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A three-scroll chaotic attractor

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Abstract

This Letter introduces a new chaotic member to the three-dimensional smooth autonomous quadratic system family, which derived from the classical Lorenz system but exhibits a three-scroll chaotic attractor. Interestingly, the two other scrolls are symmetry related with respect to the z-axis as for the Lorenz attractor, but the third scroll of this three-scroll chaotic attractor is around the z-axis. Some basic dynamical properties, such as Lyapunov exponents, fractal dimension, Poincaré map and chaotic dynamical behaviors of the new chaotic system are investigated, either numerically or analytically. The obtained results clearly show this is a new chaotic system and deserves further detailed investigation.

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Keywords: Chaos; Lorenz attractor; Rössler attractor; Three-scroll attractor

1. Introduction

In 1963, using a simplified model of Rayleigh–Bénard convection, Lorenz found the first chaotic attractor in a smooth three-dimensional (3D) autonomous system with only two nonlinear terms [1]. The Lorenz system, as the first chaotic model, revealed the complex and fundamental behaviors of the nonlinear dynamical systems. But only until recently, was the existence of the Lorenz attractor of the Lorenz attractor rigorously proved with the aid of computer [2]. Although during the past half-century, extensive studies have been focused on chaos phenomena, and many fundamental characteristic of chaos have been identified [3–7], the link between the structure of the ordinary differential equations of the chaotic flows and the topology of their solutions is still missing, and the necessary conditions on the algebraic structure of ordinary differential equations producing chaos is still lacking, thus further research on these issues is still needed. Notably, during the development history of chaos theory, considerable research interests have been made in searching for new chaotic attractors in simple three-dimensional autonomous systems. In this pursuit, two utterly different research directions were developed: one attempt is to find the simplest three-dimensional chaotic flow than any previously known, where the simplicity refers to the algebraic representation rather than the physical process described by the equations or the topological structure of the strange attractor. Along this direction, in 1976, Rössler found a three-dimensional autonomous smooth chaotic system [8], which has only one quadratic nonlinearity and produces a one-scroll or single-spiral strange attractor. Later, other autonomous chaotic flows that are algebraically simpler have also been discovered [9]. Particularly, Sprott found 19 algebraically simple chaotic systems via exhaustive computer searching [10–12]. It is notable that the Rössler attractor and the Sprott's attractors (other than the Sprott B and Sprott C attractors [13]) are all algebraically simpler than the celebrated two-scroll Lorenz attractor, because they have a single nonlinear term while the Lorenz system has two. Furthermore, except the Sprott B and Sprott C systems that have a rotation symmetry and are two-scroll like the Lorenz attractor [13], the other Sprott's attractors behave similarly in that they all tend to resemble the one-scroll or single-spiral structure of the Rössler attractor. Another research direction is to gener-

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ate chaotic attractors in three-dimensional autonomous smooth chaotic system with more complicated topological structures, due not only to the theoretical interest but also more importantly to the potential real-world applications in various chaos-based technologies and information systems. In this effort, based on the concept of the generalized Lorenz system [14,15], some similar and new chaotic systems were recently coined, such as the Chen system [16], the Lü system [17], the generalized Lorenz system family [18], and the hyperbolic type of generalized Lorenz canonical form [19]. Same as the famous Lorenz system, all these Lorenz-like systems are smooth with two quadratic or nonlinear terms, have three real equilibriums and display a two-scroll butterfly-like attractor, which is different from the attractors of the generalized Chua's circuit at least in shape. Other than this group of this generalized Lorenz system, there are some other three-dimensional smooth autonomous chaotic systems reported in the literature, although not directly related to the Lorenz system [7,20–23].

While aiming for creating simple and no smooth chaotic systems, a notable one is Chua's circuit [24], coined in 1984, which also has a double-scroll attractor. When working with the classical Lorenz system, by using piecewise-linear continuous functions [25] or hysteresis-series function [26–28], chaotic attractors typically with double scrolls, three scrolls and four scrolls were also generated. But one must note that the nonlinearities of these systems are not smooth functions.

And recently, in the study of the Rössler-like 3D autonomous smooth system with quadratic nonlinear terms, Liu and Chen [29] found a chaotic system with a quadratic nonlinear term in each equation, which can display a two-scroll and also visually four-scroll chaotic attractor. In 2004, Lü et al. found one more simple three-dimensional autonomous chaotic system [30], which is also smooth with a quadratic nonlinear term in each equation, but can generate two single-scroll chaotic attractor simultaneously, on two complex double-scroll chaotic attractor simultaneously. In the same paper, the concept of the generalized Lorenz system was then extended, and a class of generalized Lorenz-like systems was discussed.

Nevertheless, until now, a complete classification of all possible types of the known three-dimensional chaotic attractors with their corresponding topological properties is still lacking. In this pursuit, by using the concept of a knot-holder, commonly called a template, Birman and Williams presented a first try on topological analysis of dynamical systems [31]. Noting that bounding tori organize branched manifolds (also known as a template or a knot holder) in the same way that branched manifolds organize the periodic orbits embedded in a strange attractor [6,32], recently, T. Tsankov and R. Gilmore further classified strange attractors with Lyapunov dimension $D_L < 3$ by bounding tori [33]. In Ref. [30], a simple and intuitionistic classification of the three-dimensional smooth autonomous chaotic systems was given. That is graphically, or according to the chaotic behaves to the autonomous systems in the phase space, their chaotic attractors generated in the aforementioned three-dimensional smooth autonomous chaotic systems can be classified as: one-scroll or single-spiral attractor, such as the Rössler attractors [8,9], the Sprott's attractors [10–12] and the Genesio attractor [21]; two-scroll or two-spiral attractor, such as the Lorenz attractor [1], the Chen attractor [16], the Lü attractor [17], the Rucklidge's attractor [20], the Liu attractor [22], the Sprott B and Sprott C attractor [13], the Burke–Shaw attractor [7] and the Shimizu–Morioka attractor [23], and obtained research results show some of these two-scroll chaotic systems actually belong to the same topological class with the Lorenz system acting as an emblem [7,13]; four-scroll attractor, such as the Liu–Chen attractor [29], etc. And very recently, another interesting yet rather complete classification of the three-dimensional chaotic systems was given in Ref. [34], for both the smooth and nonsmooth cases.

According to the simple classification of three-dimensional autonomous chaotic systems with smooth quadratic terms presented by Lü et al. [30], that is, from the viewpoint of the chaotic attractor solutions to the dynamical systems, one can clearly see an interesting fact: in the past half-century, among the multiscroll chaotic attractors generated in three-dimensional autonomous systems by (non)parametric polynomial transformation [35,36], there are one-scroll attractors, two-scroll attractors and four-scroll attractors that have been found in the three-dimensional smooth quadratic autonomous chaotic systems. And to the best of our knowledge, in the literature, there is no report of the finding of a three-scroll attractor generated in three-dimensional autonomous chaotic systems with smooth quadratic terms. Now, a natural question is whether there is a three-dimensional autonomous chaotic system with smooth quadratic terms, which can display a three-scroll chaotic attractor? This Letter will give a positive answer to this interesting question.

The rest of the Letter is organized as following. A three-scroll chaotic system is introduced in Section 2, which derived from Lorenz system via chaotifying or anticontrolling of chaos [5,37] from its continuous-time TS fuzzy model [38] using non-resonant parametric perturbation approach [39]. In Section 3, some basic properties of this interesting chaotic system are discussed. Finally, conclusions are drawn in Section 4.

2. The new chaotic system

The celebrated Lorenz system of ordinary differential equations is described by [1]:

$$\begin{aligned}
&\dot{x}(t) = \sigma\big(y(t) - x(t)\big),\\
&\dot{y}(t) = rx(t) - x(t)z(t) - y(t),\\
&\dot{z}(t) = x(t)y(t) - bz(t)
\end{aligned}\tag{1}$$

which is chaotic when

$$\sigma = 10, \quad r = 28 \quad \text{and} \quad b = 8/3$$

and displays the famous butterfly attractor with two scrolls (or two spirals) (see Fig. 1).

In order to have a simplest chaotic system —without symmetry property from the topological aspect—the Rössler sys-

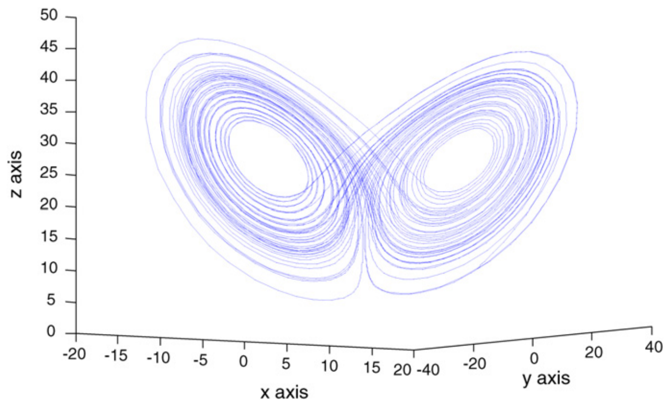


Fig. 1. The Z–X–Y 3D view of the classical Lorenz attractor.

tem was proposed as "a model of the Lorenz system". The Rössler system is described by the following equations [8]:

$$\begin{aligned}&\dot{x}(t)=-y(t)-z(t),\\&\dot{y}(t)=x(t)+ay(t),\\&\dot{z}(t)=b+\big(x(t)-c\big)z(t).\end{aligned}\tag{2}$$

When $a=0.2$, $b=0.2$ and $c=5.7$ system (2) is chaotic and exhibits an attractor with only one scroll.

Very recently [40], for the aim to explore the forming mechanism of its compound structure, we proposed a hybrid or nonresonant parametric perturbed TS fuzzy model for the classical Lorenz system as following:

Rule 1: IF $x(t)$ is about $F_{\min}$

$$\text{THEN } \dot{X}(t)=\left(A_1+\frac{1}{2}T_1A_1^2\right)X(t),$$

Rule 2: IF $x(t)$ is about $F_{\max}$

$$\text{THEN } \dot{X}(t)=\left(A_2+\frac{1}{2}T_2A_2^2\right)X(t)\tag{3}$$

where T_1 and T_2 are two controller parameters. Here

$$A_1=\begin{bmatrix}-\sigma & \sigma & 0\\ r & -1 & -M1\\ 0 & M1 & -b\end{bmatrix},\qquad A_2=\begin{bmatrix}-\sigma & \sigma & 0\\ r & -1 & -M2\\ 0 & M2 & -b\end{bmatrix}$$

and $X(t)=(x(t),y(t),z(t))^T$ with membership functions $F_{\min}$ and $F_{\max}$ given as

$$F_{\min}=\frac{-x(t)+M2}{M2-M1},\qquad F_{\max}=\frac{x(t)-M1}{M2-M1}$$

with $[M1,M2]$ is chosen as $[-30,30]$.

Via varying the nonresonant controller parameters T_1 and T_2 of the hybrid TS fuzzy model-based Lorenz system (3), a scenario is proposed illustrating that besides the compound topological structure obtained by the mirror-operation forming mechanism [26,27], the Lorenz attractor still has more abundant and complex dynamical behaviors obtained by a new forming mechanism, named the compression and pull forming mechanism [40–42]. That is, via simply varying the controller parameter when $T_1=T_2$, the Lorenz attractor can be compressed distorted along the double scrolls (see Fig. 2(a), (b)), or the connect parts of the double scrolls can become dominant gradually and pull the double scrolls highly to the vertical direction (or along the diagonal direction of the two scrolls) (see Fig. 2(c), (d)). Hence, one may ask whether there is a three-dimensional smooth autonomous chaotic system producing a chaotic attractor, not only of the similar and fundamental double-scroll topological structure of the famous Lorenz attractor, but more importantly, with another scroll along the diagonal direction of the butterfly-like two scrolls? Under the condition $M1=-M2$ for the sake of simplicity, by defuzzing the hybrid TS fuzzy Lorenz model (3) and after some tedious algebraic manipulations, we obtain a three-dimensional time-varying system of the form shown in the following:

$$\begin{aligned}&\dot{x}(t)=Ax(t)+By(t)+Cx(t)z(t),\\&\dot{y}(t)=Dx(t)+Ey(t)+Fx(t)z(t),\\&\dot{z}(t)=Gx^2(t)+Hx(t)y(t)+Iz(t),\end{aligned}\tag{4}$$

here the parameters of the system (4) are given as

$$\begin{aligned}&A=-\sigma+0.5T\big(\sigma^2+\sigma r\big),\qquad B=\sigma+0.5T\big(-\sigma^2-\sigma\big),\\&C=-0.5T\sigma,\qquad D=r+0.5T(-r-\sigma r),\\&E=-1+0.5T(1+\sigma r+M1M2),\\&F=-1+0.5T(1+b),\qquad G=0.5rT,\\&H=1-0.5T(1+b),\qquad I=-b+0.5T\big(M1M2+b^2\big)\end{aligned}$$

with $T=T_1=T_2$, $\sigma=10$, $r=28$ and $b=8/3$.

One can easily see that the system (4) is essentially a time-varying system with the varying of the variable T, and the Lorenz system (1) is just a special case of the system (4) when $T=0$. Hence, by varying the nonresonant control parameter T, abundant and complex dynamical behaviors about the compound structures of the chaotic Lorenz attractor can be observed [40], which may contribute to a better understanding of the essence of Lorenz system and its generalization.

Now in order to obtain the aforementioned three-scroll chaotic attractor from a three-dimensional smooth autonomous system, with the smooth autonomous system holding some similar basic algebraic structure of the Lorenz system, we let all the parameter values of system (4) being constant and $A=-B=a$, $F=-1$, $H=1$ as for the chaotic Lorenz system. Then, with trial-and-error numerical simulations, we found a new chaotic attractor from the following three-dimensional quadratic autonomous system with four smooth quadratic terms:

$$\begin{aligned}&\dot{x}(t)=a\big(y(t)-x(t)\big)+dx(t)z(t),\\&\dot{y}(t)=kx(t)+fy(t)-x(t)z(t),\\&\dot{z}(t)=-ex^2(t)+x(t)y(t)+cz(t)\end{aligned}\tag{5}$$

which has the strange attractor shown in Fig. 3 when $a=40$, $k=55$, $c=11/6$, $d=0.16$ and $e=0.65$, $f=20$.

From Fig. 3, one can clearly see that the three-dimensional quadratic autonomous chaotic system (5) can exhibit an attractor with three scrolls. Hence, it has a more complex topological structure than the classical two-scroll Lorenz attractor. Notably, the third scroll of the new chaotic attractor connects the other

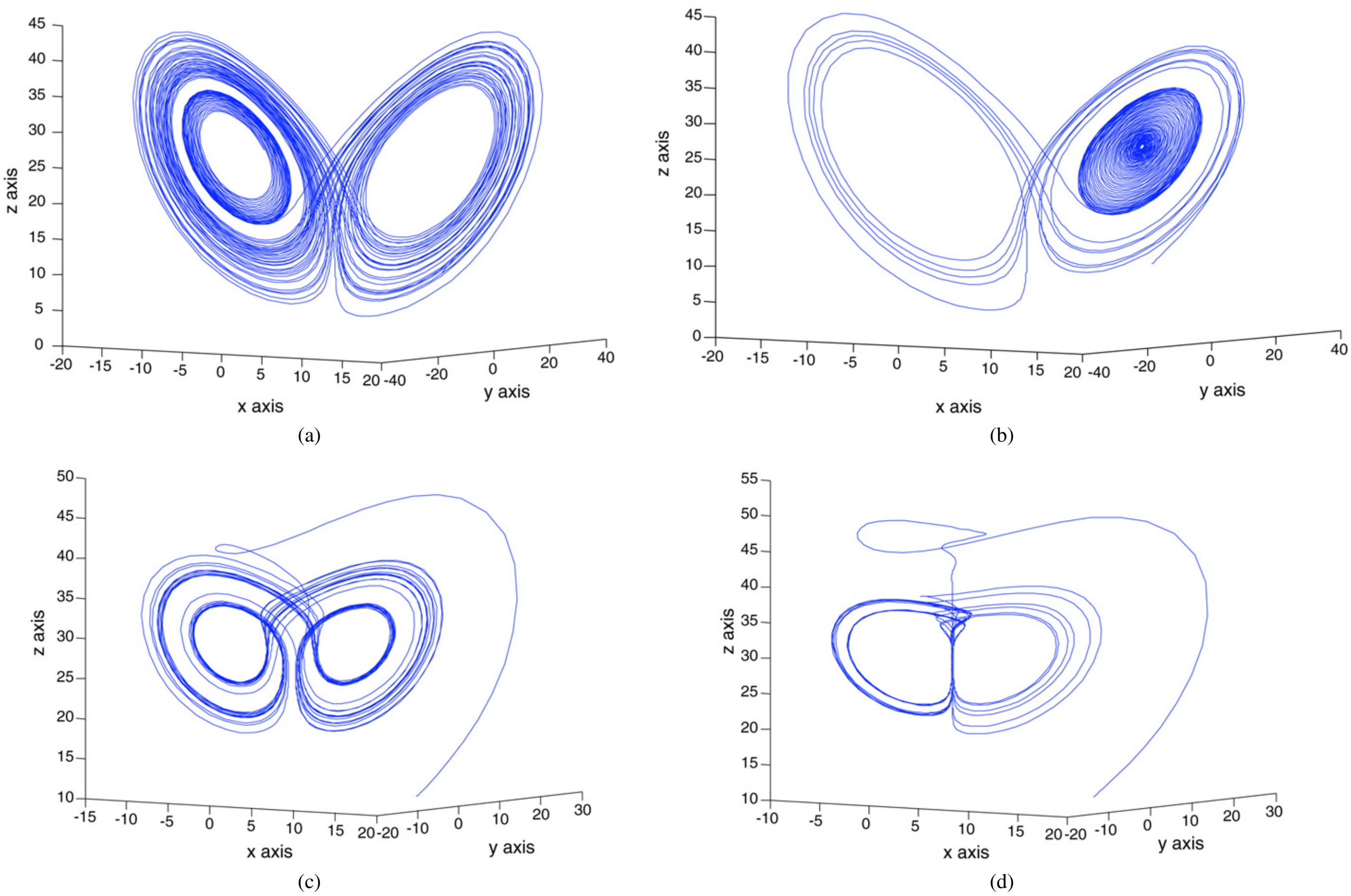


Fig. 2. The Y–X–Z 3D view of phase portraits of the hybrid TS fuzzy Lorenz system. (a) When $T_1 = T_2 = 0.0012946$; (b) when $T_1 = T_2 = 0.00123$; (c) when $T_1 = T_2 = -0.0055$; (d) when $T_1 = T_2 = -0.00558$. $(x_0, y_0, z_0) = (10, 10, 10)$.

two scrolls exactly along their diagonal direction (see Fig. 3(a), (b), (d)). It is noticeable that the autonomous chaotic system (5) is just the special case of the most general form for flows with $R_Z(\pi)$ symmetry [13] or of the generalized Lorenz system family [30], but it was obtained by a new way.

3. Some basic properties of the three-scroll chaotic system

Now, some basic properties of the system (5) are analysed.

(1) Firstly, it is easy to see the invariance of system term under the coordinate transformation $(x, y, z) \to (-x, -y, z)$, i.e. the system has a rotation symmetry around the z-axis as for the Lorenz system. But unlike the Lorenz system, where the chaotic dynamics only remain confined to positive half-space with respect to the z state variable (see Fig. 1), the new system's chaotic dynamics can extend to the negative half-space with respect to the z state variable (see Fig. 3).

(2) Since

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -a + f + c + dz \tag{6}$$

as long as $\nabla V < 0$ at $z < (a - f - c)/d$, system (5) is dissipative. The 3D dissipative systems have strange attractors with dimension between two and three, while the conservative systems fill a three-dimensional volume. The Lyapunov exponent spectrum of system (5) is found to be $L_1 = 0.23155$, $L_2 = 0$, $L_3 = -1.9872$ for initial value $(2, 2, 2)$, therefore, the Lyapunov dimension of this system is

$$\begin{aligned} D_L &= j + \frac{1}{|L_{j+1}|} \sum_{i=1}^{j} L_i = 2 + \frac{L_1 + L_2}{|L_3|} \\ &= 2 + \frac{0.23155}{|-1.9872|} = 2.1165 \end{aligned}$$

which means the system (5) is really a dissipative system, and the Lyapunov dimension of this system is fractional. The fractal nature of an attractor does not merely mean this system has nonperiodic orbits; it also causes nearby trajectories to diverge. As all strange attractors, orbits that are initiated from different initial conditions soon reach the attracting set, but two nearby orbits does not stay close to each other, they soon diverge and follow utterly different trajectories in the attractor [43]. Therefore, there is really strange chaos in this system. More interestingly, the third Lyapunov exponent L_3 of the new system is about -2 and that of the Lorenz system is about -15 [43], therefore the new chaotic system has a slower contracting rate in phase space than the Lorenz system's, which means the new system has a lower degree of orbital disorder and randomness. It can be seen that the x-time series of the Lorenz system and

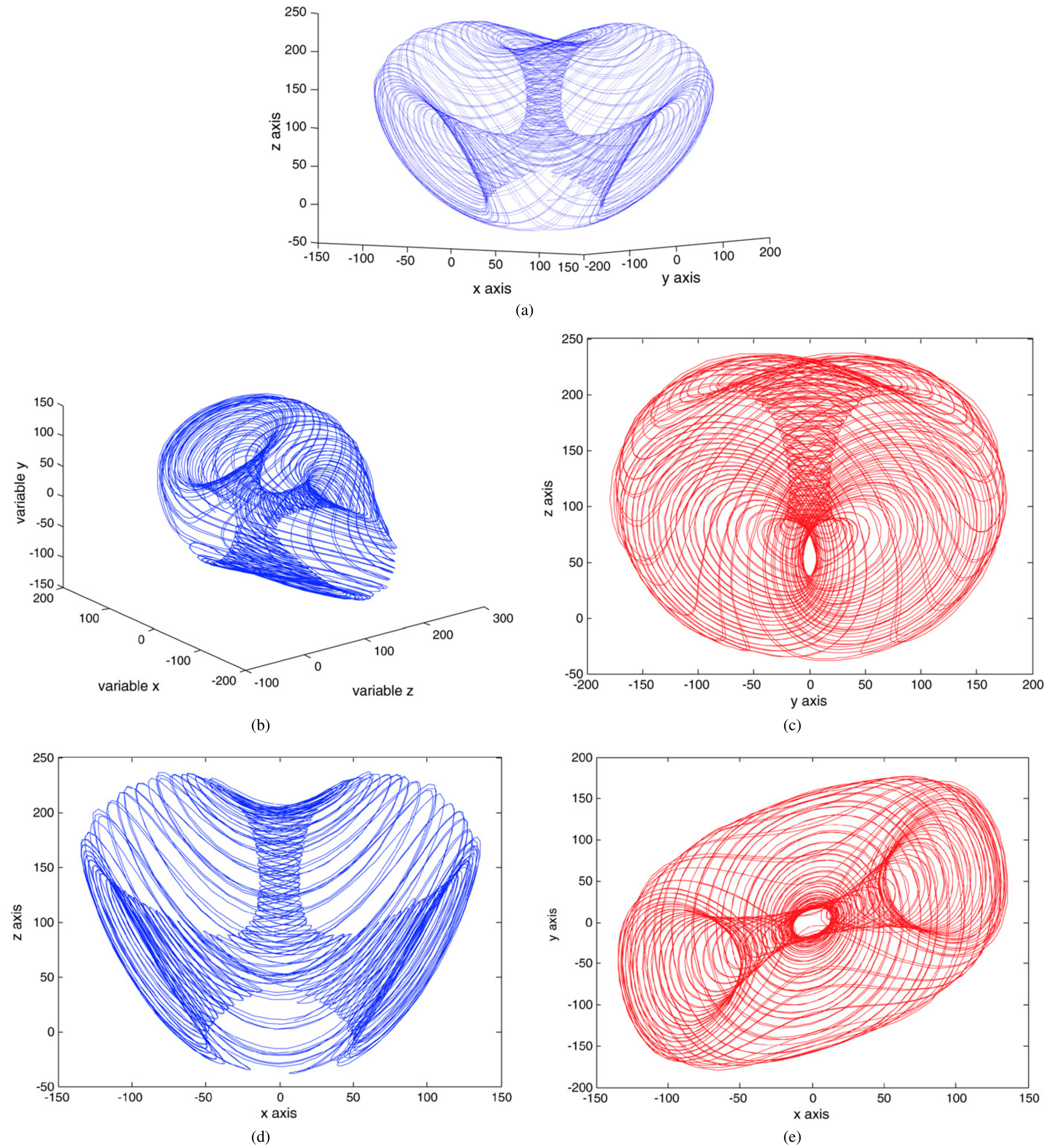


Fig. 3. Phase portraits of the new attractor. (a) The Y–X–Z 3D view of the attractor, (b) The Z–X–Y 3D view of the attractor, (c) Y–Z phase plane the attractor, (d) X–Z phase plane the attractor, (e) X–Y phase plane the attractor. $(x_0, y_0, z_0) = (2, 2, 2)$.

the x-time series of the new system and their related spectra (Fig. 4) are consistent with the theoretical analysis.

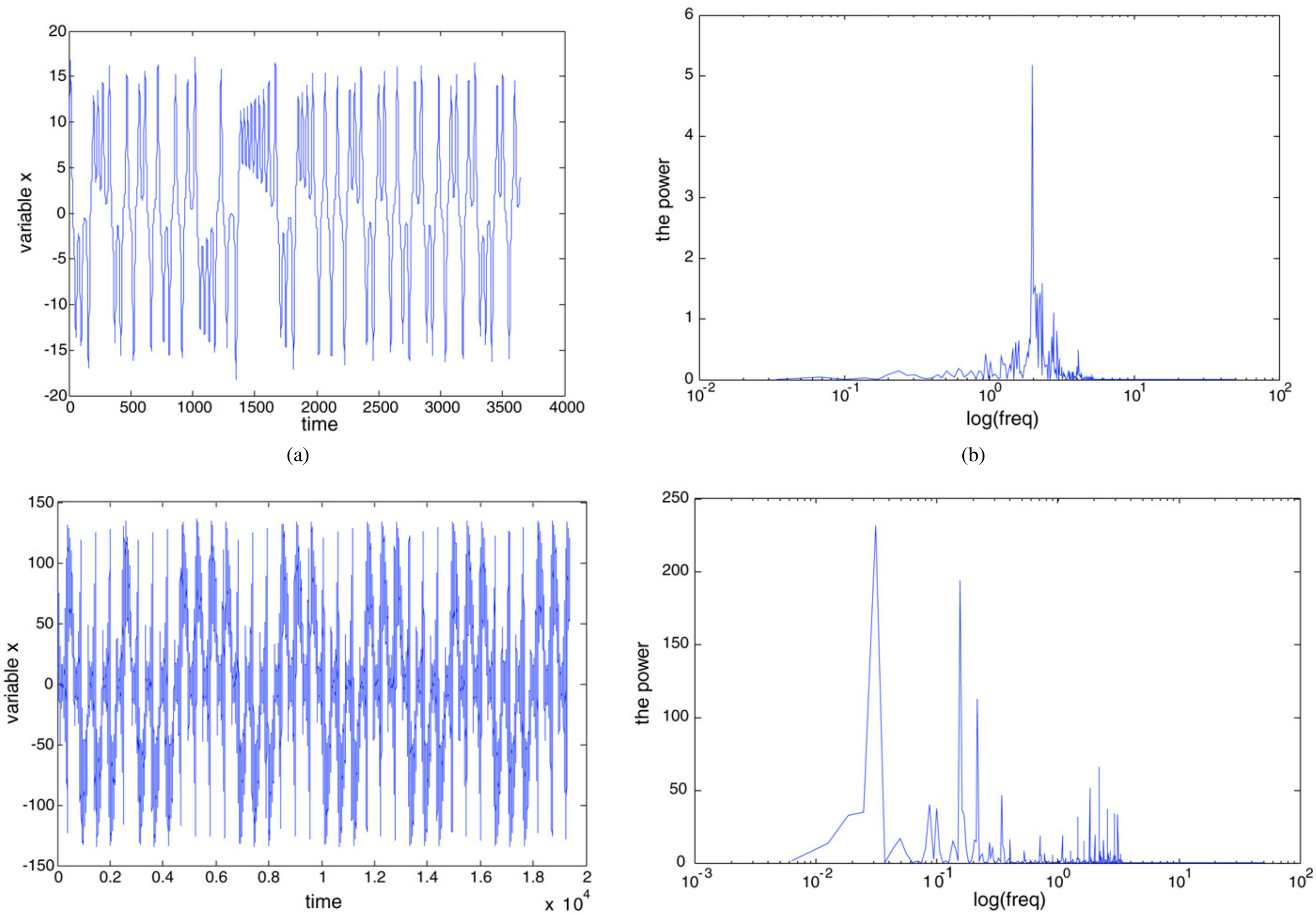
(3) The fixed points of system (5) can be easily found by solving the following system of equations:

$$a(y-x)+dxz=0, \qquad kx+fy-xz=0,$$
$$-ex^2+xy+cz=0. \tag{7}$$

The system has three fixed points, which are respectively described as follows:

$$O(0,0,0), \qquad S^-(-x_0,-y_0,z_0), \qquad S^+(x_0,y_0,z_0).$$

After operating above three nonlinear algebraic equations we obtain



(a) (b) (c) (d)

Fig. 4. (a) Time response of the state x of Lorenz system. (b) Related spectrum of the state x of Lorenz system. (c) Time response of the state x of the new system. (d) Related spectrum of the state x of the new system.

$$O(0,0,0),$$

$$S^-\left(-\sqrt{\frac{ca(k+f)}{ea+efd+kd-a}}, -\frac{a-kd}{a+fd}\sqrt{\frac{ca(k+f)}{ea+efd+kd-a}}, \frac{a(k+f)}{a+fd}\right),$$

$$S^+\left(\sqrt{\frac{ca(k+f)}{ea+efd+kd-a}}, \frac{a-kd}{a+fd}\sqrt{\frac{ca(k+f)}{ea+efd+kd-a}}, \frac{a(k+f)}{a+fd}\right).$$

For the equilibrium $O(0,0,0)$, system (5) is linearized and the Jacobian matrix is defined as

$$J_0 = \begin{bmatrix} -a+dz & a & dx \\ k-z & f & -x \\ -2ex+y & x & c \end{bmatrix} = \begin{bmatrix} -40 & 40 & 0 \\ 55 & 20 & 0 \\ 0 & 0 & 11/6 \end{bmatrix}.$$

In order to gain its eigenvalues, we let

$$|\lambda I - J_0| = 0.$$

The eigenvalues corresponding to equilibrium $O(0,0,0)$ are obtained as follows

$$\lambda_1 = -65.6776, \qquad \lambda_2 = 45.6776, \qquad \lambda_3 = 1.8333.$$

Here λ_1 is a negative real number, λ_2 and λ_3 are two positive real numbers, this means the equilibrium $O(0,0,0)$ is a saddle point. Hence the equilibrium $O(0,0,0)$ is unstable and yields a possibility for chaos.

Since $a = 40$, $k = 55$, $c = 11/6$, $d = 0.16$ and $e = 0.65$, $f = 20$ are all positive real numbers in system (5), we have $ea + efd + bd - a = -3.12 < 0$. Hence, x_0 and y_0 are complex number, that is the system (5) has one real equilibrium $O(0,0,0)$ and two complex fixed points $S^-(-x_0, -y_0, z_0)$, $S^+(x_0, y_0, z_0)$. It is known that the Lorenz system has three real fixed points and displays a double-scroll attractor, while the new chaotic system exhibits a three-scroll attractor. Hence, they are not belonging to the same topological class.

According to the results obtained above, the new chaotic attractor has three scrolls, which can be further clearly seen from the Poincaré map as (Fig. 5), that is, from the section of the attractor looking like three circles.

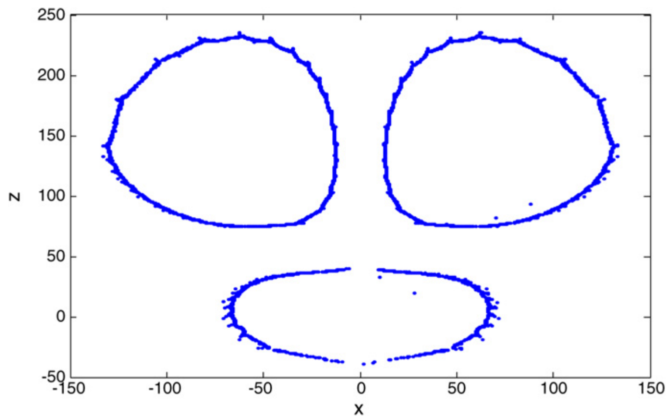


Fig. 5. The Poincaré map of X–Z plane.

4. Conclusion

Although the necessary conditions on the algebraic structure of ordinary differential equations that ensure a chaotic solution is observed in a given system are still lacking, it is believed that there are still many chaotic prototype flows quietly be there and waiting to be explored. In this Letter, we have proposed a fully new smooth autonomous chaotic system derived from the Lorenz system and capable of generating a complex three-scroll attractor. Particularly, two scrolls of the three-scroll attractor—to link with the two scrolls commonly observed in the Lorenz system—are symmetry related with respect to the z-axis, while the third—a new one—is around the z-axis. Hence, the three-scroll attractor has a different property. From this point of view, the finding of this particular example of dissipative chaotic system illustrates the necessary requirements of chaos of its type and deserves further detailed investigation on its topological structure in the near future.

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