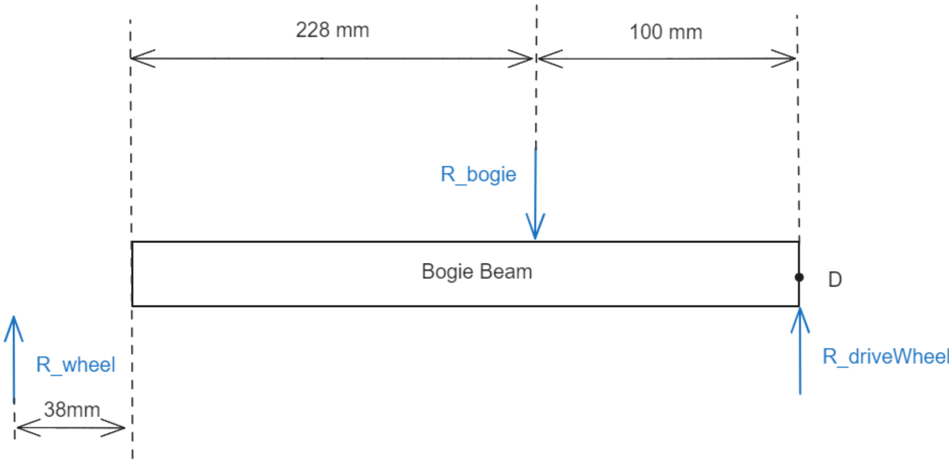

Figure 6: Free body diagram of the bogie beam, AMR driving backwards

1.4 Determining the Most Critical Case

The calculated reactions on the bogie beam for both driving conditions (the AMR driving forwards and the AMR driving backwards) can be seen in the Table 1. Despite the load being higher for the first case (driving forwards), the more critical one is the second one (driving backwards). This happens because the distance between the bearing bushing and the caster-wheel is higher, when driving backwards. This causes a larger reaction force from the drive wheel and results in a larger torque and a larger bending moment at the bearing bushing.

	Forwards	**Backwards**
R_{bogie} [N]	1554.604	1468.031
R_{wheel} [N]	536.07	401.101
$R_{\text{driveWheel}}$ [N]	1018.534	1066.93
T_{bushing} [Nmm]	60551.85	63428.989

Table 1: Reactions on the bogie beam for both driving conditions

2 Maximal Normal Stress Caused by Bending and the Maximal Shearing Stress Caused by Torsion

Based on the previous section it was determined that the most critical case is the AMR driving backwards. That is why the stresses will be calculated for that condition. Both stresses use the centeroid of the bogie beam. The bogie beam is symmetric and the centeroid is equal to the radius of the shaft.

$$c = \frac{25\,\text{mm}}{2} = 12.5\,\text{mm} \tag{15}$$

2.1 Maximal Normal Stress Caused by Bending

The maximal normal stress caused by bending is given by:

$$\sigma_m = \frac{M \cdot c}{I} \tag{16}$$

where M is the maximum bending moment, c is the centroid, and I is the area moment of inertia of the bogie beam.

The maximum bending moment in the bogie beam is located at the bearing bushing and is calculated as:

$$M = R_{\text{driveWheel}} \cdot (100\,\text{mm}) = 106693\,\text{Nmm} \tag{17}$$

The area moment of inertia is calculated as:

$$I = \frac{\pi}{4} \cdot (r_1^4 - r_2^4) = 9628.196\,\text{mm}^4 \tag{18}$$

where $r_1 = 12.5\,\text{mm}$ and $r_2 = 10.5\,\text{mm}$ are the inner and outer radii of the bogie beam.

Finally, the maximal normal stress caused by bending can be calculated as:

$$\sigma_m = \frac{(106693\,\text{Nmm}) \cdot (12.5\,\text{mm})}{(9628.196\,\text{mm}^4)} = 138.516\,\text{MPa} \tag{19}$$

2.2 Maximal Shearing Stress Caused by Torsion

The maximal shearing stress caused by torsion is given by:

$$\tau_m = \frac{T \cdot c}{J} \tag{20}$$

where T is the maximal torque applied to the bogie beam, J is the polar moment of inertia of the bogie beam, and c is the centroid of the bogie beam.

The only torque acting on the bogie beam is the T_{bushing} and that is the maximal torque applied to the bogie beam.

The polar area moment of inertia is calculated as:

$$J = \frac{\pi}{2} \cdot (r_1^4 - r_2^4) = 19256.392\,\text{mm}^4 \tag{21}$$

where $r_1 = 12.5\,\text{mm}$ and $r_2 = 10.5\,\text{mm}$ are the inner and outer radii of the bogie beam.

Finally, the maximal shearing stress caused by torsion can be calculated as:

$$\tau_m = \frac{(63428.989\,\text{Nmm}) \cdot (12.5\,\text{mm})}{(19256.392\,\text{mm}^4)} = 41.174\,\text{MPa} \tag{22}$$

3 Maximal Equivalent Normal Stress

The maximal equivalent normal stress is given by:

$$\sigma_{\text{eq}} = \sqrt{\sigma_m^2 + 3 \cdot \tau_m^2} \tag{23}$$

where σ_m is the maximal normal stress caused by bending and τ_m is the maximal shearing stress caused by torsion.

The maximal equivalent normal stress can be calculated as:

$$\sigma_{\text{eq}} = \sqrt{(138.516\,\text{MPa}) + 3 \cdot (41.174\,\text{MPa})^2} = 155.797\,\text{MPa} \tag{24}$$

4 Risk of Failure During Operation

In order to determine the risk of failure during operation the factor of safety against yield has to be calculated. This is typically calculated with the 'Von Mises' or 'maximum shrearing stress criterion'. The 'Von Mises Stress' can be calculated the same way as the previously determined maximal equivalent normal stress. From that factor of safety against yield ('Von Mises') can be directly calculated, as follows:

$$F.O.S_y = \frac{\sigma_Y}{\sigma_{\text{eq}}} = \frac{(235\,\text{MPa})}{(155.797\,\text{MPa})} = 1.508 \tag{25}$$

where σ_Y is the yield strength of the steel that the bogie beam is made of and σ_{eq} is the maximal equivalent normal stress.

The resulting factor of safety against yield, 1.508, is lower than the minimum required, 2. The 'maximum shrearing stress criterion' is more conservative than 'Von Mises', that means that the factor of safety against yield calculated with it would be even lower. This means that there is an increased risk of failure because the structure is closer to its yield point under normal operating conditions.

5 Different design with an increased strength of the bogie beam

This new design should use the same amount of material, as the previous one, and the factor of safety against yield in the bogie beam should be at least 2, for the most critical condition.

One of the ways to increase strength would be to increase the outer diameter and reduce the wall thickness, whilst ensuring that the wall thickness is sufficient. This can improve the moments of inertia, which increases the beam's resistance to bending and torsion.

The task demands that the amount of used material should stay the same. In order to do that the cross sectional area of the beam should stay the same. The area calculated from the original shape should be used to provide a link between the inner and outer radii.

$$A_1 = \pi \cdot ({r_1}^2 - {r_2}^2) = \pi \cdot \left((12.5\,\text{mm})^2 - (10.5\,\text{mm})^2\right) = 144.513\,\text{mm}^2 \tag{26}$$

$$A_2 = \pi \cdot (r_{out}^2 - r_{in}^2) \tag{27}$$

$$A_1 = A_2 \tag{28}$$

$$r_{in} = \sqrt{\frac{\pi r_{out}^2 - A_1}{\pi}} \tag{29}$$

Now after obtaining the equation linking the inner and outer radii, the equation for factor of safety against yield in the bogie beam should be determined. In order to do that, the maximal equivalent normal stress should be calculated. The change in radii values will affect the centeroid, the area moment of inertia and the polar area moment of inertia, which will in turn cause change to the maximal normal stress caused by bending and maximal shearing stresses caused by torsion. Using the minimal required value, 2, for the factor of safety against yield in the bogie beam, the minimal radii can be calculated.

The following equation shows how the minimal outer radii can be calculated:

$$\text{F.O.S}_Y(r_{out}) = \frac{\sigma_Y}{\sqrt{\left(\frac{M \cdot r_{out}}{\frac{1}{4}\pi {r_{out}}^4 - \frac{1}{4}\pi\left(\sqrt{\frac{\pi {r_{out}}^2 - A_1}{\pi}}\right)^4}\right)^2 + 3 \cdot \left(\frac{T \cdot r_{out}}{\frac{1}{2}\pi {r_{out}}^4 - \frac{1}{2}\pi\left(\sqrt{\frac{\pi {r_{out}}^2 - A_1}{\pi}}\right)^4}\right)^2}} = 2 \tag{30}$$

The inner and outer radii that satisfy the minimal required factor of safety against yield of 2 are:

$$r_{out} = 15.608\,\text{mm} \tag{31}$$

$$r_{in} = 14.057\,\text{mm} \tag{32}$$