Risk Management - Problems IV

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Time series modelling

1. Data file *ARdata.txt* contains a sample time series generated from some AR(p) process

   (a) Find the order p and fit the parameters: β_i $(i = 1, ..., p)$, α_0 using the linear regression method. In order to find p and the best fit use the "AIC" and "BIC" criteria (you can use the "AIC" and "BIC" method from Wolfram Mathematica or use the "standard" one if you prefer).

   (b) Compute the sample autocorrelation function (SACF, $\rho(t)$) and the sample partial autocorrelation function (SPACF, $\phi(t)$) and plot them fot $t = 1, ...10$. Based on the plot of SPACF check the order p (on the plot include Gaussian $N(0, (T-t)^{-1})$ bands for 95% confidence level to check when SPACF becomes statistically zero).

   (c) For the order p established in (b) fit the parameters: β_i $(i = 1, ..., p)$, α_0 using the Yule-Walker method (in the matrix form).

   (d) Using the data and the fit of point (a) and/or (c) compute the empirical noise $\hat{\eta}(t) = (y_t - \hat{y}_t)/\alpha_0$, where $\hat{y}_t = \sum_{i=1}^{p} \beta_i y_{t-i}$ and check if it has standard Gaussian $N(0, 1)$ distribution (use e.g. Kolmogorov-Smirnov test).

   (e) Using the fit of point (a) and/or (c) simulate $N = 100$ future (forecast) paths for $t = T+1, T+2, ..., T+10$. Using the generated forecast paths estimate the mean value $\langle y(t) \rangle$ and the standard deviation $\sigma(y(t))$ of $y(t)$ for each future $t = T + 1, T + 2, T + 10$ and plot them as the continuation of the sample data series (plot of mean with error bars of standard deviation).

2. Data file *MAdata.txt* contains a sample time series generated from some MA(q) process. Repeat all tasks of Problem 1 for this data series. Of course adjust to the case of an MA(q) process when necessary: e.g. remember that for the regression method you have to use TWO-STEP REGRESSION (please use $m = 5$ to estimate the realized noise in 1-st step and remember about normalizing it such that the variance is 1, in 2-nd step use this data to fit regression of $y(t)$ against the (shifted) noise variables), use SACF to estimate order q. For the Yule-Walker equations find a solution $\{\alpha_i\}$ which gives INVERTIBLE MA(q) model !

3. Data file *SARIMAdata.txt* contains a NON-STATIONARY sample time series (with some trend and periodicity).

   (a) Plot the (raw) data series and also plot SACF and SPACF for this data series

   (b) Try to detrend the data using the $\Delta^n = (1-B)^n$ operator, where B is the "backshift" operator ($By_t = y_{t-1}$ and $n = 1, 2, ...$ (for the linear, quadratic, ... trend, respectively). Plot again the detrended data and its SACF and SPACF.

   (c) Based on the bahaviour of SACF and SPACF above try to find periodicity of the data and cancel it by aplying the operator $\Delta_p = (1 - B^p)$ (where p is the period) to the (detrended) data series

   (d) Check if now the data seem to be stationary (make plots of data, SACF and PACF). If so, try to fit an ARMA(p,q) model to the stationary data. You can fit it "by hand" (using any method) or use automatic model fitting by Mathematica or some other software (but please do it "wisely", i.e. check the options).