Exampled Atom Userdef Base

$$\frac{}{\Delta' \cup \{(r,U)\} \vdash (U, [], []) : (U, [])}$$

Exampled Atom Userdef Add Example

$$\frac{\Delta' \vdash s : r \qquad \Delta' \cup \{(r,U)\} \vdash (U, sl, enll) : (U, esl)}{\Delta' \cup \{(r,U)\} \vdash (U, s :: sl, enl :: enll) : (U, s :: esl)}$$

Exampled Atom Star

$$\frac{\Delta \vdash (dr, enll) : (dr, esl) \qquad validcombine(combine, enll)}{\Delta \vdash ((dr, enll)^*, combine_{enll}(enll)) : (dr, combine_{esl}(esl))}$$

Exampled Clause

$$\frac{\Delta \vdash (a_1, enll) : (a_1, [s_{1,1}, \ldots, s_{1,m}]) \qquad \ldots \qquad \Delta \vdash (a_n, enll) : (a_n, [s_{n,1}, \ldots, s_{n,m}]) \qquad s_1 \in \Sigma^* \qquad \ldots}{\Delta \vdash ([(a_1, enll); \ldots; (a_n, enll)], [s_1; \ldots; s_{n+1}], enll) : ([a_1; \ldots; a_n], [s_1; \ldots; s_{n+1}])[s_1 s_{1,1} \ldots s_{n,1} s_{n+1}, \ldots, s_1 s_{1,m} \ldots}$$

Exampled DNF Regex

$$\frac{\Delta \vdash (cl_1, enll_1) : (cl_1, esl_1) \qquad \ldots \qquad \Delta \vdash (cl_n, enll_n) : (cl_n, esl_n)}{\Delta \vdash ([(r_1, enll_1); \ldots; (r_n, enll_n)], interleave(enll_1, \ldots, enll_n)) : [r_1; \ldots; r_n], interleave(esl_1, \ldots, esl_n)}$$

Define $parented(enll)$ to be held if $enll = [0 :: enl; \ldots; (len(enll) - 1) :: enl]$ for some enl. If an $enll$is parented, define $parent(enll)$ as that enl

Define $\Delta \vdash (r, enll) : (r, esl)$ as closed if:

- $len(enll) = len(esl)$
- $parented(enll)$ and $parent(enll)$=[]

Let $combine : [0, n] \twoheadrightarrow [0, m]$

Define $validcombine(combine, enll)$ to be held if

- $parented(enll[combine^{-1}(i)])$
- $parent(enll[combine^{-1}(i)) = parent(enll[combine^{-1}(j)) \Rightarrow i = j$

If $validcombine(combine, enll)$
Define $combine_{enll}(enll) = [parent(enll[combine^{-1}(0)]); \ldots; parent(enll[combine^{-1}(len(enll))])]$

Define $combine_{esl}(esl) = [concat(esl[combine^{-1}(0)]); \ldots; concat(esl[combine^{-1}(len(esl))])]$

Define $interleaving(interleave)$ to be held, where $interleave(l_1, \ldots, l_n) = l_{n+1}$ and $l_i \cap l_j = \emptyset$ for $i, j \in [1, n]$, if

- $elements(l_1) \cup \ldots \cup elements(l_n) = elements(l_{n+1})$
- For $i \leq n$, $pos(x, l_i) < pos(y, l_i) \Rightarrow pos(x, l_{n+1}) < pos(y, l_{n+1})$