(*Zestaw 3 Zadanie 1 *)

```
(*Solution of this problem is made usig the method called bootstrapping,
which assums that each cash flow is treated
 independently. First step of the solution is to build a cash
 flow matrix. Inverse matrix of cash flows is shown below*)
```

```
CF = {{104, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 104, 0, 0, 0, 0, 0, 0, 0, 0},
   {3, 0, 103, 0, 0, 0, 0, 0, 0, 0}, {0, 4, 0, 104, 0, 0, 0, 0, 0, 0},
   {4, 0, 4, 0, 104, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 100, 0, 0, 0, 0},
   {5, 0, 5, 0, 5, 0, 105, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 100, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0, 100, 0}, {0, 4, 0, 4, 0, 4, 0, 4, 0, 104}};
```

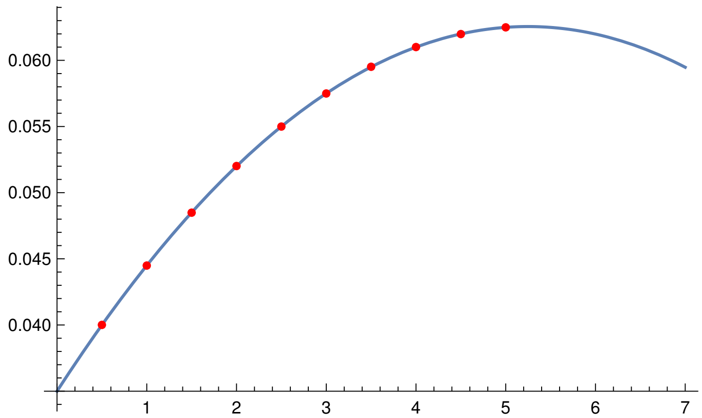
```
MatrixForm[CF]
```

$$\begin{pmatrix} 104 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 104 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 103 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 104 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 4 & 0 & 104 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 \\ 5 & 0 & 5 & 0 & 5 & 0 & 105 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 4 & 0 & 4 & 0 & 4 & 0 & 4 & 0 & 104 \end{pmatrix}$$

```
(*Price values*)

PV = {{101.98, 99.57, 98.88, 97.80, 98.62, 84.56, 99.70, 78.91, 76.29, 90.79}};

(*Time*)

t = Table[0.5 * i, {i, 1, 10}]

{0.5, 1., 1.5, 2., 2.5, 3., 3.5, 4., 4.5, 5.}

(*Now we count discount factors, starting from first and using it to second
 equation. Using discount factors solved before, we get every of them*)

Solve[101.98 == 104 * df1, df1]

{{df1 → 0.980577}}

Solve[99.57 == 104 * df2, df2]

{{df2 → 0.957404}}

Solve[98.88 == 3 * 0.9805769230769231` + 103 * df3, df3]

{{df3 → 0.93144}}

Solve[97.8 == 4 * 0.9574038461538461` + 104 * df4, df4]

{{df4 → 0.903561}}
```

```
Solve[98.62 == 4 * 0.9805769230769231` + 4 * 0.9314395070948468` + 104 * df5, df5]
{{df5 → 0.87473}}

Solve[84.56 == 100 * df6, df6]
{{df6 → 0.8456}}

Solve[99.7 == 5 * 0.9805769230769231` +
   5 * 0.9314395070948468` + 5 * 0.8747301373010857` + 105 * df7, df7]
{{df7 → 0.816822}}

Solve[78.91 == 100 * df8, df8]
{{df8 → 0.7891}}

Solve[76.29 == 100 * df9, df9]
{{df9 → 0.7629}}

Solve[90.79 == 4 * 0.9574038461538461` + 4 * 0.9035613905325444` +
   4 * 0.8456` + 4 * 0.7890999999999999` + 104 * df10, df10]
{{df10 → 0.738532}}

df = {0.9805769230769231`, 0.9574038461538461`,
   0.9314395070948468`, 0.9035613905325444`, 0.8747301373010857`, 0.8456`,
   0.8168215920251022`, 0.7890999999999999`, 0.7629`, 0.7385321062812928`};

(* Now we get y(t) from discount factors..*)

tt = Table[df[[i]] ^ (-1 / t[[i]]) - 1, {i, 1, 10}]
{0.040008, 0.0444913, 0.0484882, 0.0520131, 0.0549949,
 0.057495, 0.0595135, 0.0610039, 0.0619848, 0.0624931}


dd = Table[{t[[i]], tt[[i]]}, {i, 1, 10}]
{{0.5, 0.040008}, {1., 0.0444913}, {1.5, 0.0484882},
 {2., 0.0520131}, {2.5, 0.0549949}, {3., 0.057495}, {3.5, 0.0595135},
 {4., 0.0610039}, {4.5, 0.0619848}, {5., 0.0624931}}

(*..and make quadraric function fit y(t)*)

n = Fit[dd, {1, x, x^2}, x]
0.0349958 + 0.0105049 x - 0.00100111 x^2

lp = ListPlot[dd, PlotStyle → Red];

pl = Plot[0.034995800334064325` +
    0.0105049082569706` x - 0.0010011113804187007` x^2, {x, 0, 7}];
```

```
Show[pl, lp]
```



(*Exercise 2*)

```
t0 = {2019, 6, 3}
t1 = {2019, 12, 3}
t2 = {2020, 3, 3}
tend = {2020, 1, 1}
```

```
Out[•]= {2019, 6, 3}
Out[•]= {2019, 12, 3}
Out[•]= {2020, 3, 3}
Out[•]= {2020, 1, 1}
```

```
In[•]:= t2t1 = DateDifference[t1, t2] // QuantityMagnitude
```

```
Out[•]= 91
```

```
In[•]:= rFRA = 0.018;
rWIB3M = 0.0174; (*we take the value on 3/12*)
n = 10^7;
```

```
In[•]:= DCF = QuantityMagnitude[DateDifference[t1, t2]] / 365 // N
```

```
Out[•]= 0.249315
```

```
In[•]:= NCF = (n (rWIB3M - rFRA) DCF) / (1 + rWIB3M DCF)
```

```
Out[•]= -1489.43
```

```
(* the payment will be done on 3rd
 December 2019 and made by the buyer of the contract

 since the WIBOR3M turned out to be smaller than rFRA *)

1489.43
```

In[1]:= # (*Exercise 3*)

In[2]:= (*a*)

In[3]:=
```
y == -0.001 t^2 + 0.0105 t + 0.045; (*zero-coupon yield curve*)
t1 = 1;
t2 = 1.5;
y1 = -0.001 t1^2 + 0.0105 t1 + 0.045;
y2 = -0.001 t2^2 + 0.0105 t2 + 0.045;
```

In[8]:= $r_{FRA} == \left(\frac{(1+y2)^{t2}}{(1+y1)^{t1}}\right)^{\frac{1}{(t2-t1)}} - 1$

Out[8]= $r_{FRA} == 0.0665456$

In[9]:= (*b*)

In[10]:=
```
Clear[t]
t3 = 0.5;
t4 = 0.5 + t;
y3 = -0.001 t3^2 + 0.0105 t3 + 0.045;
y4 = -0.001 t4^2 + 0.0105 t4 + 0.045;
```

In[15]:= $f\_new == \left(\frac{(1+y4)^{t4}}{(1+y3)^{t3}}\right)^{\frac{1}{(t4-t3)}} - 1$ (*forward yield curve in 0.5 years*)

Out[15]= $f\_new == -1 + 0.9759^{\frac{1}{0.+t}} \left(\left(1.045 + 0.0105\,(0.5+t) - 0.001\,(0.5+t)^2\right)^{0.5+t}\right)^{\frac{1}{0.+t}}$

In[16]:= (*c*)
(*calculate r_{FRA} for a new curve*)

In[17]:=
```
k1 = 0.5;
k2 = 1;
```
$fone = -1 + 0.9759^{\frac{1}{0.+k1}} \left(\left(1.045 + 0.0105\,(0.5+k1) - 0.001\,(0.5+k1)^2\right)^{0.5+k1}\right)^{\frac{1}{0.+k1}}$

Out[19]= 0.0590191

In[20]:= $ftwo = -1 + 0.9759^{\frac{1}{0.+k2}} \left(\left(1.045 + 0.0105\,(0.5+k2) - 0.001\,(0.5+k2)^2\right)^{0.5+k2}\right)^{\frac{1}{0.+k2}}$

Out[20]= 0.0627757

In[21]:= $r_{FRA2} == \left(\frac{(1+ftwo)^{k2}}{(1+fone)^{k1}}\right)^{\frac{1}{(k2-k1)}} - 1$

Out[21]= $r_{FRA2} == 0.0665456$

In[22]:= (*d*)
(*calculate r_{FRA} for the second curve, but now it is shifted by ± 0.001*)

In[23]:=
```
Clear[t]
f_shifted[t_] =
```
$$-1+0.9759^{\frac{1}{0.+t}}\left(\left(1.045+0.0105\,(0.5+t)-0.001\,(0.5+t)^2\right)^{0.5+t}\right)^{\frac{1}{0.+t}}+0.001$$

Out[24]= $-0.999+0.9759^{\frac{1}{0.+t}}\left(\left(1.045+0.0105\,(0.5+t)-0.001\,(0.5+t)^2\right)^{0.5+t}\right)^{\frac{1}{0.+t}}$

In[25]:= `(* +0.001 shift curves*)`

In[26]:= $\text{gone} = -0.999+0.9759^{\frac{1}{0.+k1}}\left(\left(1.045+0.0105\,(0.5+k1)-0.001\,(0.5+k1)^2\right)^{0.5+k1}\right)^{\frac{1}{0.+k1}}$

Out[26]= 0.0600191

In[27]:= $\text{gtwo} = -0.999+0.9759^{\frac{1}{0.+k2}}\left(\left(1.045+0.0105\,(0.5+k2)-0.001\,(0.5+k2)^2\right)^{0.5+k2}\right)^{\frac{1}{0.+k2}}$

Out[27]= 0.0637757

In[28]:= $r_{FRAg} == \left(\frac{(1+\text{gtwo})^{k2}}{(1+\text{gone})^{k1}}\right)^{\frac{1}{(k2-k1)}}-1$

Out[28]= $r_{FRAg} == 0.0675456$

In[29]:= `(* -0.001 shift curves*)`

In[30]:= $\text{hone} = -1.001+0.9759^{\frac{1}{0.+k1}}\left(\left(1.045+0.0105\,(0.5+k1)-0.001\,(0.5+k1)^2\right)^{0.5+k1}\right)^{\frac{1}{0.+k1}}$

Out[30]= 0.0580191

In[31]:= $\text{htwo} = -1.001+0.9759^{\frac{1}{0.+k2}}\left(\left(1.045+0.0105\,(0.5+k2)-0.001\,(0.5+k2)^2\right)^{0.5+k2}\right)^{\frac{1}{0.+k2}}$

Out[31]= 0.0617757

In[32]:= $r_{FRAh} == \left(\frac{(1+\text{htwo})^{k2}}{(1+\text{hone})^{k1}}\right)^{\frac{1}{(k2-k1)}}-1$

Out[32]= $r_{FRAh} == 0.0655456$

(*exercise 4*)

```
-0, 001 * t^2 + 0, 0105 * t + 0, 045

In[∘]:= f (05) = -0.001 * 0.5 * 0.5 + 0.0105 * 0.5 + 0.045
Set: Tag Times in 5 (0.045 + 0.0105 t - 0.001 t^2) is Protected.
Out[∘]= 0.05

In[∘]:= f (10) = -0.001 + 0.0105 + 0.045
Set: Tag Times in 10 (0.045 + 0.0105 t - 0.001 t^2) is Protected.
Out[∘]= 0.0545

In[∘]:= f (15) = -0.001 * 1.5^2 + 0.0105 * 1.5 + 0.045
Set: Tag Times in 15 (0.045 + 0.0105 t - 0.001 t^2) is Protected.
Out[∘]= 0.0585

In[∘]:= f (20) = -0.001 * 2^2 + 0.0105 * 2 + 0.045
Set: Tag Times in 20 (0.045 + 0.0105 t - 0.001 t^2) is Protected.
Out[∘]= 0.062

In[∘]:= f (25) = -0.001 * 2.5^2 + 0.0105 * 2.5 + 0.045
Set: Tag Times in 25 (0.045 + 0.0105 t - 0.001 t^2) is Protected.
Out[∘]= 0.065

In[∘]:= r = (1 - (1 / (1.065^2))) / ((1 / (1.05^0.5)) +
        (1 / (1.0545^1)) + (1 / (1.0585^1.5)) + (1 / (1.062^2)) + (1 / (1.065^2.5)))
Out[∘]= 0.0258191
```

rirs = 2.58 %

Based on the last equation from lecture 4, page 32 I compute fixed interest rate (r_IRS). r _IRS is equel 2, 58 %. Then I compare r_IRS with current WIBOR6M which is 1, 79 %. Because r_IRS is bigger then r_ZM IRS is a good deal. If WIBIR6M would be smaller by 1 % it still will be a good deal. However in case that r_ZM would be bigger by 1 % them current WIBOR6M r_IRS would be smaller than r_ZM.

1 Exsercise 5 / set 3

I am starting form the point, in which, as a bank, I own 0 PLN and 0 USD

- $S_{ASK}^{USD/PLN} = [\frac{PLN}{USD}]$
- $X_{ASK} = [\frac{PLN}{USD}]$
- $X_{BID} = [\frac{PLN}{USD}]$

1.1 X_{ASK} pricing

Procedure now:

1. First I need to borrow P PLN at y_{ASK}^{PLN} for T = 3 months
2. I buy $K = P \cdot \frac{1}{S_{ASK}^{USD/PLN}}$ USD at the current FX rate $S_{ASK}^{USD/PLN}$
3. I invest K USD dollars for T = 3 months at y_{BID}^{USD} interest rate

Procedure in three months:

1. I have now $K(T) = P \cdot \left(\frac{1}{S_{ASK}^{USD/PLN}}\right) \cdot (1 + y_{BID}^{USD})^T$ USD
2. The client want to buy USD by PLN, so he would pay:

$$X_{ASK} \cdot P \cdot \left(\frac{1}{S_{ASK}^{USD/PLN}}\right) \cdot \left(1 + y_{BID}^{USD}\right)^T$$

3. I need to pay off the loan: $P \cdot (1 + y_{ASK}^{PLN})^T$ PLN

If we want to met the static arbitrage conditions, X_{ASK} should be priced, so that the money in PLN paid by the client is equal to the money in PLN I need to pay off the loan:

$$P \cdot (1 + y_{ASK}^{PLN})^T = X_{ASK} \cdot P \cdot \left(\frac{1}{S_{ASK}^{USD/PLN}}\right) \cdot \left(1 + y_{BID}^{USD}\right)^T$$

$$X_{ASK} = S_{ASK}^{USD/PLN} \cdot \left(\frac{1 + y_{ASK}^{PLN}}{1 + y_{BID}^{USD}}\right)^T$$

For:

- $y_{ASK}^{USD} = 2.78\%$

- $y_{BID}^{PLN} = 1.75\%$
- $S_{ASK} = 3.8020 \left[\frac{PLN}{USD}\right]$

$$X_{ASK} = 3.8020 \left[\frac{PLN}{USD}\right] \cdot \left(\frac{1+1.75\%}{1+2.78\%}\right)^3 = 3.79244$$

1.2 X_{BID} pricing

Procedure now:

1. First I need to borrow K USD at y_{ASK}^{USD} for $T = 3$ months
2. I buy $K \cdot S_{BID}^{USD/PLN}$ PLN at the current FX rate $S_{BID}^{USD/PLN}$
3. I invest $K \cdot S_{BID}^{USD/PLN}$ PLN for $T = 3$ months at the interest rate y_{BID}^{PLN}

Procedure in three months:

1. I have now $K \cdot S_{BID}^{USD/PLN} \cdot (1 + y_{BID}^{PLN})^T$ PLN
2. The client want to sell me USD at X_{BID} to get $K \cdot S_{BID}^{USD/PLN} \cdot (1 + y_{BID}^{PLN})^T$ PLN
3. I need to pay off the loan: $K \cdot (1 + y_{ASK}^{USD})^T$ USD

If we want to met the static arbitrage conditions, X_{BID} should be priced, so that the money in USD paid by the client is equal to the money in USD I need to pay off the loan:

$$X_{BID} \cdot K \cdot (1 + y_{ASK}^{USD})^T = K \cdot S_{BID}^{USD/PLN} \cdot (1 + y_{BID}^{PLN})^T$$

$$X_{BID} = S_{BID}^{USD/PLN} \cdot \left(\frac{1 + y_{BID}^{PLN}}{1 + y_{ASK}^{USD}}\right)^T$$

For:

- $y_{ASK}^{USD} = 2.80\%$
- $y_{BID}^{PLN} = 1.72\%$
- $S_{BID} = 3.8010 \left[\frac{PLN}{USD}\right]$

$$X_{BID} = 3.8010 \left[\frac{PLN}{USD}\right] \cdot \left(\frac{1+1.72\%}{1+2.80\%}\right)^3 = 3.79098$$

1 Exercise 6 / set 3

Using STATIC ARBITRAGE arguments prove the 3rd(European put bands),6th(American call-put parity) and 7th(European call price is a monotonic function of X) relations concerning option prices from Lecture 5, page 36

$$max[PV(X) - S; 0] \leq p \leq PV(X)$$

X – eXercise price PV(X) – present value of the eXercise price

We rewrite the first inequality to remove max function:

1. $0 < p$ (trivial)
2. $PV(X) - S \leq p$
3. $p \leq PV(X)$

1.1 Proof 2

Action now: one constructs now (t=0) a portfolio:

- buy 1 put option @ $-p$
- buy 1 share @ $-S(0)$
- take a loan: $PV(X)$

Cash flow: $CF(0) = -p - S(0) + PV(X)$

Action at T >0: We have two cases to consider: $S(T) > X$:

- sell the share @ $S(T)$ (do not execute the option)
- pay off the loan: $-X$

Cash flow: $CF(T) = S(T) - X > 0$

or $S(T) > X$:

- sell the share @ $S(T)$
- execute the option: $X - S(T)$
- pay off the loan: $-X$

Cash flow: $CF(T) = S(T) - (S(T) - X) - X = 0$

In each case the future cashflow is non-negative: $CF(T) \geq 0$ so the cash flow at $t = 0$ has to be non-positive: $CF(0) = -p - S(0) + PV(X) \leq 0$ (otherwise we have static arbitrage). So:

$$PV(X) - S(0) \leq p$$

1.2 Proof 3

Action now: one constructs now (t=0) a portfolio:

- deposit $PV(X)$
- sell the put option @ p

Cash flow: $CF(0) = -PV(X) + p$

Action at T >0:

- we have the money from the loan X
- we need to pay off the option: $max[X - S(T); 0]$ (the institution who bought the option from me, now have the right to execute it)

As $X > 0$ and $S(T) > 0$ (the price of an asset at any time cannot be lower then 0). Cash flow: $CF(T) = X - max[X - S(T); 0] = \{+S(T) or X\} \geq 0$

$$CF(T) \geq 0$$

so

$$CF(0) = -PV(X) + p \leq 0$$

$$p \leq PV(X)$$