exercise3

June 10, 2020

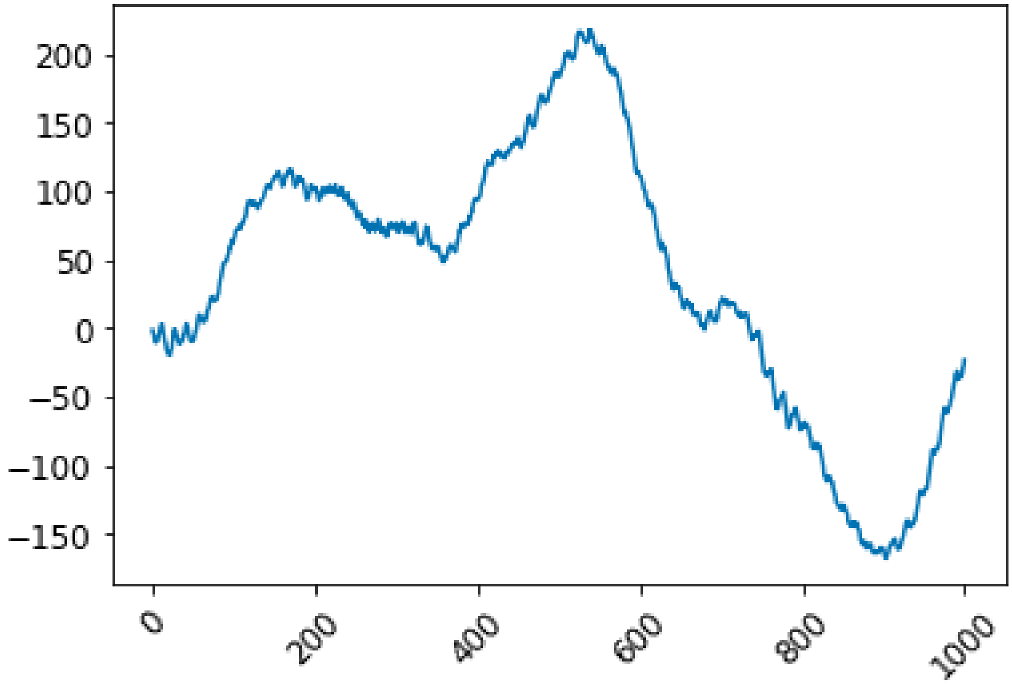
[1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from sklearn.linear_model import LinearRegression
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

plt.style.use('seaborn-colorblind')

import statsmodels.api as sm
import warnings
warnings.simplefilter('ignore')

%matplotlib inline
```

1 a)

Plot the (raw) data series and also plot SACF and SPACF for this data series

[2]:
```
data = []

with open("SARIMAdata.txt", "r") as f:
    for line in f:
        data.append(float(line))

data = np.array(data)

plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.plot(data)
```

[2]:
```
[<matplotlib.lines.Line2D at 0x7fc87fb6d310>]
```



[3]:
```
def dividing_data_into_subseries(data, k=2):
    data_k = []
    for i in range(len(data)-k):
        tmp = [ ]
        for j in range(k):
            tmp.append(data[j+i])
        tmp.append(data[k+i])
        data_k.append(tmp)

    return np.array(data_k).T
```

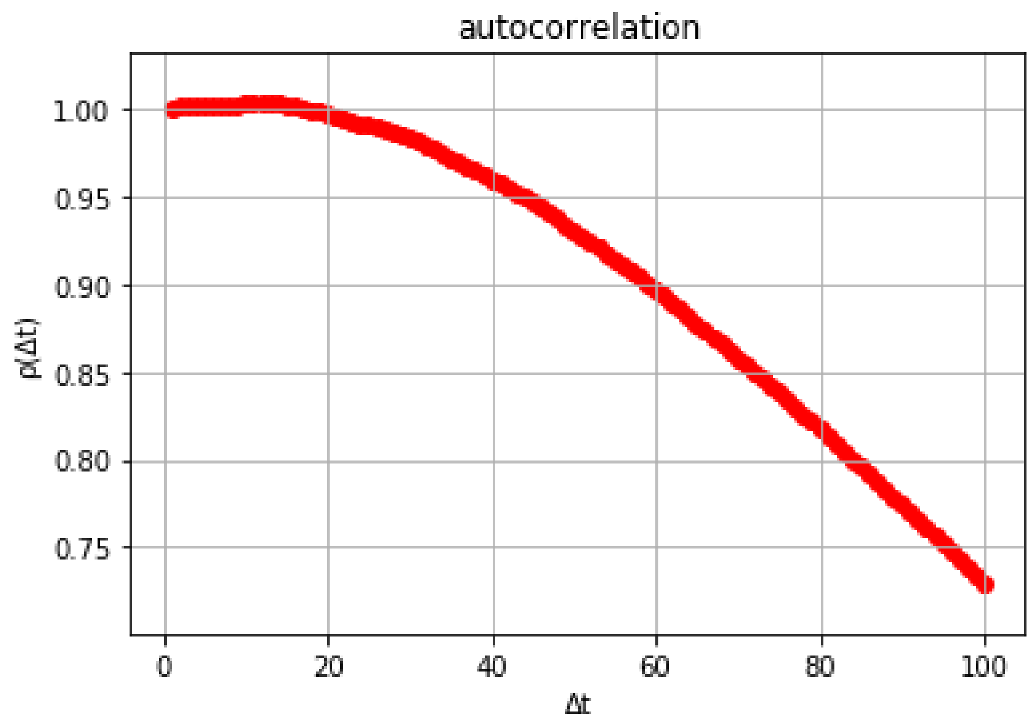
[4]:
```
def get_autocorrelation_coeff(data, delta):

    data_k = dividing_data_into_subseries(data, k=delta)
    denominator = (data**2).mean()
    nominator = (data_k[0]*data_k[-1]).mean()

    return nominator/denominator
```

[5]:
```
def get_autocorrelation(p,data):

    autocorrelation_tab = []
```

```
    for delta in range(1,p+1):
        autocorrelation = get_autocorrelation_coeff(data, delta)
        autocorrelation_tab.append(autocorrelation)

    return autocorrelation_tab
```

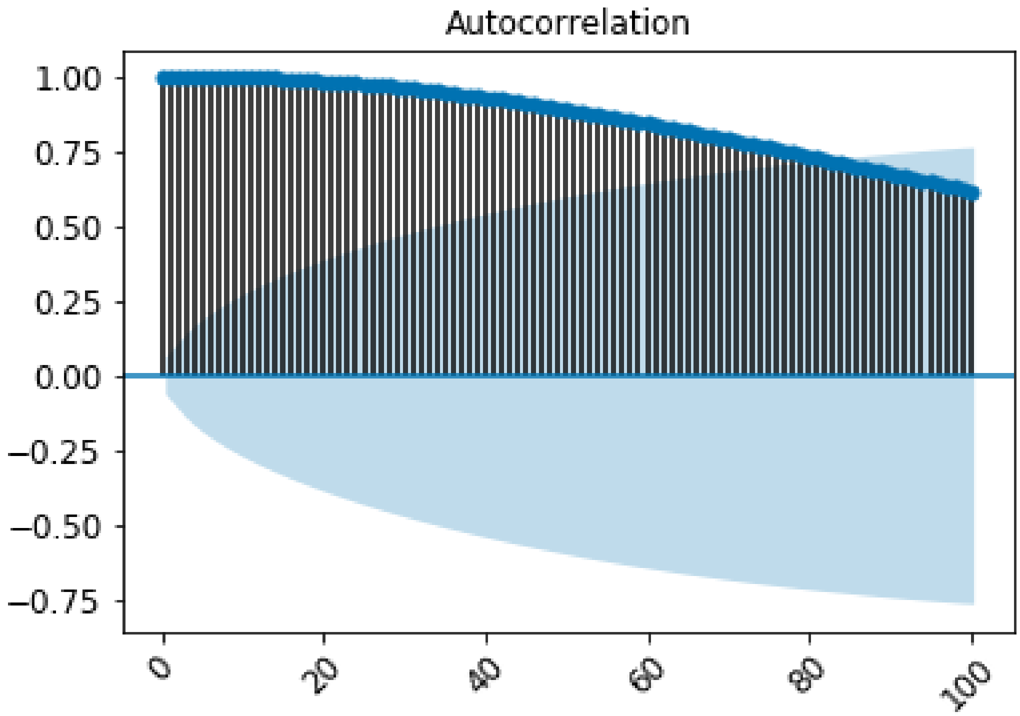
1.0.1 sample autocorrelation function (SACF)

[6]:
```
p=100

p_tab = list(range(1,p+1))
autocorrelation_tab = get_autocorrelation(p,data)

plt.title('autocorrelation')
plt.scatter(p_tab, autocorrelation_tab, color ='red')
plt.ylabel(u'\u03C1(\u0394t)')
plt.xlabel(u'\u0394t')
plt.grid()
```



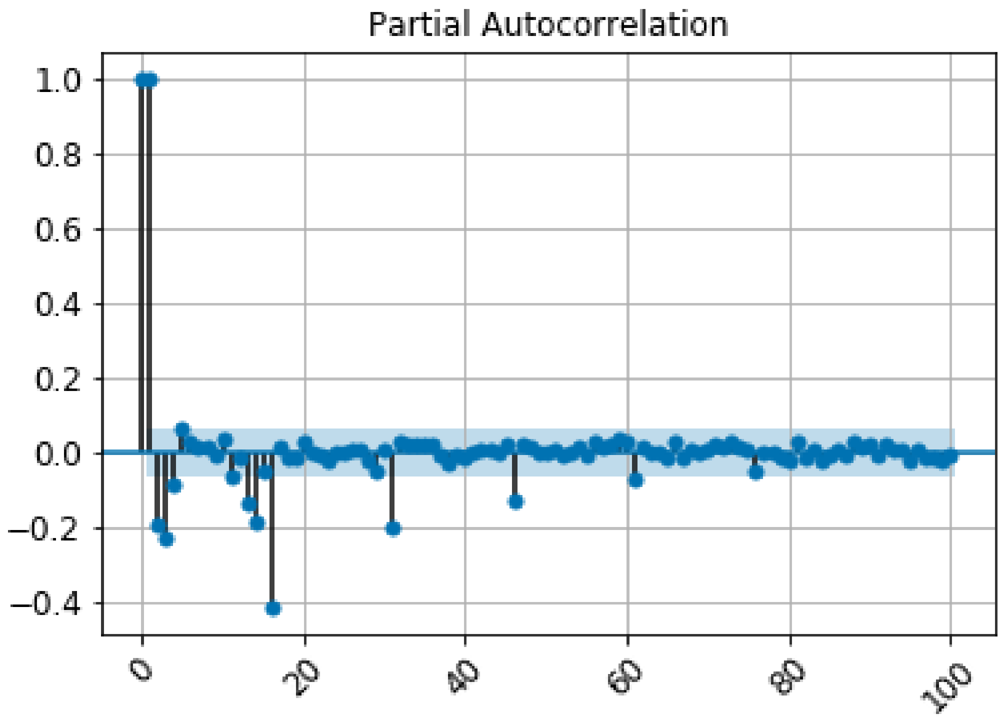
[7]:
```
plot_acf(data, lags = p, unbiased=True)
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
```

```
plt.show()
```



1.0.2 Partial autocorrelation function (PACF)

[8]:
```
plot_pacf(data, lags = p, method = 'ywmle')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.grid()
plt.show()
```



2 b)

Try to detrend the data using the $\Delta n = (1-B)^n$ operator, where B is the "backshift" operator \$ By_t= y_{t−1} \$ and $n = 1, 2, \ldots$ (for the linear, quadratic, … trend, respectively). Plot again the detrended data and its SACF and SPACF.

[9]:
```
df = pd.DataFrame(data, columns = ['y_t'])

df['y_(t1)'] = df['y_t'].shift(-1)
df['y_(t2)'] = df['y_t'].shift(-2)
df['y_(t3)'] = df['y_t'].shift(-3)
df.dropna(inplace = True)

df['Delta_1'] = df['y_(t3)'] - df['y_(t2)']
df['Delta_2'] = df['y_(t3)'] - 2*df['y_(t2)']+ df['y_(t1)']
df['Delta_3'] = df['y_(t3)'] - 3*df['y_(t2)']+ 3*df['y_(t1)']- df['y_t']

df.head(6)
```

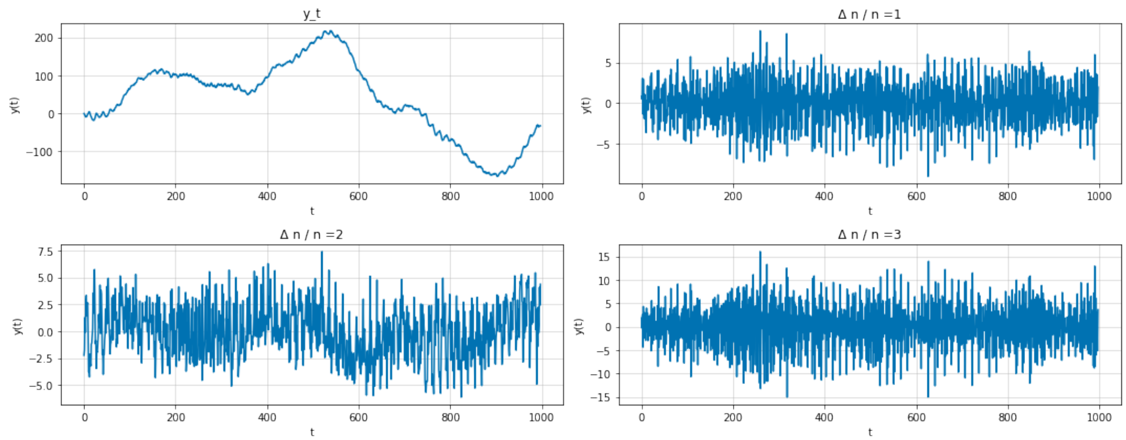
[9]:

	y_t	y_(t1)	y_(t2)	y_(t3)	Delta_1	Delta_2	Delta_3
0	-1.538553	-3.592503	-6.659171	-8.904765	-2.245593	0.821075	1.833793
1	-3.592503	-6.659171	-8.904765	-10.665135	-1.760371	0.485223	-0.335852

```
2  -6.659171   -8.904765 -10.665135   -9.393670   1.271465   3.031836   2.546614
3  -8.904765 -10.665135   -9.393670   -9.477606  -0.083935  -1.355401  -4.387237
4 -10.665135   -9.393670   -9.477606   -6.621387   2.856218   2.940154   4.295554
5  -9.393670   -9.477606   -6.621387   -3.287828   3.333559   0.477341  -2.462813
```

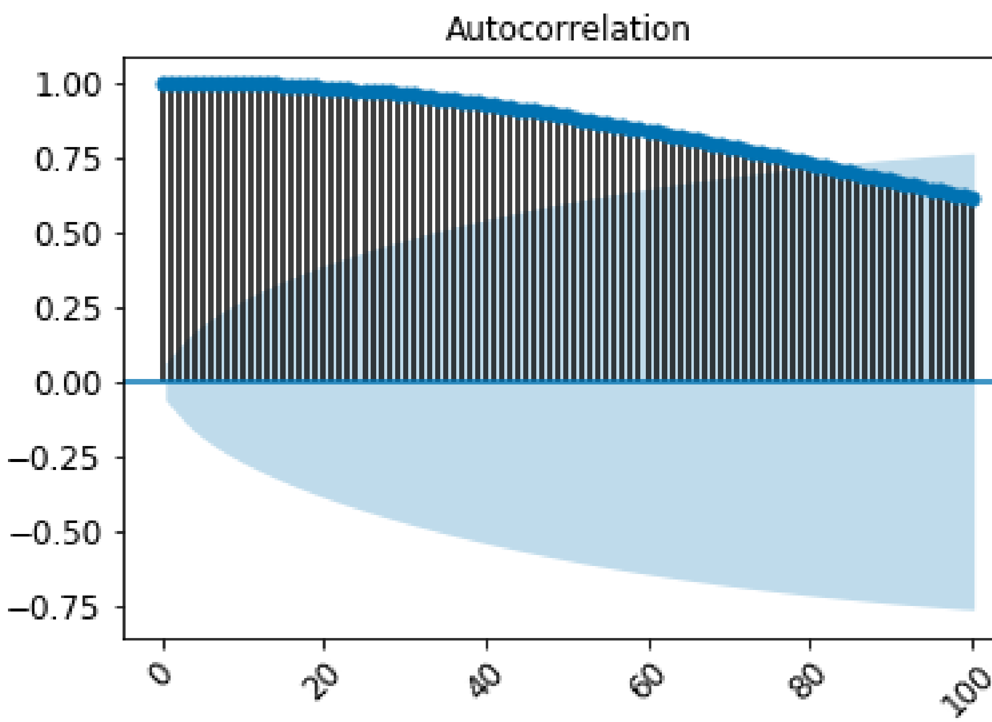
2.0.1 Plots Δn for n=1,2,3

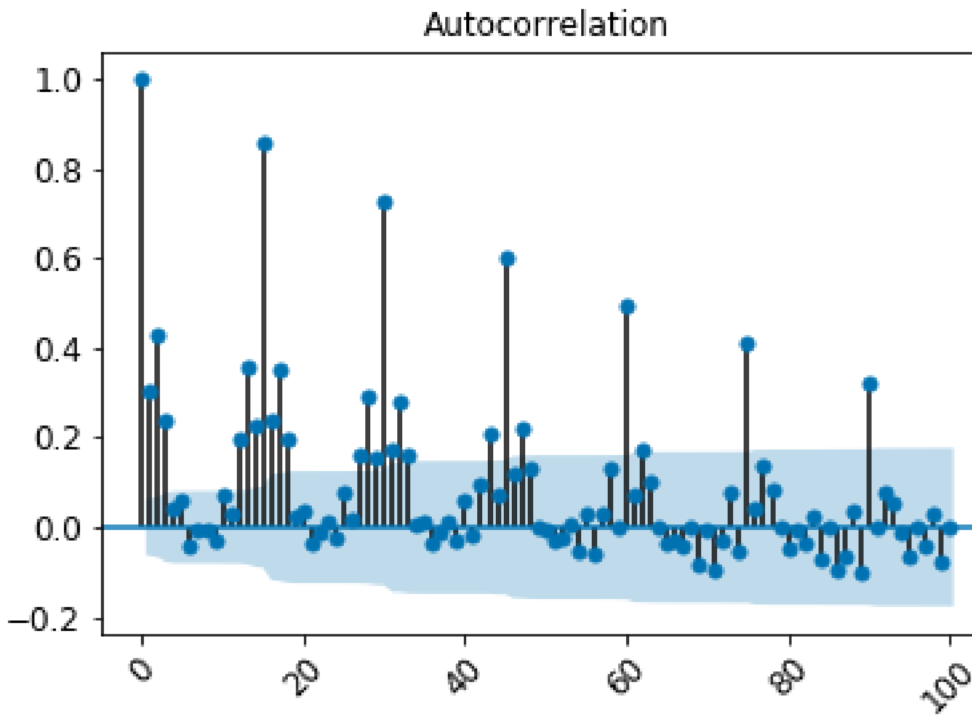
[10]:
```python
fig, axs = plt.subplots(2, 2, figsize=(15,6), sharex = False)

for row in axs:
    for ax in row:
        ax.grid(alpha = 0.5)
        ax.set_xlabel('t')
        ax.set_ylabel('y(t)')

axs[0][0].plot(df['y_t'])
axs[1][0].plot(df['Delta_1'])
axs[0][1].plot(df['Delta_2'])
axs[1][1].plot(df['Delta_3'])


axs[0][0].set_title('y_t')
axs[0][1].set_title(u'\u0394 n / n =1')
axs[1][0].set_title(u'\u0394 n / n =2')
axs[1][1].set_title(u'\u0394 n / n =3')

fig.tight_layout()
```

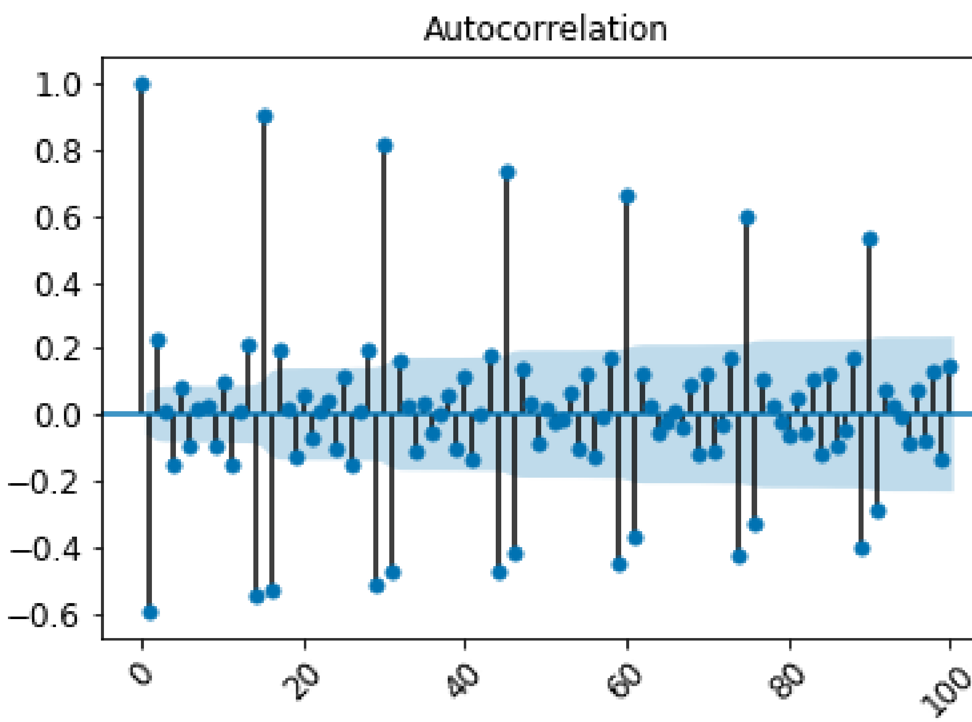


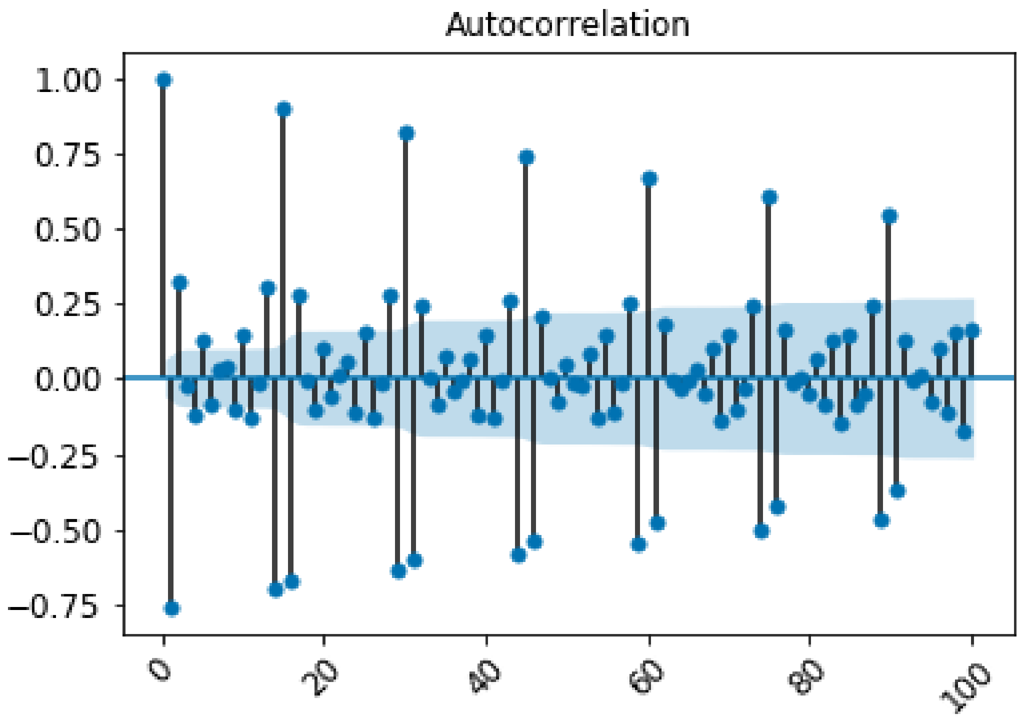
2.0.2 autocorrelation for Δn

[11]:
```
plot_acf(df['y_t'], lags = p, unbiased=True)
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_acf(df['Delta_1'], lags = p, unbiased=True)
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_acf(df['Delta_2'], lags = p, unbiased=True)
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_acf(df['Delta_3'], lags = p, unbiased=True)
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()
```



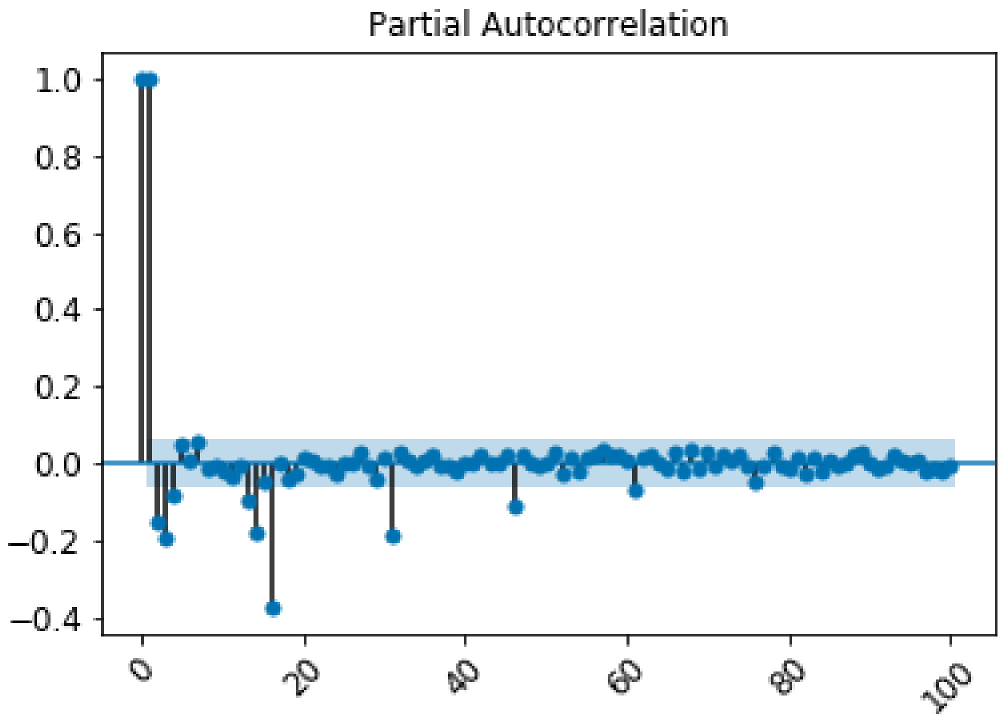

Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100

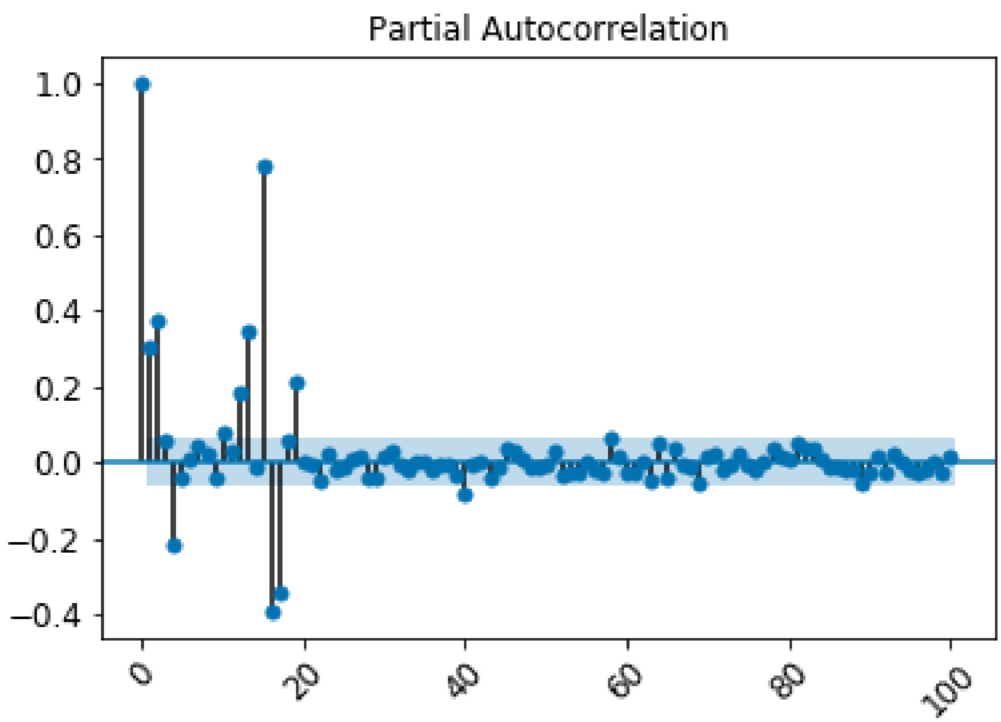

Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
20
40
60
80
100

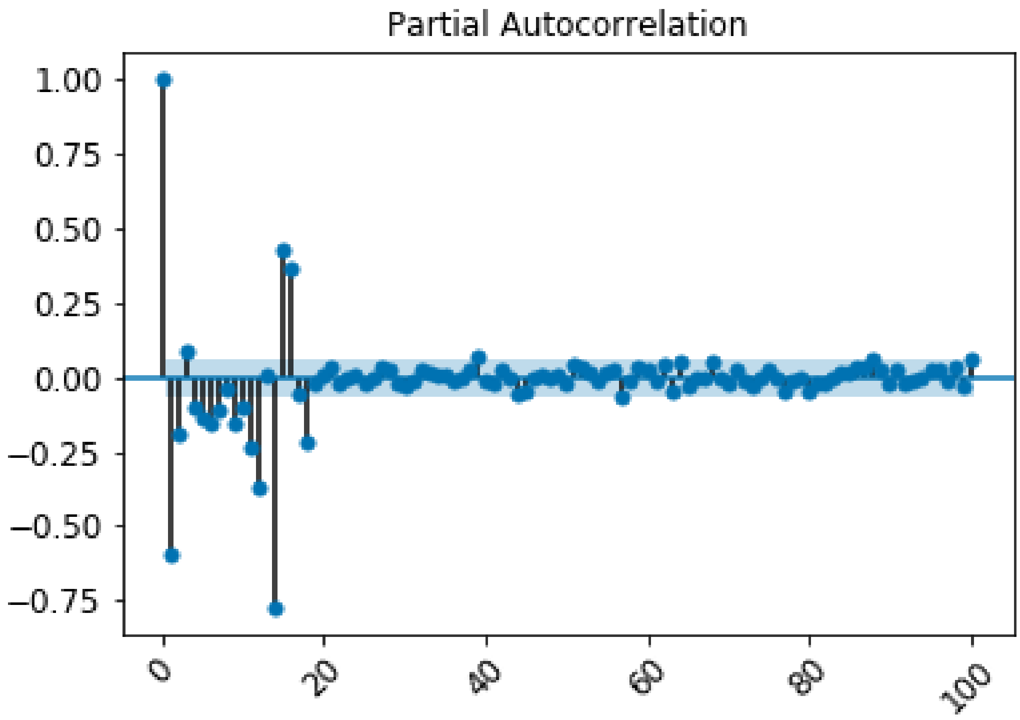


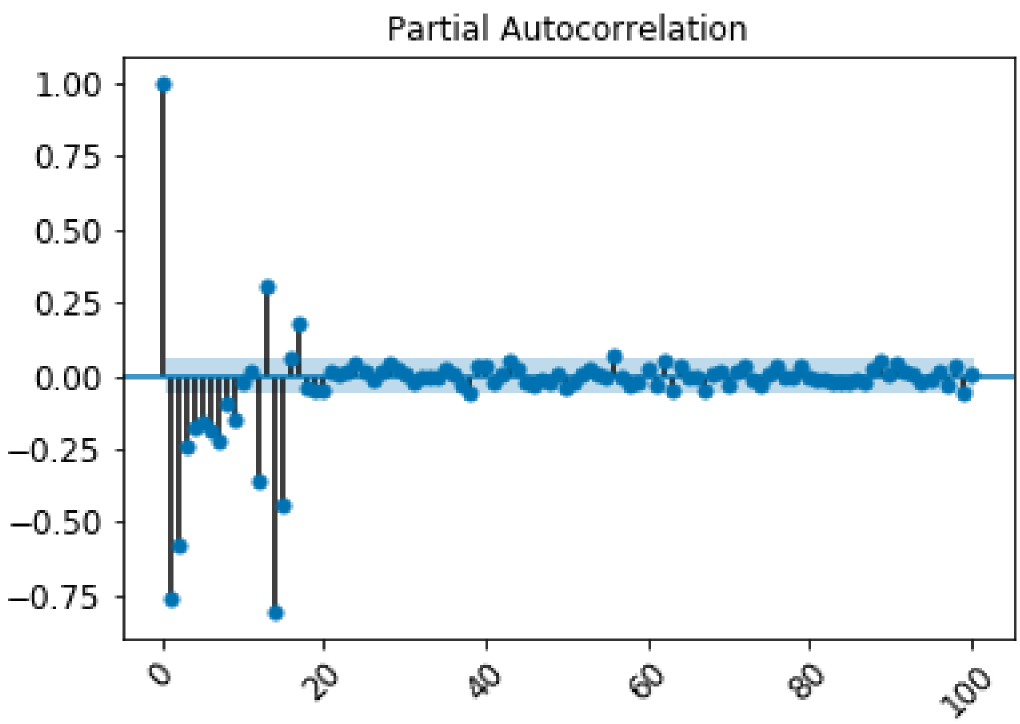
2.0.3 partial autocorrelation Δn

[12]:
```
plot_pacf(df['y_t'], lags = p, method = 'ywmle')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_pacf(df['Delta_1'], lags = p, method = 'ywmle')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_pacf(df['Delta_2'], lags = p, method = 'ywmle')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_pacf(df['Delta_3'], lags = p, method = 'ywmle')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()
```


Partial Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0
20
40
60
80
100


Partial Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0
20
40
60
80
100


Partial Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0
20
40
60
80
100


Partial Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0
20
40
60
80
100

Based on the (partial) autocorrelation, trend is removed for $n \geq 1$. For further analysis we take Δn for $n = 1$

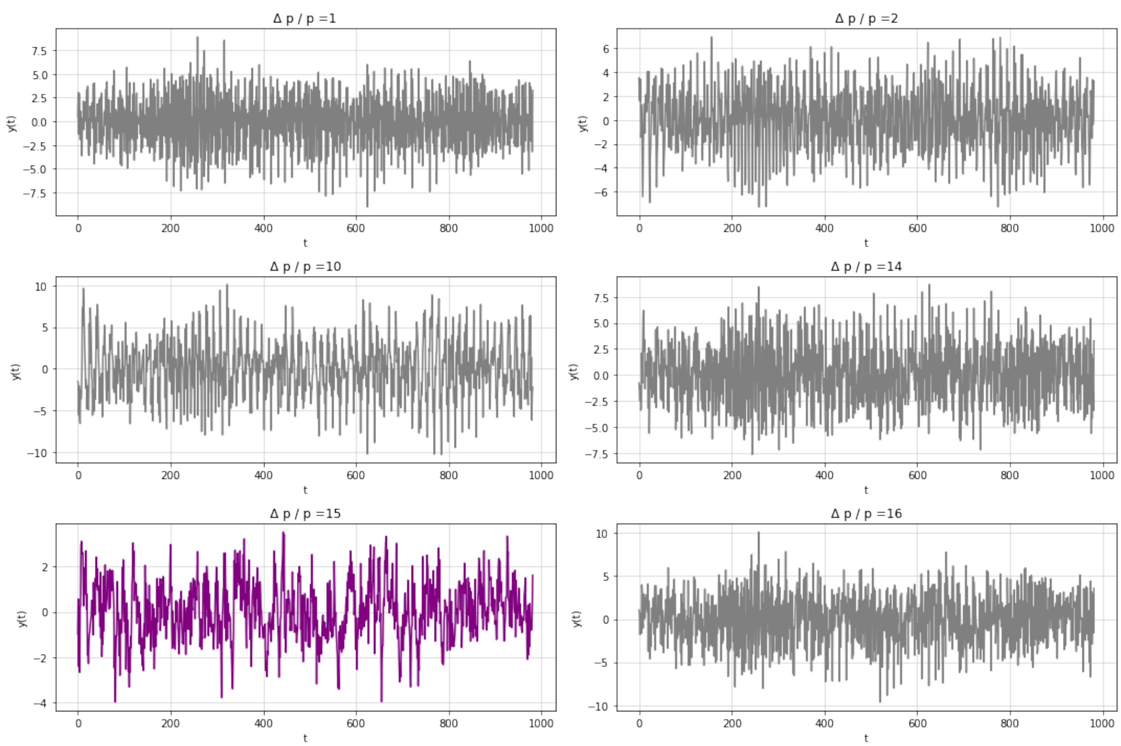
3 c)

Based on the bahaviour of SACF and SPACF above try to find periodicity of the data and cancel it by aplying the operator $\Delta p = (1-B^p)$ (where p is the period) to the (detrended) data series

[13]:
```
detrended_data = np.array(df['Delta_1'])
df_detrended = pd.DataFrame(detrended_data, columns = ['y_t'])

df_detrended['y_(t1)'] = df_detrended['y_t'].shift(-1)
df_detrended['y_(t2)'] = df_detrended['y_t'].shift(-2)
df_detrended['y_(t10)'] = df_detrended['y_t'].shift(-10)
df_detrended['y_(t14)'] = df_detrended['y_t'].shift(-14)
df_detrended['y_(t15)'] = df_detrended['y_t'].shift(-15)
df_detrended['y_(t16)'] = df_detrended['y_t'].shift(-16)

df_detrended.dropna(inplace = True)

df_detrended['Delta_1'] = df_detrended['y_(t1)'] - df_detrended['y_t']
df_detrended['Delta_2'] = df_detrended['y_(t2)'] - df_detrended['y_t']
df_detrended['Delta_10'] = df_detrended['y_(t10)'] - df_detrended['y_t']
df_detrended['Delta_14'] = df_detrended['y_(t14)'] - df_detrended['y_t']
df_detrended['Delta_15'] = df_detrended['y_(t15)'] - df_detrended['y_t']
df_detrended['Delta_16'] = df_detrended['y_(t16)'] - df_detrended['y_t']

df_detrended.head(6)
```

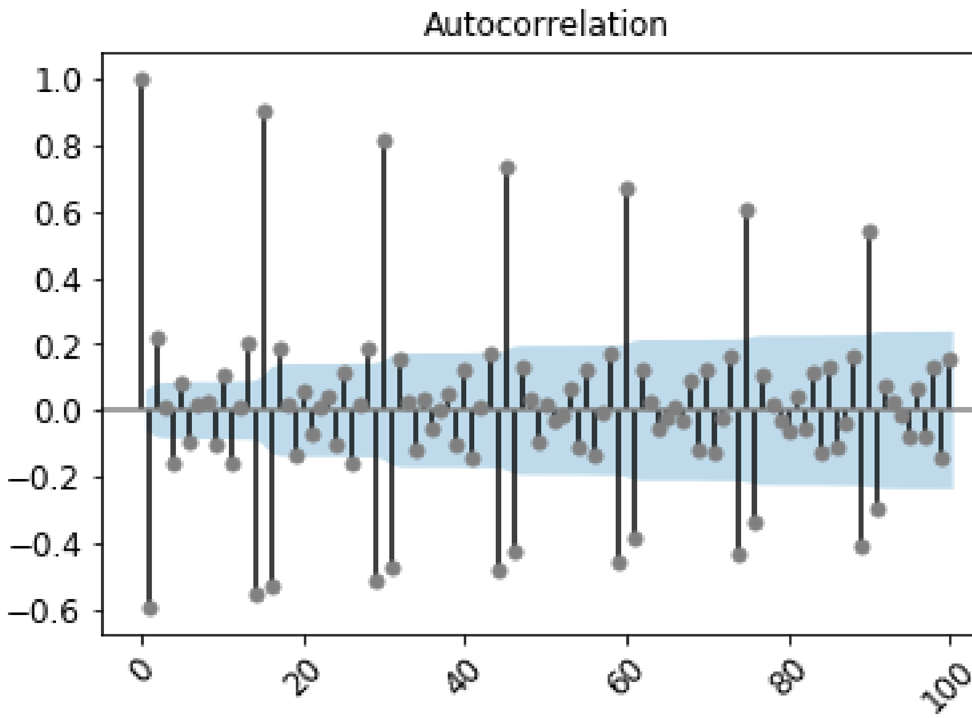
[13]:
```
        y_t    y_(t1)    y_(t2)   y_(t10)   y_(t14)   y_(t15)   y_(t16)  \
0 -2.245593 -1.760371  1.271465 -3.776604 -3.016050 -3.228688 -1.193939
1 -1.760371  1.271465 -0.083935 -3.475717 -3.228688 -1.193939 -1.126277
2  1.271465 -0.083935  2.856218 -4.261620 -1.193939 -1.126277 -0.523359
3 -0.083935  2.856218  3.333559 -2.136829 -1.126277 -0.523359  0.199432
4  2.856218  3.333559  1.411047 -3.016050 -0.523359  0.199432  3.526334
5  3.333559  1.411047  2.468733 -3.228688  0.199432  3.526334  1.679568

    Delta_1   Delta_2  Delta_10  Delta_14  Delta_15  Delta_16
0  0.485223  3.517059 -1.531011 -0.770457 -0.983094  1.051654
1  3.031836  1.676436 -1.715347 -1.468317  0.566432  0.634094
2 -1.355401  1.584753 -5.533085 -2.465404 -2.397742 -1.794824
3  2.940154  3.417495 -2.052893 -1.042342 -0.439423  0.283367
4  0.477341 -1.445171 -5.872269 -3.379577 -2.656786  0.670115
5 -1.922512 -0.864826 -6.562247 -3.134127  0.192774 -1.653992
```

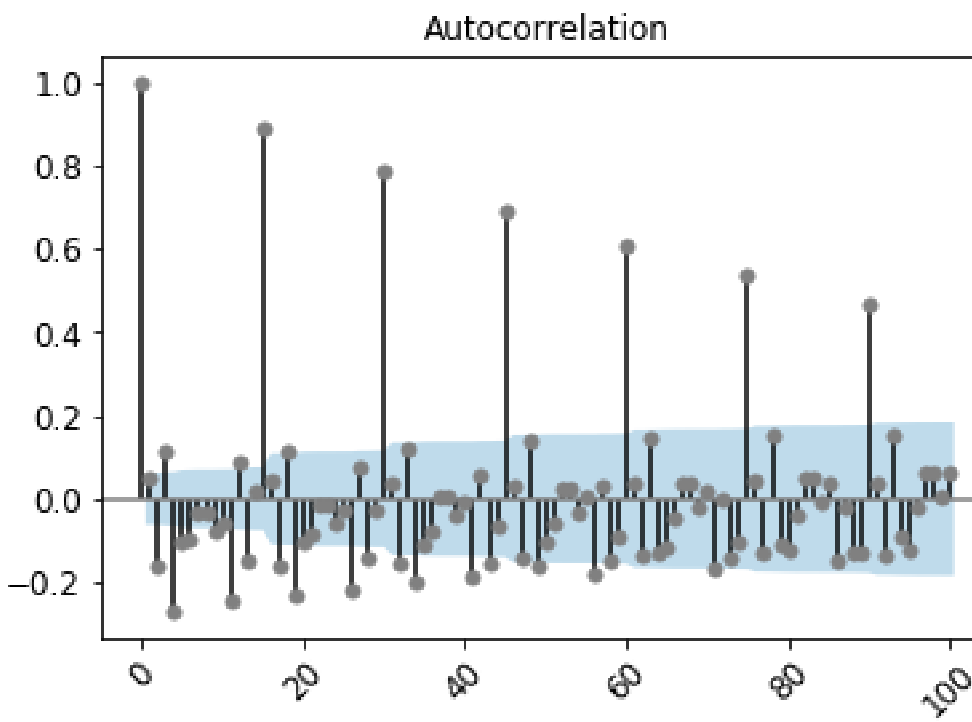
[14]:
```
fig, axs = plt.subplots(3, 2, figsize=(15,10), sharex = False)

for row in axs:
    for ax in row:
        ax.grid(alpha = 0.5)
        ax.set_xlabel('t')
        ax.set_ylabel('y(t)')

axs[0][0].plot(df_detrended['Delta_1'], color = 'gray')
axs[0][1].plot(df_detrended['Delta_2'], color = 'gray')
axs[1][0].plot(df_detrended['Delta_10'], color = 'gray' )
axs[1][1].plot(df_detrended['Delta_14'], color = 'gray')
axs[2][0].plot(df_detrended['Delta_15'], color = 'purple')
axs[2][1].plot(df_detrended['Delta_16'], color = 'gray')

axs[0][0].set_title(u'\u0394 p / p =1')
axs[0][1].set_title(u'\u0394 p / p =2')
axs[1][0].set_title(u'\u0394 p / p =10')
axs[1][1].set_title(u'\u0394 p / p =14')
axs[2][0].set_title(u'\u0394 p / p =15')
axs[2][1].set_title(u'\u0394 p / p =16')

fig.tight_layout()
```

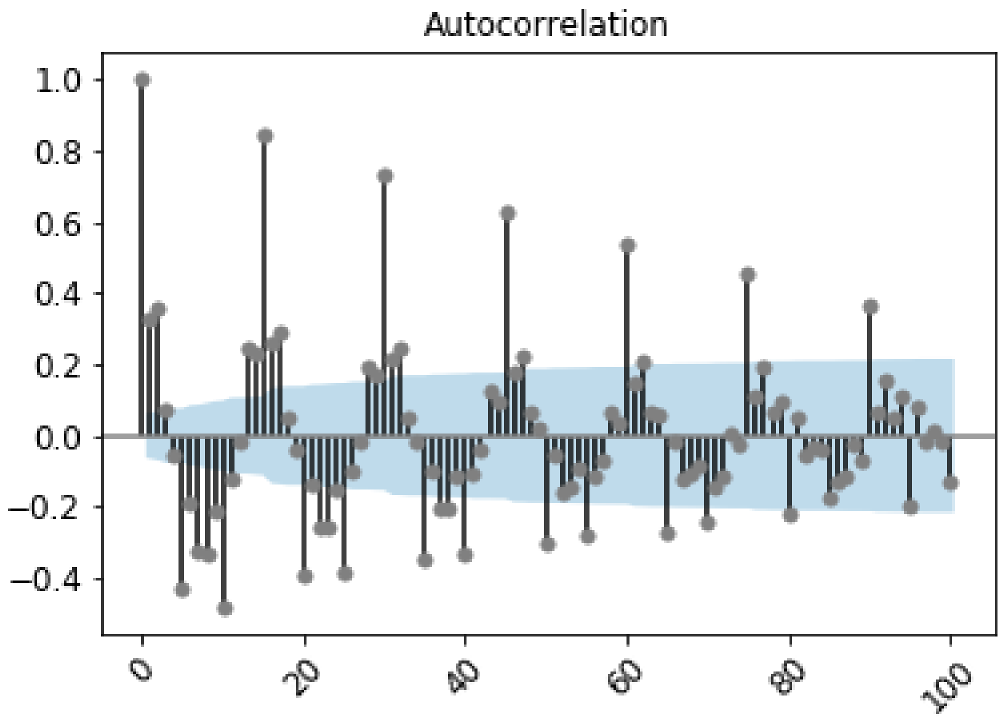


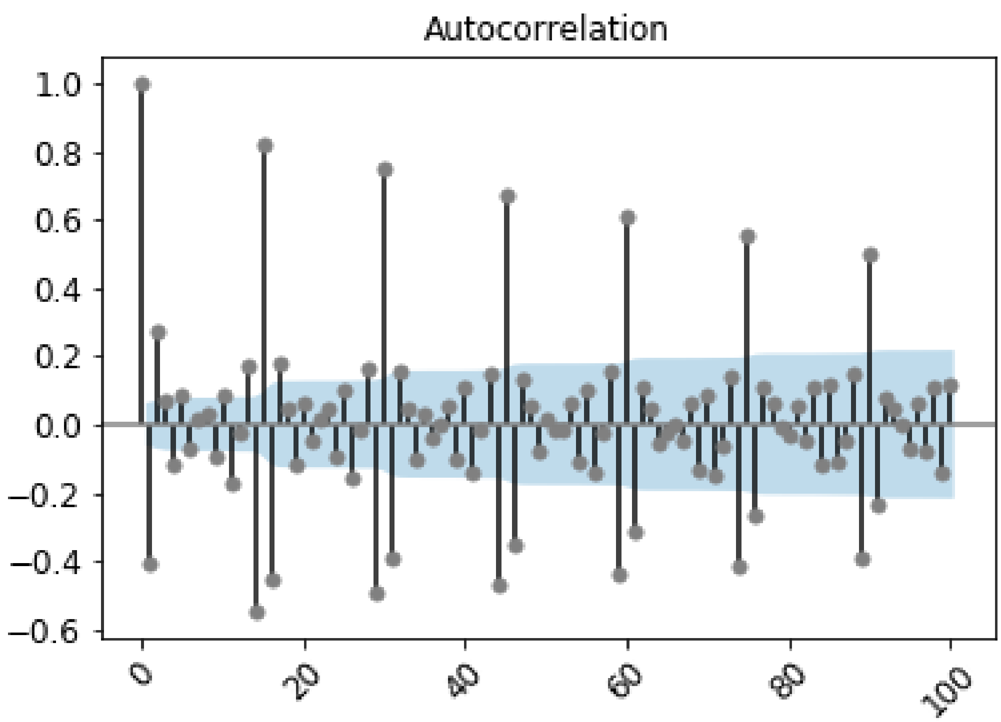
[15]:

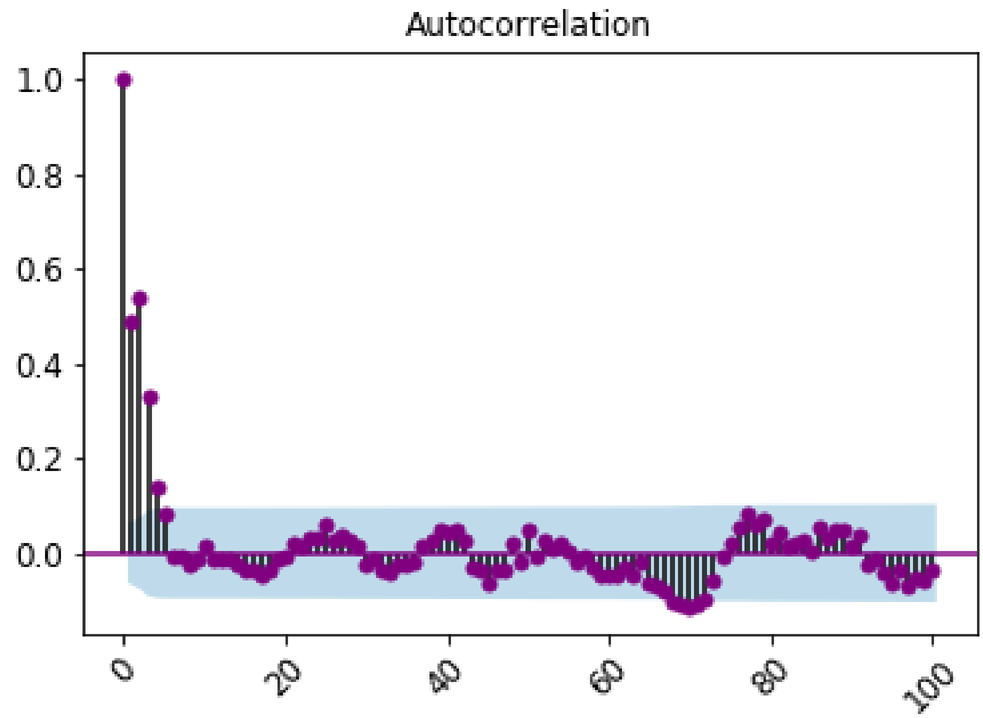
```
plot_acf(df_detrended['Delta_1'], lags = p, unbiased=True, color ='gray')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_acf(df_detrended['Delta_2'], lags = p, unbiased=True, color ='gray')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_acf(df_detrended['Delta_10'], lags = p, unbiased=True, color ='gray')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_acf(df_detrended['Delta_14'], lags = p, unbiased=True, color ='gray')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_acf(df_detrended['Delta_15'], lags = p, unbiased=True, color ='purple')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_acf(df_detrended['Delta_16'], lags = p, unbiased=True, color ='gray')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()
```

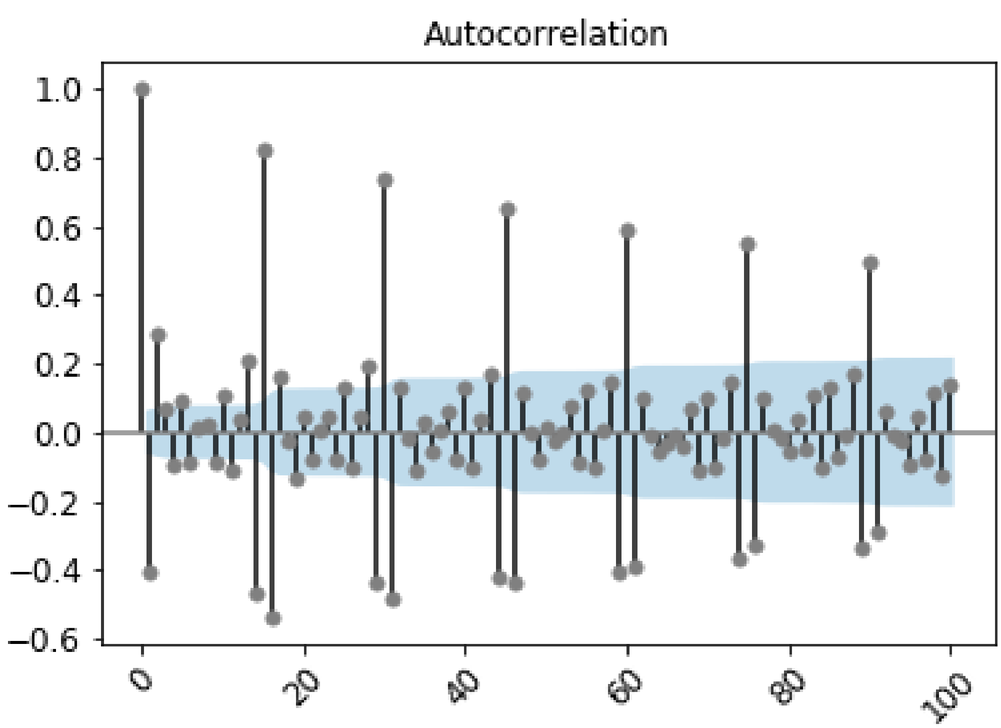

Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
20
40
60
80
100


Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100


Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0
20
40
60
80
100

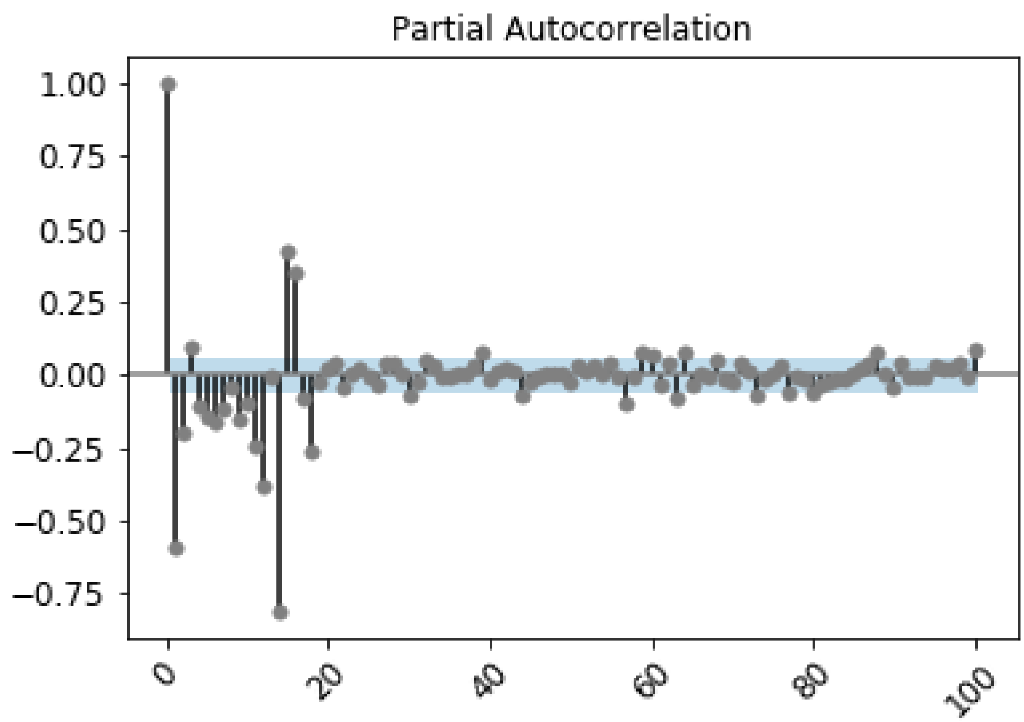

Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
20
40
60
80
100


Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100

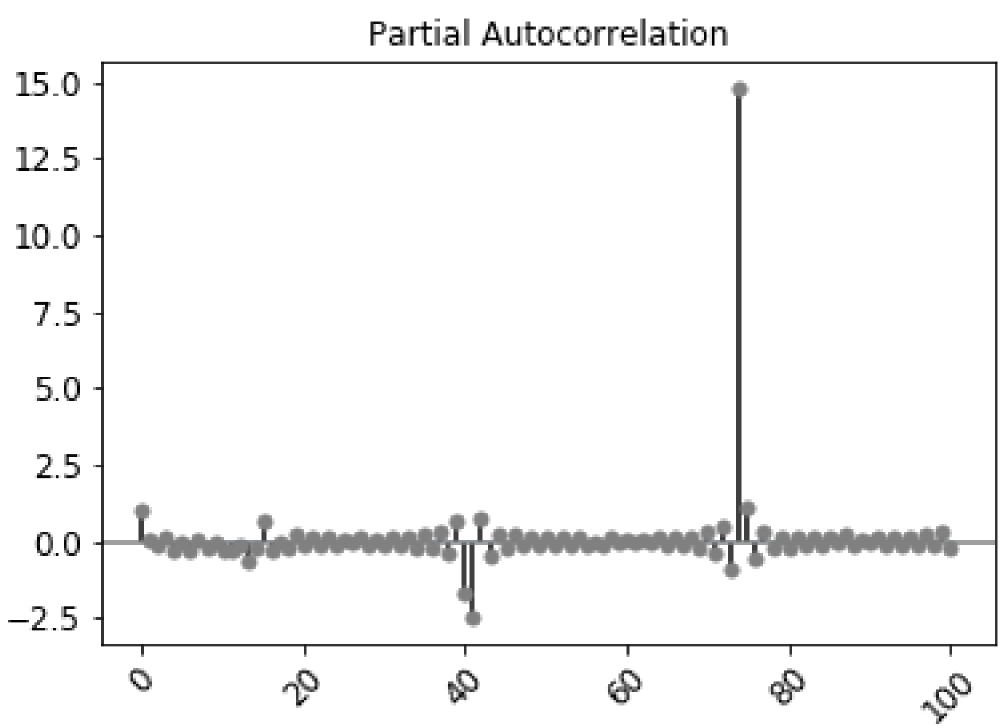

Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
20
40
60
80
100

[16]:

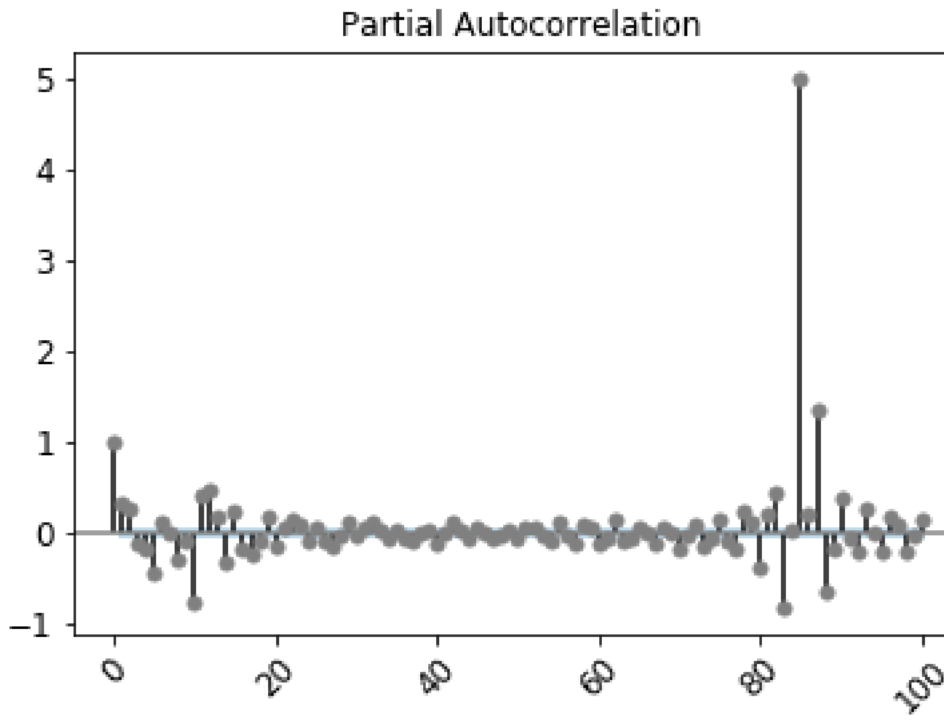
```
plot_pacf(df_detrended['Delta_1'], lags = p, color ='gray')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_pacf(df_detrended['Delta_2'], lags = p, color ='gray')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_pacf(df_detrended['Delta_10'], lags = p, color ='gray')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_pacf(df_detrended['Delta_14'], lags = p, color ='gray')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_pacf(df_detrended['Delta_15'], lags = p, color='purple')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()

plot_pacf(df_detrended['Delta_16'], lags = p, color ='gray')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()
```

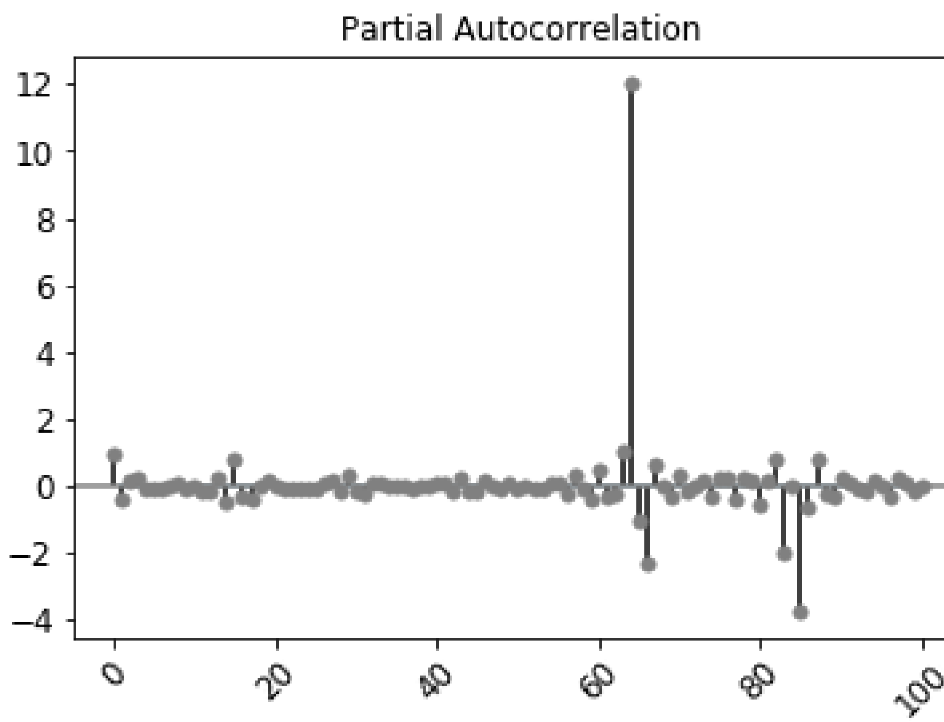


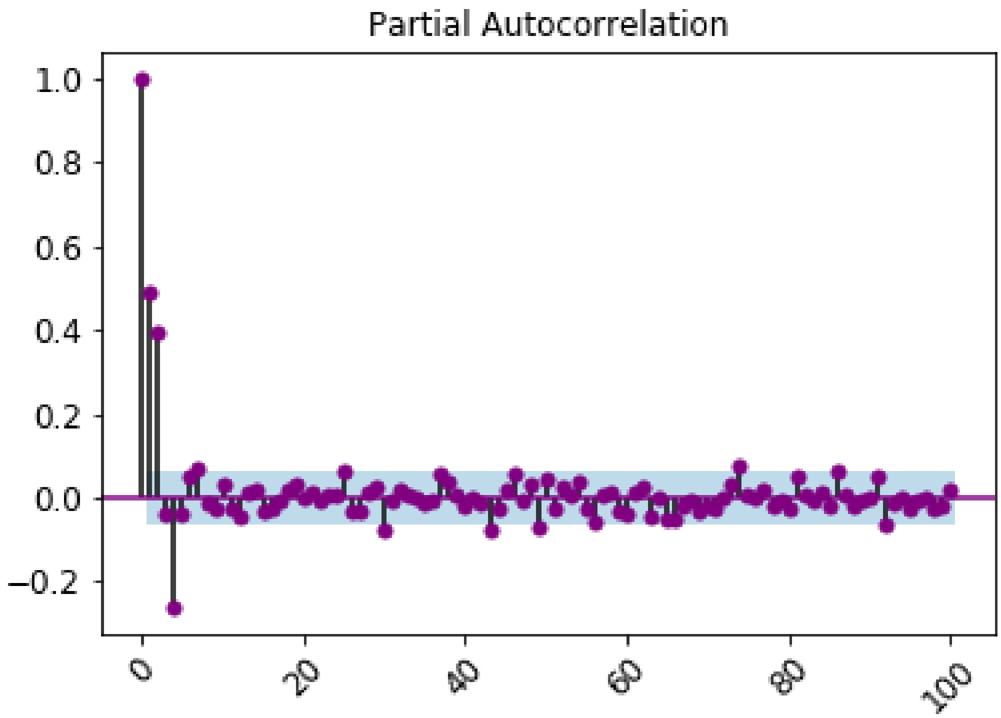
Partial Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0
20
40
60
80
100

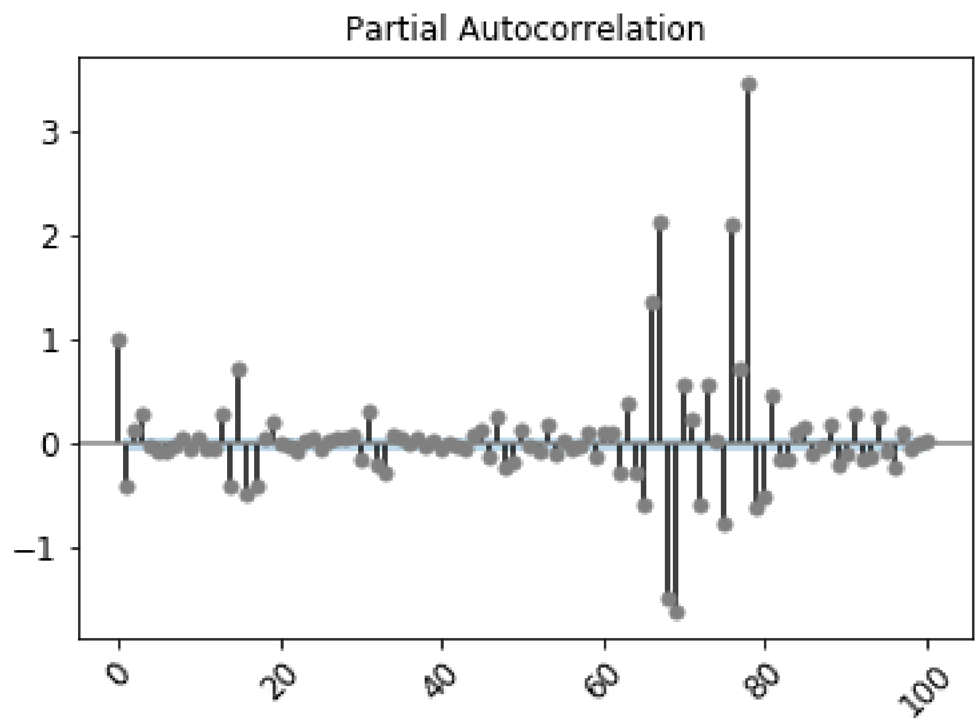


Partial Autocorrelation
15.0
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
0
20
40
60
80
100


Partial Autocorrelation
5
4
3
2
1
0
−1
0
20
40
60
80
100


Partial Autocorrelation
12
10
8
6
4
2
0
−2
−4
0
20
40
60
80
100


Partial Autocorrelation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100


Partial Autocorrelation
3
2
1
0
−1
0
20
40
60
80
100

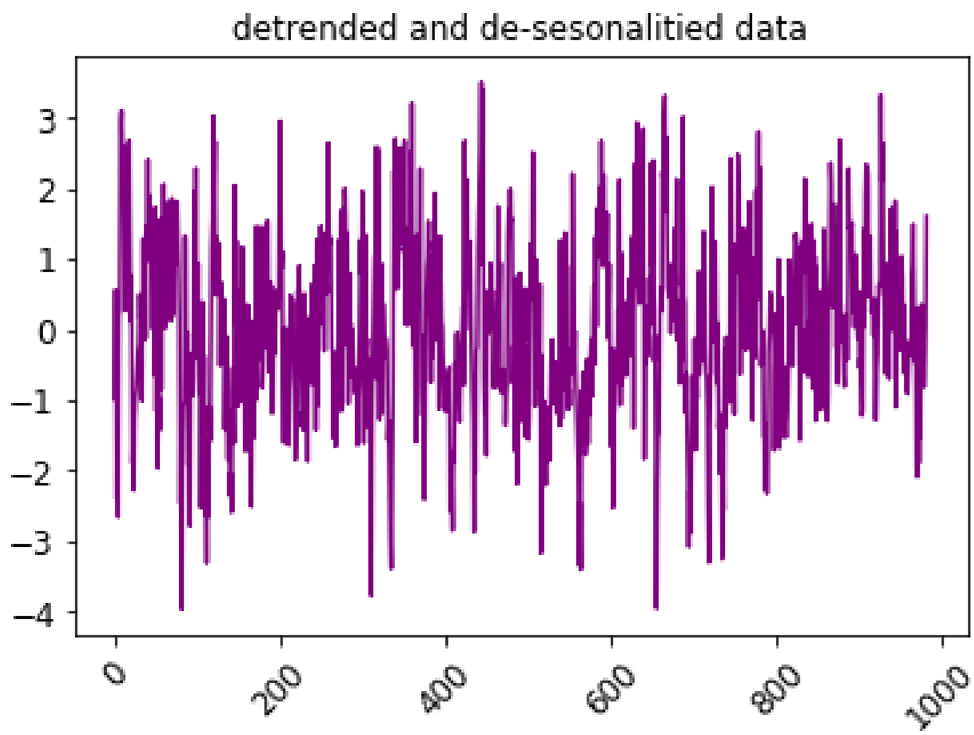
Based on the above plots and (partial) autocorrelation plots we may assume that the seasonality of the given process is $s = 15$

4 d)

Check if now the data seem to be stationary (make plots of data,SACF and PACF). If so, try to fit an ARMA(p,q) model to the stationary data. You can fit it “by hand” (using any method).

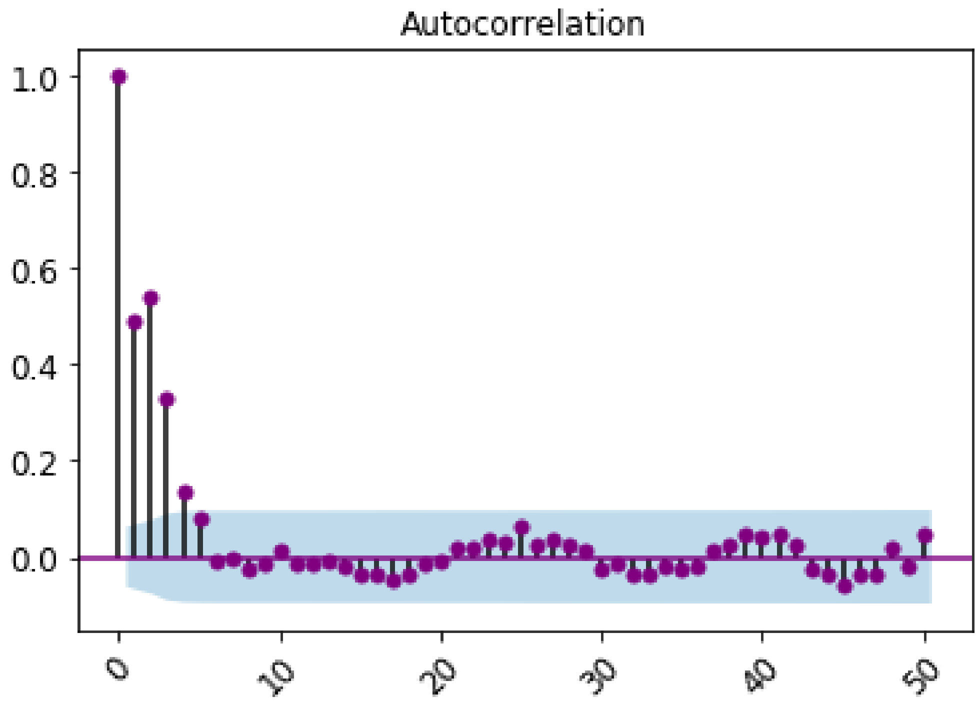
[17]:
```
stationary_data = np.array(df_detrended['Delta_15'])

plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.title('detrended and de-sesonalitied data')
plt.plot(stationary_data, color = 'purple')
```

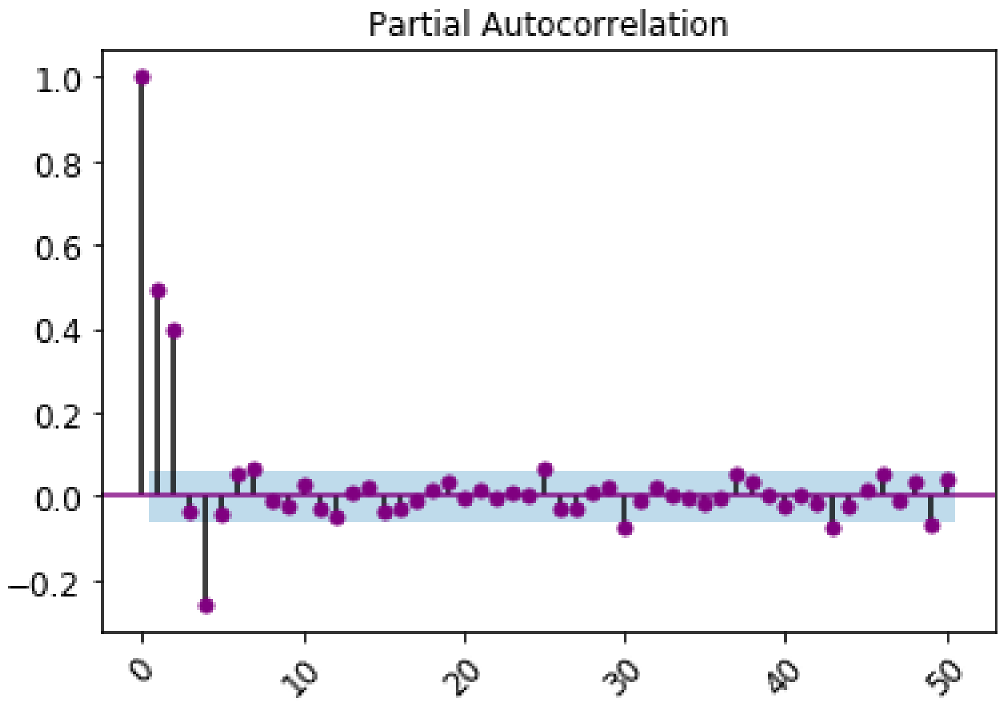
[17]:
```
[<matplotlib.lines.Line2D at 0x7fc87c39da90>]
```



[18]:
```
plot_acf(stationary_data, lags = 50, color ='purple')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()
```

```
plot_pacf(stationary_data, lags = 50, method = 'ywmle', color ='purple')
plt.xticks(fontsize=12, rotation=45)
plt.yticks(fontsize=12)
plt.show()
```





4.0.1 Fitting ARMA(p,q) parameters: brute force method

[19]:
```
def check_for_ARMA(dta, p_max, q_max):

    aic = pd.DataFrame(np.zeros((p_max, q_max), dtype=float))
    bic = pd.DataFrame(np.zeros((p_max, q_max), dtype=float))

    for p in range(p_max):
        for q in range(q_max):
            if p == 0 and q == 0:
                continue
            try:
                mod = sm.tsa.ARMA(dta, (p,q)).fit()
                aic.iloc[p,q] = mod.aic
                bic.iloc[p,q] = mod.bic

            except:
                aic.iloc[p,q] = np.nan
                bic.iloc[p,q] = np.nan

    return aic, bic
```

[20]:
```
aic, bic = check_for_ARMA(stationary_data, 8,8)
```

[21]:
```
aic
```

[21]:
```
             0            1            2            3            4  \
0     0.000000  3121.414085  2883.964991  2786.558951  2776.197036
1  2993.874463  2904.104194  2759.531219  2760.785492  2762.750864
2  2829.352262  2830.690012  2760.785612  2760.580913  2762.500622
3  2829.822751  2809.766897  2762.668326  2762.451596  2763.527109
4  2763.486663  2764.402487  2759.755975  2761.239230  2763.180891
5  2763.888881  2765.247829  2761.376258  2763.159397  2765.151716
6  2763.121653  2762.906776  2763.366550  2765.241870  2764.395350
7  2760.443927  2762.363914  2760.739563  2767.067318  2769.194259

             5            6            7
0  2759.859177  2759.230023  2760.748800
1  2759.422782  2761.095922  2761.884872
2  2759.513955  2763.331850  2758.686371
3  2760.495566  2763.270972  2760.627119
4  2762.544137  2763.096287          NaN
5  2762.855972  2761.353132  2763.196281
6  2756.629488  2765.324456  2757.040264
7  2763.260941  2765.164456  2766.884745
```

[36]:
```
bic
```

[36]:
```
             0            1            2            3            4  \
0     0.000000  3136.082859  2903.523356  2811.006907  2805.534584
1  3008.543237  2923.662560  2783.979176  2790.123040  2796.978003
2  2848.910627  2855.137969  2790.123160  2794.808052  2801.617353
3  2854.270708  2839.104445  2796.895466  2801.568326  2807.533431
4  2792.824211  2798.629627  2798.872706  2805.245551  2812.076805
5  2798.116020  2804.364559  2805.382579  2812.055310  2818.937220
6  2802.238384  2806.913098  2812.262463  2819.027375  2823.070446
7  2804.450249  2811.259827  2814.525068  2825.742414  2832.758946

             5            6            7
0  2794.086316  2798.346753  2804.755122
1  2798.539512  2805.102244  2810.780785
2  2803.520276  2812.227763  2812.471875
3  2809.391479  2817.056477  2819.302214
4  2816.329642  2821.771383          NaN
5  2821.531068  2824.917819  2831.650560
6  2820.194175  2833.778734  2830.384133
7  2831.715219  2838.508326  2845.118206
```

[30]:
```
p = 6
q = 5

model = sm.tsa.ARMA(stationary_data, (p,q)).fit()
print("aic = {:}\t bic = {:}".format(model.aic, model.bic))
print("\n\n parameters: {:}".format(model.params))
```

```
aic = 2756.6294883777073         bic = 2820.1941753863152


 parameters: [ 0.03747872  0.48944801 -0.56304202 -0.02537633  0.32499946
-0.01869447
 -0.13383007 -0.20155135  1.0553327   0.29869201  0.01640507  0.22729577]
```

[37]:
```
p = 2
q = 2

model = sm.tsa.ARMA(stationary_data, (p,q)).fit()
print("aic = {:}\t bic = {:}".format(model.aic, model.bic))
print("\n\n parameters: {:}".format(model.params))
```

```
aic = 2760.785611714453  bic = 2790.1231595645795


 parameters: [ 0.03586934  0.64866779 -0.05760555 -0.35864322  0.48898433]
```

4.1 Results:

A heuristic analisys based on hand-waving gives the following results: * trend: $n = 1$ * seasonalisty $s = 15$ * ARMA(p,q) parameters: $p = 6$, $q = 5$ (based on AIC) or $p = 2$, $q = 2$ (based on BIC)

[ ]:
```

```