HW3_Q1

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4/20/2020

Predictive Model Building

I will first start by tweaking the data around and also taking a look at summaries of a handful of the variables.

```
library(remotes)
library(rlang)
library(LICORS)
library(foreach)
library(tidyverse)
```

```
## -- Attaching packages ----------------------------------------------------------------------------------

## <U+2713> ggplot2 3.3.0     <U+2713> purrr   0.3.3
## <U+2713> tibble  2.1.3     <U+2713> dplyr   0.8.4
## <U+2713> tidyr   1.0.0     <U+2713> stringr 1.4.0
## <U+2713> readr   1.3.1     <U+2713> forcats 0.4.0

## -- Conflicts ---------------------------------------------------------------------------------------------
## x purrr::%@%()         masks rlang::%@%()
## x purrr::accumulate()  masks foreach::accumulate()
## x purrr::as_function() masks rlang::as_function()
## x dplyr::filter()      masks stats::filter()
## x purrr::flatten()     masks rlang::flatten()
## x purrr::flatten_chr() masks rlang::flatten_chr()
## x purrr::flatten_dbl() masks rlang::flatten_dbl()
## x purrr::flatten_int() masks rlang::flatten_int()
## x purrr::flatten_lgl() masks rlang::flatten_lgl()
## x purrr::flatten_raw() masks rlang::flatten_raw()
## x purrr::invoke()      masks rlang::invoke()
## x dplyr::lag()         masks stats::lag()
## x purrr::list_along()  masks rlang::list_along()
## x purrr::modify()      masks rlang::modify()
## x purrr::prepend()     masks rlang::prepend()
## x purrr::splice()      masks rlang::splice()
## x purrr::when()        masks foreach::when()
```

```
library(dplyr)
library(mosaic)
```

```
## Loading required package: lattice

## Loading required package: ggformula

## Loading required package: ggstance

##
```

```
## Attaching package: 'ggstance'

## The following objects are masked from 'package:ggplot2':
##
##     geom_errorbarh, GeomErrorbarh

##
## New to ggformula?  Try the tutorials:
##  learnr::run_tutorial("introduction", package = "ggformula")
##  learnr::run_tutorial("refining", package = "ggformula")

## Loading required package: mosaicData

## Loading required package: Matrix

##
## Attaching package: 'Matrix'

## The following objects are masked from 'package:tidyr':
##
##     expand, pack, unpack

## Registered S3 method overwritten by 'mosaic':
##   method                           from
##   fortify.SpatialPolygonsDataFrame ggplot2

##
## The 'mosaic' package masks several functions from core packages in order to add
## additional features.  The original behavior of these functions should not be affected by this.
##
## Note: If you use the Matrix package, be sure to load it BEFORE loading mosaic.

##
## Attaching package: 'mosaic'

## The following object is masked from 'package:Matrix':
##
##     mean

## The following objects are masked from 'package:dplyr':
##
##     count, do, tally

## The following object is masked from 'package:purrr':
##
##     cross

## The following object is masked from 'package:ggplot2':
##
##     stat

## The following objects are masked from 'package:stats':
##
##     binom.test, cor, cor.test, cov, fivenum, IQR, median, prop.test,
##     quantile, sd, t.test, var

## The following objects are masked from 'package:base':
##
##     max, mean, min, prod, range, sample, sum
```

```r
greendata_raw = read.csv("~/GitHub/SDS323_Spring2020/hw3/q1/greenbuildings.csv")
#View(greendata_raw)
```

```r
greendata = na.omit(greendata_raw)
#View(greendata)

#median rent for green and non-green buildings; green buildings seem to have higher rent compared to no
greendata %>%
  group_by(green_rating) %>%
  summarize(median_rent = median(Rent))
```

```
## # A tibble: 2 x 2
##   green_rating median_rent
##          <int>       <dbl>
## 1            0        25
## 2            1        27.6
```

```r
#percent class a and class b, separated by green and non-green; green buildings are mostly class a
greendata %>%
  group_by(green_rating) %>%
  summarize(pct_a = mean(class_a) * 100, pct_b = mean(class_b) * 100,
            pct_c = 100 - pct_a - pct_b)
```

```
## # A tibble: 2 x 4
##   green_rating pct_a pct_b pct_c
##          <int> <dbl> <dbl> <dbl>
## 1            0  36.1  48.5 15.4
## 2            1  79.5  19.4  1.03
```

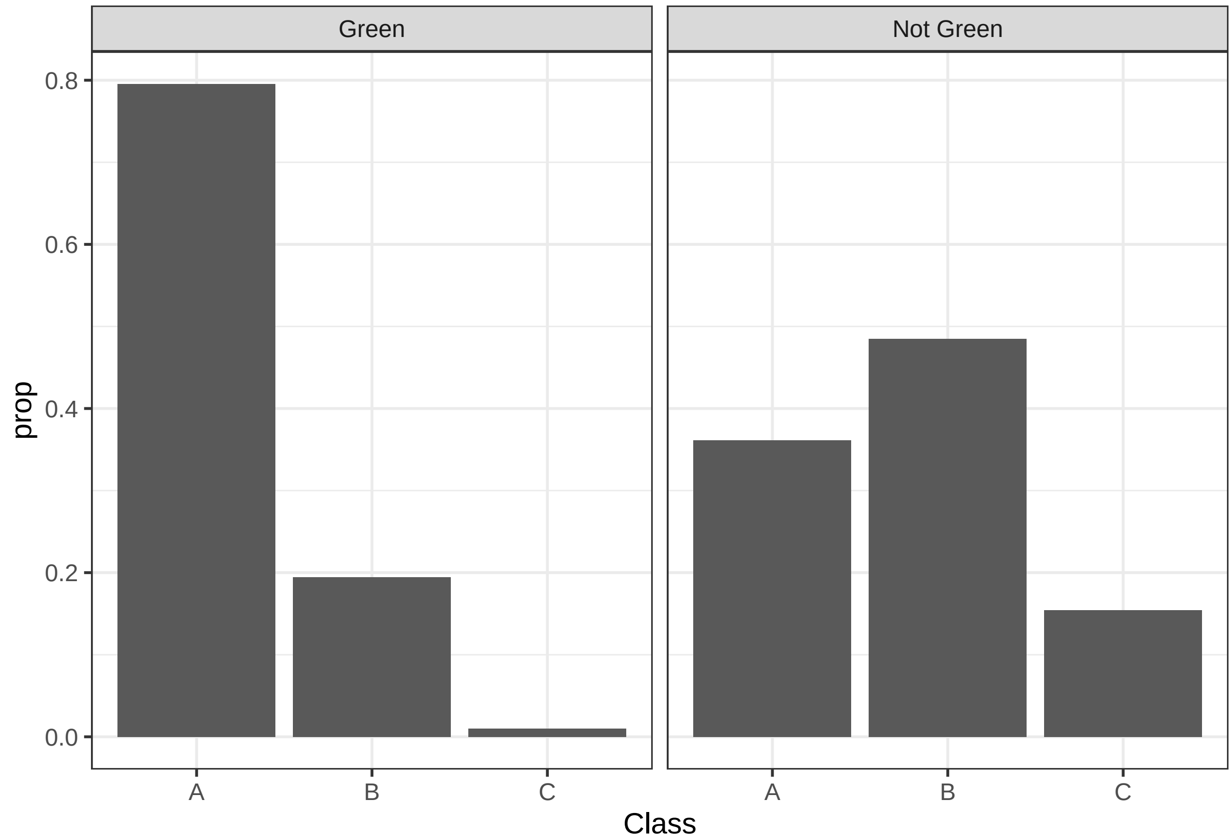
```r
#Changing green rating to string variable
greendata$GreenRating <- rep(0,nrow(greendata))
greendata$GreenRating[which(greendata$green_rating==1)] <- "Green"
greendata$GreenRating[which(greendata$green_rating==0)] <- "Not Green"

#Creating ClassC Variable
greendata$ClassC <- rep(0,nrow(greendata))
greendata$ClassC <- greendata$class_a + greendata$class_b

#Creating Class Variable
greendata$Class <- rep(0,nrow(greendata))
greendata$Class[which(greendata$class_a==1)] <- "A"
greendata$Class[which(greendata$class_b==1)] <- "B"
greendata$Class[which(greendata$ClassC==0)] <- "C"

#Changing Amenities variable
greendata$amenities[greendata$amenities == "1"] <- "Yes"
greendata$amenities[greendata$amenities == "0"] <- "No"

#graph that shows proportions of green/not green buildings in each class
ggplot(data = greendata) +
  geom_bar(mapping = aes(x = Class, y = ..prop.., group = 1)) +
  facet_wrap(GreenRating~.) +
  theme_bw()
```



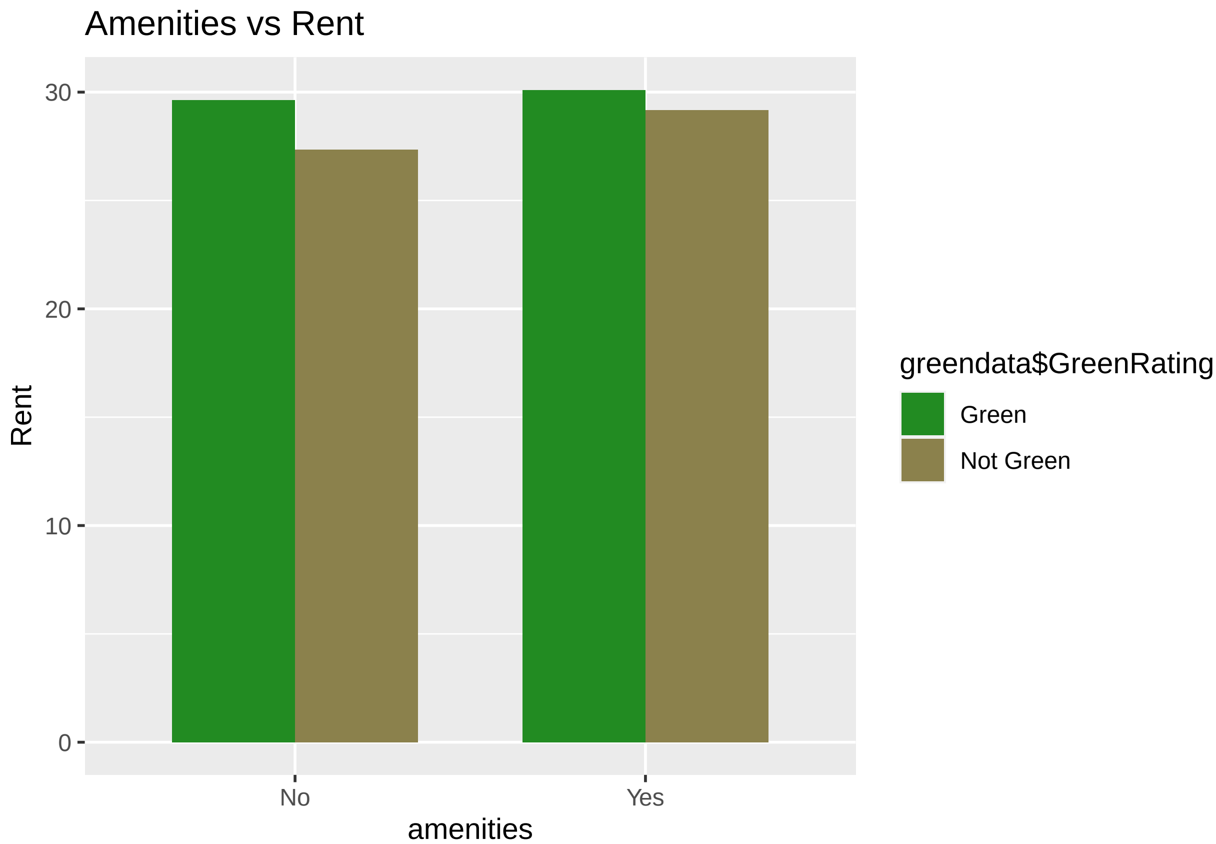

```
#RENT VS AMENITIES
fill <- c("forestgreen", "lightgoldenrod4")
greendata$amenities[greendata$amenities == "1"] <- "Yes"
greendata$amenities[greendata$amenities == "0"] <- "No"

AmenitiesChart<- ggplot(data=greendata, aes(x=amenities, y=Rent, fill=greendata$GreenRating)) +
  ggtitle("Amenities vs Rent") +
  geom_bar(stat="summary", fun.y = "mean", position=position_dodge(),
           width = 0.7)+  scale_fill_manual(values=fill)
```

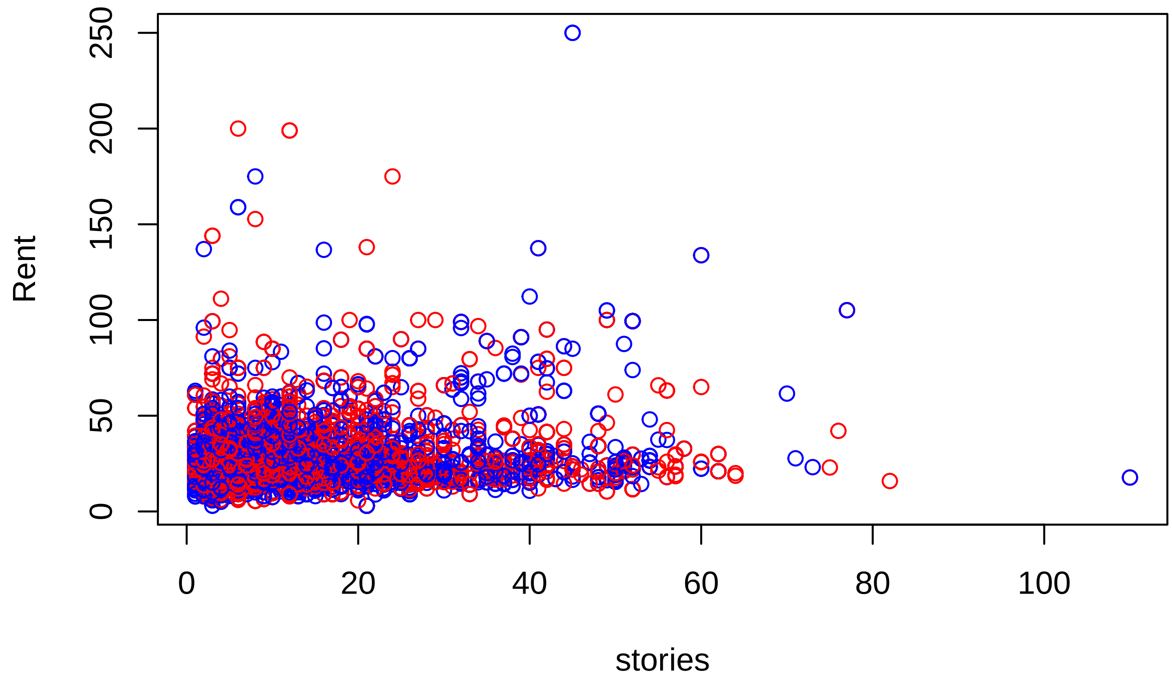
```
## Warning: Ignoring unknown parameters: fun.y
```

```
AmenitiesChart
```

```
## No summary function supplied, defaulting to `mean_se()`
```




```
plot(Rent ~ stories, data = greendata, col=c("red", "blue"))
```

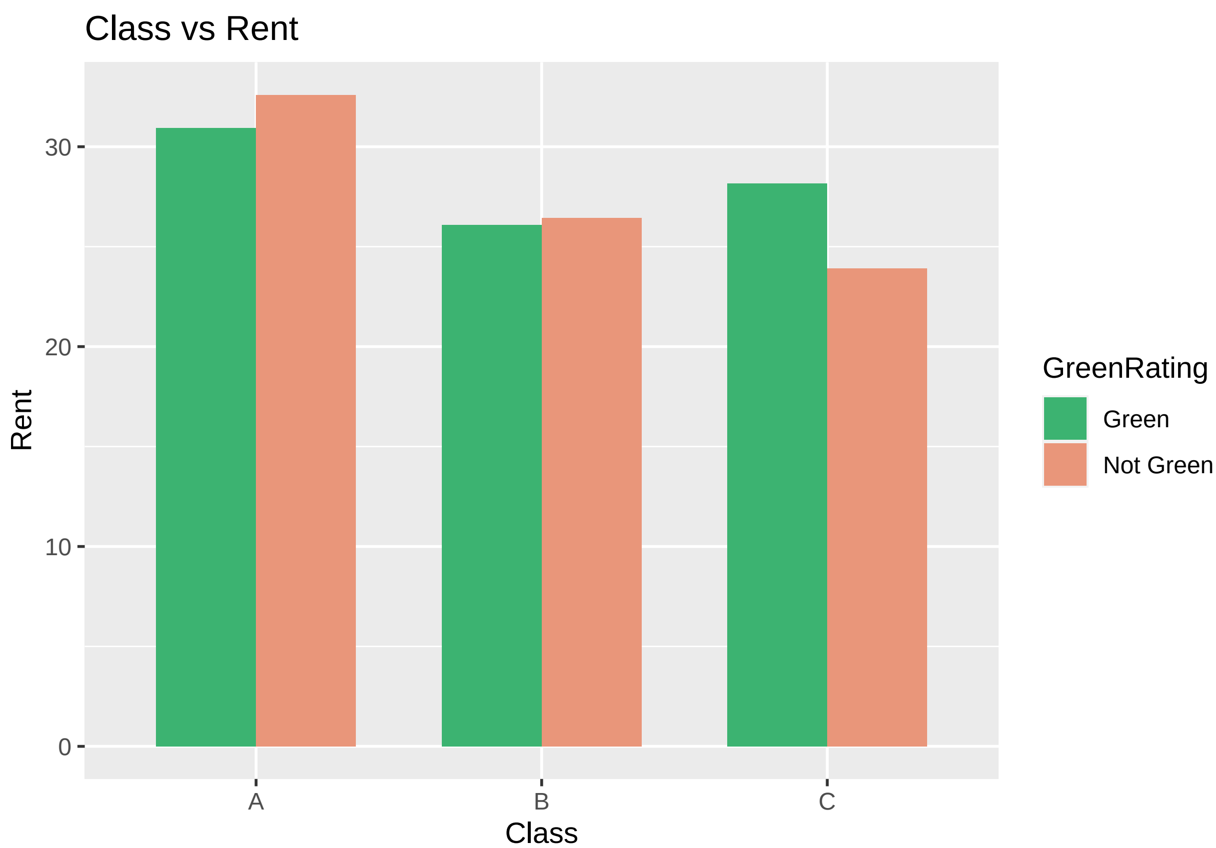



```
#Rent by Class Bar Chart
fill <- c("mediumseagreen", "darksalmon")
ClassChart1<- ggplot(data=greendata, aes(x=Class, y=Rent, fill=GreenRating)) +
  ggtitle("Class vs Rent") +
  geom_bar(stat="summary", fun.y = "mean",position = position_dodge(),
           width =0.7) +  scale_fill_manual(values=fill)
```

```
## Warning: Ignoring unknown parameters: fun.y
```

```
ClassChart1
```

```
## No summary function supplied, defaulting to `mean_se()`
```



To create the best model to predict price, I wanted to change the data a bit and understand the data through summaries. Through doing this, I was able to create a dummy variable for class and see that the rent was higher for green buildings when accounting for other variables such as class.

After understanding the data a bit more clearly, I will now try to fit a model that can predict rent.

```
#Begin linear regression models

greenlm1 <- lm(Rent~green_rating+Class+amenities, data = greendata)
summary(greenlm1)
```

```
##
## Call:
## lm(formula = Rent ~ green_rating + Class + amenities, data = greendata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -24.413  -8.936  -2.796   5.727 217.727
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   33.4125     0.4012  83.288   <2e-16 ***
## green_rating  -1.2570     0.6116  -2.055   0.0399 *
## ClassB        -6.4769     0.3934 -16.465   <2e-16 ***
## ClassC        -9.2276     0.5639 -16.364   <2e-16 ***
## amenitiesYes  -1.1392     0.3685  -3.091   0.0020 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 14.73 on 7815 degrees of freedom
## Multiple R-squared:  0.04873,    Adjusted R-squared:  0.04824
## F-statistic: 100.1 on 4 and 7815 DF,  p-value: < 2.2e-16
```

```r
#Adding cluster rent greatly increased the R^2
greenlm2 <- lm(Rent~green_rating+Class+amenities+cluster_rent, data = greendata)
summary(greenlm2)
```

```
##
## Call:
## lm(formula = Rent ~ green_rating + Class + amenities + cluster_rent,
##     data = greendata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -50.582  -3.735  -0.655   2.329 185.804
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.36368    0.41398   0.878   0.3797
## green_rating  0.66579    0.39891   1.669   0.0952 .
## ClassB       -2.86761    0.25867 -11.086  < 2e-16 ***
## ClassC       -4.64416    0.37007 -12.549  < 2e-16 ***
## amenitiesYes  1.27863    0.24125   5.300 1.19e-07 ***
## cluster_rent  1.06529    0.01035 102.944  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.6 on 7814 degrees of freedom
## Multiple R-squared:  0.5963, Adjusted R-squared:  0.596
## F-statistic:  2308 on 5 and 7814 DF,  p-value: < 2.2e-16
```

```r
#age doesn't add much value to the R^2
greenlm3 <- lm(Rent~green_rating+Class+amenities+cluster_rent+age , data = greendata)
summary(greenlm3)
```

```
##
## Call:
## lm(formula = Rent ~ green_rating + Class + amenities + cluster_rent +
##     age, data = greendata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -50.382  -3.719  -0.691   2.308 185.816
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.471250   0.429102   1.098    0.272
## green_rating  0.624025   0.401312   1.555    0.120
## ClassB       -2.762644   0.281159  -9.826  < 2e-16 ***
## ClassC       -4.471507   0.412064 -10.851  < 2e-16 ***
## amenitiesYes  1.275094   0.241277   5.285 1.29e-07 ***
## cluster_rent  1.065541   0.010352 102.934  < 2e-16 ***
## age          -0.003857   0.004049  -0.953    0.341
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.6 on 7813 degrees of freedom
## Multiple R-squared:  0.5963, Adjusted R-squared:  0.596
## F-statistic:  1924 on 6 and 7813 DF,  p-value: < 2.2e-16
```

```r
#A model with just cluster rent and green rating
greenlm4 <- lm(Rent~green_rating+cluster_rent, data = greendata)
summary(greenlm4)
```

```
##
## Call:
## lm(formula = Rent ~ green_rating + cluster_rent, data = greendata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -49.735  -3.693  -0.654   2.477 188.044
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.53196    0.31032  -4.937 8.11e-07 ***
## green_rating  2.45702    0.39376   6.240 4.61e-10 ***
## cluster_rent  1.08120    0.01044 103.553  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.803 on 7817 degrees of freedom
## Multiple R-squared:  0.5788, Adjusted R-squared:  0.5787
## F-statistic:  5371 on 2 and 7817 DF,  p-value: < 2.2e-16
```

```r
#A large model
greenlm5 <- lm(Rent~green_rating+cluster_rent+renovated+Class+net+cluster+size+empl_gr, data = greendata
summary(greenlm5)
```

```
##
## Call:
## lm(formula = Rent ~ green_rating + cluster_rent + renovated +
##     Class + net + cluster + size + empl_gr, data = greendata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -52.952  -3.657  -0.453   2.369 176.294
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.365e+00  4.310e-01  -3.166 0.001550 **
## green_rating  7.624e-01  3.945e-01   1.933 0.053329 .
## cluster_rent  1.060e+00  1.051e-02 100.900  < 2e-16 ***
## renovated    -4.708e-01  2.273e-01  -2.072 0.038328 *
## ClassB       -1.764e+00  2.652e-01  -6.650 3.14e-11 ***
## ClassC       -3.237e+00  3.701e-01  -8.745  < 2e-16 ***
## net          -2.543e+00  5.890e-01  -4.318 1.60e-05 ***
## cluster       9.752e-04  2.768e-04   3.524 0.000428 ***
## size          6.442e-06  4.104e-07  15.697  < 2e-16 ***
## empl_gr       2.597e-03  1.317e-02   0.197 0.843716
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.462 on 7810 degrees of freedom
## Multiple R-squared:  0.6079, Adjusted R-squared:  0.6075
## F-statistic:  1346 on 9 and 7810 DF,  p-value: < 2.2e-16
```

```
#Remove the highly insignifcant variables
greenlm6 <- lm(Rent~green_rating+cluster_rent+Class+net+cluster+size, data = greendata)
summary(greenlm6)
```

```
##
## Call:
## lm(formula = Rent ~ green_rating + cluster_rent + Class + net +
##     cluster + size, data = greendata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -52.877  -3.658  -0.455   2.388 175.980
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.542e+00  4.140e-01  -3.725 0.000196 ***
## green_rating  8.171e-01  3.937e-01   2.075 0.037978 *
## cluster_rent  1.062e+00  1.045e-02 101.613  < 2e-16 ***
## ClassB       -1.851e+00  2.619e-01  -7.067 1.72e-12 ***
## ClassC       -3.325e+00  3.675e-01  -9.047  < 2e-16 ***
## net          -2.524e+00  5.890e-01  -4.284 1.86e-05 ***
## cluster       1.002e-03  2.763e-04   3.626 0.000289 ***
## size          6.375e-06  4.087e-07  15.598  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.464 on 7812 degrees of freedom
## Multiple R-squared:  0.6077, Adjusted R-squared:  0.6074
## F-statistic:  1729 on 7 and 7812 DF,  p-value: < 2.2e-16
```

```
#The interaction variable does not add significant value to predicting rent
greenlm7 <- lm(Rent~green_rating*Class+cluster_rent+net+cluster+size, data = greendata)
summary(greenlm7)
```

```
##
## Call:
## lm(formula = Rent ~ green_rating * Class + cluster_rent + net +
##     cluster + size, data = greendata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -52.951  -3.626  -0.457   2.351 175.926
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)         -1.464e+00  4.157e-01  -3.521 0.000432 ***
## green_rating         3.811e-01  4.487e-01   0.849 0.395651
## ClassB              -1.979e+00  2.698e-01  -7.333 2.47e-13 ***
## ClassC              -3.428e+00  3.708e-01  -9.245  < 2e-16 ***
```

```
## cluster_rent         1.062e+00  1.045e-02 101.631  < 2e-16 ***
## net                 -2.510e+00  5.890e-01  -4.262 2.05e-05 ***
## cluster              1.002e-03  2.763e-04   3.628 0.000288 ***
## size                 6.359e-06  4.087e-07  15.558  < 2e-16 ***
## green_rating:ClassB  1.767e+00  9.513e-01   1.857 0.063330 .
## green_rating:ClassC  4.073e+00  3.616e+00   1.126 0.260141
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.462 on 7810 degrees of freedom
## Multiple R-squared:  0.6079, Adjusted R-squared:  0.6075
## F-statistic:  1346 on 9 and 7810 DF,  p-value: < 2.2e-16
```

```r
#models 2, 4, 6 seem good to proceed with!
```

Next, I went into the process of creating a model that would be able to predict rent . I did this by creating several linear regression models that used Rent as the dependent variable. My process that I used to select models is by starting with a small model and then adding different variables to see how R^2 was affected. I then selected three models that had favorable R^2 and that were limited to significant variables and not insignificant variables. I would then evaluate the RMSE's of these models to see which produced the smallest when predicting out of sample rent.

Let's look at predictive power with train and test sets

```r
# easy averaging over train/test splits
set.seed(0824)
n = nrow(greendata)
n_train = round(0.8*n)  # round to nearest integer
n_test = n - n_train

rmse = function(y, yhat) {
  sqrt(mean(y-yhat)^2)
}

rmse_vals = do(1000)*{

  # re-split into train and test cases with the same sample sizes
  train_cases = sample.int(n, n_train, replace=FALSE)
  test_cases = setdiff(1:n, train_cases)
  green_train = greendata[train_cases,]
  green_test = greendata[test_cases,]


  # Fit to the training data
  greenlm2t <- lm(Rent~green_rating+Class+amenities+cluster_rent, data = green_train)

  greenlm4t <- lm(Rent~green_rating+cluster_rent, data = green_train)

  greenlm6t <- lm(Rent~green_rating+cluster_rent+Class+net+cluster+size, data = green_train)


  # Predictions out of sample
  yhat_test2 = predict(greenlm2t, green_test)
  yhat_test4 = predict(greenlm4t, green_test)
  yhat_test6 = predict(greenlm6t, green_test)
```

```r
  c(rmse(greendata$Rent, yhat_test2),
    rmse(greendata$Rent, yhat_test4),
    rmse(greendata$Rent, yhat_test6)
    )
}

rmse_vals
```

```
##                V1           V2          V3
## 1    5.463413e-01 0.5111330606 0.582882050
## 2    1.042186e-01 0.0093188908 0.080985204
## 3    3.920333e-01 0.4431290002 0.349059002
## 4    4.286310e-01 0.4133935489 0.393193269
## 5    4.753788e-02 0.0811236473 0.057708595
## 6    1.608754e-02 0.0242433624 0.080898269
## 7    1.009346e-01 0.1460309573 0.108952948
## 8    1.257816e-01 0.1146094136 0.088682493
## 9    5.078142e-02 0.0035049810 0.061473951
## 10   1.597849e-01 0.1386989958 0.193099119
## 11   2.082012e-01 0.1934901043 0.165771571
## 12   2.542076e-01 0.2225742574 0.211765306
## 13   1.351863e-01 0.2229772626 0.155086099
## 14   3.398525e-02 0.0816230029 0.010169563
## 15   2.697404e-01 0.3359854447 0.252305425
## 16   1.353973e-01 0.2007632691 0.141307522
## 17   6.474456e-02 0.0792590756 0.061404189
## 18   3.317592e-02 0.0103463722 0.048021625
## 19   1.589556e-01 0.1799395445 0.096237246
## 20   1.408377e-01 0.0931545391 0.245686180
## 21   1.756743e-01 0.1574482868 0.225265384
## 22   2.116920e-01 0.2478364870 0.162312611
## 23   6.527499e-02 0.1081138217 0.093423294
## 24   1.149841e-02 0.0853623023 0.030955487
## 25   8.938052e-01 0.9084178328 0.854565443
## 26   4.059131e-01 0.3701290831 0.359056856
## 27   8.094656e-02 0.0409395360 0.077080912
## 28   2.997329e-01 0.2064116292 0.354683252
## 29   4.277500e-01 0.3892676486 0.413445640
## 30   6.328010e-02 0.1449265969 0.074829269
## 31   1.035875e-01 0.0844376985 0.126078291
## 32   2.226124e-01 0.2532684732 0.186630356
## 33   1.451777e-01 0.0632059567 0.084438885
## 34   2.272354e-01 0.1854849754 0.260480387
## 35   4.494578e-02 0.0357745530 0.026266556
## 36   5.776371e-02 0.1061008026 0.051006501
## 37   1.398842e-01 0.1670434042 0.078733070
## 38   1.926712e-01 0.2566905265 0.258753790
## 39   3.274543e-01 0.3999958582 0.316967988
## 40   1.499719e-01 0.2154855192 0.159913561
## 41   5.180670e-01 0.4493306292 0.506671724
## 42   1.163679e-01 0.2489306279 0.225849177
## 43   1.815340e-02 0.0639051299 0.012884675
## 44   8.956467e-02 0.0038841713 0.039986228
```

```
## 45   7.254878e-02 0.0333744545 0.041517785
## 46   1.893101e-01 0.0920322917 0.239108081
## 47   6.569660e-01 0.6230757911 0.620315468
## 48   3.969894e-01 0.3491542506 0.389207895
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## 52   2.777026e-01 0.3165669150 0.309049767
## 53   4.903749e-01 0.4572273133 0.476473061
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## 74   1.467646e-01 0.1205441721 0.099823455
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## 84   2.121580e-01 0.1903824452 0.196168952
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## 700  4.058736e-01 0.3790981037 0.345061375
## 701  6.671753e-01 0.6673787354 0.581085570
## 702  9.182237e-02 0.0879419583 0.095350019
## 703  1.867070e-01 0.1223963926 0.220480959
## 704  9.598256e-01 0.9083353197 1.034825082
## 705  1.643090e-01 0.1471391081 0.207906141
## 706  1.638800e-01 0.2979634702 0.073986168
## 707  3.117684e-01 0.3509982984 0.326760139
## 708  3.414664e-01 0.2669805430 0.232049240
## 709  1.719674e-01 0.1756404955 0.051209158
## 710  3.011864e-02 0.0293962605 0.032742477
## 711  1.147877e-01 0.1007911320 0.040456349
## 712  2.115082e-01 0.1407637952 0.134153080
## 713  1.939464e-01 0.2544267584 0.182434004
## 714  6.385052e-02 0.0563572725 0.023703660
## 715  6.274563e-02 0.0937588040 0.099367539
## 716  3.612061e-01 0.2765125915 0.362898151
## 717  1.927663e-02 0.0163167524 0.004019923
## 718  1.363851e-01 0.1055003968 0.192864143
## 719  3.285051e-01 0.3099088909 0.317248345
## 720  4.772361e-02 0.0202567446 0.025085664
## 721  5.334864e-02 0.0061613572 0.077468721
## 722  1.442507e-01 0.1909922794 0.096119758
## 723  1.155579e-02 0.0093358469 0.005211485
## 724  9.478158e-02 0.1026825640 0.102952558
## 725  3.546414e-01 0.3356307247 0.342614713
## 726  3.856053e-01 0.3463311383 0.413188152
## 727  5.659058e-03 0.0369165332 0.011870922
## 728  6.296781e-02 0.0668160641 0.165054366
## 729  7.182414e-02 0.1288721159 0.111470891
## 730  2.002374e-01 0.2363716904 0.154283873
## 731  6.100037e-02 0.0983004516 0.133393268
## 732  2.967522e-01 0.3449395233 0.351817963
## 733  5.201345e-03 0.0578994863 0.003739281
## 734  2.914366e-01 0.3028445935 0.299363926
## 735  4.730224e-01 0.4186389041 0.484620605
## 736  2.560202e-01 0.3130735282 0.225136719
## 737  8.313701e-02 0.1531237554 0.023860153
## 738  2.833302e-01 0.2720487961 0.219707836
## 739  1.257378e-01 0.1654850593 0.143292154
## 740  2.311879e-02 0.0266463081 0.052950416
## 741  1.193907e-01 0.0825654538 0.042214061
## 742  5.286967e-02 0.0139540919 0.080439560
## 743  2.399862e-01 0.2258499639 0.253579237
## 744  3.409786e-01 0.2736844228 0.385926607
## 745  3.278133e-01 0.3308832960 0.288714651
## 746  2.466372e-01 0.1761762393 0.217919488
```

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## 747  1.711709e-01 0.1117356349 0.133829937
## 748  5.608316e-01 0.5272609351 0.562417995
## 749  2.276794e-01 0.3258656043 0.285131674
## 750  1.860856e-01 0.1243729387 0.153616697
## 751  3.177329e-01 0.3323645442 0.326240384
## 752  5.698828e-02 0.0658918797 0.037666850
## 753  1.589009e-01 0.0864810110 0.192146837
## 754  1.589849e-02 0.0113974159 0.019753480
## 755  3.830468e-01 0.3884230460 0.407155702
## 756  3.074636e-01 0.2264145793 0.360877656
## 757  2.915027e-01 0.2841946477 0.303808692
## 758  2.717311e-01 0.3580564294 0.162599785
## 759  1.494464e-01 0.2396782764 0.119509274
## 760  4.387875e-01 0.4574098985 0.448244939
## 761  3.944045e-01 0.3114960217 0.357004793
## 762  4.120675e-02 0.1263979013 0.138231650
## 763  1.982544e-01 0.0884620330 0.251200946
## 764  1.498870e-02 0.0598036470 0.099617843
## 765  7.063712e-01 0.6390595545 0.682589272
## 766  1.035823e-01 0.1828697090 0.145752994
## 767  1.750645e-01 0.3185778243 0.124913328
## 768  2.827545e-01 0.3177089980 0.248776365
## 769  2.017647e-01 0.3144863543 0.181349121
## 770  5.419017e-02 0.0074904869 0.037762726
## 771  4.168680e-01 0.2805012358 0.429171865
## 772  1.122847e-01 0.0618327039 0.148380146
## 773  6.853682e-02 0.0460697592 0.192207091
## 774  2.193049e-01 0.1684947195 0.356286970
## 775  1.980530e-01 0.1835153200 0.191227011
## 776  2.107372e-01 0.3351923858 0.147557397
## 777  2.278495e-01 0.1841711535 0.290334737
## 778  4.494556e-02 0.1415918484 0.022709765
## 779  1.586473e-01 0.0935376711 0.216359735
## 780  7.706545e-02 0.0872387403 0.057772066
## 781  7.875790e-02 0.0493644515 0.131288507
## 782  6.711814e-02 0.1053139437 0.052985565
## 783  2.885668e-01 0.2193274811 0.332663519
## 784  3.361253e-01 0.2651516289 0.363567486
## 785  1.774581e-01 0.1361722759 0.159064187
## 786  1.705087e-01 0.0699291311 0.159654641
## 787  2.966776e-01 0.3243344216 0.258380291
## 788  1.595269e-01 0.2292875026 0.132821036
## 789  5.679881e-02 0.0428741832 0.024449847
## 790  1.598756e-02 0.0292976599 0.010411959
## 791  1.684255e-02 0.0107903763 0.008485324
## 792  4.949508e-02 0.1030574574 0.033265575
## 793  1.425539e-01 0.0984366814 0.168023979
## 794  2.621815e-02 0.0049679174 0.077506933
## 795  3.769368e-02 0.0508443350 0.109439976
## 796  2.263202e-01 0.2014790158 0.314461714
## 797  2.732487e-01 0.2073479530 0.292412532
## 798  3.669298e-01 0.3350933504 0.374427905
## 799  3.013520e-01 0.2876801486 0.303487496
## 800  4.614017e-01 0.3370561741 0.438938530
```

```
## 801  1.990641e-02 0.0579532575 0.035823123
## 802  3.627454e-01 0.4092806866 0.331623844
## 803  5.452230e-01 0.4519193109 0.523911268
## 804  5.076956e-02 0.0496399114 0.042906736
## 805  2.200986e-01 0.1796547997 0.279799753
## 806  3.335454e-01 0.2922095887 0.363609998
## 807  1.609192e-01 0.2226974630 0.245740625
## 808  2.754360e-01 0.3224019970 0.313572173
## 809  4.592632e-02 0.0301564556 0.011406085
## 810  6.068730e-02 0.1364926324 0.040529897
## 811  1.459635e-01 0.2463656768 0.177071001
## 812  1.473358e-02 0.0749281391 0.079044808
## 813  2.565660e-01 0.3231039006 0.214031437
## 814  1.546226e-01 0.2206292172 0.178899240
## 815  2.281420e-01 0.2301869509 0.193262348
## 816  3.947862e-01 0.3844919299 0.356210315
## 817  3.653947e-02 0.0705881211 0.061086009
## 818  2.429111e-01 0.2367335245 0.264787471
## 819  5.081841e-03 0.1568610522 0.031308347
## 820  1.804191e-01 0.1102397801 0.169303415
## 821  2.739595e-01 0.2335934447 0.284861400
## 822  1.574887e-01 0.2545648432 0.178194706
## 823  3.280910e-02 0.0283527272 0.089257348
## 824  1.261239e-01 0.0698150171 0.175464079
## 825  1.528106e-01 0.0905653918 0.098473939
## 826  9.542370e-02 0.1282830627 0.150591735
## 827  1.358634e-02 0.0162877164 0.004930346
## 828  6.525353e-03 0.0327612248 0.022528856
## 829  6.618695e-01 0.7310419898 0.687402190
## 830  1.768465e-02 0.1084706860 0.009305726
## 831  1.178634e-01 0.1258394244 0.100352248
## 832  4.175911e-02 0.2211840544 0.006988659
## 833  1.577126e-01 0.1410071484 0.174214152
## 834  1.694897e-01 0.1592238733 0.200166782
## 835  9.413248e-02 0.0616960712 0.082019389
## 836  1.167769e-01 0.1719044956 0.099842534
## 837  2.228248e-01 0.2932154918 0.209002850
## 838  1.280012e-01 0.1262325866 0.158276981
## 839  2.528270e-02 0.0104284754 0.075920243
## 840  3.211627e-01 0.4098275710 0.316993240
## 841  2.829273e-01 0.3396907174 0.260675137
## 842  2.062059e-01 0.1997915330 0.238888561
## 843  3.364225e-01 0.2888074009 0.340042382
## 844  1.115440e-01 0.1023254358 0.108928395
## 845  2.092822e-01 0.2310440067 0.170399765
## 846  3.902376e-01 0.3224823164 0.462287988
## 847  9.791359e-02 0.0490084213 0.053365425
## 848  5.727057e-02 0.0010128486 0.083013709
## 849  3.451974e-01 0.3574481570 0.378292463
## 850  2.522095e-02 0.0447779484 0.052143407
## 851  2.495601e-01 0.2372792567 0.295207272
## 852  1.164191e-01 0.1594390112 0.134664864
## 853  3.624563e-01 0.3012817283 0.327258931
## 854  2.118715e-01 0.2988098104 0.200389857
```

```
## 855  5.024318e-02 0.0782331693 0.034901032
## 856  1.293365e-01 0.2264003516 0.187523833
## 857  3.905591e-01 0.3854858104 0.427975639
## 858  1.874480e-01 0.2250856965 0.240786719
## 859  3.705691e-01 0.3693920989 0.341557199
## 860  3.377839e-02 0.0337074496 0.076690851
## 861  2.366535e-01 0.1517932924 0.218624037
## 862  9.955761e-02 0.2730078586 0.021751420
## 863  4.109391e-01 0.4706311282 0.443574892
## 864  1.183272e-02 0.1029320701 0.103623705
## 865  8.419560e-02 0.0382708691 0.181287339
## 866  1.267780e-01 0.0958990468 0.056785627
## 867  1.361488e-01 0.1614081259 0.082215053
## 868  1.294380e-01 0.1392808754 0.053288754
## 869  3.447901e-01 0.4124319593 0.232869686
## 870  3.062431e-02 0.0566833942 0.025495362
## 871  2.813385e-01 0.2678270041 0.299279158
## 872  2.042556e-01 0.1396194698 0.233805026
## 873  2.522824e-01 0.1412106316 0.296430045
## 874  2.451699e-01 0.2256612262 0.211148325
## 875  3.432360e-01 0.3464718152 0.400271092
## 876  1.930034e-01 0.0949352322 0.170632239
## 877  3.652745e-01 0.2655147621 0.310522185
## 878  8.223025e-01 0.7810913305 0.816596377
## 879  1.214346e-01 0.0521979034 0.070838062
## 880  5.036574e-01 0.4914569206 0.551067983
## 881  2.037301e-01 0.2518426732 0.194665966
## 882  2.028469e-01 0.2299428547 0.172703149
## 883  4.334314e-01 0.3198171677 0.256244199
## 884  2.176398e-01 0.2883407086 0.156916692
## 885  1.408695e-01 0.1610025818 0.128855645
## 886  1.070501e-01 0.0612631501 0.029232267
## 887  2.524927e-02 0.0593213158 0.032322579
## 888  3.368234e-01 0.3672940617 0.291200406
## 889  9.619372e-02 0.1066515627 0.041796526
## 890  4.455837e-01 0.3869159774 0.394537161
## 891  2.702040e-01 0.1821267336 0.301622610
## 892  2.095881e-01 0.1800879179 0.181868052
## 893  2.558584e-01 0.2122675657 0.248487341
## 894  4.638015e-02 0.0280139136 0.006792678
## 895  4.574854e-01 0.5124382496 0.478919827
## 896  5.362383e-01 0.5232201426 0.491784243
## 897  2.013071e-01 0.1281677165 0.172266938
## 898  2.194088e-01 0.1224556756 0.172889988
## 899  1.747461e-01 0.1688935969 0.152930742
## 900  1.626313e-01 0.1990881654 0.104233102
## 901  3.341242e-01 0.4249762207 0.282421404
## 902  6.970261e-01 0.6132029344 0.710181224
## 903  8.536995e-02 0.0991648496 0.092393871
## 904  7.278635e-02 0.0494353088 0.102850498
## 905  3.757498e-02 0.0020308222 0.030856076
## 906  2.511044e-01 0.0869679640 0.292982985
## 907  2.196047e-01 0.2043124394 0.239344751
## 908  3.097013e-01 0.3407332659 0.302065041
```

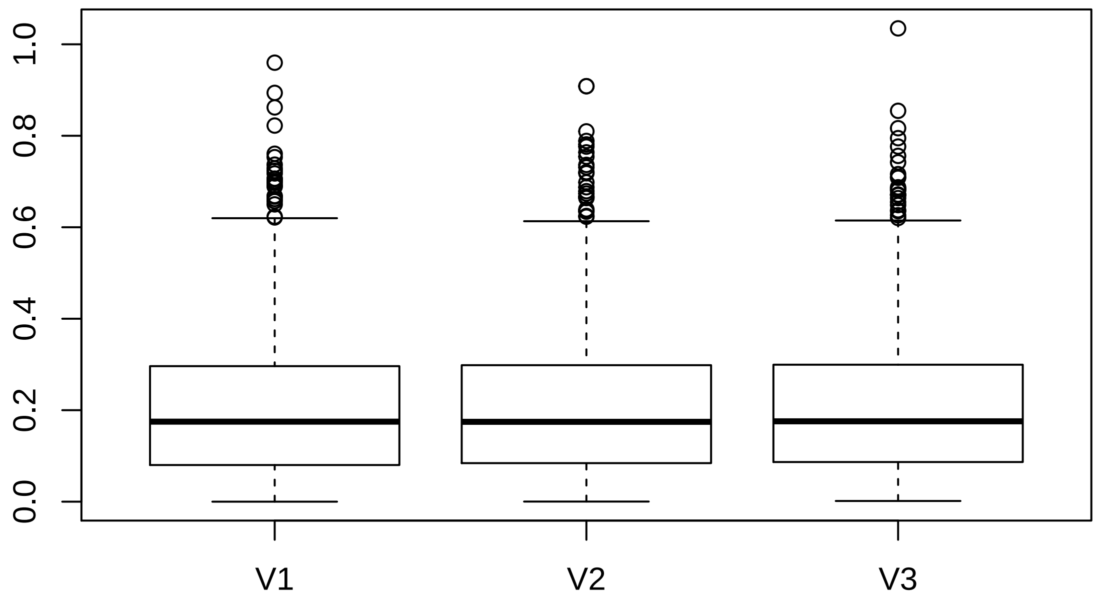
```
## 909  3.041959e-01 0.2832428374 0.270562989
## 910  1.203156e-01 0.0728261141 0.136346575
## 911  2.551203e-01 0.2393140822 0.275796063
## 912  3.490950e-01 0.3254896047 0.335780548
## 913  5.259731e-03 0.0448253636 0.008367020
## 914  3.446388e-01 0.3473505245 0.361965477
## 915  1.532940e-01 0.1899710616 0.110242427
## 916  2.438738e-01 0.3229341860 0.222272283
## 917  1.453938e-01 0.1015586975 0.086805092
## 918  3.415559e-01 0.3159102700 0.310875887
## 919  3.323781e-01 0.3501928929 0.364914437
## 920  1.789681e-01 0.2409396004 0.205817414
## 921  8.404515e-02 0.0486982349 0.166719581
## 922  1.913768e-01 0.1715032735 0.180484881
## 923  2.592755e-01 0.2273327853 0.222962662
## 924  2.694034e-01 0.2571861799 0.252186531
## 925  5.112714e-02 0.0646433015 0.009354752
## 926  2.206322e-01 0.2048572233 0.228618674
## 927  1.469733e-01 0.1667283582 0.057534654
## 928  3.809866e-01 0.3122744110 0.328747250
## 929  4.641562e-01 0.4188500755 0.506938877
## 930  4.577662e-01 0.4649331832 0.378439982
## 931  3.728071e-02 0.0660935202 0.010009909
## 932  1.070998e-01 0.0845204727 0.005085603
## 933  6.971607e-02 0.0932137020 0.007163795
## 934  1.257764e-01 0.0937740840 0.099235213
## 935  1.660986e-02 0.0069573368 0.061075792
## 936  7.437366e-02 0.1090200321 0.069745513
## 937  1.323060e-01 0.0898987864 0.162652701
## 938  2.881968e-01 0.2659155298 0.264409049
## 939  4.341331e-02 0.0606097905 0.004658902
## 940  2.955039e-01 0.3741949289 0.393537899
## 941  3.031813e-01 0.2440726440 0.265734583
## 942  4.754307e-01 0.4688029904 0.521298477
## 943  1.259863e-01 0.1471379090 0.111281940
## 944  2.536472e-01 0.2129997280 0.321154108
## 945  1.077272e-01 0.1020943979 0.105974931
## 946  6.811087e-02 0.1631510609 0.096581856
## 947  9.422216e-02 0.0784811454 0.053808152
## 948  2.352760e-01 0.1870541071 0.260417232
## 949  1.022092e-02 0.0312927328 0.080205694
## 950  1.049770e-01 0.1177569482 0.071395936
## 951  8.718566e-02 0.1257450883 0.098043989
## 952  5.189768e-02 0.0125644281 0.007104151
## 953  6.500023e-01 0.7205461883 0.614778813
## 954  1.755857e-01 0.2234571228 0.283559899
## 955  1.666704e-01 0.2228477412 0.178715147
## 956  6.573467e-02 0.0785988919 0.077173391
## 957  7.342671e-02 0.1148052200 0.067760675
## 958  1.122874e-01 0.1076317715 0.091554692
## 959  2.041937e-01 0.1599640568 0.248528916
## 960  3.666736e-01 0.4335652511 0.337830596
## 961  3.710909e-01 0.3611339787 0.354810967
## 962  2.223217e-01 0.1204890474 0.264438775
```

```
## 963  4.987909e-01 0.4516355857 0.504268191
## 964  6.111737e-01 0.5450048707 0.708125420
## 965  6.214775e-01 0.6362910412 0.582031056
## 966  6.058671e-01 0.5220636875 0.599715775
## 967  6.535725e-02 0.0934880284 0.125781448
## 968  1.766915e-01 0.1366687964 0.160845095
## 969  3.191399e-01 0.2578018101 0.353850364
## 970  3.075008e-01 0.2602895631 0.338410434
## 971  5.586699e-02 0.1140904178 0.089587908
## 972  1.844659e-01 0.1295457891 0.253499520
## 973  2.184332e-01 0.1429711522 0.242556579
## 974  1.572808e-01 0.0490239954 0.177886463
## 975  8.668253e-02 0.2164237802 0.085419748
## 976  2.115731e-01 0.1191756399 0.222983547
## 977  5.659712e-01 0.4970833771 0.605955976
## 978  6.027432e-01 0.5993710786 0.635152412
## 979  1.091472e-01 0.1450389034 0.153797572
## 980  6.834153e-03 0.0381199251 0.011813744
## 981  4.937408e-01 0.4492947938 0.436127633
## 982  1.852040e-01 0.1968018539 0.122646850
## 983  4.215814e-01 0.5195511643 0.392096629
## 984  2.447772e-03 0.0430285229 0.054388340
## 985  2.024483e-01 0.0966201120 0.244195510
## 986  6.948827e-02 0.0270996619 0.031771655
## 987  2.198382e-01 0.2048760004 0.214246774
## 988  2.754424e-02 0.0205040436 0.022884091
## 989  7.179465e-02 0.0250848071 0.112771578
## 990  2.038407e-01 0.1988197651 0.205514097
## 991  6.194117e-02 0.1063620423 0.025492035
## 992  2.238728e-01 0.1427071390 0.217597354
## 993  1.368566e-01 0.1686300241 0.119085874
## 994  1.522401e-01 0.1394443451 0.178413201
## 995  5.028059e-01 0.5237207490 0.476671399
## 996  5.730950e-02 0.0448750685 0.130494184
## 997  4.192571e-02 0.0690635454 0.083950996
## 998  4.050812e-02 0.0070896167 0.031416356
## 999  1.312266e-02 0.0097286421 0.005407245
## 1000 4.237990e-02 0.0993582587 0.063242597
```

```r
colMeans(rmse_vals)
```

```
##        V1        V2        V3
## 0.2083847 0.2081181 0.2116004
```

```r
boxplot(rmse_vals)
```




```
#greenlm4 seems to be the model with the lowest RMSE
```

The models I proceeded with were models 2, 4, and 6. I wanted to test the predictive power of these models to see which would be considered the "best." I did this by separating the data into a test and train set. I then used the models built with the train set to predict rent from the test set. I ran code to do this 1000 times for each model and give the average of the RMSE's. As a result, I found that model 4 had the lowest RMSE. This shows that it has the smallest error of predicting rent. But it is worth considering, that the RMSE for all three selected models are similar and all quite low.

```
greenlm6 <- lm(Rent~green_rating+cluster_rent+Class+net+cluster+size, data = greendata)
summary(greenlm6)
```

```
##
## Call:
## lm(formula = Rent ~ green_rating + cluster_rent + Class + net +
##     cluster + size, data = greendata)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -52.877  -3.658  -0.455   2.388 175.980
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -1.542e+00  4.140e-01  -3.725 0.000196 ***
## green_rating  8.171e-01  3.937e-01   2.075 0.037978 *
## cluster_rent  1.062e+00  1.045e-02 101.613  < 2e-16 ***
## ClassB       -1.851e+00  2.619e-01  -7.067 1.72e-12 ***
```

```
## ClassC       -3.325e+00  3.675e-01  -9.047  < 2e-16 ***
## net          -2.524e+00  5.890e-01  -4.284 1.86e-05 ***
## cluster       1.002e-03  2.763e-04   3.626 0.000289 ***
## size          6.375e-06  4.087e-07  15.598  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.464 on 7812 degrees of freedom
## Multiple R-squared:  0.6077, Adjusted R-squared:  0.6074
## F-statistic:  1729 on 7 and 7812 DF,  p-value: < 2.2e-16
```

Based off of this I would select model 6 as the "best" model. Model 6 had the highest R^2 which shows that it does the best explaining the variance in Rent compared to the other models. It also had a low RMSE when predicting rent which shows its capability to predict. Using this model, we can see that the average rent increases by $8.17 when the building is green (controlling for the significant variables of: cluster_rent, class, net, cluster, and size).