TABLE 3.1
Models derived in this chapter. Type refers to (1) recursion equation in discrete time, (2) difference equation in discrete time, and (3) differential equation in continuous time.

Model	Type	Equation	
Exponential growth	1	$n(t+1) = R\,n(t)$	(3.1b)
	2	$\Delta n = (R-1)\,n(t)$	(3.2)
	3	$\frac{dn}{dt} = r\,n(t)$	(3.3)
Logistic growth	1	$n(t+1) = n(t) + r\,n(t)\left(1 - \frac{n(t)}{K}\right)$	(3.5a)
	2	$\Delta n = r\,n(t)\left(1 - \frac{n(t)}{K}\right)$	(3.5b)
	3	$\frac{dn}{dt} = r\,n(t)\left(1 - \frac{n(t)}{K}\right)$	(3.5c)
Haploid selection	1	$p(t+1) = \frac{W_A\,p(t)}{W_A\,p(t) + W_a\,q(t)}$	(3.8c)
	2	$\Delta p = \frac{(W_A - W_a)\,p(t)\,q(t)}{W_A\,p(t) + W_a\,q(t)}$	(3.9)
	3	$\frac{dp}{dt} = s\,p(t)\,q(t)$	(3.11b)
Diploid selection	1	$p(t+1) = p(t)^2\frac{W_{AA}}{\overline{W}} + p(t)\,q(t)\frac{W_{Aa}}{\overline{W}}$	(3.13a)
Competition equations	1	$n_1(t+1) = n_1(t) + r_1\,n_1(t)\left(1 - \frac{n_1(t) + \alpha_{12}\,n_2(t)}{K_1}\right)$ $n_2(t+1) = n_2(t) + r_2\,n_2(t)\left(1 - \frac{n_2(t) + \alpha_{21}\,n_1(t)}{K_2}\right)$	(3.14)
	3	$\frac{dn_1}{dt} = r_1\,n_1(t)\left(1 - \frac{n_1(t) + \alpha_{12}\,n_2(t)}{K_1}\right)$ $\frac{dn_2}{dt} = r_2\,n_2(t)\left(1 - \frac{n_2(t) + \alpha_{21}\,n_1(t)}{K_2}\right)$	(3.15)
Consumer-resource equations	3	$\frac{dn_1}{dt} = f(n_1) - g(n_1, n_2)$ $\frac{dn_2}{dt} = \varepsilon\,g(n_1, n_2) - h(n_2)$	(3.16)
SIR equations	3	$\frac{dS}{dt} = b - d\,S(t) - a\,c\,S(t)\,I(t) + \sigma\,R(t)$ $\frac{dI}{dt} = a\,c\,S(t)\,I(t) - \delta\,I(t) - \rho\,I(t)$ $\frac{dR}{dt} = \rho\,I(t) - \sigma\,R(t) - d\,R(t)$	(3.19)

might involve inflow and outflow of abiotic resources (as in a chemostat; see Problem 2.4) or immigration, emigration, births, and deaths of biotic resources. Furthermore, the rate of change of the resource might depend on the current density of resources, or it might not. Table 3.3 lists some possible choices for $f(n_1)$. To simplify the notation, we drop the (t) notation, but we must remember that n_1 and n_2 are functions of time.

The term $g(n_1,n_2)$ represents the rate of consumption of the resource by the consumer. In the simplest case, a "mass-action" rate of consumption is assumed, as in the flu model of Chapter 2. That is, the total rate of contact between consumers and resources within the community is assumed to equal $c\, n_1\, n_2$. At each contact, the probability that the consumer successfully uses the resource is a. Hence, $g(n_1,n_2) = a\, c\, n_1\, n_2$, which is known as a "linear" or "type-I" consumption rate (sometimes referred to as a type-I functional response). Table 3.3 lists other common choices for $g(n_1,n_2)$. After having gathered a resource, the consumer converts it into the biomass needed for offspring production. In the flow diagram and in equation (3.16), the conversion factor by which resource units are turned into consumers is given by ε. For example, one prey might represent only a fraction ε of the resources needed to produce one predator.

Finally, $h(n_2)$ represents the rate at which the number of consumers changes in the absence of resources (i.e., assuming $n_1 = 0$). Typically, it is assumed that

TABLE 3.3
Consumer-resource models. Examples of functions that can be used in the consumer-resource model (3.16), where n_1 refers to the level of resources (e.g., number of prey) and n_2 refers to the level of consumers (e.g., number of predators).

Function	Description
$f(n_1) = \theta$	Inflow of resources at a constant rate
$f(n_1) = -\psi$	Outflow of resources at a constant rate
$f(n_1) = r\, n_1$	Constant per capita growth of resource species
$f(n_1) = rn_1\left(1 - \dfrac{n_1}{K}\right)$	Per capita growth of resource species declines linearly with resource level (logistic)
$f(n_1) = r\, n_1\, e^{-\alpha n_1}$	Per capita growth of resource species declines exponentially with resource level
$g(n_1, n_2) = a\, c\, n_1\, n_2$	Linear (type I) rate of resource consumption
$g(n_1,n_2) = \dfrac{a\, c\, n_1}{b + n_1} n_2$	Saturating (type II) rate of resource consumption
$g(n_1,n_2) = \dfrac{a\, c\, n_1^k}{b + n_1^k} n_2$	Generalized (type III) rate of resource consumption
$h(n_2) = \delta\, n_2$	Constant per capita death rate of consumer
$h(n_2) = (\delta\, n_2)\, n_2$	Per capita death rate of consumer increases linearly with consumer population size