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1 exercise

Figure 1: Answers for ex.1 - all values given in EUR

	30/360 US	30E/360	ACT/ACT (ICMA)	ACT/360 (fixed)	ACT/360
coupons in 2018	1000	1000	1000	1000	1013.89
coupons in 2019	1000	1000	1000	1000	1013.89
coupons in 2020	1000	1002.78	1000	1002.74	1016.67
accrued interest	641.667	647.222	642.077	643.836	652.778
dirty price	10 621.7	10 627.2	10 622.1	10 623.8	10 632.8

In[1]:= (** EXERCISE 1 A **)

```
In[2]:= P = 10000;
r = 0.1;
frequency = 1.0;
```

```
In[5]:= (** count convention: 30/360 US **)

(* 28.02.2017 - 28.02.2018 *)
ΔYears = 2018 - 2017;
ΔMonths = 2 - 2;
ΔDays = 30 - 30;
dayCount = ΔYears * 360 + ΔMonths * 30 + ΔDays ;
accruedIntrest = P * r * dayCount / 360

(* 28.02.2018 - 28.02.2019 *)
ΔYears = 2019 - 2018;
ΔMonths = 2 - 2;
ΔDays = 30 - 30;
dayCount = ΔYears * 360 + ΔMonths * 30 + ΔDays ;
accruedIntrest = P * r * dayCount / 360

(* 28.02.2019 - 28.02.2020 *)
ΔYears = 2020 - 2019;
ΔMonths = 2 - 2;
ΔDays = 30 - 30;
dayCount = ΔYears * 360 + ΔMonths * 30 + ΔDays ;
accruedIntrest = P * r * dayCount / 360
```

```
Out[9]= 1000.

Out[14]= 1000.

Out[19]= 1000.
```

In[20]:=
```
(** count convention: 30E/360 **)

(* 28.02.2017 - 28.02.2018 *)
ΔYears = 2018 - 2017 ;
ΔMonths = 2 - 2 ;
ΔDays = 28 - 28;
dayCount = ΔYears * 360 + ΔMonths * 30 + ΔDays ;
accruedIntrest = P * r * dayCount / 360

(* 28.02.2018 - 28.02.2019 *)
ΔYears = 2019 - 2018 ;
ΔMonths = 2 - 2 ;
ΔDays = 28 - 28;
dayCount = ΔYears * 360 + ΔMonths * 30 + ΔDays ;
accruedIntrest = P * r * dayCount / 360

(* 28.02.2019 - 28.02.2020 *)
ΔYears = 2020 - 2019 ;
ΔMonths = 2 - 2 ;
ΔDays = 29 - 28;
dayCount = ΔYears * 360 + ΔMonths * 30 + ΔDays
accruedIntrest = P * r * dayCount / 360
```

Out[24]= 1000.

Out[29]= 1000.

Out[33]= 361

Out[34]= 1002.78

In[35]:=

In[36]:=
```
(** count convention: ACT/ACT (ICMA) **)

(* 28.02.2017 - 28.02.2018 *)
accruedIntrest = P * r

(* 28.02.2018 - 28.02.2019 *)
accruedIntrest = P * r

(* 28.02.2019 - 28.02.2020 *)
accruedIntrest = P * r
```

Out[36]= 1000.

Out[37]= 1000.

Out[38]= 1000.

In[45]:=
```
(** count convention: ACT 365 (fixed) **)

(* 28.02.2017 - 28.02.2018 *)

dayCount = DateDifference[{2017, 2, 28}, {2018, 2, 28}]
accruedIntrest = P * r * dayCount[[1]]/ 365

(* 28.02.2018 - 28.02.2019 *)

dayCount = DateDifference[{2018, 2, 28}, {2019, 2, 28}]
accruedIntrest = P * r * dayCount[[1]] / 365

(* 28.02.2019 - 28.02.2020 *)
dayCount = DateDifference[{2019, 2, 28}, {2020, 2, 29}]
accruedIntrest = P * r * dayCount[[1]] / 365
```

Out[45]= 365 days

Out[46]= 1000.

Out[47]= 365 days

Out[48]= 1000.

Out[49]= 366 days

Out[50]= 1002.74

In[39]:=
```
(** count convention: ACT 360 **)

(* 28.02.2017 - 28.02.2018 *)

dayCount = DateDifference[{2017, 2, 28}, {2018, 2, 28}]
accruedIntrest = P * r * dayCount[[1]] / 360

(* 28.02.2018 - 28.02.2019 *)

dayCount = DateDifference[{2018, 2, 28}, {2019, 2, 28}]
accruedIntrest = P * r * dayCount[[1]] / 360

(* 28.02.2019 - 28.02.2020 *)
dayCount = DateDifference[{2019, 2, 28}, {2020, 2, 29}]
accruedIntrest = P * r * dayCount[[1]] / 360
```

Out[39]= 365 days

Out[40]= 1013.89

Out[41]= 365 days

Out[42]= 1013.89

Out[43]= 366 days

Out[44]= 1016.67

```
☺ (** EXERCISE 1 B **)

(* Consider a 10-year bond with maturity
  on 29th February 2020. The bond has nominal
value of 10 000 EUR and coupon rate of 10
 %.Coupons are paid annually in the end of
February (EOM). For a transaction done on the 17
 th October 2019 the clean price was 99,80.
Compute accrued interest and dirty price
 (cash flow) paid on the spot date (D+2). *)
```

(** The spot date(D+2) in case of 17
th of October (Thursday) is 21th of October (Monday)

date1 = 28.02.2019 – starting date for the accrual
date2 = 21.10.2019 –
date through which interest is being accrued. (settlement date of the trade)
date3 = 29.02.2019
**)

In[106]:=

```
P = 10 000 ;
r = 0.1 ;
frequency = 1.0 ;
CleanPrice = 0.9980 * P
```

Out[109]= 9980.

```
(** count convention: 30/360 US **)
ΔYears = 2019 - 2019 ;
ΔMonths = 10 - 2 ;
ΔDays = 21 - 30;
dayCount = ΔYears * 360 + ΔMonths * 30 + ΔDays ;
accruedIntrest = P * r * dayCount / 360
DirtyPrice = CleanPrice + accruedIntrest
```

Out[98]= 641.667

Out[99]= 10 621.7

In[100]:=
```
(** count convention: 30E/360 **)
ΔYears = 2019 - 2019 ;
ΔMonths = 10 - 2 ;
ΔDays = 21 - 28;
dayCount = ΔYears * 360 + ΔMonths * 30 + ΔDays ;
accruedIntrest = P * r * dayCount / 360
DirtyPrice = CleanPrice + accruedIntrest
```

Out[104]= 647.222

Out[105]= 10 627.2

In[113]:=
```
(** count convention: ACT/ACT (ICMA) **)

dayDifference12 = DateDifference[{2019, 2, 28}, {2019, 10, 21}]
dayDifference13 = DateDifference[{2019, 2, 28}, {2020, 2, 29}]
accruedIntrest = P * r * dayDifference12[[1]] / (dayDifference13[[1]] * frequency)
DirtyPrice = CleanPrice + accruedIntrest
```

Out[113]= 235 days

Out[114]= 366 days

Out[115]= 642.077

Out[116]= 10 622.1

In[117]:=
```
(** count convention: ACT 365 (fixed) **)

dayDifference = DateDifference[{2019, 2, 28}, {2019, 10, 21}]
accruedIntrest = P * r * dayDifference[[1]] / 365
DirtyPrice = CleanPrice + accruedIntrest
```

Out[117]= 235 days

Out[118]= 643.836

Out[119]= 10 623.8

In[120]:=
```
(** count convention: ACT 360 **)

dayDifference = DateDifference[{2019, 2, 28}, {2019, 10, 21}]
accruedIntrest = P * r * dayDifference[[1]] / 360
DirtyPrice = CleanPrice + accruedIntrest
```

Out[120]= 235 days

Out[121]= 652.778

Out[122]= 10 632.8

(*Exercise 2*)

```
(*PS0420*) (*adding 2 days for transaction to be settled on the spot date,
but also adding 2 more for the spot date not to be on weekend*)
DateDifference[{2019, 04, 25}, {2019, 10, 21},
 "Year", DayCountConvention → "ActualActualICMA"]
```

Out[•]= 0.489071 yr

```
In[•]:= (*we calculate accrued interest,
which contains DCF - from the latest coupon date to the spot date*)
```

```
In[•]:= (*accrued interest = DCF x coupon rate x principal*)
0.489071 * 0.015 * 1000
```

Out[•]= 7.33607

```
(*just checking if the formula above is correct*)
179 / 366 * 0.015 * 1000
```

Out[•]= 7.33607

```
In[•]:= (*dirty price*)
7.33607 + 1000 * 1.0019
```

Out[•]= 1009.24

```
In[•]:= (*YTM*)
```

```
In[•]:= DateDifference[{2019, 10, 21}, {2020, 04, 25},
 DayCountConvention → "ActualActualICMA"]
```

Out[•]= 187 days

In[•]:= Solve$\left[1009.24 == \frac{15 + 1000}{(1 + y)^{\frac{187}{365}}}, y\right]$

Out[•]= {{y → 0.0111701}}

```
In[•]:= (*PS0421*)
DateDifference[{2019, 04, 25}, {2019, 10, 21},
 "Year", DayCountConvention → "ActualActualICMA"]
```

Out[•]= 0.489071 yr

```
In[•]:= (*accrued interest*)
0.489071 * 0.02 * 1000
```

Out[•]= 9.78142

In[•]:=
```
(*dirty price*)
9.78142 + 1000 * 1.0086
```

Out[•]= 1018.38

In[•]:=
```
(*YTM*)
```

In[•]:=
```
DateDifference[{2019, 10, 21}, {2020, 04, 25},
 DayCountConvention → "ActualActualICMA"]
```

Out[•]= 187 days

In[•]:=

$$\text{Solve}\left[1018.38 == \frac{20}{(1+y)^{\frac{187}{365}}} + \frac{20+1000}{(1+y)^{\frac{187}{365}+1}}, y, \text{Reals}\right]$$

... Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[•]= {{y → 0.0141705}}

In[•]:=
```
(*PS0721*)
DateDifference[{2019, 07, 25}, {2019, 10, 21},
 "Year", DayCountConvention → "ActualActualICMA"]
```

Out[•]= 0.240437 yr

In[•]:=
```
(*accrued interest*)
0.240437 * 0.0175 * 1000
```

Out[•]= 4.20765

In[•]:=
```
(*dirty price*)
4.20765 + 1000 * 1.0046
```

Out[•]= 1008.81

```
(*YTM*)
```

In[•]:=
```
DateDifference[{2019, 10, 21}, {2020, 07, 25},
 DayCountConvention → "ActualActualICMA"]
```

Out[•]= 278 days

In[•]:=

$$\text{Solve}\left[1008.81 == \frac{17.5}{(1+y)^{\frac{278}{365}}} + \frac{17.5+1000}{(1+y)^{\frac{278}{365}+1}}, y, \text{Reals}\right]$$

... Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[•]= {{y → 0.0147994}}

In[•]:=
```
(*PS0422*)
DateDifference[{2019, 04, 25}, {2019, 10, 21},
 "Year", DayCountConvention → "ActualActualICMA"]
```

Out[•]= 0.489071 yr

In[•]:=
```
(*accrued interest*)
0.489071 * 0.0225 * 1000
```

Out[•]= 11.0041

In[•]:=
```
(*dirty price*)
11.0041 + 1000 * 1.0175
```

Out[•]= 1028.5

```
(*YTM*)
```

In[•]:=
```
DateDifference[{2019, 10, 21}, {2020, 04, 25},
 DayCountConvention → "ActualActualICMA"]
```

Out[•]= 187 days

In[•]:= Solve$\left[1028.5 == \frac{22.5}{(1+y)^{\frac{187}{365}}} + \frac{22.5}{(1+y)^{\frac{187}{365}+1}} + \frac{22.5+1000}{(1+y)^{\frac{187}{365}+2}}, y, \text{Reals}\right]$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[•]= {{y → 0.0153176}}

In[•]:=
```
(*OK0720*)
(*accrued interest = 0*)
```

In[•]:=
```
(*dirty price*)
1000 * 0.9912
```

Out[•]= 991.2

```
(*YTM*)
```

In[•]:=
```
DateDifference[{2019, 10, 21}, {2020, 07, 25},
 DayCountConvention → "ActualActualICMA"]
```

Out[•]= 278 days

In[•]:= Solve$\left[991.2 == \frac{1000}{(1+y)^{\frac{278}{365}}}, y\right]$

Out[•]= {{y → 0.0116727}}

(*Exercise 3*)

```
(*PS0420*)
DateDifference[{2019, 10, 21}, {2020, 04, 25}, "Year",
 DayCountConvention -> "ActualActualICMA"]
```

Out[•]= 0.510929 yr

```
In[•]:= T = (0.510929 * 15) / 15
```

Out[•]= 0.510929

```
In[•]:= Du = ((0.510929 * (15 + 1000) / (1 + 0.0111701) ^ 0.510929)) / 1009.24
```

Out[•]= 0.510937

```
In[•]:= MD = 0.510937 / (1 + 0.0111701)
```

Out[•]= 0.505293

```
In[•]:= FinancialBond[{"FaceValue" → 1000, "Coupon" → 0.015, "Maturity" → {2020, 4, 25}},
  {"InterestRate" → 0.0111701, "Settlement" → {2019, 10, 21},
   "DayCountBasis" → "ActualICMA"}, "Rules"]
```

Out[•]= {Value → 1001.92, FullValue → 1009.26, AccruedInterest → 7.33607,
 Duration → 0.510929, ModifiedDuration → 0.505285, Convexity → 0.755016,
 CouponPeriodDays → 366, CouponToSettlementDays → 179, SettlementToCouponDays → 187,
 NextCouponDate → Day: Sat 25 Apr 2020, PreviousCouponDate → Day: Thu 25 Apr 2019,
 RemainingCoupons → 1, AccruedFactor → $\frac{179}{366}$}

```
In[•]:= (*PS0421*)
DateDifference[{2019, 10, 21}, {2020, 04, 25},
 "Year", DayCountConvention → "ActualActualICMA"]
```

Out[•]= 0.510929 yr

```
In[•]:= T = (0.510929 * 20 + 1.510929 * (20 + 1000)) / (2 * 20 + 1000)
```

Out[•]= 1.4917

```
In[•]:= Du = ((0.510929 * 20 / (1 + 0.0141705) ^ 0.510929) +
      1.510929 * (20 + 1000) / (1 + 0.0141705) ^ 1.510929) / 1018.38
```

Out[•]= 1.49146

```
In[•]:= MD = 1.49146 / (1 + 0.0141705)
```

Out[•]= 1.47062

```
In[•]:= FinancialBond[{"FaceValue" → 1000, "Coupon" → 0.020, "Maturity" → {2021, 4, 25}},
  {"InterestRate" → 0.0141705, "Settlement" → {2019, 10, 21},
   "DayCountBasis" → "ActualICMA"}, "Rules"]
```

```
Out[•]= {Value → 1008.62, FullValue → 1018.4, AccruedInterest → 9.78142,
 Duration → 1.49143, ModifiedDuration → 1.47059, Convexity → 3.63127,
 CouponPeriodDays → 366, CouponToSettlementDays → 179, SettlementToCouponDays → 187,
 NextCouponDate → Day: Sat 25 Apr 2020 , PreviousCouponDate → Day: Thu 25 Apr 2019 ,
 RemainingCoupons → 2, AccruedFactor → 179/366}

In[•]:=
(*PS0721*)
DateDifference[{2019, 10, 21}, {2020, 07, 25},
 "Year", DayCountConvention → "ActualActualICMA"]

Out[•]= 0.759563 yr

In[•]:= T = (0.759563 * 17.5 + 1.759563 * (17.5 + 1000)) / (2 * 17.5 + 1000)

Out[•]= 1.74265

In[•]:= Du = ((0.759563 * 17.5 / (1 + 0.0147994) ^0.759563) +
    1.759563 * (17.5 + 1000) / (1 + 0.0147994) ^1.759563) / 1008.81

Out[•]= 1.74246

In[•]:= MD = 1.74246 / (1 + 0.0147994)

Out[•]= 1.71705

In[•]:=
FinancialBond[{"FaceValue" → 1000, "Coupon" → 0.0175, "Maturity" → {2021, 7, 25}},
 {"InterestRate" → 0.0147994, "Settlement" → {2019, 10, 21},
  "DayCountBasis" → "ActualICMA"}, "Rules"]

Out[•]= {Value → 1004.63, FullValue → 1008.84, AccruedInterest → 4.20765,
 Duration → 1.74241, ModifiedDuration → 1.717, Convexity → 4.65641,
 CouponPeriodDays → 366, CouponToSettlementDays → 88,
 SettlementToCouponDays → 278, NextCouponDate → Day: Sat 25 Jul 2020 ,
 PreviousCouponDate → Day: Thu 25 Jul 2019 , RemainingCoupons → 2, AccruedFactor → 44/183}

(*PS0422*)

In[•]:= DateDifference[{2019, 10, 21}, {2020, 04, 25},
 "Year", DayCountConvention → "ActualActualICMA"]

Out[•]= 0.510929 yr

In[•]:= Dur[n_, cpn_, t_, y_] :=
 (Sum[cpn (t + i) / (1 + y) ^ (t + i), {i, 0, n}] + 1000 (n + t) / (1 + y) ^ (n + t)) /
  (Sum[cpn / (1 + y) ^ (t + i), {i, 0, n}] + 1000 / (1 + y) ^ (n + t))
```

In[∘]:=
```
Dur[2, 22.5, 0.510929, 0.015317]
```
Out[∘]= 2.44614

In[∘]:=
```
T = (0.510929 * 22.5 + 1.510929 * 22.5 + 2.510929 * (1000 + 22.5)) / (3 * 22.5 + 1000)
```
Out[∘]= 2.4477

In[∘]:=
```
Du = ((0.510929 * 22.5 / (1 + 0.0153176) ^ 0.510929) +
    (1.510929 * 22.5 / (1 + 0.0153176) ^ 1.510929) +
    (2.510929 * (22.5 + 1000) / (1 + 0.0153176) ^ 2.510929)) / 1028.5
```
Out[∘]= 2.44619

In[∘]:=
```
MD = 2.44619 / (1 + 0.0153176)
```
Out[∘]= 2.40929

In[∘]:=
```
FinancialBond[{"FaceValue" → 1000, "Coupon" → 0.0225, "Maturity" → {2022, 4, 25}},
  {"InterestRate" → 0.015324, "Settlement" → {2019, 10, 21},
   "DayCountBasis" → "ActualICMA"}, "Rules"]
```
Out[∘]=
```
{Value → 1017.5, FullValue → 1028.51, AccruedInterest → 11.0041,
 Duration → 2.44614, ModifiedDuration → 2.40922, Convexity → 8.27807,
 CouponPeriodDays → 366, CouponToSettlementDays → 179, SettlementToCouponDays → 187,
 NextCouponDate → Day: Sat 25 Apr 2020, PreviousCouponDate → Day: Thu 25 Apr 2019,
 RemainingCoupons → 3, AccruedFactor → 179/366}
```

```
(*OK0720*)
```

In[∘]:=
```
DateDifference[{2019, 10, 21}, {2020, 07, 25},
  "Year", DayCountConvention → "ActualActualICMA"]
```
0.759563 yr

In[∘]:=
```
CF = (1000 - 991.2) / 991.2
```
Out[∘]= 0.00887813

In[∘]:=
```
T = (0.759563 * (8.9 + 1000)) / (8.9 + 1000)
```
Out[∘]= 0.759563

In[∘]:=
```
Du = ((0.759563 * (8.9 + 1000)) / ((1 + 0.0116727) ^ 0.759563)) / 1000
```
Out[∘]= 0.759598

In[∘]:=
```
MD = 0.759598 / (1 + 0.0116727)
```
Out[∘]= 0.750834

In[∘]:=
```
FinancialBond[
 {"FaceValue" → 991.2, "Coupon" → 0.00887813, "Maturity" → {2020, 7, 25}},
 {"InterestRate" → 0.0116727, "Settlement" → {2019, 10, 21},
  "DayCountBasis" → "ActualICMA"}, "Rules"]
```

Out[∘]=
```
{Value → 989.108, FullValue → 991.224, AccruedInterest → 2.11585,
 Duration → 0.759563, ModifiedDuration → 0.750799, Convexity → 1.30584,
 CouponPeriodDays → 366, CouponToSettlementDays → 88,
 SettlementToCouponDays → 278, NextCouponDate → Day: Sat 25 Jul 2020 ,
 PreviousCouponDate → Day: Thu 25 Jul 2019 , RemainingCoupons → 1, AccruedFactor → 44/183}
```

(*exercise 4*)

```
FV = 1000; (*face value*)
r = 0.05; (*coupon rate*)
CPN = FV r (*cash flow associated with each coupon*)
```

Out[•]= 50.

```
In[•]:= (*structure of cashflows*)
CF[0.5] = CPN
CF[1.5] = CPN
CF[2.5] = CPN + FV
```

Out[•]= 50.

Out[•]= 50.

Out[•]= 1050.

```
(*a formula for the principal value*)
PV[y_] := CF[0.5] / (1 + y) ^ (0.5) + CF[1.5] / (1 + y) ^ (1.5) + CF[2.5] / (1 + y) ^ (2.5)
```

```
In[•]:= currentDP = PV[0.06]
```

Out[•]= 1002.04

Exact calculations

```
In[•]:= exactDPa = PV[0.061]
exactDPb = PV[0.059]
exactDPc = PV[0.065]
exactDPd = PV[0.055]
```

Out[•]= 999.818

Out[•]= 1004.27

Out[•]= 990.99

Out[•]= 1013.28

In[∘]:=
```
(*exact differences*)
exactDPa - currentDP
exactDPb - currentDP
exactDPc - currentDP
exactDPd - currentDP
```

Out[∘]= -2.22483

Out[∘]= 2.23209

Out[∘]= -11.0522

Out[∘]= 11.2336

Approximate calculations

```
(*instead of typing the formula for the modified duration,
it is faster to differentiate the formula for PV*)
D[PV[y], y]
```

Out[∘]= $-\frac{2625.}{(1+y)^{3.5}} - \frac{75.}{(1+y)^{2.5}} - \frac{25.}{(1+y)^{1.5}}$

In[∘]:= fDer[y_] := $-\frac{2625.`}{(1+y)^{3.5`}} - \frac{75.`}{(1+y)^{2.5`}} - \frac{25.`}{(1+y)^{1.5`}}$

```
(*assuming that YTM changes by 0.1%, no matter which direction *)
fDer[0.06] 0.001
```

Out[∘]= -2.22846

```
fDer[0.06] 0.005(*assuming that YTM changes by 0.5%, no matter which direction*)
```

Out[∘]= -11.1423

1. Exercise 5

A small change of the current bond's price, relatively to the current bond's price is equal to minus modified duration times a small sudden change in the market yields.

$$\frac{\Delta PV}{PV} = -MD\Delta y \tag{1}$$

If the bank wants to be independent of the changes of the market yields, it needs to have the same amount of money times modified duration offered and to finance itself. We take these values with oposite signs and get an equation.

$$\sum_i MD_i \Delta y = 1 * 1\Delta y + 1 * 3\Delta y - 2 * 1.5\Delta y + P(1 - 10)\Delta y = 0, \tag{2}$$

where value P is computed in billion PLN and modified duration is computed in years. So P=1/9 billion PLN.

1 EXERCISE 6

In the Lecture we showed that Convexity: C >= 0. As a result the profit caused by the decrease in YTM by Dy >= loss caused by the increase in YTM by the same Dy (if all other conditions are equal investors should choose bonds with highest convexity C). Suppose an investor can choose between 2 bonds: bond A has maturity in 4 years and it pays annual coupon of 4%, bond B is zero-coupon and it has 3.75 years to maturity. YTMs of both bonds are equal 9.29%. a. compute modified duration (MD) and Convexity (C) for both bonds b. check the effect of a decrease/increase of YTM by 5% on prices of the bonds

SOLUTION:

Bank A:

- fixed-coupon (annual coupon of $\eta = 4\%$)
- maturity in $T = 4$ years
- yield to maturity: $YTM = y = 9.29\%$

Bank B:

- zero-coupon
- maturity in $T = 3.75$ years
- yield to maturity: $YTM = 9.29\%$

First of all, we need to calculate the ratio of the present value PF to the face value FV based on the given value YTM

case A: fixed-coupon bounds:

$$PV = \sum_t \frac{CPN}{(1+y)^t} + \frac{FV}{(1+y)^T} \tag{1}$$

$$PV = \sum_t \frac{FV \cdot \eta}{(1+y)^t} + \frac{FV}{(1+y)^T} \tag{2}$$

$$I_A = \frac{PV}{FV} = \sum_t \frac{\eta}{(1+y)^t} + \frac{1}{(1+y)^T} = 0.829704 \tag{3}$$

case B: zero-coupon bounds:

$$PV = \frac{FV}{(1+y)^T} \tag{4}$$

$$I_B = \frac{PV}{FV} = \frac{1}{(1+y)^T} = 0.716677 \tag{5}$$

1.1 Modified duration

case A: fixed-coupon bounds:

$$PV(y) = \sum_t \frac{CPN}{\left(1+y\right)^t} + \frac{FV}{\left(1+y\right)^T} \tag{6}$$

$$\frac{\Delta PV}{PV} \approx -\frac{1}{1+y}\frac{1}{PV}\sum_t \frac{FV \cdot \eta \cdot t}{\left(1+y\right)^t} + \frac{FV \cdot T}{\left(1+y\right)^T} \tag{7}$$

$$\frac{\Delta PV}{PV} \approx \left(-\frac{1}{1+y}\frac{1}{I_A}\sum_t \frac{\eta \cdot t}{\left(1+y\right)^t} + \frac{T}{\left(1+y\right)^T}\right)\Delta y = -\frac{D}{1+y}\Delta y = -MD \cdot \Delta y \tag{8}$$

$$MD = -\frac{1}{1+y}\frac{1}{I_A}\sum_t \frac{\eta \cdot t}{\left(1+y\right)^t} + \frac{T}{\left(1+y\right)^T} = 3.43125 \tag{9}$$

case B: zero-coupon bounds:

$$PV(y) = \frac{FV}{\left(1+y\right)^T} \tag{10}$$

$$\frac{\Delta PV}{PV} \approx -\frac{T}{1+y} \cdot \Delta y = -\frac{D}{1+y}\Delta y = -MD \cdot \Delta y \tag{11}$$

$$MD = \frac{T}{1+y} = \frac{4}{1+0.0929} = 3.431237 \tag{12}$$

1.2 Convexity

case A: fixed-coupon bounds:

$$C = \frac{1}{PV}\frac{\partial^2 PV(y)}{\partial y^2} = \frac{1}{(1+y)^2}\frac{1}{PV}\left(\sum_t \frac{t(t+1)}{(1+y)^t} FV \cdot \eta + T(T+1)\frac{FV}{(1+y)^T}\right) \tag{13}$$

$$C = \frac{1}{(1+y)^2}\frac{1}{I_A}\left(\sum_t \frac{t(t+1)}{(1+y)^t} \cdot \eta + T(T+1)\frac{1}{(1+y)^T}\right) = 15.3592 \tag{14}$$

case B: zero-coupon bounds:

$$C = \frac{1}{PV}\frac{\partial^2 PV(y)}{\partial y^2} = \frac{1}{(1+y)^2}\frac{1}{PV}\left(T(T+1)\frac{FV}{(1+y)^T}\right) \tag{15}$$

$$C = \frac{1}{(1+y)^2}\frac{1}{I_B}\left(T(T+1)\frac{1}{(1+y)^T}\right) = 14.913 \tag{16}$$

1.3 The effect of a decrease/increase of YTM by 5% on prices of the bonds

case A: fixed-coupon bounds:

$$\frac{PV(y)}{FV} = \sum_t \frac{\eta}{(1+y)^t} + \frac{1}{(1+y)^T} \tag{17}$$

$$I_{A,4.29} = \frac{PV(y = 4.29\%)}{FV} = 0.989545 \tag{18}$$

$$I_{A,9.29} = \frac{PV(y = 9.29)}{FV} = 0.829704 \tag{19}$$

$$I_{A,14.29} = \frac{PV(y = 14.29\%)}{FV} = 0.701953 \tag{20}$$

In case of fixed-coupon bounds, a decrease (y = 4.29) and increase (y = 14.29) of YTM by 5% increase the clean price of the bounds by

$$\frac{I_{A,4.29}}{I_{A,9.29}} = \frac{0.989545}{0.829704} \cdot 100\% = 119.265\%$$

and decrease the clean price of the bounds by

$$\frac{I_{A,14.29}}{I_{A,9.29}} = \frac{0.701953}{0.829704} \cdot 100\% = 84.603\%$$

, respectively.

case B: zero-coupon bounds:

$$\frac{PV(y)}{FV} = \frac{1}{(1+y)^T} \tag{21}$$

$$I_{B,4.29} = \frac{PV(y = 4.29\%)}{FV} = 0.85426 \tag{22}$$

$$I_{B,9.29} = \frac{PV(y = 9.29\%)}{FV} = 0.716677 \tag{23}$$

$$I_{B,14.29} = \frac{PV(y = 14.29\%)}{FV} = 0.605995 \tag{24}$$

In case of fixed-coupon bounds, a decrease ($y = 4.29\%$) and increase ($y = 14.29\%$) of YTM by 5% increase the clean price of the bounds by

$$\frac{I_{A,4.29}}{I_{A,9.29}} = \frac{0.85426}{0.716677} \cdot 100\% = 119.197\%$$

and decrease the clean price of the bounds by

$$\frac{I_{A,14.29}}{I_{A,9.29}} = \frac{0.605995}{0.716677} \cdot 100\% = 84.556\%$$

, respectively.

Conclusion: In case of fixed-coupon bounds, clean price of the bound is (slightly) more sensitive on YTM's changes.