Projektovanje algoritama

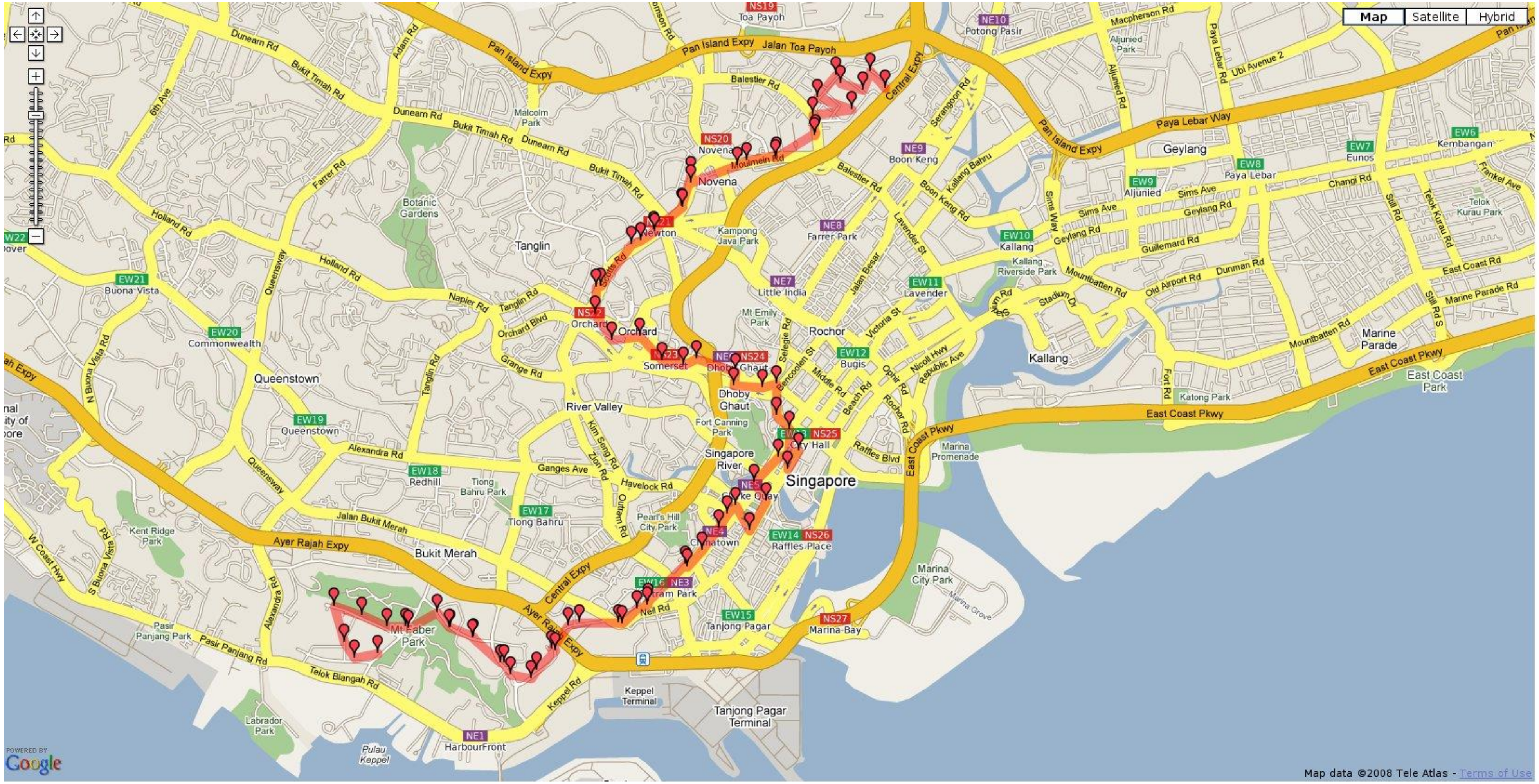
L11. Problem najkraće putanje

Današnje teme

- Problem najkraće putanje
- Bellman-Ford algoritam
- Najkraće putanje u usmerenim acikličnim grafovima
- Dijkstra algoritam

- **Lektira:**
  - 24. Single-Source Shortest Paths [str. 643-664]

Problem



Map
Satellite
Hybrid
Toa Payoh
Pan Island Expy
Jalan Toa Payoh
Potong Pasir
Macpherson Rd
Dunearn Rd
Bukit Timah Rd
Adam Rd
Balestier Rd
Central Expy
Malcolm Park
Novena
Moulmein Rd
Boon Keng
Paya Lebar Way
Geylang
Paya Lebar
Eunos
Kembangan
Aljunied
Botanic Gardens
Farrer Rd
Holland Rd
Newton
Kampong Java Park
Farrer Park
Tanglin
Kallang
Sims Ave
Geylang Rd
Guillemard Rd
Buona Vista
Napier Rd
Tanglin Rd
Orchard
Little India
Lavender
Mountbatten Rd
Old Airport Rd
Dunman Rd
East Coast Rd
Marine Parade Rd
Commonwealth
Orchard Blvd
Mt Emily Park
Rochor
Somerset
Dhoby Ghaut
Bugis
Marine Parade
Queenstown
Grange Rd
River Valley
Fort Canning Park
Katong Park
East Coast Pkwy
East Coast Park
Alexandra Rd
Singapore River
City Hall
Raffles Blvd
Marina Promenade
Singapore
Redhill
Tiong Bahru Park
Ganges Ave
Havelock Rd
Clarke Quay
Tiong Bahru
Pearl's Hill City Park
Kent Ridge Park
Jalan Bukit Merah
Ayer Rajah Expy
Bukit Merah
Chinatown
Raffles Place
Marina City Park
Outram Park
Neil Rd
Mt Faber Park
Pasir Panjang Park
Pasir Panjang Rd
Tanjong Pagar
Marina Bay
Telok Blangah Rd
Keppel Rd
Keppel Terminal
Tanjong Pagar Terminal
Labrador Park
Pulau Keppel
HarbourFront
Google
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Problem najkraće putanje - tipovi

- Problem najkraće putanje iz jednog čvora (single source)
- Problem najkraće putanje do jednog čvora (single destination)
- Problem najkraće putanje jednog para čvorova (single pair)
- Problem najkraće putanje svih parova čvorova (all pairs)

Problem najkraće putanje - osobine

- **Optimalna podstruktura najkraće putanje** – delovi najkraće putanje su sami po sebi najkraće putanje između odgovarajućih čvorova unutar najkraće putanje.
- **Negativna težina ivica** – je dozvoljena u problemu najkraće putanje.
- **Ciklusi** – nisu deo najkraće putanje.
  - **Pozitivni ciklusi** – uvećavaju putanju.
  - **Negativni ciklusi** – dovode do nepostojanja najkraće putanje.
  - **0-ciklusi** – mogu da se izbace.

Problem najkraće putanje - priprema

```
INITIALIZE-SINGLE-SOURCE(G,s)
  for each vertex v ∈ G.V
    v.d = Inf
    v.p = NIL
  s.d = 0
```

$\boldsymbol{T(V) = \theta(V)}$

d – gornja granica dužine najkraće putanje do datog čvora
p – čvor prethodnik u najkraćoj putanji do datog čvora

Problem najkraće putanje - relaksacija

```
RELAX(u,v,w)
   if v.d > u.d + w(u,v)
      v.d = u.d + w(u,v)
      v.p = u
```

$$T(n) = \theta(1)$$

d – gornja granica dužine najkraće putanje do datog čvora
p – čvor prethodnik u najkraćoj putanji do datog čvora

Bellman-Ford algoritam

```
BELLMAN-FORD(G,w,s)
  INITIALIZE-SINGLE-SOURCE(G,s)
  for i = 1 to len(G.V)-1
    for each edge (u,v) ∈ G.E
      RELAX(u,v,w)
  for each edge (u,v) ∈ G.E
    if v.d > u.d + w(u,v)
      return FALSE
  return TRUE
```

$$T(V,E) = \theta(VE)$$

Problem najkraće putanje u DAG

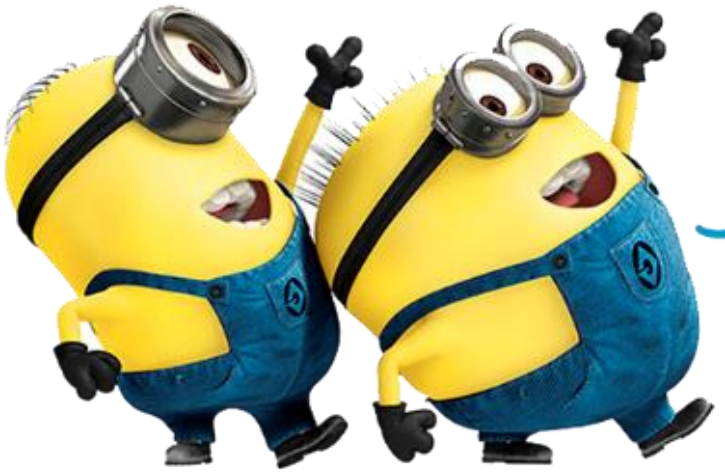
```
DAG-SHORTEST-PATHS(G,w,s)
  topologically sort vertices of G
  INITIALIZE-SINGLE-SOURCE(G,s)
  for each vertex u in topological order
    for each vertex v ∈ G.Adj[u]
      RELAX(u,v,w)
```

$$\boldsymbol{T(V,E) = \theta(V+E)}$$

Dijkstra algoritam

```
DIJKSTRA(G,w,s)
   INITIALIZE-SINGLE-SOURCE(G,s)
   S = ∅
   Q = G.V
   while Q ≠ ∅
      u = EXTRACT-MIN(Q)
      S = S + {u}
      for each vertex v ∈ G.Adj[u]
         RELAX(u,v,w)
```

$$\boldsymbol{T(V,E) = \theta\big((V+E)\lg V\big)}$$



thank you!

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