Financial Instruments and Pricing
Problems 3

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1 Ex 1

The bonds given in the exercise give bootstrap structure

$$\begin{pmatrix} 104 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 104 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 103 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 104 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 4 & 0 & 104 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 \\ 5 & 0 & 5 & 0 & 5 & 0 & 105 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 4 & 0 & 4 & 0 & 4 & 0 & 4 & 0 & 104 \end{pmatrix}$$

which yields the structure of yields as follows:

t	y(t)
0.5	4.0008
1	4.44913
1.5	4.84882
2	5.20131
2.5	5.49949
3	5.7495
3.5	5.95135
4	6.10039
4.5	6.19848
5	6.24931

Fitting yield curve to these value yields

$$y(t) = 0.0349958 + 0.0105049t - 0.00100111t^2 \tag{1.1}$$

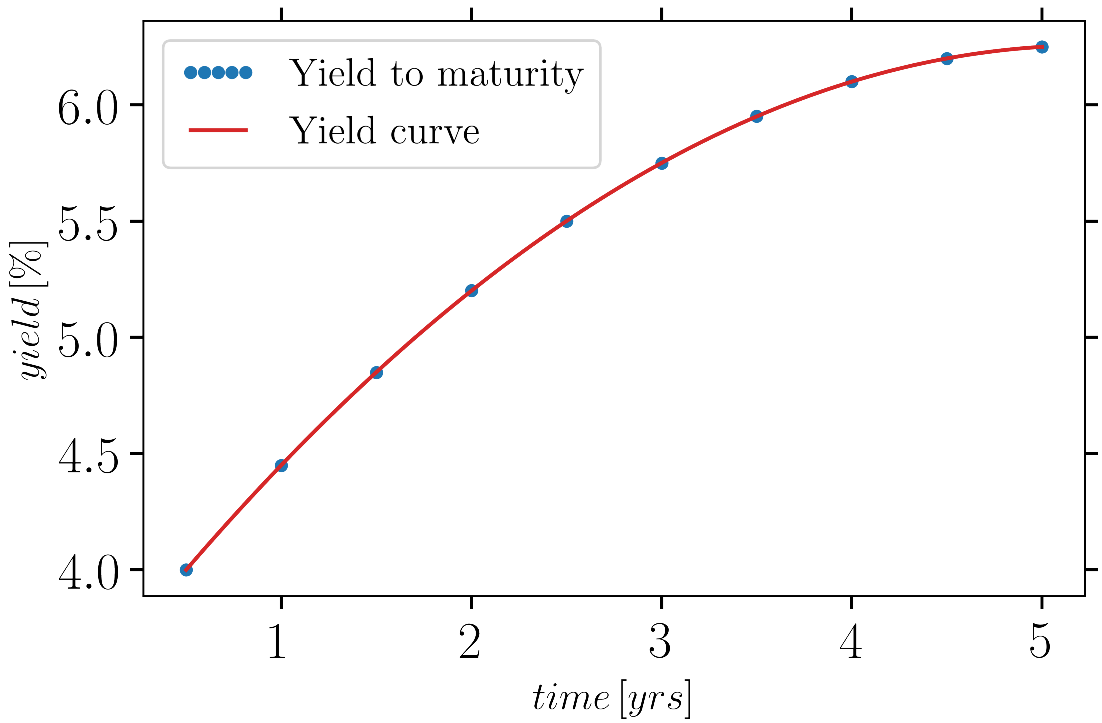


Figure 1: Such caption

2 Ex 2

We take WIBOR3M 1.81% for date 6 months from now and fix it. We compute the net cash flow using the expression :

$$NCF = \frac{[FRA](WIBOR3 - r_{FRA})\frac{\Delta t}{365}}{(1 + WIBOR3\frac{\Delta t}{365})} \tag{2.1}$$

where :
FRA = $10mlnPLN$
$r_{FRA} = 1.93\%$
$\Delta t = 90$ for dates (01.12.18) to (01.03.19)

We obtain :

$$NCF = -2945.76[PLN] \tag{2.2}$$

tu byłem

3 Ex 3

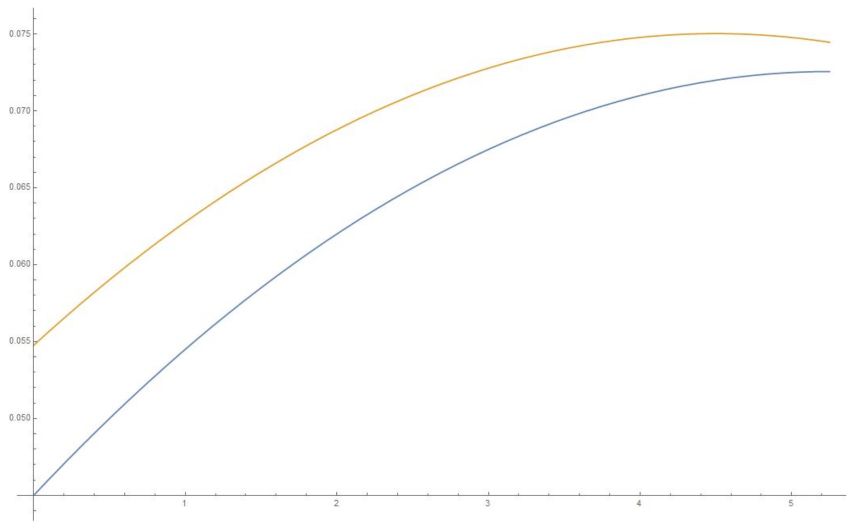
a) fixed rate for FRA "1y x 1.5y" :

$$FRA_{1y\times 1.5y}(t_1,t_2) = (\frac{(1+y(t_2))^{t_2}}{(1+y(t_1))^{t_1}} - 1)\frac{1}{t_2 - t_1} \tag{3.1}$$

$$FRA_{1y\times 1.5y}(t_1 = 1, t_2 = 1.5) = 0.0654739 \tag{3.2}$$

b) forward yield curve in 0.5 years :

$$y_{0.5}(t_1 = 0.5, t_2 = t + 0.5) = (\frac{(1+y(t_2))^{t_2}}{(1+y(t_1))^{t_1}})^{\frac{1}{t_2 - t_1}} - 1 \tag{3.3}$$



red curve : forward yield curve in 0.5 years
blue curve : original yeld curve

c)

$$FRA_{1y\times 1.5y}(t_1 = 0.5, t_2 = 1) = 0.0654739 \tag{3.4}$$

d)

$$FRA_{y+0.001} = 0.0664419 \tag{3.5}$$
$$FRA_{y-0.001} = 0.0645054 \tag{3.6}$$

4 Ex 4

Using the expression for r_{IRS} :

$$1 = \sum_{i=0.5}^{T} \frac{(r_{IRS}+1)^{0.5}-1}{(1+y(t_i))^{t_i}} + \frac{1}{(1+y(T))^T} \tag{4.1}$$

a) We calculate the numerical value for r_{ZM} :

$$r_{IRS} = 0.0645733 \tag{4.2}$$

b) for $r_{ZM}+1\%$:

$$r_{IRS} = 0.0745733 \tag{4.3}$$

c) for $r_{ZM}-1\%$:

$$r_{IRS} = 0.0545733 \tag{4.4}$$

5 Ex 5

a) In order to sell 1$ forward we need to:

a. We need to buy $\frac{1}{(1+y)_{BID}^{USD}(T))^T}$\$ at exchange rate $e^{ASK}(0)$ and invest them to have 1$ at time $t=1$

b. We get the loan of $e^{ASK}(0)\frac{1}{\left(1+y_{ASK}^{USD}(T)\right)^T}$ zł

c. at time $t=1$ our loan is worth $\left(\left(1+y_{ASK}^{PLN}(T)\right)^T\right)\left(e^{ASK}(0)\frac{1}{\left(1+y_{BID}^{USD}(T)\right)^T}\right)$

For this reason the ask forward exchange rate ist giben by

$$e^{ASK}(T) = \frac{\left(1+y_{ASK}^{PLN}(T)\right)^T}{\left(1+y_{BID}^{USD}(T)\right)^T} e^{ASK}(0) \tag{5.1}$$

In order to get $e^{BID}(t)$

a. we short sell (borrow) one future dolar $\frac{1\$}{(1+y^{USD}(T))^T}$ for $\frac{e^{BID}(0)}{\left(1+y_{ASK}^{USD}(T)\right)^T}$ zł

b. In the future we have $\left(\left(1+y_{BID}^{PLN}(T)\right)^T\right)\left(\frac{e^{BID}(0)}{\left(1+y_{ASK}^{USD}(T)\right)^T}\right)$ zł to buy 1$ to repay our short selling

Thus, by similar arguments inverted, one can calculate the bid exchange rate as

$$e^{BID}(T) = \frac{\left(1+y_{BID}^{PLN}(T)\right)^T}{\left(1+_{ASK}^{USD}(T)\right)^T} e^{BID}(0) \tag{5.2}$$

b) Using the data from exercise

(T = 3M and: S^{USDPLN} : $3.8010–3.8020, 3M\_PLN\_DEPOS : 1.72\% - 1.75\%, 3M\_USD\_DEPOS. : 2.78\% -$ 2.80%)

S^{USDPLNFUTURE} : 3.79098–3.79244

6 Ex 6

6.1 First proof

$$c_2 \leq c_1 \quad \text{if} \quad X_2 > X_1$$

Let's take

$$-c_2 + c_1$$

as portfolio. This means that we buy call option 2 and short call option 1.
Upon closing it has value

$$\max(S - X_2; 0) - \max(S - X_1; 0)$$

that has three options

$$\begin{aligned} S \leq X_1: &\quad 0 \leq 0 \\ S \in (X_1,\, X_2): &\quad -S + X_1 < 0 \\ S > X_2: &\quad X_1 - X_2 < 0 \end{aligned}$$

In this way we see that all these cashflows will be less than zero. In order to eliminate the arbitrage, we need to have

$$-c_2 + c_1 \geq 0 \Rightarrow c_1 \geq c_2$$

QED

6.2 Second proof

$$\max(X - S; 0) \leq P \leq X$$

gives

$$\begin{array}{lr} 1) & X - P \geq 0 \\ 2) & P \geq 0 \\ 3) & P - X + S \geq 0 \end{array}$$

ad 2) $P \geq 0$ we buy put option

a. $\max(X - S(T); 0) \geq 0$

b. $CF(T) = X - S(T) \geq 0 \longrightarrow CF(0) = -P < 0$

Now it can be easly seen that $CF(0) = X - S(T) \leq 0$.
ad 1) $X - P \geq 0$ we borrow X money and short put option

a. Deposit X and short A-Put P: $-X + P$

b. Closing positions: $FV(X, t) - \max(X - S(t); 0)$

c. if $X < S(t) : FV(X, t) > 0$

d. else $X \geq S(t) : FV(X,t) - X + S(t) \geq 0$ for any t

e. In particular for $t = 0$: $S(0) > 0$, which proves 1)

ad 3)

a. Short put option P, deposit/borrow $S(0) - X$: $P + S - X$

b. Close positions $-\max(X - S(t); 0) + FV(X - S(0), t)$

c. Consider $t = 0$ and $X > S$: -X + S + X - S = 0

d. Else $X \leq S \Rightarrow X - S \leq 0$

e. Since at the end we lose money, the initial cashflow is positive:

$$P + S - X \geq 0$$

6.3 Third proof

At the beginning we take portfolio: short put option, buy call option, short sell stock on the market and deposit $PV(X)$ money, so the initial cashflow is

$$p - c + S(0) - PV(X)$$

On closing this portfolio by buying stock, closing the deposit and closing the contracts we have

$$-\max(X - S(T); 0) + \max(S(T) - X; 0) - S(T) + X = 0$$

which necesitates that initial cashflow is also zero and proves european pc parity:

$$p - c = PV(X) - S(0)$$