CA-2 (Assignment)

Q1 If $R = \{(a,b), (b,c), (c,a)\}$ is relation in $\{a,b,c\}$, Find R^+ & R^*.

→ Find R^+ ~~(Transitive closure)~~:

$R \circ R = \{(a,b), (b,c), (c,a)\} \circ \{(a,b), (b,c), (c,a)\}$

$R^2 = \{(a,c), (b,a), (c,b)\}$

$R^3 = R^2 \circ R$

$= \{\{(a,c), (b,a), (c,b)\} \circ \{(a,b), (b,c), (c,a)\}$

$= \{(a,a), (b,b), (c,c)\} = R$

$R^+ = R \cup R^2 \cup R^3$

$= \{(a,b), (b,c), (c,a), (a,c), (b,a), (c,b), (a,a), (b,b), (c,c)\}$

$R^* = R^+$

(2) $R = \{(1,2), (2,3), (2,4)\}$ be a relation in $\{1,2,3,4\}$ Find R^+

→ $R^2 = R \circ R$

$= \{(1,2), (2,3)\ (2,4)\} \circ \{(1,2), (2,3), (2,4)\}$

$= \{(1,3), (1,4)\}$

$R^3 = R^2 \circ R = \{(1,3), (1,4)\} \circ \{(1,2), (2,3), (2,4)\}$

$= \phi$

$R^+ = R \cup R^2 \cup R^3$

$= \{(1,2)\ (2,3), (2,4)\} \cup \{(1,3)\ (1,4)\} \cup \{\phi\}$

$= \{(1,2), (2,3), (2,4), (1,3), (1,4)\}$

Q3 Convert grammar to language of the following

(a) $S \rightarrow \varepsilon$, $S \rightarrow aS$

Step 1: smallest string (applying (i))

$L_1(G) = S \xrightarrow{(i)} \varepsilon$

Step 2: 2nd smallest string

$S \xrightarrow{(ii)} aS \xrightarrow{(i)} a$ (applying (ii) followed by (i))

$L_2(G) = a$

Step 3: 3rd smallest string

$S \xrightarrow{(ii)^2} aaS \xrightarrow{(i)} aa$

$L_3(G) = aa$

Step n: nth smallest string

$S \xrightarrow{(ii)^*} a^n S \xrightarrow{(i)} a^n$

$L_n(G) = a^n$

$L(G) = L_1(G) \cup L_2(G) \cdots L_n(G)$

$= \{\varepsilon, a, aa \ldots a^n\}$

$= \{a^n \mid n \geq 0\}$

(b) $S \rightarrow \varepsilon$, $S \rightarrow Sb$

Step 1: the smallest string

$L_1(G) = S \xrightarrow{(i)} \varepsilon$

$L_1(G) = \{\varepsilon\}$

Step 2: 2nd smallest string

$S \xrightarrow{(ii)} Sb \xrightarrow{(i)} b$

$L_2(G) = \{b\}$

Step 3: 3rd Smallest String

$S \xrightarrow{(ii)^2} Sbb \xrightarrow{(i)} bb$

$L_3(G) = \{bb\}$

Step n : nth smallest string

$S \xrightarrow{(ii)^*} Sb^n \xrightarrow{(i)} b^n$

$L_n(G) = \{b^n\}$

$L(G) = L_1(G) \cup L_2(G) \ldots \cup L_n(G)$

$= \{\varepsilon, b, bb \ldots b^n\}$

$= \{b^n \mid n \geq 0\}$

(c) $S \longrightarrow a$

$S \longrightarrow aS$

Step 1: smallest string

$S \xrightarrow{(i)} a$

$L_1(G) = a$

Step 2: 2nd smallest string

$S \xrightarrow{ii} aS \xrightarrow{(i)} aa$ $L_2(G) = \{aa\}$

Step 3: 3rd smallest string

$S \xrightarrow{(ii)2} aaS \xrightarrow{(i)} aaa$

$L_2(G) = \{a.aa\}$

$L(G) = L_1(G) \cup L_2(G) \ldots L_n(G)$

$= \{a, aa, aaa \ldots a^n\}$

$= \{a^n \mid n \geq 1\}$

(d) $S \longrightarrow b$, $S \longrightarrow Sb$

Given $S \longrightarrow b$

$S \longrightarrow Sb$

Step 1 : smallest string

$S \xrightarrow{(i)} b$

$L_1(G) = \{b\}$

Step 2 : 2nd smallest string

$S \xrightarrow{(ii)} Sb \xrightarrow{(i)} bb$

$L_2(G) = \{b, b\}$

$L_3(G) = S \xrightarrow{(ii)^2} Sbb \xrightarrow{(i)} bbb$

$L_3(G) = bbb$

Step n: nth string

$L_n(G) = S \xrightarrow{(ii)^*} Sb^{n-1} \xrightarrow{(i)} b^n$

$L(G) = L_1(G) \cup L_2(G) \dots L_n(G)$

$= \{b, bb, bbb \dots b^n\}$

$= \{b^n \mid n \geq 1\}$

(e) $S \longrightarrow \varepsilon, \; S \longrightarrow aSb$

Step 1: Smallest string

$S \xrightarrow{(i)} \varepsilon$ Applying (i)

$L_1(G) = \varepsilon$

Step 2: $S \xrightarrow{ii} aSb \xrightarrow{(i)} ab$

$L_2(G) = ab$ applying (ii) followed by (i),

Step 3: $S \xrightarrow{(ii)^2} aaSbb \xrightarrow{(i)} aabb$

Step n: $S \xrightarrow{(ii)^*} a^nSb^n \xrightarrow{(i)} a^nb^n$

$L(G) = L_1(G) \cup L_2(G) \dots L_n(G)$

$= \{\varepsilon, ab, aabb \dots a^nb^n\}$

$= \{a^nb^n \mid n \geq 0\}$

(g) $S \longrightarrow \varepsilon, \; S \longrightarrow aS, \; S \longrightarrow Sb$

Step 1: Smallest string

$S \xrightarrow{(i)} \varepsilon$

$L_1(G) = \{\varepsilon\}$

Step 2: 2nd smallest string

$S \xrightarrow{ii} aS \xrightarrow{(i)} a$

$S \xrightarrow{(iii)} ab \xrightarrow{(i)} b$

Applying rule (iii) (ii) & (i)

$S \xrightarrow{(iii)} Sb \xrightarrow{(ii)} aSb \xrightarrow{(i)} ab$

~~Step 3~~ $L_2(G) = ab$

$L(G) = L_1(G) \cup L_2(G) \ldots$

$= \{a^n, b^n, a^p b^q \mid m, n \geq 0, p+q = n\}$

(4) $S \rightarrow ab, \; S \rightarrow aS, \; S \rightarrow Sb$

Step 1: $S \xrightarrow{(i)} ab$, $\cancel{S \xrightarrow{(i)} aS}$, $\cancel{S \xrightarrow{(i)} Sb}$

$L_1(G) = ab$ $\cancel{= L_1(G) = a}$

Step 2: $S \xrightarrow{(ii)} aS \xrightarrow{(i)} aab$

$L_2 G = a^2 b$

$S \xrightarrow{(iii)} Sb \xrightarrow{(i)} abb$

$L_2(G) = ab^2$

~~Step n: S~~

$L(G) = L_1(G) \cup L_2(G) \ldots L_n(G)$

$= \{ab, a^2b, ab^2, a^3b, ab^3 \ldots\}$

$= \{a^n b^m \mid n \geq 1, m \geq 1\}$

Q4 Convert Language to grammar of the following

a) $L(G) = \{a^n \mid n \geq 0\}$

$L(G) = \{\varepsilon, a, aa, aaa \ldots\}$

$P = S \rightarrow \varepsilon$

$S \rightarrow aS$

$G = \{\{S\}, \{a\}, P, S\}$

b) $L(G) = \{a^n \mid n \geq 1\}$

$L(G) = \{a, aa, aaa \ldots\}$

$P = S \rightarrow a$

$S \rightarrow aS$

$G = \{\{S\}, \{a\}, P, S\}$

c) $L(G) = \{a^n b^n \mid n >= 0\}$

$L = \{\varepsilon, ab, aabb, aaabbb \ldots\}$

$P = S \longrightarrow \varepsilon$

$S \longrightarrow aSb$

$G = \{\{S\}, \{a\}, P, S\}$

(d) $\{a^n b^n \mid n \geq 1\}$

$L = \{ab, a^2b^2, a^3b^3 \ldots\}$

$P = S \rightarrow ab$

$S \rightarrow aSb$

(e) $L(G) = \{ a^m b^n \mid m >= 0,\ n >= 0 \}$

$L = \{ \varepsilon, a, b, ab, a^2b, a^2, b^2 \dots \}$

$P = S \rightarrow aS$
$S \rightarrow Sb$
$S \rightarrow \varepsilon$

(f) $L(G) = \{ a^m b^n \mid m >= 1,\ n >= 1 \}$

$= \{ ab, a^2b, ab^2, a^2b^2 \dots \}$

$P = S \rightarrow ab$
~~$S \rightarrow aSb$~~
$S \rightarrow aS$
$S \rightarrow Sb$

Q Draw a table to corelate different types of grammar Languages and corresponding machines used in Automata theory.

Grammar	Language	Machines
Type - 0	No Restriction Language	Turning Machine
Type - 1	context Sensitive Language	Linear bounded Automata (LBA)
Type - 2	Context free Language	Push down automata (PDA)
Type - 3	Regular Language	Finite Automata