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Lecture 2

Bonds

Financial instruments and pricing

Fall 2018

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Bonds

The lecture about bonds will be split in three parts:

- ❖General overview
  (definitions and taxonomy)
- ❖Cash flow calculus
  (pricing and day-count conventions)
- ❖Pricing and risk
  (yield and sensitivity)

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Bonds – general overview

❖ The bond is a debt security, under which the issuer owes the holders a debt and (depending on the terms of the bond) is obliged to pay them interest (the coupon) or to repay the principal at a later date, termed the maturity date. (from Wikipedia)

Bonds – general overview

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  - In English, the word "bond" (PL: obligacja) relates to the etymology of "bind". In the sense "instrument binding one to pay a sum to another" (from Wikipedia)

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Bonds – general overview: securities

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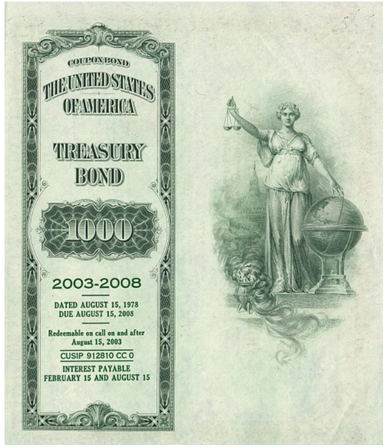
  - Bonds and stocks / shares (PL: akcje) are both securities (PL: papiery wartościowe). The major differences:
    - (capital) stockholders have an equity stake in a company (i.e. they are owners), they are entitled to profits (dividends)
    - bondholders have a creditor stake in the company (i.e., they are lenders), they are entitled to get interest (coupons)
    - bondholders have priority over stockholders: by law, if a company goes bankrupt they are paid before shareholders (i.e. bonds carry less risk than shares) *
    - bonds usually have a defined term, or maturity (PL: data wykupu), after which the bond is redeemed, whereas stocks typically remain outstanding indefinitely
  - E.g. LOANS are NOT securities, but just contracts (different legal status than bonds)
  - As securities bonds can both have material (document) form or be dematerialized.



* The priority may depend on the type of a bond, e.g. subordinated bonds (PL: obligacje podporządkowane) have lower priority than normal bonds but still higher priority than shares.

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Bonds – general overview: issuers

- The bond is a debt security, under which the issuer owes the holders a debt and (depending on the terms of the bond) is obliged to pay them interest (the coupon) or to repay the principal at a later date, termed the maturity date. (from Wikipedia)
  - A bond is a form of loan: the issuer (PL: emitent) of the bond is the borrower (debtor)
  - Bonds provide the issuer with external funds to finance his financial needs
  - Issuers:
    - Government (i.e. (State) Treasury Bonds, T-Bonds, T-Notes, T-Bills). That may be both federal (e.g. US Treasury) and local governments (e.g. State of California)
    - State agencies. There is a difference between US Fed. Gov. Ag. (e.g. Fed. Hausing Administr.) and US Government-Sponsored Enterprises (e.g. Fannie Mae)
    - Municipalities. These are usually big cities with their own budgets.
    - Banks. These bonds are more risky than usual deposits !
    - Companies, i.e. corporate bonds, may vary in credit risk from „investment grade" (rating** AAA – BBB) to „junk bonds" („high-yield bonds")

    Credit / Default RISK*
  - The primary market: auctions of T-Bonds, other issuers usually use banks to find investors

*Even Government Bonds are NOT completely (default) risk-free when issued in other currency

** Major Rating Agencies are: Standard & Poor's (AAA, AA+, AA, AA-, …) and Moody's (Aaa, Aa1, Aa2, Aa3, …)

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Bonds – general overview: holders / trading

- The bond is a debt security, under which the issuer owes the holders a debt and (depending on the terms of the bond) is obliged to pay them interest (the coupon) or to repay the principal at a later date, termed the maturity date. (from Wikipedia)
  - A bond is a form of loan: the holder of the bond is the lender (creditor)
  - Bond is usually profitable (e.g. one gets interest) and less risky than shares
  - There are many types of the holders, ranging from central banks, commercial banks, insurance companies, investment / pension funds, corporates, … to private individuals
  - Very often bonds are negotiable, i.e. the ownership of the bond can be transferred in the secondary market

Liquidity RISK

    - Exchange traded: highly regulated and centralized market. Settlement through specialized Clearing Institutions. (trading / pricing / settlement rules & conventions, commissions !)
    - Over-The-Counter (i.e. OTC): less but still regulated local markets organized by banks / securities firms acting as dealers who quote BID – ASK prices. Settlement through Transfer Agents. (trading / pricing / settlement rules & conventions, NO commissions !)
    - Bonds traded privately (usually banks / investment firms mediate) *

  - Bearer bonds (PL: o. na okaziciela) vs Registered / named holder bonds (PL: o. imienne)

* Different legal requirements may apply for „private" and „public" securities

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Bonds – general overview: maturity

- The bond is a debt security, under which the issuer owes the holders a debt and (depending on the terms of the bond) is obliged to pay them interest (the coupon) or to repay the principal at a later date, termed the maturity date. (from Wikipedia)
  - A bond is a form of loan: the principal (PL: kapitał), i.e. the nominal / face / par value must be repaid (PL: wartość nominalna)
  - One usually distinguishes between:
    - Short-term bonds (maturity ≤ 1Y), which are part of "money markets"(PL: rynek pieniężny), i.e. they are issued to finance current liquidity needs (e.g. current production). Short-term bonds are sometimes called "bills", e.g. US T-Bills
    - Medium-term bonds (1Y < maturity ≤ 10Y), which are part of "capital markets" (PL: rynek kapitałowy), i.e. they are issued to finance longer-term investment needs (e.g. new factory) Medium-term bonds are sometimes called "notes", e.g. US T-Notes
    - Long-term bonds (maturity > 10Y), which are also part of "capital markets", e.g. US T-Bonds

  - Perpetual bonds have no maturity (they are not popular now but existed in the history, e.g. British or US Consols)
  - Serial bonds (not very popular) are redeemed in instalments (e.g. each year some part of the principal is repaid)

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Bonds – general overview: interest (coupons)

- The bond is a debt security, under which the issuer owes the holders a debt and (depending on the terms of the bond) is obliged to pay them interest (the coupon) or to repay the principal at a later date, termed the maturity date. (from Wikipedia)
  - A bond is a form of loan: usually bondholders get interest (called coupons)
  - One can distinguish:

- Zero coupon bonds * don’t pay interest. They are issued at a discount to nominal (par) value
- Fixed rate bonds * with nominal interest rate r% = const. (as we will see the value of a coupon may still vary slightly from period to period due to various day-count conventions)
- Floating rate bonds ** with interest rate linked to some reference rate, e.g. r% = LIBOR + premium, or indexed to some economic data, e.g. inflation rate (e.g. Polish “saving” T-bonds) or GDP growth rate. Sometimes not only interest but also principal is indexed.
- Other, e.g. structured bonds (in fact they are mixtures of bonds & options) with coupons depending on some external “events” / “data” (e.g. stock index / currency / share prices, ….)

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* Will focus on that

** How to forecast future interest rates will be discussed later

Bonds – general overview: other conditions

- The bond is a debt security, under which the issuer owes the holders a debt and (depending on the terms of the bond) is obliged to pay them interest (the coupon) or to repay the principal at a later date, termed the maturity date. (from Wikipedia)
  - Bonds may be issued in foreign currency
    - It might be easier to sell the bonds abroad (bigger market)
    - Cost of financing in foreign currency may be lower (and, as will be discussed later, all payments may be swapped to domestic currency)
  - Bonds may have additional conditions, e.g.
    - Callable bonds, where the issuer has an option to redeem the bonds before maturity
    - Puttable bonds*, where the holder has the right, but not the obligation, to demand early repayment of the principal. The put option is exercisable on one or more specified dates. **
  - There may be some non-standard bonds, e.g.
    - Covertible / exchangeable bonds (PL: obligacje zamienne) which can be converted into shares of the issuer / other company (they are in fact a mixture of bonds and warrants / options for the shares)

* In many bonds the holders have right to demand early redemption in the event of the issuer default (these are NOT „puttable bonds" as this is rather legal than financial issue)

** E.g. a holder of Polish "savings" T-Bonds can demand early redemption at par at any date (at some cost)

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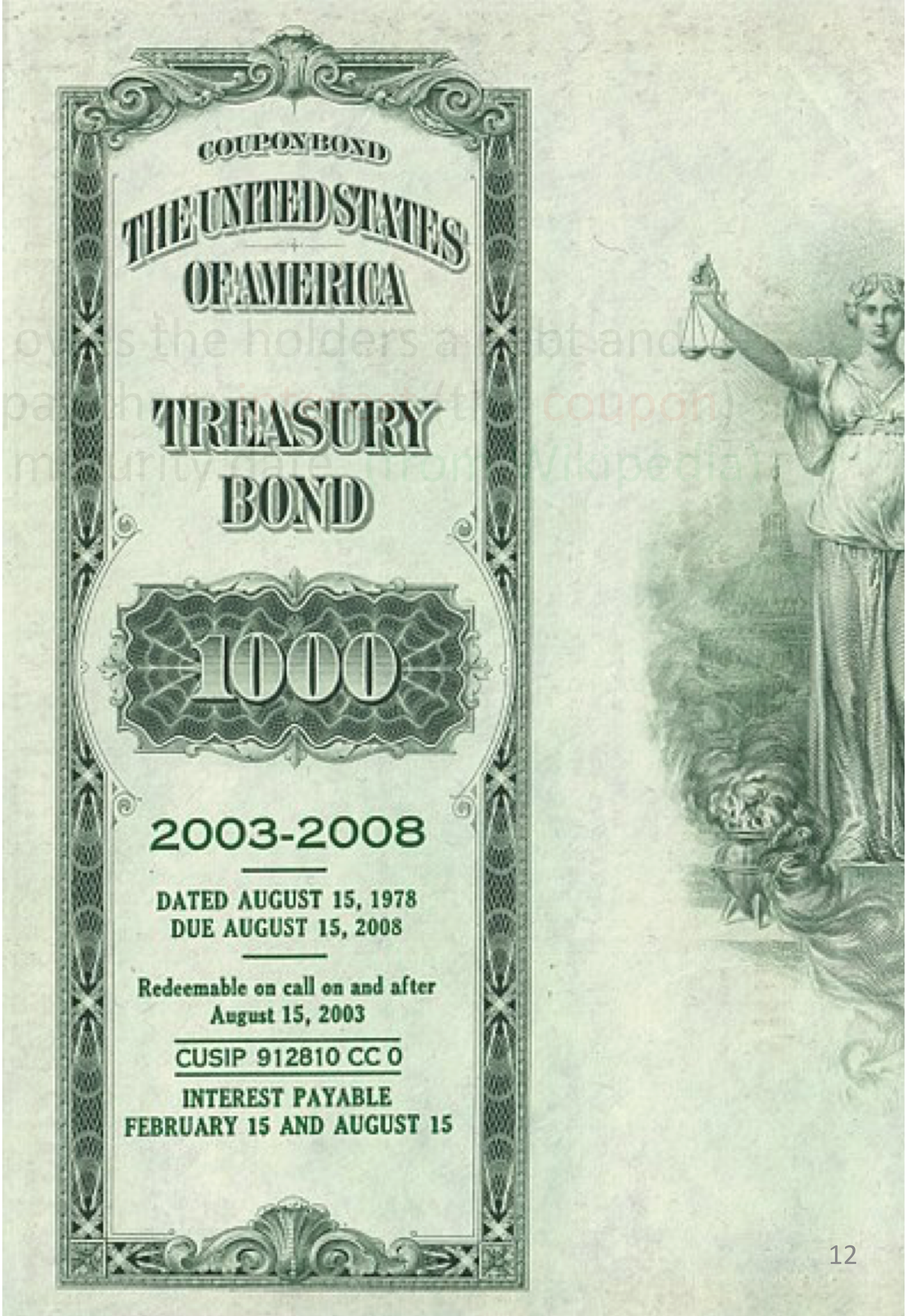
Bonds – general overview

- The bond is a debt security, under which the issuer owes the holders a debt and (depending on the terms of the bond) is obliged to pay them interest (the coupon) or to repay the principal at a later date, termed the maturity date. (from Wikipedia)
- In order to price / analyse the zero / fixed coupon bonds one has to know:
  - Coupons: CPN(t) (these are CF(t)) dependend on
    - (nominal) interest rate: r%
    - CPN (payment) days: t (interest frequency: n)
    - Day-count conventions (how to convert r%, t and n into coupon CF(t))
  - Face Value (also called the Nominal / Principal / Par Value, PL: wartość nominalna): FV
  - Maturity: T (final redemption day)



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Bonds – general overview: Summary

❖The bond is a debt security, under which the issuer owes the holders a debt and (depending on the terms of the bond) is obliged to pay them interest (the coupon) or to repay the principal at a later date, termed the maturity date. (from Wikipedia)

Legal form:
- Material /Dematerilized
- Bearer / Named / Registered
- Private / Public

Maturity (T):
- Money Market
  - Short term (≤ 1Y)
- Capital Market
  - Medium-term (≤ 10Y)
  - Long-term (> 10Y)
  - Perpetuities (∞)

Duration Risk

Interest (CPN):
- Zero Coupon
- Fixed Coupon
- Floating Rate
- Other (e.g. structured)

r% , n, PV, T, Conventions

Other features:
- Currency
- Callable / Puttable
- Convertible / Exchangable

Issuers:
- Treasury
- Agency
- Local Gov.
- Municipality
- Bank
- Corporate

Credit RISK ↑ (RATING↓)

Bond markets

Primary market:
- Auctions
- Public issues
- Private issues

Secondary market:
- Exchange
- OTC
- Private contacts
- Non-negotiable

Liquidity RISK

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Bonds

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- ❖General overview (definitions and taxonomy)
- ❖Cash flow calculus (pricing and day-count conventions)
- ❖Pricing and risk (yield and sensitivity)

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Bonds – cash flow calculus

Let's start with simple example: on 18th July 2018 the US Treasury bond with a 1 3/8 coupon rate that matures in September 30, 2019 was quoted as "98.733 – 98.738"

- ❖ How much (and when) will we pay for the bond if we want to buy it: P = -CF(0) = ???
- ❖ What will be future cash flows related to Coupons & Face Value: CF(t) = CPN(t) = ???

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P = \$ 987.38 + \$ 4.17 = \$ 991.55 and it is payed on July 20th !

Bonds Order Verification

Description:	US Treasury Bds 1.375% 09/30/2019
Maturity Date:	September 30, 2019 (1 year, 2 months and 11 days from today)
Action:	Buy
Face Value:	\$1,000
Order Type:	\$98.738 Limit Fill or Kill
Timing:	Day Only
Settlement Date:	07/20/2018

Market Price:	\$987.38
Estimated Markup:	\$0.00
Principal Amount:	**\$987.38**
Accrued Interest:	\$4.17
Estimated Total Cost:	**\$991.55**

S&P Rating NR	Moody's Rating Aaa	Coupon Rate 1.375	Coupon Frequency Semi-annually
Quoted Price \$98.738	Yield To Maturity 2.450	Yield To Worst ---	CUSIP 9128282X7
Callable No	Next Call Date ---	Next Call Price ---	

„The devil is in the detail"

Bonds – cash flow calculus: SPOT, BID-ASK

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- Usually (for the Exchanges and OTC markets) transaction day (D) ≠ settlement day
  - There must be some time for "back-office" to verify the transaction, transfer money, …
  - For most markets (but not always *) the settlement day is the, so called, SPOT = D + 2 (work days !)
  - In our case: SPOT = D + 2 = 20th July

* E.g. the shortest loans/deposits are "OverNight" O/N (i.e. from D to D+1) or "Tom-Next" T/N (i.e. from D+1 to D+2)

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BID ASK

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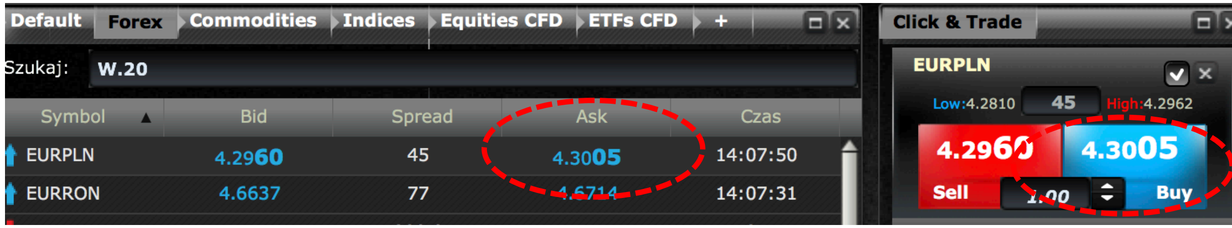
  - o In our case: SPOT = D + 2 = 20th July
- ❖ Usually on the market you see two prices: the one at which you can immediately BUY (ASK / OFFER) and the one at which you can immediately SELL (BID): BID (SELL) < ASK (BUY)
  - o The difference is called the spread = ASK-BID which is (part of) the transaction cost
  - o In our case the BUY price (ASK) is: "98.738"



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Bonds – cash flow calculus: POINTS, PAR

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  P = $ 987.38 + $ 4.17 = $ 991.55 and it is payed on July 20th !

- Now come the bond trading / quoting CONVENTIONS *:
  - Bonds are quoted in POINTS, i.e. in % of PAR (nominal) value: "98.738" means 98.738 % of $ 1000 (the par value of the above bond). The PAR of course can be different for each bond !
  - In some markets, e.g. in US, bonds are quoted in points + (non decimal) fraction of a point (e.g. 1/8, 1/16 or 1/32). The price: "98:12" can e.g. mean 98 12/32 or 98 12/16 % of PAR.
  - Fixed coupon bonds are usually issued at (around) PAR. Then the price can go lower or higher but eventually (at maturity) it goes again to PAR. So one usually says about a quote:
    if P < 100 (at "discount"), if P = 100 (at "par"), if P > 100 (at "premium")

* These conventions are valid for a vast majority of markets but there still can be some exceptions ! (even in a single country one must be VERY CAREFULL when comparing prices over different markets and check it all in very detail)

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Bonds – cash flow calculus: ACCRUED INT.

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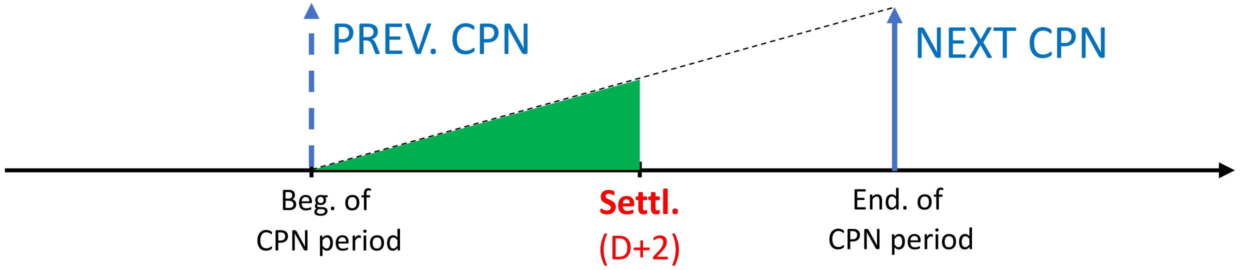
  - Bonds are quoted in "clean prices". In order to compute the settlement "dirty price" / "full price" one must add interest accrued to the settlement day! ("BUY CLEAN, PAY DIRTY")

Clean price + Accrued int. = Dirty price



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- Accrued interest (PL: narosłe odsetki) is interest that would be paid if coupon was due at the settlement. Usually it is not, so the buyer will receive full coupon in the next coupon day and he must compensate the seller for the interest that accrued from last coupon to the settlement day.
- Accrued interest grow (linearly) with time to the value of full coupon from prev. to next coupon day
- NOTE: to get interest one must buy before the "Ex-coupon" day (can be ≠ coupon (payment) day)

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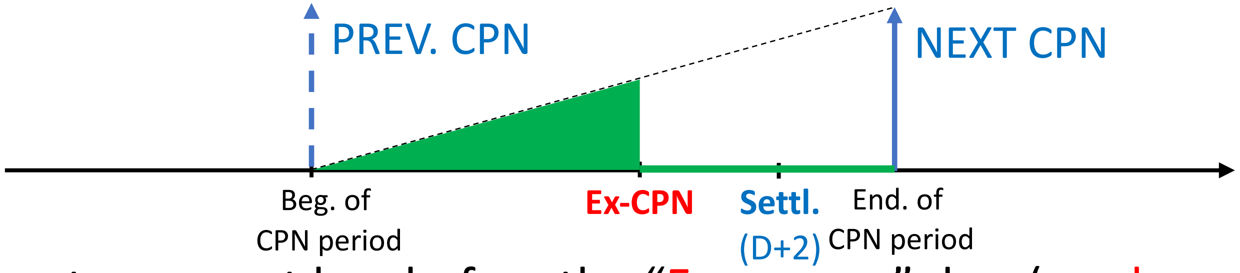

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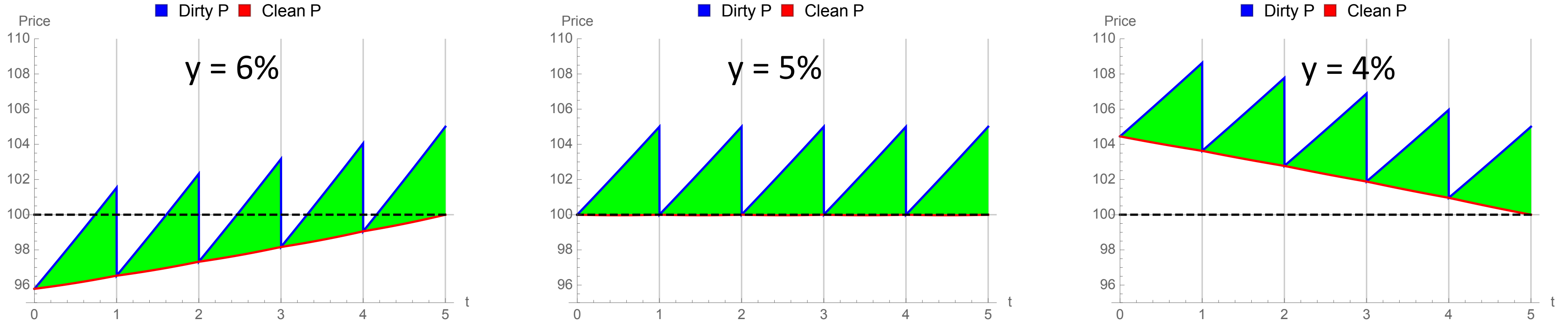

Example: a 5 Y bond with r = 5% coupon, and yield: y = 6% (left), y = 5% (center), y = 4% (right)

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Maturity Date:	September 30, 2019 (1 year, 2 months and 11 days from today)
Action:	Buy
Face Value:	$1,000
Order Type:	$98.738 Limit Fill or Kill
Timing:	Day Only
Settlement Date:	07/20/2018

Market Price:	$987.38
Estimated Markup:	$0.00
Principal Amount:	**$987.38**
Accrued Interest:	$4.17
Estimated Total Cost:	**$991.55**

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Quoted Price $98.738	Yield To Maturity 2.450	Yield To Worst ---	CUSIP 9128282X7
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Accr. Int. (20/07/18) = $ 4.17

CF(30/09/18) = CPN = $ 6.88
CF(31/03/19) = CPN = $ 6.88
CF(30/09/19) = CPN + FV = $ 1006.88

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Bonds – cash flow calculus: CONVENTIONS

Let's start with simple example: on 18th July 2018 the US Treasury bond with a 1 3/8 coupon rate that matures in September 30, 2019 was quoted as "98.733 – 98.738"

- ❖ What will be future cash flows related to Coupons & Face Value: CF(t) = CPN(t) = ???
- ❖ This is: how to convert FaceValue FV, nom. rate r%, cpn freq. n and cpn dates t into Interest ?

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Maturity Date:	September 30, 2019 (1 year, 2 months and 11 days from today)
Action:	Buy
Face Value:	$1,000
Order Type:	$98.738 Limit Fill or Kill
Timing:	Day Only
Settlement Date:	07/20/2018

Market Price:	$987.38
Estimated Markup:	$0.00
Principal Amount:	$987.38
Accrued Interest:	$4.17
Estimated Total Cost:	**$991.55**

S&P Rating	Moody's Rating	Coupon Rate	Coupon Frequency
NR	Aaa	1.375	Semi-annually
Quoted Price	Yield To Maturity	Yield To Worst	CUSIP
$98.738	2.450	---	9128282X7
Callable	Next Call Date	Next Call Price	
No	---	---	

Accr. Int. (20/07/18) = $ 4.17

CF(30/09/18) = CPN = $ 6.88
CF(31/03/19) = CPN = $ 6.88
CF(30/09/19) = CPN + FV = $ 1006.88

„The devil is in the detail"

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Bonds – cash flow calculus: CONVENTIONS

Let's start with simple example: on 18th July 2018 the US Treasury bond with a 1 3/8 coupon rate that matures in September 30, 2019 was quoted as "98.733 – 98.738"

- ❖ What will be future cash flows related to Coupons & Face Value: CF(t) = CPN(t) = ???
- ❖ This is: how to convert FaceValue FV, nom. rate r%, cpn freq. n and cpn dates t into Interest ?

Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = $ 1 000 × 1.375 % × ???

Accr. Int. (20/07/18) = $ 4.17

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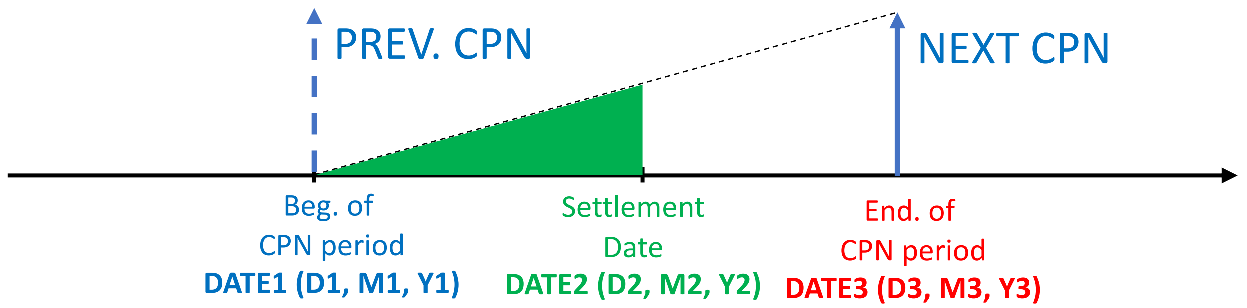
CF(30/09/19) = CPN + FV = $ 1006.88

„The devil is in the detail"

- ❖ DAY COUNT CONVENTIONS !
  - A lot of conventions arose overtime in various markets
  - Most of them were invented before the use of computers so were developed in order to facilitate computations
  - Lack of central authority deciding about the standards but need for standards / precise definitions / documentation, especially due to globalization (ISDA, ICMA)
  - One must be very careful !, e.g. "ACT/ACT" ISDA ≠ "ACT/ACT" ICMA

Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = \$ 1 000 × 1.375 % × ???

DAY COUNT CONVENTIONS *

❖ 30 / 360 Methods

ΔD = DATE2 - DATE1 =
(Y2-Y1) × 360 + (M2-M1) × 30 + D2-D1

- Some adjustments if D is end of month, e.g. D=31 → D=30
- DayCountFactor = ΔD / 360

❖ Actual Methods

ΔD = DATE 2 – DATE1 =
Actual # of days between the dates

- Some adjustments if Leap Year
- DayCountFactor = ΔD/ "Year", where: "Year" = Actual or Fixed # days

Accr. Int. (20/07/18) = \$ 4.17

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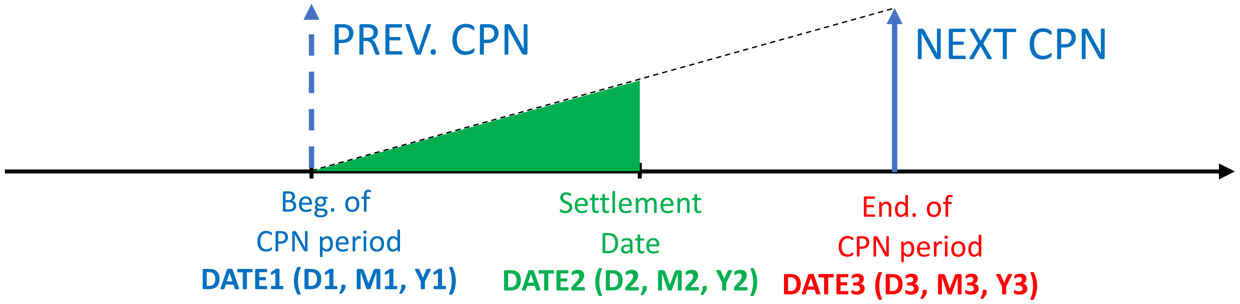
Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

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ΔD = DATE2 - DATE1 =
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- DayCountFactor = ΔD / 360

❖ Actual Methods

ΔD = DATE 2 – DATE1 =
Actual # of days between the dates

- Some adjustments if Leap Year
- DayCountFactor = ΔD/ "Year", where: "Year" = Actual or Fixed # days

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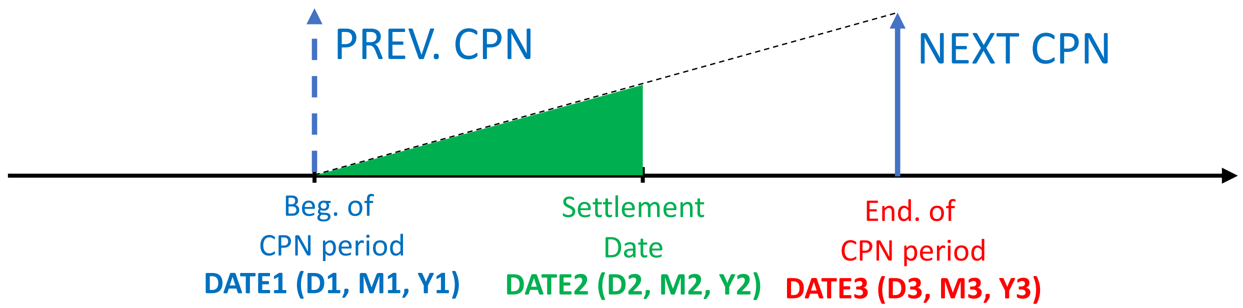
Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = $ 1 000 × 1.375 % × ???

DAY COUNT C

❖ 30 / 360 Methods

ΔD = DATE2 - DATE1 =
(Y2-Y1) × 360 + (M2-M1) × 30 + D2-D1

- Some adjustments if D is end of month, e.g. D=31 → D=30
- DayCountFactor = ΔD / 360

E.g. „30 / 360 Bond Basis" rules: *

- D1 = MIN (D1, 30)
- If D1 = 30 then D2 = MIN (D2,30)

ORDER !

	Accr. Int.	CPN1	CPN2	CPN3
DATE1	31.03.2018	31.03.2018	30.09.2018	31.03.2019
DATE2	20.07.2018	30.09.2018	31.03.2019	30.09.2019
D1	30	30	30	30
D2	20	30	30	30
ΔD	110	180	180	180
DayCountFact.	**110/360**	**180/360**	**180/360**	**180/360**
Interest	**4.20**	**6.88**	**6.88**	**6.88**

* See, e.g. https://en.wikipedia.org/wiki/Day_count_convention or http://data.cbonds.info/files/cbondscalc/Calculator.pdf

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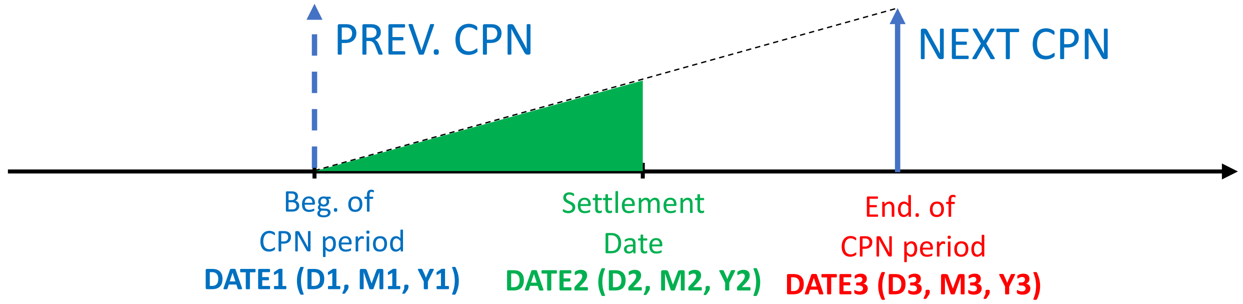
Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = $ 1 000 × 1.375 % × ???

DAY COUNT C

❖ 30 / 360 Methods

ΔD = DATE2 - DATE1 =
(Y2-Y1) × 360 + (M2-M1) × 30 + D2-D1

- Some adjustments if D is end of month, e.g. D=31 → D=30
- DayCountFactor = ΔD / 360

E.g. „30 / 360 US" rules: *

- If EOM & DATE1 = Feb END & DATE2 = Feb END then D2 = 30
- If EOM & DATE1=Feb END then D1 = 30
- If D1 = 30 or D1 = 31 then D2 = MIN (D2, 30)
- D1 = MIN (D1, 30)

ORDER !

	Accr. Int.	CPN1	CPN2	CPN3
DATE1	31.03.2018	31.03.2018	30.09.2018	31.03.2019
DATE2	20.07.2018	30.09.2018	31.03.2019	30.09.2019
D1	30	30	30	30
D2	20	30	30	30
ΔD	110	180	180	180
DayCountFact.	**110/360**	**180/360**	**180/360**	**180/360**
Interest	**4.20**	**6.88**	**6.88**	**6.88**

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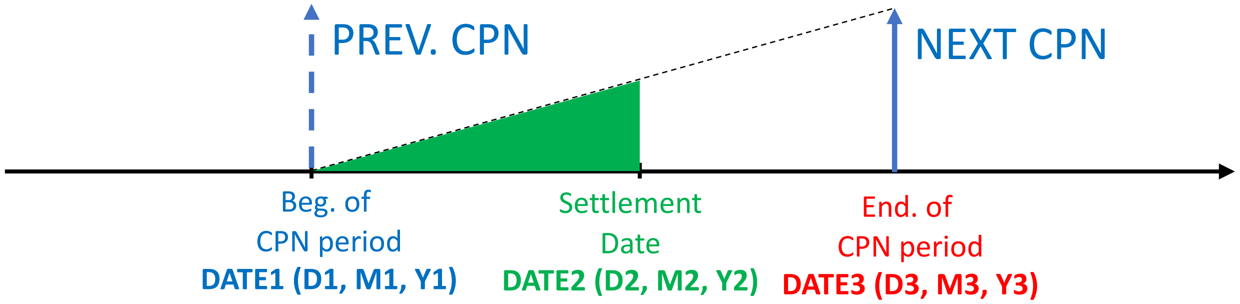
Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = \$ 1 000 × 1.375 % × ???

DAY COUNT C

❖ 30 / 360 Methods

ΔD = DATE2 - DATE1 =
(Y2-Y1) × 360 + (M2-M1) × 30 + D2-D1

○ Some adjustments if D is end of month, e.g. D=31 → D=30

○ DayCountFactor = ΔD / 360

E.g. „30E / 360" rules: *

○ D1 = MIN (D1, 30)

○ D2 = MIN (D2, 30)

ORDER !

	Accr. Int.	CPN1	CPN2	CPN3
DATE1	31.03.2018	31.03.2018	30.09.2018	31.03.2019
DATE2	20.07.2018	30.09.2018	31.03.2019	30.09.2019
D1	30	30	30	30
D2	20	30	30	30
ΔD	110	180	180	180
DayCountFact.	**110/360**	**180/360**	**180/360**	**180/360**
Interest	**4.20**	**6.88**	**6.88**	**6.88**

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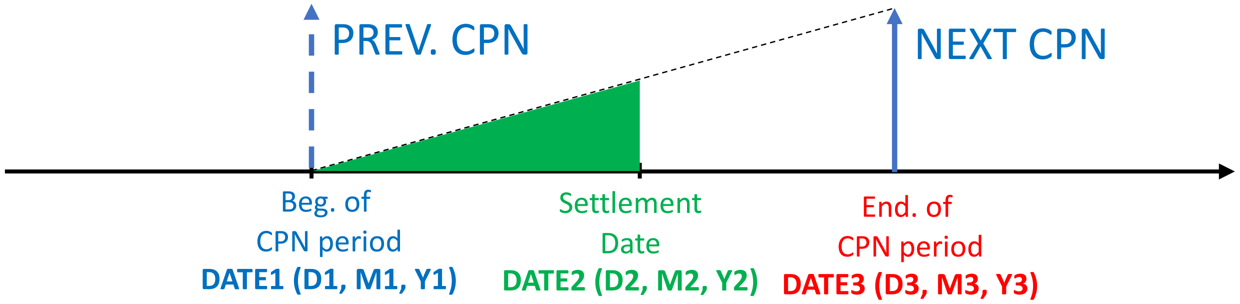
Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = $ 1 000 × 1.375 % × ???

DAY COUNT C

❖ 30 / 360 Methods

ΔD = DATE2 - DATE1 =
(Y2-Y1) × 360 + (M2-M1) × 30 + D2-D1

- Some adjustments if D is end of month, e.g. D=31 → D=30
- DayCountFactor = ΔD / 360

E.g. „30E / 360 ISDA" rules: *

- If D1 = MONTH END then D1 = 30
- If D2 = MONTH END & DATE2 ≠ MATURITY in Feb then D2 = 30

ORDER !

	Accr. Int.	CPN1	CPN2	CPN3
DATE1	31.03.2018	31.03.2018	30.09.2018	31.03.2019
DATE2	20.07.2018	30.09.2018	31.03.2019	30.09.2019
D1	30	30	30	30
D2	20	30	30	30
ΔD	110	180	180	180
DayCountFact.	**110/360**	**180/360**	**180/360**	**180/360**
Interest	**4.20**	**6.88**	**6.88**	**6.88**

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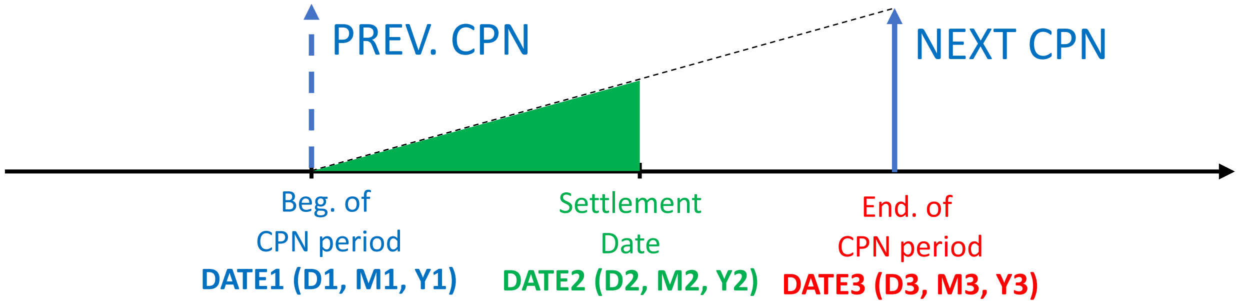
Bonds – cash flow calculus: CONVENTIONS



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Interest = $ 1 000 × 1.375 % × ???

DAY COUNT CONVENTIONS *

❖ 30 / 360 Methods

ΔD = DATE2 - DATE1 =
(Y2-Y1) × 360 + (M2-M1) × 30 + D2-D1

- Some adjustments if D is end of month, e.g. D=31 → D=30
- DayCountFactor = ΔD / 360

❖ Actual Methods

ΔD = DATE 2 – DATE1 =
Actual # of days between the dates

- Some adjustments if Leap Year
- DayCountFactor = ΔD/ "Year", where: "Year" = Actual or Fixed # days

Accr. Int. (20/07/18) = $ 4.17

CF(30/09/18) = CPN = $ 6.88
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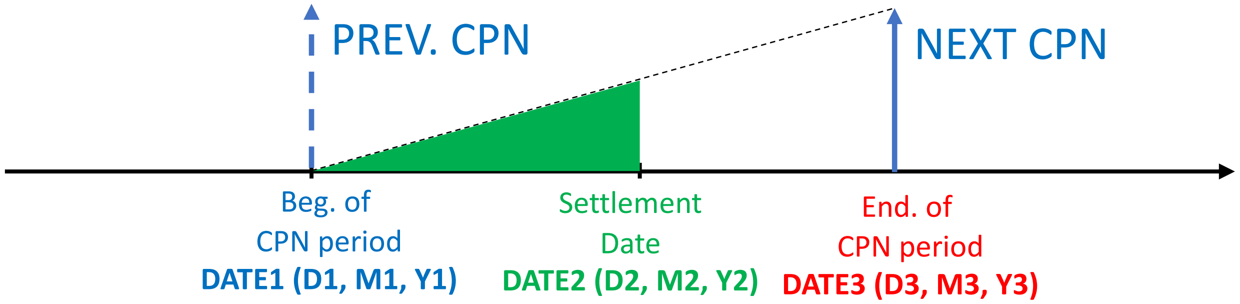
Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = $ 1 000 × 1.375 % × ???

E.g. „ACT/ACT ISMA" rules: *

- ΔD = DATE2 – DATE1
- „Year" = n (DATE3 – DATE1), where:
- DATE3 = END OF CPN PERIOD
- n = CPN Freq.

CONVENTIONS *

❖ Actual Methods

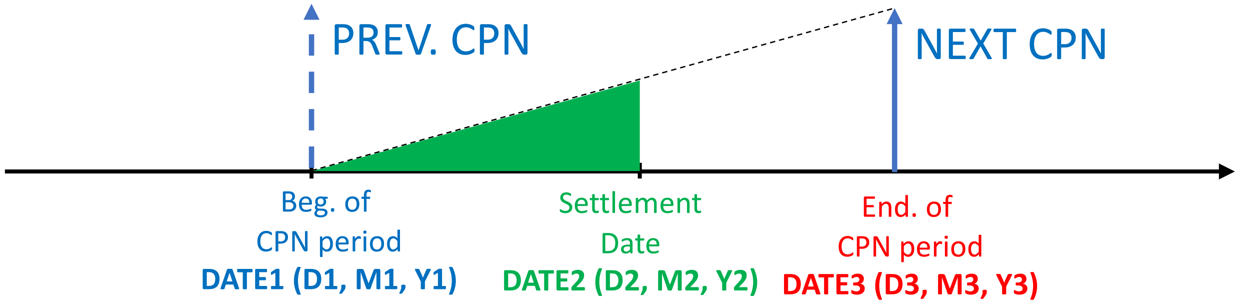
ΔD = DATE 2 – DATE1 =
Actual # of days between the da

- Some adjustments if Leap Yea
- DayCountFactor = ΔD/ "Year", where: "Year" = Actual or Fixed # days

	Accr. Int.	CPN1	CPN2	CPN3
DATE1	31.03.2018	31.03.2018	30.09.2018	31.03.2019
DATE2	20.07.2018	30.09.2018	31.03.2019	30.09.2019
ΔD	111	183	182	183
„Year"	183 × 2	183 × 2	182 × 2	183 × 2
DayCountFact.	**111/366**	**1/2**	**1/2**	**1/2**
Interest	**4.17**	**6.88**	**6.88**	**6.88**

* See, e.g. https://en.wikipedia.org/wiki/Day_count_convention or http://data.cbonds.info/files/cbondscalc/Calculator.pdf

Bonds – cash flow calculus: CONVENTIONS



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E.g. „ACT/ACT ISMA" rules: *

- ΔD = DATE2 – DATE1
- „Year" = n (DATE3 – DATE1), where:
- DATE3 = END OF CPN PERIOD
- n = CPN Freq.

This one is used for US T-Bonds !

CONVENTIONS *

❖ Actual Methods

ΔD = DATE 2 – DATE1 =
Actual # of days between the dates

- Some adjustments if Leap Year
- DayCountFactor = ΔD/ "Year", where: "Year" = Actual or Fixed # days

Accr. Int. (20/07/18) = $ 4.17

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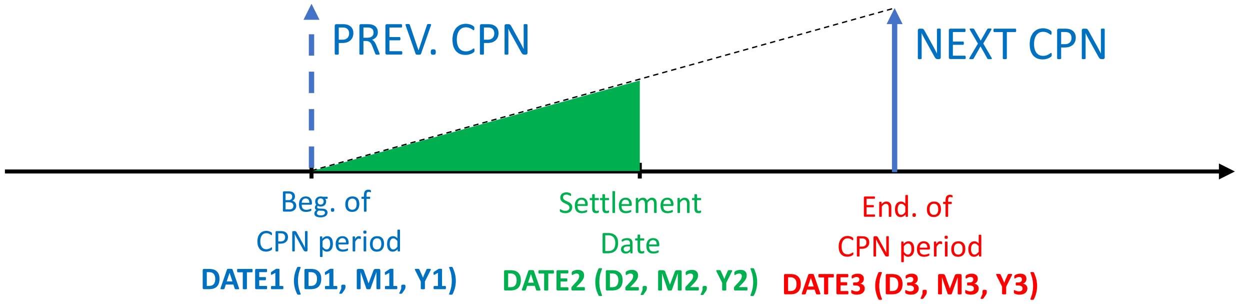
Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = $ 1 000 × 1.375 % × ???

CONVENTIONS *

E.g. „ACT/ACT ISDA" rules: *

○ DayCountFactor =

DAYS NOT IN LEAP YEAR / 365

\+

DAYS IN LEAP YEAR / 366

❖ Actual Methods

ΔD = DATE 2 – DATE1 =

Actual # of days between the da

○ Some adjustments if Leap Yea

○ DayCountFactor = ΔD/ "Year",

where: "Year" = Actual or Fixed # days

	Accr. Int.	CPN1	CPN2	CPN3
DATE1	31.03.2018	31.03.2018	30.09.2018	31.03.2019
DATE2	20.07.2018	30.09.2018	31.03.2019	30.09.2019
#D. NOT Leap	111	183	182	183
#D. Leap	0	0	0	0
DayCountFact.	**111/365**	**183/365**	**182/365**	**183/365**
Interest	**4.18**	**6.89**	**6.86**	**6.89**

* See, e.g. https://en.wikipedia.org/wiki/Day_count_convention or http://data.cbonds.info/files/cbondscalc/Calculator.pdf

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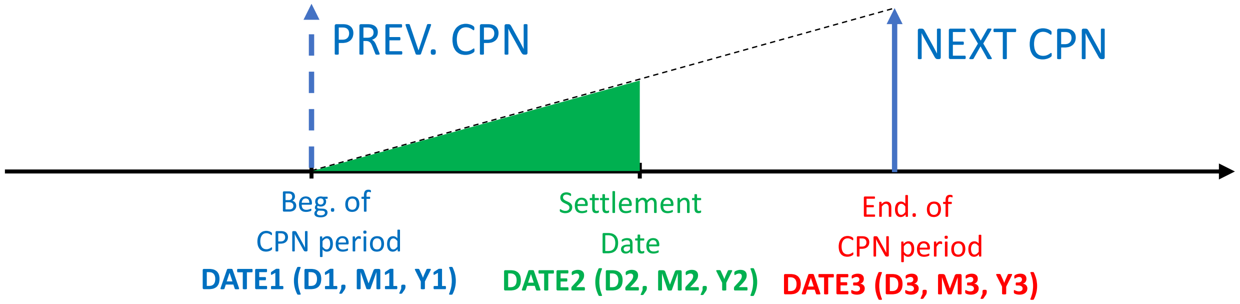
Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = $ 1 000 × 1.375 % × ???

E.g. „ACT/365 Fixed” rules: *

- ΔD = DATE2 – DATE1
- „Year” = 365

CONVENTIONS *

❖ Actual Methods

ΔD = DATE 2 – DATE1 =
Actual # of days between the da

- Some adjustments if Leap Yea
- DayCountFactor = ΔD/ ”Year”,
  where: “Year” = Actual or Fixed # days

	Accr. Int.	CPN1	CPN2	CPN3
DATE1	31.03.2018	31.03.2018	30.09.2018	31.03.2019
DATE2	20.07.2018	30.09.2018	31.03.2019	30.09.2019
ΔD	111	183	182	183
„Year”	365	365	365	365
DayCountFact.	**111/365**	**183/365**	**182/365**	**183/365**
Interest	**4.18**	**6.89**	**6.86**	**6.89**

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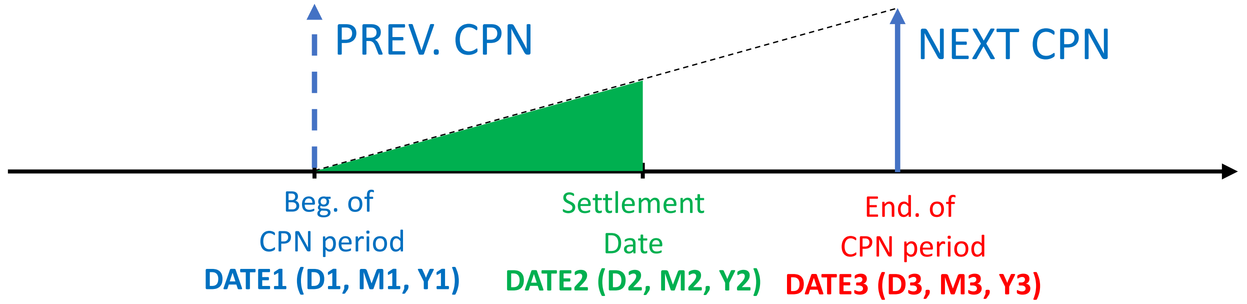
Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = $ 1 000 × 1.375 % × ???

E.g. „ACT/360" rules: *

- ΔD = DATE2 – DATE1
- „Year" = 360

CONVENTIONS *

❖ Actual Methods

ΔD = DATE 2 – DATE1 =
Actual # of days between the da

- Some adjustments if Leap Yea
- DayCountFactor = ΔD/ "Year", where: "Year" = Actual or Fixed # days

	Accr. Int.	CPN1	CPN2	CPN3
DATE1	31.03.2018	31.03.2018	30.09.2018	31.03.2019
DATE2	20.07.2018	30.09.2018	31.03.2019	30.09.2019
ΔD	111	183	182	183
„Year"	360	360	360	360
DayCountFact.	**111/360**	**183/360**	**182/360**	**183/360**
Interest	**4.24**	**6.99**	**6.95**	**6.99**

* See, e.g. https://en.wikipedia.org/wiki/Day_count_convention or http://data.cbonds.info/files/cbondscalc/Calculator.pdf

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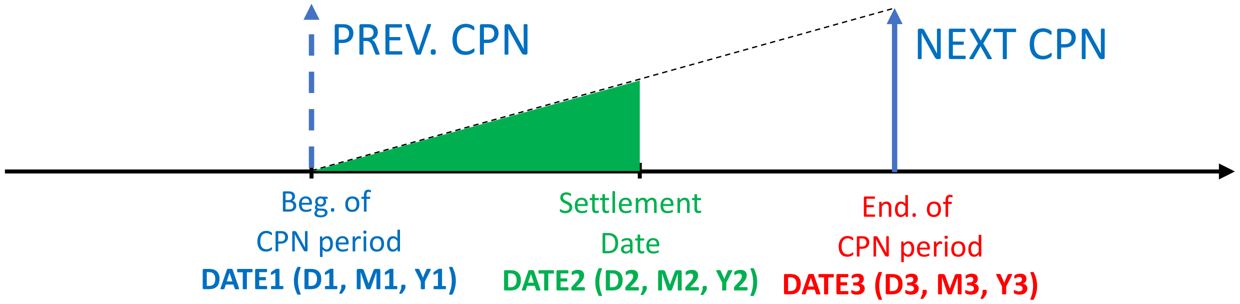
Bonds – cash flow calculus: CONVENTIONS



Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

Interest = $ 1 000 × 1.375 % × ???

DAY COUNT CONVENTIONS *

❖ 30 / 360 Methods

30/360	20/07/18	30/09/18	31/03/19	30/09/19
30/360 BondBasis	4,20	6,88	6,88	6,88
30/360 US	4,20	6,88	6,88	6,88
30 E/360	4,20	6,88	6,88	6,88
30 E/360 ISDA	4,20	6,88	6,88	6,88

❖ Actual Methods

ACTUAL	20/07/18	30/09/18	31/03/19	30/09/19
ACT/ACT ISMA	4,17	6,88	6,88	6,88
ACT/ACT ISDA	4,18	6,89	6,86	6,89
ACT/365 Fixed	4,18	6,89	6,86	6,89
ACT/360	4,24	6,99	6,95	6,99

Accr. Int. (20/07/18) = $ 4.17

CF(30/09/18) = CPN = $ 6.88
CF(31/03/19) = CPN = $ 6.88
CF(30/09/19) = CPN + FV = $ 1006.88

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Bonds – cash flow calculus: Summary

- Bonds are typically quoted in POINTS (% of PAR value): BID (SELL) < ASK (BUY)
  - **NOTE**: in some markets (e.g. US) bonds may be quoted in non-decimal parts of a POINT (e.g. 1/32)
- Bonds are usually quoted in CLEAN PRICES
- On the Settlement Date (usually the SPOT = D+2) the buyer pays / the seller receives

DIRTY PRICE = CLEAN PRICE + ACCRUED INTEREST

  - **NOTE**: be careful about ex-coupon dates !
- To convert FaceValue FV, nom. rate r%, cpn freq. n and cpn dates t into CPN or Accr. Int.

Interest (CPN) = Principal (FV) × CouponRate (r% p.a.) × DayCountFactor (fraction of a year)

- 30 / 360 Methods

  ΔD = DATE2 - DATE1 =

  (Y2-Y1) × 360 + (M2-M1) × 30 + D2-D1
  - Some adjustments if D is end of month, e.g. D=31 → D=30
  - DayCountFactor = ΔD / 360

- Actual Methods

  ΔD = DATE 2 – DATE1 =

  Actual # of days between the dates
  - Some adjustments if Leap Year
  - DayCountFactor = ΔD/ ”Year”, where: “Year” = Actual or Fixed # days

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Bonds

The lecture about bonds will be split in three parts:

- ❖General overview (definitions and taxonomy)
- ❖Cash flow calculus (pricing and day-count conventions)
- ❖Pricing and risk (yield and sensitivity)

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Bonds – yield to maturity (YTM)

Assume we know all CFs from the bond (including the initial one, i.e. CF(0) = - PV, the dirty price). We can now use formula (1) from Lecture 1:

$$\sum_t \frac{CF(t)}{(1+y)^t} = 0$$

Problem with Leap Years:
- One can adjust t for leap years
- Or just leave it, as the difference in y will typically be tiny

❖ Remember that t and T are in years (from the settlement day, usually the SPOT date):

t = (CF date – SPOT date) / 365

❖The effective rate y computed this way is called the Yield To Maturity (YTM) *

* For puttable / callable bonds one can have as well Yield To Put / Call, where only all CFs until the possible early redemption date are taken into account.

Bonds – yield to maturity (YTM)

Assume we know all CFs from the bond (including the initial one, i.e. CF(0) = - PV, the dirty price). We can now use formula (1) from Lecture 1:

$$\sum_t \frac{CF(t)}{(1+y)^t} = 0$$

- Remember that t and T are in years (from the settlement day, usually the SPOT date):
  t = (CF date – SPOT date) / 365
- The effective rate y computed this way is called the Yield To Maturity (YTM) *
- For a zero coupon bond one simply has:

$$PV = \frac{FV}{(1+y)^T}$$

Thus the YTM is:

$$y = (FV/PV)^{1/T} - 1$$

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Bonds – yield to maturity (YTM)

Assume we know all CFs from the bond (including the initial one, i.e. CF(0) = - PV, the dirty price). We can now use formula (1) from Lecture 1:

$$\sum_t \frac{CF(t)}{(1+y)^t} = 0$$

- ❖ Remember that t and T are in years (from the settlement day, usually the SPOT date):
  t = (CF date – SPOT date) / 365
- ❖ The effective rate y computed this way is called the Yield To Maturity (YTM) *
- ❖ For a fixed coupon bond YTM (y) can be solved numerically from:

$$PV = \sum_t \frac{CPN(t)}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

In general can be adjusted for Day Count Conventions

Bonds – yield to maturity (YTM)

Assume we know all CFs from the bond (including the initial one, i.e. CF(0) = - PV, the dirty price). We can now use formula (1) from Lecture 1:

$$\sum_t \frac{CF(t)}{(1+y)^t} = 0$$

❖ Remember that t and T are in years (from the settlement day, usually the SPOT date):

t = (CF date – SPOT date) / 365

❖ The effective rate y computed this way is called the Yield To Maturity (YTM) *

❖ For a fixed coupon bond YTM (y) can be solved numerically from:

$$PV = \sum_t \frac{CPN(t)}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

Assumed to be constant !

Here Δt is a fraction of the yr to next CPN:
Δt = (Next CPN date – SPOT date) / 365

❖ If one assumes (only a tiny error): *CPN = const.* and *n payments a year* in regular periods

$$PV = \sum_{i=0}^{\#CPNs-1} \frac{CPN}{(1+y)^{\Delta t + i/n}} + \frac{FV}{(1+y)^{\Delta t + (\#CPNs-1)/n}}$$

Bonds – yield to maturity (YTM)

Let's go back to the example: on 18th July 2018 the US Treasury bond with a 1 3/8 coupon rate that matures in September 30, 2019 was quoted as "98.733 – 98.738"

❖ What is the YTM ?

Bonds Order Verification

Description:	US Treasury Bds 1.375% 09/30/2019
Maturity Date:	September 30, 2019 (1 year, 2 months and 11 days from today)
Action:	Buy
Face Value:	$1,000
Order Type:	$98.738 Limit Fill or Kill
Timing:	Day Only
Settlement Date:	07/20/2018

Market Price:	$987.38
Estimated Markup:	$0.00
Principal Amount:	**$987.38**
Accrued Interest:	$4.17
Estimated Total Cost:	**$991.55**

S&P Rating NR	Moody's Rating Aaa	Coupon Rate 1.375	Coupon Frequency Semi-annually
Quoted Price $98.738	Yield To Maturity 2.450	Yield To Worst ---	CUSIP 9128282X7
Callable No	Next Call Date ---	Next Call Price ---	

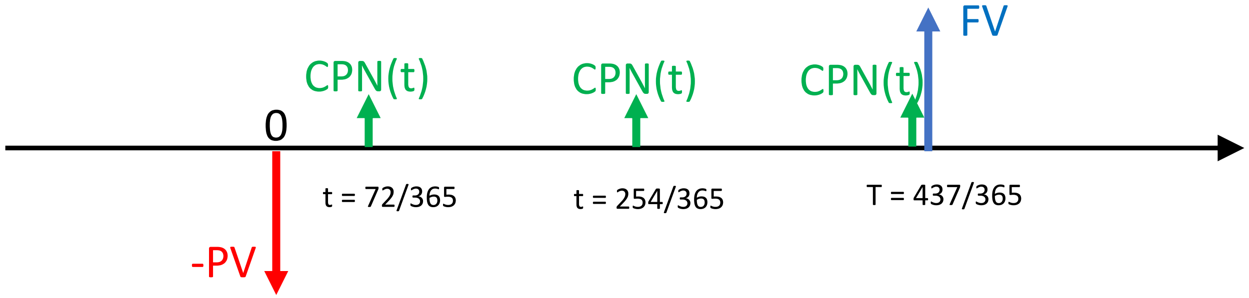
t = 0 ≡ Settl. Day (SPOT) = 20/07/18

PV = -CF(20/07/18) = $ 991.55
CPN(t = 30/09/18) = $ 6.88
CPN(t = 31/03/19) = $ 6.88
CPN(T = 30/09/19) = $ 6.88
FV(T = 30/09/19) = $ 1 000

Bonds – yield to maturity (YTM)

Let's go back to the example: on 18th July 2018 the US Treasury bond with a 1 3/8 coupon rate that matures in September 30, 2019 was quoted as "98.733 – 98.738"

- What is the YTM ?
- Exact: y = 2.466 %



In general can vary !

$$PV = \sum_t \frac{CPN(t)}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

$$991.55 = \frac{6.88}{(1+y)^{72/365}} + \frac{6.88}{(1+y)^{254/365}} + \frac{6.88 + 1000}{(1+y)^{437/365}}$$

t = 0 ≡ Settl. Day (SPOT) = 20/07/18

PV = -CF(20/07/18) = $ 991.55
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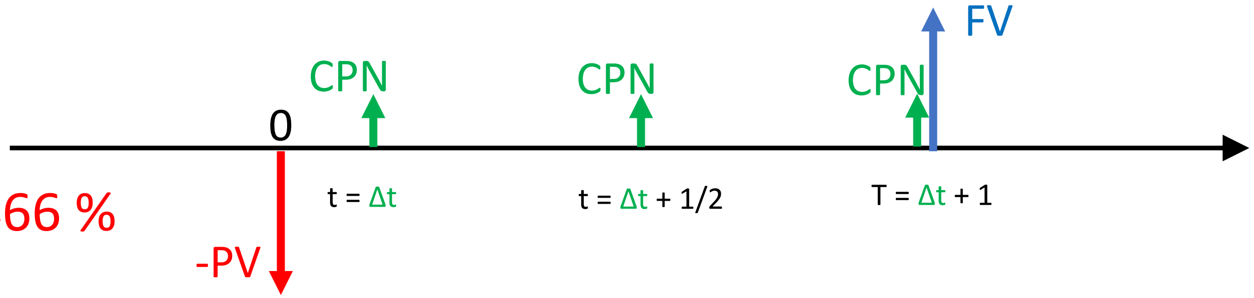
Bonds – yield to maturity (YTM)

Let's go back to the example: on 18th July 2018 the US Treasury bond with a 1 3/8 coupon rate that matures in September 30, 2019 was quoted as "98.733 – 98.738"

- ❖ What is the YTM ?
- ❖ Exact: y = 2.466 %
- ❖ Approximate: y = 2.466 %



Assumed to be const. !

$$PV = \sum_{i=0}^{\#CPNs-1} \frac{CPN}{(1+y)^{\Delta t + i/n}} + \frac{FV}{(1+y)^{\Delta t + (\#CPNs-1)/n}}$$

$$991.55 = \frac{6.88}{(1+y)^{72/365}} + \frac{6.88}{(1+y)^{72/365+1/2}} + \frac{6.88+1000}{(1+y)^{72/365+1}}$$

t = 0 ≡ Settl. Day (SPOT) = 20/07/18

PV (t = 0) = \$ 991.55
CPN(t = Δt + 0) = \$ 6.88 72d
CPN(t = Δt + 1/2) = \$ 6.88 72d+1/2y
CPN(T = Δt + 1) = \$ 6.88 72d+1y
FV(T = Δt + 1) = \$ 1 000

n=2 *#CPNs* = 3 Δt = 72/365

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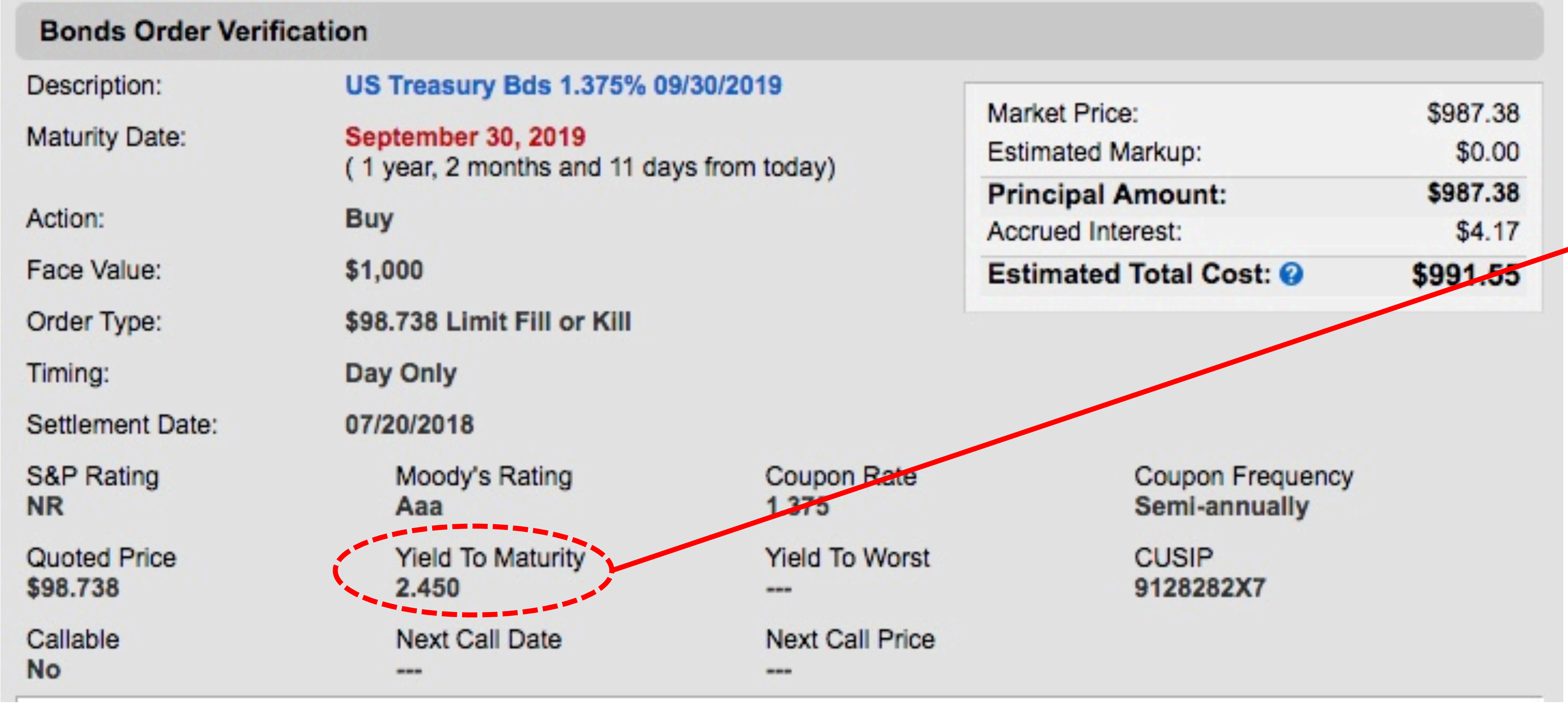
Bonds – yield to maturity (YTM)

Let's go back to the example: on 18th July 2018 the US Treasury bond with a 1 3/8 coupon rate that matures in September 30, 2019 was quoted as "98.733 – 98.738"

- What is the YTM ?
- Exact: y = 2.466 %



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Timing:	Day Only
Settlement Date:	07/20/2018

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Estimated Markup:	$0.00
Principal Amount:	**$987.38**
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S&P Rating NR	Moody's Rating Aaa	Coupon Rate 1.375	Coupon Frequency Semi-annually
Quoted Price $98.738	Yield To Maturity 2.450	Yield To Worst ---	CUSIP 9128282X7
Callable No	Next Call Date ---	Next Call Price ---	

BUT: one has „YTM = 2.450 %":
In some markets (e.g. in US) one uses („incorrect") definition of YTM if CPN freq. $n > 1$:

$$(1+\text{"YTM"}/n)^n = 1 + y$$

It is sometimes called "Nominal Yield"

„YTM" = 2.4508 %

„The devil is in the detail"

Bonds – sensitivity

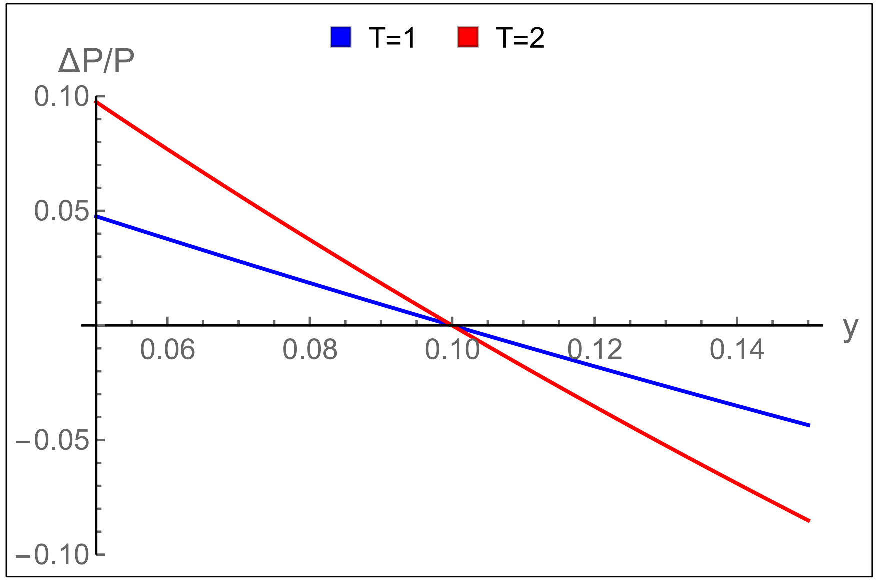
- Now we know how to compute YTM for a bond. As YTM is financial measure of "profitability" it should be approximately the same for a class of comparable bonds, i.e. with equal credit / default risk and equal "life-time" *
- One can therefore "inverse" the question, i.e. ask how the bond's (dirty) price PV will depend on the yield y which is dictated by "the market": PV = PV(y)
- Yield To Maturity, as the name suggests, is the effective rate of return for a "long-term" investor who intends to keep the bond to maturity (of course if the issuer doesn't default)
- But most investors will sell the bond earlier and thus will realize a different rate of return (profit or loss)
- Therefore "short-term" investors are rather interested in $\Delta PV / PV$, where ΔPV is a change in the current bond's price PV
- More precisely, we will ask a question: how a (small) sudden change in market yields $y \rightarrow y + \Delta y$ will affect market prices of bonds and thus $\Delta PV / PV$

* It is straightforward to assume that but the exact reason will be discussed later.

Bonds – sensitivity: (Modified) Duration

- More precisely, we will ask a question: how a (small) sudden change in market yields $y \to y + \Delta y$ will affect market prices of bonds and thus $\Delta PV / PV$
- Again, for a zero coupon bond:

$$PV(y) = \frac{FV}{(1+y)^T}$$



- So the price is inversely proportional to the yield, and the price will FALL if y↑ and it will INCREASE if y↓
- The effect is bigger for longer T !
- For small Δy one has: $(1+y+\Delta y)^{-T} \approx (1+y)^{-T}(1 - T\,\Delta y / (1+y))$, so:

$$\frac{\Delta PV}{PV} \approx -\frac{T}{(1+y)}\Delta y$$

Maturity T is the DURATION (D) of the bond.
T/(1+y) also has dim. of TIME (it's OK as [Δy] is: % / p.a.)
It is called the MODIFIED DURATION (MD)

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Bonds – sensitivity: (Modified) Duration

- More precisely, we will ask a question: how a (small) sudden change in market yields $y \rightarrow y + \Delta y$ will affect market prices of bonds and thus $\Delta PV / PV$
- Let's repeat the same for the fixed coupon bond:

$$PV(y) = \sum_t \frac{CPN(t)}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

- Using (the first term in) Taylor's series expansion in Δy one has:

$$\Delta PV = PV(y+\Delta y) - PV(y) \approx \frac{\partial PV(y)}{\partial y}\Delta y =$$

$$= \left(\sum_t CPN(t)\frac{\partial}{\partial y}\frac{1}{(1+y)^t} + FV\frac{\partial}{\partial y}\frac{1}{(1+y)^T}\right)\Delta y = \left(\sum_t CPN(t)\frac{-t}{(1+y)^{t+1}} + FV\frac{-T}{(1+y)^{T+1}}\right)\Delta y =$$

$$= -\frac{1}{(1+y)}\left(\sum_t t\,\frac{CPN(t)}{(1+y)^t} + T\,\frac{FV}{(1+y)^T}\right)\Delta y$$

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Bonds – sensitivity: (Modified) Duration

- More precisely, we will ask a question: how a (small) sudden change in market yields y → y + Δy will affect market prices of bonds and thus ΔPV / PV
- Let's repeat the same for the fixed coupon bond:

$$PV(y) = \sum_t \frac{CPN(t)}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

- Thus:

DURATION

MD

$$\frac{\Delta\, PV}{PV} \approx -\frac{1}{(1+y)} \frac{1}{PV}\left(\sum_t t\,\frac{CPN(t)}{(1+y)^t} + T\,\frac{FV}{(1+y)^T}\right)\Delta y = -\frac{D}{(1+y)}\Delta y$$

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Bonds – sensitivity: (Modified) Duration

- More precisely, we will ask a question: how a (small) sudden change in market yields $y \rightarrow y + \Delta y$ will affect market prices of bonds and thus $\Delta PV / PV$
- In general (for zero / fixed coupon bonds):

$$\frac{\Delta PV}{PV} \approx -\frac{D}{(1+y)}\Delta y = -MD\,\Delta y$$

- Duration:

$$D = \frac{1}{PV}\left(\sum_t t\,\frac{CPN(t)}{(1+y)^t} + T\,\frac{FV}{(1+y)^T}\right)$$

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Bonds – sensitivity: (Modified) Duration

- More precisely, we will ask a question: how a (small) sudden change in market yields $y \rightarrow y + \Delta y$ will affect market prices of bonds and thus $\Delta PV / PV$
- In general (for zero / fixed coupon bonds):

$$\frac{\Delta\, PV}{PV} \approx -\frac{D}{(1+y)}\Delta y = -\, MD\ \Delta y$$

These WEIGHTS sum up to PV !!!

- Duration:

$$D = \frac{1}{PV}\left(\sum_t t\, \overbrace{\frac{CPN(t)}{(1+y)^t}}^{\text{PV(CF(t))}} + T\, \overbrace{\frac{FV}{(1+y)^T}}^{\text{PV(CF(T))}}\right)$$

- This again has dimension of TIME !
- Duration is the effective (weighted by PV(CFs)) average „life-time" of the bond !
- CPN payments DECREASE the Duration (effective „life-time"): we get some cash earlier !
- (Modified) Duration is some proxy of investment RISK (related to yield curve changes)!

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Bonds – sensitivity: (Modified) Duration

- More precisely, we will ask a question: how a (small) sudden change in market yields $y \to y + \Delta y$ will affect market prices of bonds and thus $\Delta PV / PV$
- In general (for zero / fixed coupon bonds):

$$\frac{\Delta PV}{PV} \approx -\frac{D}{(1+y)}\Delta y = -MD\,\Delta y$$

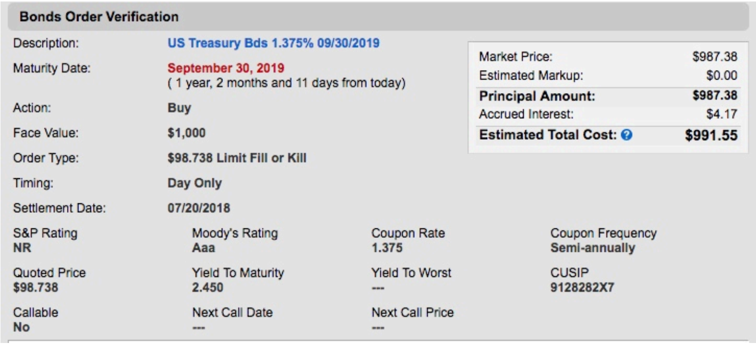
- Duration:

$$D = \frac{1}{PV}\left(\sum_t t\,\frac{CPN(t)}{(1+y)^t} + T\,\frac{FV}{(1+y)^T}\right)$$

$$\mathrm{T} = 437/365 = 1.197\ yr$$

$$\mathrm{D} = \frac{1}{991.55}\left(\frac{72}{365}\frac{6.88}{(1+y)^{72/365}} + \frac{254}{365}\frac{6.88}{(1+y)^{254/365}} + \frac{437}{365}\frac{6.88+1000}{(1+y)^{437/365}}\right) = 1.187\ yr$$

$$\mathrm{MD} = \frac{1}{(1+y)}1.187 = 1.158\ yr$$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Maturity Date:	September 30, 2019 (1 year, 2 months and 11 days from today)
Action:	Buy
Face Value:	$1,000
Order Type:	$98.738 Limit Fill or Kill
Timing:	Day Only
Settlement Date:	07/20/2018

Market Price:	$987.38
Estimated Markup:	$0.00
Principal Amount:	$987.38
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S&P Rating NR	Moody's Rating Aaa	Coupon Rate 1.375	Coupon Frequency Semi-annually
Quoted Price $98.738	Yield To Maturity 2.450	Yield To Worst ---	CUSIP 9128282X7
Callable No	Next Call Date ---	Next Call Price ---	

„Correct" y =2.466 % 20/07/18

PV = -CF(20/07/18) = $ 991.55
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Bonds – sensitivity: (Modified) Duration

❖ More precisely, we will ask a question: how a (small) sudden change in market yields $y \to y + \Delta y$ will affect market prices of bonds and thus $\Delta PV / PV$

❖ In general (for zero / fixed coupon bonds):

$$\frac{\Delta PV}{PV} \approx -\frac{D}{(1+y)}\Delta y = -MD\,\Delta y$$

❖ Duration:

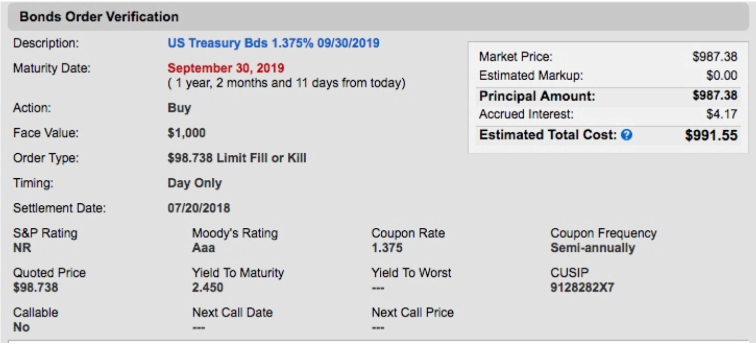
$$PV(y) = \sum_t \frac{CPN(t)}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

$$D = \frac{1}{PV}\left(\sum_t t\,\frac{CPN(t)}{(1+y)^t} + T\,\frac{FV}{(1+y)^T}\right)$$

$$T = 437/365 = 1.197\ yr$$

$$D = \frac{1}{991.55}\left(\frac{72}{365}\,\frac{6.88}{(1+y)^{72/365}} + \frac{254}{365}\,\frac{6.88}{(1+y)^{254/365}} + \frac{437}{365}\,\frac{6.88+1000}{(1+y)^{437/365}}\right) = 1.187\ yr$$

$$MD = \frac{1}{(1+y)}1.187 = 1.158\ yr$$



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„Correct" y =2.466 % 20/07/18

PV = $ 991.55

$y \to y + 1\%$	$y \to y - 1\%$
Exact:	
PV = $ 980.18	PV = $ 1003.16
ΔPV = – $ 11.37	ΔPV = + $ 11.61
(– 1.146%)	(+ 1.171%)

Approximate:

ΔPV=−991.55×1.158× ±1% = ∓$11.49

(− 1.158 × ±1% = ∓1.158%)

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Bonds – sensitivity: (Modified) Duration

- More precisely, we will ask a question: how a (small) sudden change in market yields y → y + Δy will affect market prices of bonds and thus ΔPV / PV
- One may also ask what about the whole portfolio (P) of (zero / fixed coupon) bonds. If we assume that the change in yield Δy is the same for all bonds in the portfolio: *

$$\Delta PV\left(P = \sum_i n_i PV_i\right) = \sum_i n_i \,\Delta PVi \approx -\sum_i n_i PV_i MD_i \,\Delta y$$

Total (weighted) MD
of the portfolio

- If the portfolio consists of both assets (i.e. bonds we hold) and liabilities (e.g. issued bonds) the duration of assets > 0 and duration of liabilities < 0 (all CFs → -CFs). One can therefore construct a portfolio of ZERO MD, which is immune to small Δy changes!

* Bonds may vary e.g. in Maturity (Duration risk), so in general y and Δy don't have to be the same even for the bonds of the same issuer / credit risk class ! (will be discussed later: the yield curve)

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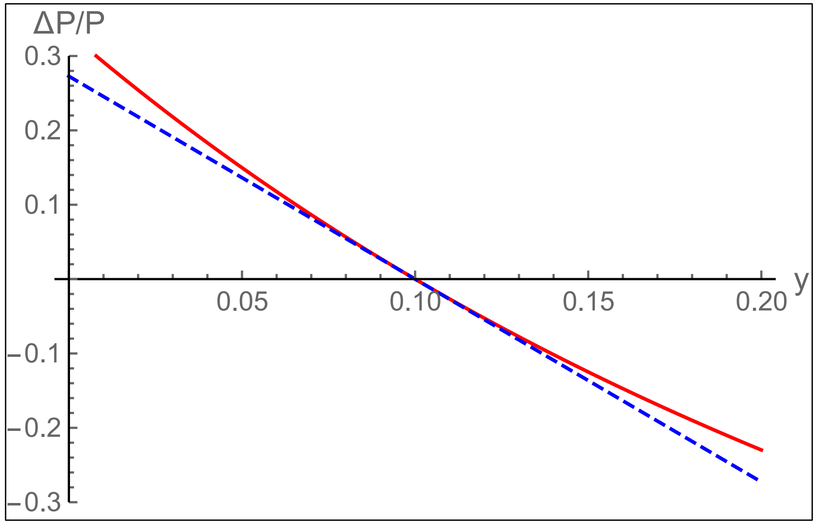
Bonds – sensitivity: Convexity

- More precisely, we will ask a question: how a (bigger) sudden change in market yields y → y + Δy will affect market prices of bonds and thus ΔPV / PV
- (Modified) Duration quantifies only the leading linear impact of Δy change

$$\frac{\Delta\, PV}{PV} \approx -\frac{D}{(1+y)}\Delta y = -\,MD\,\Delta y$$



- In fact bond prices PV(y) are CONVEX functions of y

$$PV(y) = \sum_t \frac{CPN(t)}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

$$\frac{\partial^2 PV(y)}{\partial y^2} = \frac{1}{(1+y)^2}\left(\sum_t t(t+1)\frac{CPN(t)}{(1+y)^t} + T(T+1)\,\frac{FV}{(1+y)^T}\right) > 0$$

Bonds – sensitivity: Convexity

- More precisely, we will ask a question: how a (bigger) sudden change in market yields y → y + Δy will affect market prices of bonds and thus ΔPV / PV
- (Modified) Duration quantifies only the leading linear impact of Δy change

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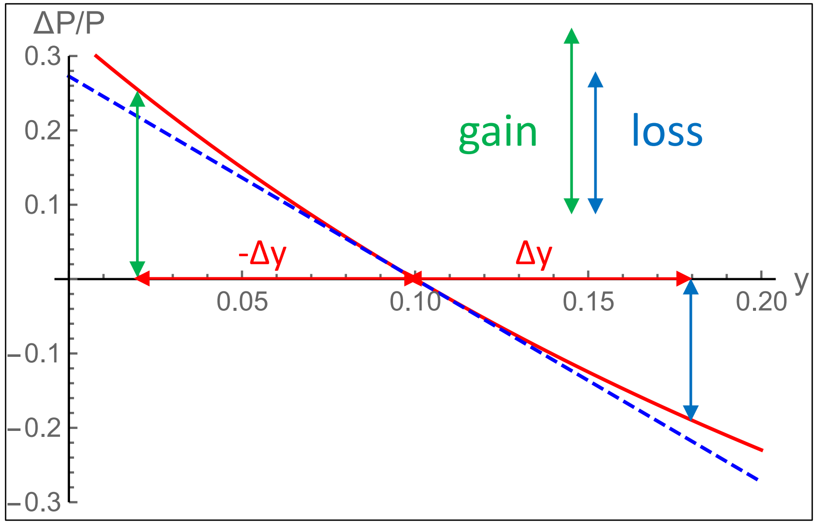

- In fact bond prices PV(y) are CONVEX functions of y
- This plays in favour of bondholders !
  Possible losses if y↑ < possible gains if y↓
  (when the "amplitude" |Δy| of y changes is the same)
- This effect is quantified by CONVEXITY (C)

$$C \equiv \frac{1}{PV}\frac{\partial^2 PV(y)}{\partial y^2} = \frac{1}{(1+y)^2}\frac{1}{PV}\left(\sum_t t(t+1)\frac{CPN(t)}{(1+y)^t} + T(T+1)\,\frac{FV}{(1+y)^T}\right)$$

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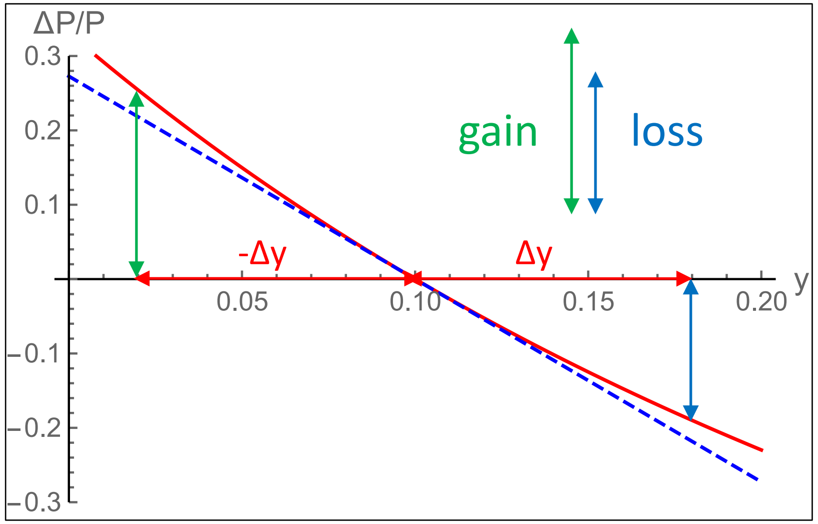
Bonds – sensitivity: Convexity

- More precisely, we will ask a question: how a (bigger) sudden change in market yields $y \rightarrow y + \Delta y$ will affect market prices of bonds and thus $\Delta PV / PV$
- Using (the first two terms in) Taylor's expansion of ΔPV in Δy one gets (more precise):

$$\frac{\Delta\, PV}{PV} \approx -\, MD\, \Delta y + \frac{1}{2}\, C\, (\Delta y)^2$$

- Convexity is usually higher for fixed coupon bonds compared to zero-coupon bonds of the same y and MD



$$MD \equiv -\frac{1}{PV}\frac{\partial PV(y)}{\partial y} = \frac{1}{(1+y)}\frac{1}{PV}\left(\sum_t t\,\frac{CPN(t)}{(1+y)^t} + T\,\frac{FV}{(1+y)^T}\right)$$

$$C \equiv \frac{1}{PV}\frac{\partial^2 PV(y)}{\partial y^2} = \frac{1}{(1+y)^2}\frac{1}{PV}\left(\sum_t t(t+1)\,\frac{CPN(t)}{(1+y)^t} + T(T+1)\,\frac{FV}{(1+y)^T}\right)$$

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- More precisely, we will ask a question: how a (bigger) sudden change in market yields $y \to y + \Delta y$ will affect market prices of bonds and thus $\Delta PV / PV$
- Using (the first two terms in) Taylor's expansion of ΔPV in Δ

$$\frac{\Delta PV}{PV} \approx -MD\,\Delta y + \frac{1}{2}\,C\,(\Delta y)^2$$

$$PV(y) = \sum_t \frac{CPN(t)}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

$$MD \equiv -\frac{1}{PV}\frac{\partial PV(y)}{\partial y} = \frac{1}{(1+y)}\frac{1}{PV}\left(\sum_t t\,\frac{CPN(t)}{(1+y)^t} + T\,\frac{FV}{(1+y)^T}\right) = 1.158\ yr$$

$$C \equiv \frac{1}{PV}\frac{\partial^2 PV(y)}{\partial y^2} = \frac{1}{(1+y)^2}\frac{1}{PV}\left(\sum_t t(t+1)\frac{CPN(t)}{(1+y)^t} + T(T+1)\frac{FV}{(1+y)^T}\right) =$$

$$= 2.48\ yr^2$$

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„Correct" y =2.466 % 20/07/18

PV = $ 991.55

$y \to y + 1\%$ $y \to y - 1\%$

Exact:

$\Delta PV/PV = -1.146\%$ $\Delta PV/PV = +1.171\%$

Approximate (only MD):

$\Delta PV/PV = -\,1.158 \times \pm 1\% = \mp 1.158\%$

Better approx. (MD & C):

$\Delta PV/PV = \Delta PV/PV + ½ \times 2.48 \times (1\%)^2$

$\Delta PV/PV = -1.146\%$ $\Delta PV/PV = +1.171\%$

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Bonds – yield & sensitivity: Summary

$$\sum_t \frac{CF(t)}{(1+y)^t} = 0$$

- The general pricing formula for zero / fixed coupon bonds

$$PV = \sum_t \frac{CPN(t)}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

Can be solved either for the Yield To Maturity: y (if we know PV) or for PV (if we know YTM)

- **NOTE 1:** PV is the DIRTY PRICE !
- **NOTE 2:** t=0 is the Settlement Date ! (usually SPOT = D+2).
- **NOTE 3:** be careful about conventions !, e.g. the ”Nominal Yield”: $(1+\text{"YTM"}/n)^n = 1 + y$

- One can also use a simplified formula

$$PV = \sum_{i=0}^{\#CPNs-1} \frac{CPN}{(1+y)^{\Delta t + i/n}} + \frac{FV}{(1+y)^{\Delta t + (\#CPNs-1)/n}}$$

Δt = time to next CPN !

Bonds – yield & sensitivity: Summary

$$\sum_t \frac{CF(t)}{(1+y)^t} = 0$$

❖ The (percentage) change in PV caused by the yield change $y \to y + \Delta y$ can be approximated by:

$$\frac{\Delta\, PV}{PV} \approx -\, MD\ \Delta y + \frac{1}{2}\, C\ (\Delta y)2$$

❖ MD is the MODIFIED DURATION

$$MD \equiv -\frac{1}{PV}\frac{\partial PV(y)}{\partial y} = \frac{D}{(1+y)}$$

where D is DURATION, i.e. some proxy of the bond's "average lifetime" (weighted by PV(CFs))

$$D \equiv \frac{1}{PV}\left(\sum_t t\,\frac{CPN(t)}{(1+y)^t} + T\ \frac{FV}{(1+y)^T}\right)$$

❖ C is the CONVEXITY

As bond prices PV are convex functions of y: $C > 0$, which is in favour of bondholders

$$C \equiv \frac{1}{PV}\frac{\partial^2 PV(y)}{\partial y^2} = \frac{1}{(1+y)^2}\,\frac{1}{PV}\left(\sum_t t(t+1)\,\frac{CPN(t)}{(1+y)^t} + T(T+1)\ \frac{FV}{(1+y)^T}\right)$$

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Summary: what to remember

$$\sum_t \frac{CF(t)}{(1+y)^t} = 0$$

Bond
Basic Bond Taxonomy
Clean/Dirty Price/Accrued Interest
Settlement Date (SPOT)
Day Count Convention (30/360 vs ACT)
YTM
(Modified) Duration
Convexity

$$\frac{\Delta\, PV}{PV} \approx -\, MD\ \Delta y + \frac{1}{2}\ C\ (\Delta y)2$$