

CONSTANT EXAMPLE

$$s \in \Sigma^*$$

$$\Delta \vdash (s, enll) : s(s, \dots, s)$$

USERDEF BASE

$$\Delta' \cup \{(r, U)\} \vdash (U, [], []) : (U, [])$$

USERDEF EXAMPLE

$$\frac{\Delta' \vdash s : r \quad \Delta' \cup \{(r, U)\} \vdash (U, sl, enll) : (U, esl)}{\Delta' \cup \{(r, U)\} \vdash (U, s :: sl, enl :: enll) : (U, s :: esl)}$$

CONCAT EXAMPLE

$$\frac{\Delta \vdash (r_1, enll) : (r_1, [s_{1,1}, \dots, s_{1,n}]) \quad \Delta \vdash (r_2, enll) : (r_2, [s_{2,1}, \dots, s_{2,n}])}{\Delta \vdash ((r_1, enll)(r_2, enll), enll) : r_1 r_2, [s_{1,1} s_{2,1}, \dots, s_{1,n} s_{2,n}]}$$

OR EXAMPLE

$$\frac{\Delta \vdash (r_1, enll_1) : (r_1, esl_1) \quad \Delta \vdash (r_2, enll_2) : (r_2, esl_2)}{\Delta \vdash ((r_1, enll_1)|(r_2, enll_2), interleave(enll_1, enll_2)) : (r_1 | r_2, interleave(esl_1, esl_2))}$$

STAR

$$\frac{\Delta \vdash (r, enll) : (r, esl) \quad validcombine(combine, enll)}{\Delta \vdash ((r, enll)^*, combine_{enll}(enll)) : (r, combine_{esl}(esl))}$$

Define $parented(enll)$ to be held if $enll = [0 :: enl; \dots; (len(enll) - 1) :: enl]$ for some enl . If an $enll$ is parented, define $parent(enll)$ as that enl

Define $\Delta \vdash (r, enll) : (r, esl)$ as closed if:

- $len(enll) = len(esl)$
- $parented(enll)$ and $parent(enll) = []$

Let $combine : [0, n] \rightarrow [0, m]$

Define $validcombine(combine, enll)$ to be held if

- $parented(enll[combine^{-1}(i)])$
- $parent(enll[combine^{-1}(i)]) = parent(enll[combine^{-1}(j)]) \Rightarrow i = j$

If $validcombine(combine, enll)$

Define $combine_{enll}(enll) = [parent(enll[combine^{-1}(0)]); \dots; parent(enll[combine^{-1}(len(enll))])]$

Define $combine_{esl}(esl) = [concat(esl[combine^{-1}(0)]); \dots; concat(esl[combine^{-1}(len(esl))])]$

Define $interleaving(interleave)$ to be held, where $interleave(l_1, l_2) = l_3$ and $l_1 \cap l_2 = \emptyset$, if

- $elements(l_1) \cup elements(l_2) = elements(l_3)$
- $pos(x, l_1) < pos(y, l_1) \Rightarrow pos(x, l_3) < pos(y, l_3)$
- $pos(x, l_2) < pos(y, l_2) \Rightarrow pos(x, l_3) < pos(y, l_3)$