Financial Instruments and Pricing

Fall 2019

Set 1
(Time Value of Money)

1. In the beginning of October the bank account XYZ showed a balance of 100 000 PLN
   a) Below is the list of inflows to / outflows from the account in October:
      - 5 X : inflow 17 000 PLN
      - 12 X: inflow 35 000 PLN
      - 21 X: outflow -55 000 PLN

      Compute interest added to the account in the end (last day) of October, assuming the interest rate is 2% p.a. (per annum).

      In November there were some additional inflows to / outflows from the account:
      - 9 XI : outflow -25 000 PLN
      - 30 XI: inflow 100 000 PLN

      Compute interest added to the account in the end (last day) of November.

2. What time t (in years) is needed to double the principal sum on a bank deposit with interest r % p.a. assuming:
   a) no interest is added (capitalized) before the end of the deposit (simple interest)
   b) interest is capitalized once a month, quarter, year, and in general n times a year (compound interest)
   c) Continuous compounding of interest

   Derive general formula. Compute t for r = 6% p.a.

3. What should be the nominal interest rate r offered by a bank to ensure that savings of its clients rise at least as fast as the inflation rate i = 2.5% p.a. Assume yearly, half-yearly, quarterly, monthly, and continuous capitalization of interest.

4. Assume the annual effective interest rate (yield) y = … % / year. What is the equivalent half-annual, monthly, weekly, daily, continuous effective interest rate (yield). Derive general formula and make computations for y = 5 % p.a.

5. Show that if interest on a deposit is capitalized more often than once a year (n > 1) then the effective interest rate y > the nominal rate r (one must assume r > 0).

6. Some time ago Polish banks offered special "tax-free" deposits with daily capitalization of interest. The deposits enabled one to avoid taxation of interest (i.e. no 19% tax rate was deducted from interest paid). Assume that Bank A offered a 91-day "tax-free" deposit with daily capitalization of interest and interest rate r = 6% p.a. What should be the interest rate of a standard (no „tax-free") 91-day deposit offered by Bank B that would make its offer competitive compared to Bank A. Make exact calculations and also approximate calculations using continuous compounding of interest.

7. What amount should be put aside on the bank account each quarter in order to accumulate 10 000 PLN after 2 years of saving. Assume interest rate of r =4% p.a. and
    a) no capitalization of interest (simple interest)
    b) quarterly capitalization of interest (compound interest).

8. Bank A offers one-year deposit with monthly capitalization of interest and the interest rate is growing each month: from 1% p.a. in the first month, 2% p.a. in the second month, …, to 12% p.a. in the last month. The bank advertises the deposit as "the best deposit with an average interest rate of 6.5%" (arithmetic mean). Bank B offers a standard deposit with monthly capitalization of interest and constant interest rate 6,5% p.a. Bank C offers a deposit with monthly capitalization of interest and the interest rate is falling each month: from 12% p.a. in the first month, …, to 1% p.a. in the last month.
    a) Which bank should we choose? (in each case compute effective interest rate)
    b) Would the above change if the banks were paying out interest on a monthly basis instead of adding it to the principal amount of the deposit ?

9. A Bank offers a loan of 10 000 PLN for one year with interest rate 10% p.a. and quarterly amortization. A client can choose between the system of equal principal payments (each quarter 1/4 of the initial principal value of the loan + accrued interest is paid) or equal total payments (each quarter client pays the same constant amount).
    a) Compute payments done by the client (CF) in both systems of amortization
    b) Prepare Amortization Schedule (a table showing current principal value of the loan in the beginning / end of each quarter and a split of the payment (CF) into Principal payment and Interest payment)
    c) Compute total amount of interest paid to the Bank in both systems
    d) Which system is more favourable for the Bank / Client (check what is the effective interest rate in each case) ?

10. A Bank granted 20-year mortgage of 200 000 PLN. The mortgage is amortized monthly as an annuity, i.e. in a system of equal total payments (principal + interest = const.). The interest rate is floating (variable) and can change every 3 months. It is set as the WIBOR3M rate + 2% p.a. (the interest rate is recalculated every 3 months based on the current WIBOR3M rate). At the initial moment (day when the credit was granted) the WIBOR3M rate was 4% p.a.
    e) Compute (equal) monthly payments set for the first three months
    f) Compute the outstanding amount of the mortgage (i.e. the remaining balance of the principal amount) after 3 months
    g) After 3 months the WIBOR3M rate increased to 5% p.a. Compute new (equal) monthly payments set for next three-months.

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Useful Wolfram Mathematica functions:

- DayCount[]
- TimeValue[]
- EffectiveInterest[]
- Annuity[]
- AnnuityDue[]
- CashFlow[]