Financial Instruments and Pricing
Problems 1

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1 Ex 1

In this exercise we assume that an amount of money P which is on the account only D days of the month accumulates interests equal to:

$$I = P \cdot r \cdot \frac{D}{365} \tag{1.1}$$

a. Cash flows in October account for 194.904 PLN $\approx$ 194.90 PLN of interest.

b. Cash flows in November account for 125.68 PLN of interest.

2 Ex 2

a. Formula when we assume no interest is added

$$t = \frac{100}{r}$$

so for $r = 6\%$ p.a. we have $t = 16\frac{2}{3}$ years.

b. Formula for capitalization n times a year

$$t = \frac{1}{n} \log_{\left(1+\frac{r}{n}\right)}(2)$$

so that we have:

- monthly: 11.5813 years
- quarterly: 11.6389 years
- yearly: 11.8957 years

c. In the case of the continuous capitalization we use that

$$\lim_{n\to\infty}(1 + \frac{r}{n})^n = e^r,$$

which leads to:

$$t = \frac{1}{r}\ln 2$$

For r = 6% p.a., t= 11.5525 Years

3 Ex 3

General solution for 1 year period with n capitalizations is given by the equation:

$$\frac{\left(1+\frac{r}{n}\right)^n}{1+i} = 1$$

where i is the inflation rate, r the interest and n is the frequency of the capitalization. In the continuous case the formula takes a form:

$$\frac{e^r}{1+i} = 1.$$

Results:

- Yearly: 2.5%,
- half-yearly: 2.4845%,
- quarterly: 2.4769 %,
- Monthly: 2.4718 %,
- Continuous: 2.4692 %.

4 Ex 4

General equation for the annual effective rate of interest is given by the equation:

$$\frac{r}{n} \equiv r_n = (1+y)^{\frac{1}{n}} - 1$$

and in the continuous case

$$r_\infty = \log(1+y)$$

where r_∞ is defined from equation

$$\lim_{n\to\infty}\left(1+\frac{r_\infty}{n}\right)^n = e^{r_\infty}$$

and is taken to be in units of p.p.a.
Which gives the equivalent rates:

- half-yearly: 2.46951%
- monthly: 0.407412%
- weekly (for 52 weeks in year): 0.0938713%
- daily: 0.0133681%
- continuous : 4.87902% p.a.

5 Ex 5

The problem is easily solved by analysing the equation

$$\begin{aligned}
1 + y &= \left(1 + \frac{r}{n}\right)^n \\
y &= \left(1 + \frac{r}{n}\right)^n - 1 \\
&= 1 + n \cdot \frac{r}{n} + O\big(r^2\big) - 1 \\
&= r + O(r^2)
\end{aligned}$$

if we note that $O(r^2) > 0$, then it is obvious that $y > r$, QED.

Alternative solution:

Initial equation reads:

$$1 + y = \left(1 + \frac{r}{n}\right)^n,$$

from this we have:

$$1 + \frac{r}{n} = \sqrt[n]{1+y} = \sqrt[n]{1^{n-1}(1+y)} \leq \frac{(n-1)\cdot 1 + (1+y)}{n} = 1 + \frac{y}{n},$$

where we have used the inequality between arithmetic and geometric averages. Finally we have:

$$r < y \quad \text{for } n > 1.$$

6 Ex 6

First we assume that the tax t is taken after the interest is added to the account, ie. after the 91 days. Thus it is easily seen that the general formula for competitive rate deposit r against no-taxation rate r_t will be given by soltuion of an equation

$$(1-t)\left(\left(1 + \frac{r}{365}\right)^{91} - 1\right) = \left(\left(1 + \frac{r_t}{365}\right)^{91} - 1\right)$$

To get the answer we solve the above for $t = 19\%$, $r_t = 6\%$. Thus we conclude that for a competitive interest rate we need to have

$$r = 7.39465\%$$

When we apply the continous capitalization our equation takes the form:

$$(1-t)(e^{91r/365} - 1) = e^{91r_t/356} - 1, \tag{6.1}$$

and the numerical solution for r is:

$$r = 7.39451\% \tag{6.2}$$

7 Ex 7

In the first part the equation may be written as:

$$\sum_{i=1}^{8} x\left(1+\frac{r}{n}\cdot i\right) = 10000 \text{ PLN}. \tag{7.1}$$

Then using numerical values $r = 4\%$ and $n = 4$ we get:

$$x = 1196.17 \text{ PLN}. \tag{7.2}$$

b) We are interested in a solution of the equation:

$$\sum_{i=1}^{8} x\left(1+\frac{r}{n}\right)^{i} = 10000 \text{ PLN}, \tag{7.3}$$

where I assume that the last money we send to the bank account spends there only 1 month (so the first works for 8 months). The solution is unnecessarily complicated to be shown here. Using numerical values $r = 4\%$ and $n = 4$ we obtain:

$$x = 1194.95 \text{ PLN}. \tag{7.4}$$

8 Ex 8

Assuming monthly capitalization, in both situations with growing and decreasing interest rate from 1% p.a. to 12% p.a (banks A and C). effective Yield is going to be

$$y_A = y_C = p_0 \prod_{i=1}^{12}(1+\frac{i}{100\cdot 12}) = 1.06692 p_0,$$

in compare to bank B offer with constant interest rate 6.5% p.a. with Yield

$$y_B = p_0(1+\frac{6.5}{100\cdot 12})^{12} = 1.06697 p_0$$

a) So obviously we should choose bank B.
b) In this case all situations considered are equal and total gain is going to be 6.5% of starting deposit.

$$y_A = y_B = y_C = Y = p_0(1+0.065)$$

9 Ex 9

a), b) and c) In the case of equal principal payments we calculated client's cashflows, prepared Amortization Schedule, in which we included total amount of interest paid to the bank.

Period (quartals)	Principal Value	Payment CF	Interest CF	Principal CF	Final Principal
1	10 000.00	2 750.00	250.00	2 500.00	7 500.00
2	7 500.00	2 687.50	187.50	2 500.00	5 000.00
3	5 000.00	2 625.00	125.00	2 500.00	2 500.00
4	2 500.00	2 562.50	62.50	2 500.00	-
Total	-	-	625	-	-

in the case of equal payments we get similar table

Period (quartals)	Principal Value	Payment CF	Interest CF	Principal CF	Final Principal
1	10 000.00	2 658.18	250.00	2 408.18	7 591.82
2	7 591.82	2 658.18	189.78	2 468.38	5 123.44
3	5 123.44	2 658.18	128.09	2 530.09	2 593.35
4	2 593.35	2 658.18	64.83	2 593.35	-
Total	-	-	632.70	-	-

d) From the perspective of the bank both systems are equally favourable. In idealistic world, where client can reinvest all of his borrowed money for the same rate as bank, it should also be identically for him. Unfortunatelly that is not the case, and the second option, in which he is paying less interest to bank is better for the client.

10 Ex 10

e) and f) are collected in the table below. The loan is assumed to have $20 \cdot 12 = 240$ payments at rate $r = WIBOR3M + 2\% = 6\%$, and only three of them are given as after this period the rate will be reevaluated. The answers to questions are given **in bold** with additional information contained in the table.

Period (months)	Principal Value	Payment CF	Interest CF	Principal CF	Final Principal
1	200000.00	**1432.86**	1000.00	432.86	199567.14
2	199567.14	**1432.86**	997.84	435.03	199132.11
3	199132.11	**1432.86**	995.66	437.20	**198694.91**

g) Here we assume that we start with new credit, having 237 payments at rate $r = 5\% + 2\% = 7\%$ and with a starting value indicated by the final value after first 3-month period. Again, the monthly payments are given in bold

Period (months)	Principal Value	Payment CF	Interest CF	Principal CF	Final Principal
1	198694.91	**1549.45**	1159.05	390.40	198304.51
2	198304.51	**1549.45**	1156.78	392.68	197911.83
3	197911.83	**1549.45**	1154.49	394.97	197516.86