Financial Instruments and Pricing

Fall 2019

Set 3&4&5
(The yield curve, Interest Rate Derivatives & Arbitrage)

1. Construct the zero-coupon yield curve y(t) using the bootstrapping procedure for bonds presented in the table below. Assume that coupons of each bond are paid annually (last coupon is always on maturity). Fit a quadratic function to the bootstrapped y(t) data points, i.e. assume $y(t) = a\,t^2 + b\,t + c$.

Name	Time to maturity (yrs)	Nominal %	Price	Accrued int.	Dirty price	YTM
AAA	0,5	4	99,98	2,00	101,98	4,00%
BBB	1	4	99,57	-	99,57	4,45%
CCC	1,5	3	97,38	1,50	98,88	4,84%
DDD	2	4	97,80	-	97,80	5,19%
EEE	2,5	4	96,62	2,00	98,62	5,47%
FFF	3	-	84,56	-	84,56	5,75%
GGG	3,5	5	97,20	2,50	99,70	5,90%
HHH	4	-	78,91	-	78,91	6,10%
JJJ	4,5	-	76,29	-	76,29	6,20%
KKK	5	4	90,79	-	90,79	6,20%

<u>Note</u>: disregard the Day-Count-Conventions and transaction-spot date differences, i.e. assume coupons are simply equal to the nominal rate.

2. Compute the net cash flow from the (PLN denominated) FRA"6x9" contract made on 3rd June 2019 with value date on 3rd December 2019 and maturity date on 3rd March 2020. Assume that the nominal of the FRA is PLN 10 mln, FRA rate was fixed at r_{FRA}=1.80%, and the floating rate is WIBOR (with ACT/365 convention). Choose the appropriate rate r_{ZM} from the table below:

Date	WIBOR3M	WIBOR6M
3/06/2019	1.72%	1.79%
3/09/2019	1.72%	1.79%
3/12/2019	1.74%	1.81%
3/03/2020	1.85%	1.92%

   When (on which date) the payment will be done and who will pay net CF (the party paying or the party receiving fixed interest rate)?

3. <u>Pricing FRA.</u> Let's assume the following zero-coupon yield curve: $y(t) = -0.001\,t^2 + 0.0105\,t + 0.045$ (this is NOT in [%], so e.g. y(t) = 0.02 = 2% !)
   a. Compute fixed rate for FRA "1y x 1.5y" contract (t1 = 1, t2 = 1.5 years).
   b. Compute the (currently expected) forward yield curve in 0.5 years: $y_{0,5}(t)$.

c. Assume that 0.5 year has passed and current yield curve y(t) is exactly equal to the previously expected forward yield curve $y_{0,5}(t)$ from point b. (market expectations were perfectly correct). The FRA contract from point a (the one signed 0.5 year ago) now has only 0.5 year to the value date and 1 year to maturity: t1 = 0.5 , t2 = 1. Compute current fixed rate for such a FRA contract (as if it was just traded).

d. Compute what would be the result of point c. if after 0.5 year (as in point c) $y(t) = y_{0,5}(t) + 0,001$ ("market" expectations in t=0 underestimated yield curve in 0.5 years) and what if $y(t) = y_{0,5}(t) - 0,001$ ("market" expectations in t=0 overestimated yield curve in 0.5 years).

Note: disregard the problem of day count conventions and simply assume the FRA rate is the forward yield: $r_{FRA} \equiv f(t1,t2)$.

4. Interest rate Swap (IRS). Let's assume the same zero-coupon yield curve y(t) as in Problem 3, i.e. $y(t) = -0.001\, t^2 + 0.0105\, t + 0.045$. (this is NOT in [%] !)
Compute fixed rate r_{IRS} for IRS contract swaping fixed interest vs floating interest, each one with semi-annual frequency (Freq. = 0.5 year) and maturity in T = 2.5 years. Assume that floating interest is payed according to:
   a. r_{ZM} (e.g. WIBOR6M)
   b. r_{ZM} + 1% (e.g. WIBOR6M + 1%)
   c. r_{ZM} - 1% (e.g. WIBOR6M - 1%)

   Note: disregard the problem of day count conventions and simply assume the IRS rate is the (annual) effective yield, i.e. fixed interest payed on 1 PLN nominal value for 0.5 yrs is equal to $(1+ r_{IRS})^{0.5}-1$

5. Using STATIC ARBITRAGE arguments find expressions (BID and ASK) for the "fair" price X of a FX forward, i.e. one wants to BUY / SELL one currency vs other currency @ FX rate X with delivery in T. Assume that the current (direct) FX rate: S^{USDPLN} and interest rates (yields) in both currencies: $y^{PLN}(T)$ and $y^{USD}(T)$ are known.
   a. Find general formulae for X^{BID} and X^{ASK}
   b. Calculate X^{BID} and X^{ASK} if T = 3M and: S^{USDPLN} : 3.8010 – 3.8020, 3M_PLN_DEPOS: 1.72% - 1.75%, 3M_USD_DEPOS.: 2.78% - 2.80%

   Note: Figure out how "a bank" can create the FX forward from the current spot FX rate, and combination of a loan and a deposit in those currencies ?

6. Using STATIC ARBITRAGE arguments prove the 3rd (European put bands), 6th (American call-put parity) and 7th (European call price is a monotonic function of X) relations concerning option prices from Lecture 5, page 36

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