Passivity-Based Asynchronous Sliding Mode Control for Delayed Singular Markovian Jump Systems

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***Abstract*—This paper is concerned with the problem of passivity-based asynchronous sliding mode control for a class of uncertain singular Markovian jump systems with time delay and nonlinear perturbations. The asynchronous control strategy is employed due to the nonsynchronization between the controller and the system modes. Considering the singularity of the system, a novel integral-type sliding surface is constructed, and then the asynchronous sliding controller is synthesized to ensure that the sliding mode dynamics satisfy the reaching condition. Sufficient conditions are presented such that the corresponding sliding mode dynamics are admissible and robustly passive. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed technique.**

***Index Terms*—Asynchronous sliding mode control (SMC), passivity, singular Markovian jump systems (MJSs).**

I. Introduction

Markovian jump systems (MJSs) have been attracted extensively attention since they can give better description for various systems with stochastic abrupt structural changes such as the failure or maintenance of the components, the switching of multiple modes, and the mutation of the nonlinear object. Especially, singular MJSs are widely used to describe the economic systems, circuits systems, and robot systems. Some resent works on the MJSs and singular MJSs can be found in [1]–[11] and the references therein.

However, it should be pointed out that most of the aforementioned works reported in the literature assumed that the information of system modes are fully accessible for the controller, which means that the controller mode should be synchronized to the system mode. Unfortunately, this assumption is difficult to be satisfied in practical applications due to the packet dropout and stochastic perturbation, which may bring asynchronization between the modes of controller and system. Hence, the importance of asynchronous control/filtering for MJSs has already begun receiving more attentions [12]. In addition, passivity theory provides an important framework for the control of complex dynamic systems using an asynchronous description.

Since sliding mode control (SMC) has the advantage of strong robustness to uncertainty and disturbance, it has been widely applied to practical engineering. Consequently, SMC has also been extensively investigated, see, for example, [14]–[18]. However, due to the special structure of the singular MJSs, one cannot obtain the regular form through model transformation. Thus, an appropriate sliding surface function need to be built to avoid some difficulties caused by singular matrix in deriving the sliding mode dynamics. On the other hand, the work become more difficult and complex due to the asynchronization phenomenon between the modes of the controller and the system. Since the singular MJSs with double stochastic process, a novel SMC method, that is asynchronous SMC, needs to be developed systematically to describe the asynchronization phenomenon better and mitigate the drawback of the existing methods, which motivates the work in this paper.

The contributions and significance of this paper lie in the following points.

1) In this paper, an asynchronous controller is designed under the framework of hidden Markov model, which is characterized by a conditional probability matrix between the system modes and the controller.
2) Due to the system we considered is singular one, a set of mode-dependent sliding surfaces is constructed and a novel asynchronous SMC law is synthesized to ensure the reachability of the asynchronous sliding mode dynamics.
3) The passivity theory has been applied to eliminate the nonlinear perturbations and model uncertainty to ensure the stability of the sliding mode dynamics.

Notations: The notations used throughout this paper are fairly standard. $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the minimum and maximum eigenvalues of symmetric matrix P, respectively. $\varphi_{n,d} \triangleq C([-d,0],\mathbb{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-d,0]$ into $\mathbb{R}^n$. $\ell_2[0,\infty)$ represents the space of square integrable functions on $[0,\infty)$. $(\Omega,\mathbb{F},\Pr)$ is a probability space, where Ω is the sample space, $\mathbb{F}$ is the σ-algebra of subsets of the sample space, and $\Pr$ is the probability measure on $\mathbb{F}$.

II. System Description and Preliminaries

Consider a class of nonlinear singular delayed Markovian jump system as follows:

$$\begin{aligned} E\dot{x}_t &= \mathbf{A}(r_t)x_t + \mathbf{A}_d(r_t)x_{t-d} + B_\omega(r_t)\omega_t \\ &\quad + B(r_t)[u_t + f(x_t,t)] \\ z_t &= \mathbf{C}(r_t)x_t + \mathbf{C}_d(r_t)x_{t-d} + D_w(r_t)\omega_t \\ x_t &= \varphi_t,\ t\in[-d,\ 0] \end{aligned} \tag{1}$$

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where x_t and $x_{t-d} \in \mathbb{R}^n$ are state and delayed state vector, respectively; $u_t \in \mathbb{R}^m$ and $z_t \in \mathbb{R}^q$ are the control input and controlled output, respectively; $\omega_t \in \mathbb{R}^p$ is the disturbance input which belongs to $\ell_2[0,\infty)$; $\varphi_t \in \varphi_{n,d}$ is a compatible vector-valued initial function; and the nonlinear perturbation function $f(x_t,t) \in \mathbb{R}^m$ is unknown but satisfying

$$\| f(x_t,t) \| \leq \eta \| x_t \| \tag{2}$$

where $\eta > 0$ is a known constant.

The process $\{r_t, t \geq 0\}$ is a Markov process taking values in a finite set $\mathcal{N} = \{1,2,\ldots,N\}$ with generator matrix $\Pi = \{\pi_{ij}\}$, $i,j \in \mathcal{N}$. The transition probability from mode i at time t to mode j at time $t+\delta$ is given by

$$\Pr\{r_{t+\delta} = j | r_t = i\} = \begin{cases} \pi_{ij}\delta + o(\delta), & \text{if } j \neq i \\ 1 + \pi_{ii}\delta + o(\delta), & \text{if } j = i \end{cases} \tag{3}$$

where $\delta > 0$ and $\lim_{\delta \to 0} o(\delta)/\delta = 0$; $\pi_{ij} > 0, j \neq i$ and $\pi_{ii} = -\sum_{j\neq i} \pi_{ij}$ for all $i \in \mathcal{N}$.

Then, for $r_t = i$, system (1) can be described as

$$\begin{aligned} E\dot{x}_t &= \mathbf{A}_i x_t + \mathbf{A}_{di} x_{t-d} + B_{\omega i}\omega_t + B_i[u_t + f(x_t,t)] \\ z_t &= \mathbf{C}_i x_t + \mathbf{C}_{di} x_{t-d} + D_{wi}\omega_t \\ x_t &= \varphi_t,\ t \in [-d,\ 0] \end{aligned} \tag{4}$$

where $E \in \mathbb{R}^{n\times n}$ may be singular, that is, $\operatorname{rank} E = r \leq n$. $\mathbf{A}_i = A_i + \Delta A_i$, $\mathbf{A}_{di} = A_{di} + \Delta A_{di}$, $\mathbf{C}_i = C_i + \Delta C_i$, and $\mathbf{C}_{di} = C_{di} + \Delta C_{di}$. Here, $A_i, A_{di}, B_{\omega i}, B_i, C_i, C_{di}$, and D_{wi} are known real constant matrices with appropriate dimensions, and $\Delta A_i, \Delta A_{di}, \Delta C_i$, and ΔC_{di} are unknown matrices representing system parameter uncertainties, which are assumed to be norm bounded, i.e.,

$$\begin{bmatrix} \Delta A_i & \Delta A_{di} \\ \Delta C_i & \Delta C_{di} \end{bmatrix} = \begin{bmatrix} M_{1i} \\ M_{2i} \end{bmatrix} F_t \begin{bmatrix} N_{1i} & N_{2i} \end{bmatrix} \tag{5}$$

where M_{1i}, M_{2i}, N_{1i}, and N_{2i} are known constant matrices, and F_t is an unknown time-varying matrix satisfying $F_t F_t^T \leq I$.

The following definition is introduced for the unforced linear delayed singular MJSs. For the definition of admissibility for the singular systems, refer [19] for details.

Definition 1: The delayed singular system (1) with $u_t = 0$ is said to be robustly passive if for all $t^* > 0$ and under zero initial conditions, there exists a $\gamma > 0$ such that

$$\mathcal{E}\left\{2\int_0^{t^*} \omega_s^T z_s \mathrm{d}s\right\} \geq -\gamma \int_0^{t^*} \omega_s^T \omega_s \mathrm{d}s \quad \forall t^* > 0 \tag{6}$$

where $\mathcal{E}$ stands for the expectation operator.

The purpose of this paper is to design an asynchronous sliding mode controller for system (1) such that the closed-loop system is admissible and passive with passivity rate γ.

III. Main Results

A. Sliding Surface Synthesis

In this paper, an asynchronous integral-type sliding surface function is synthesized by taking the singular matrix E and the asynchronization between the controller mode and system mode into account. First, a stochastic variable σ_t will be introduced to represent the mode of controller, which takes values in a finite set $\mathcal{M} = \{1,2,\ldots,M\}$ and depends on r_t by the following conditional probability:

$$\Pr\{\sigma_t = \phi | r_t = i\} = \mu_{i\phi} \tag{7}$$

where $0 \leq \mu_{i\phi} \leq 1$ for all $i \in \mathcal{N}$ and $\phi \in \mathcal{M}$. $\sum_{\phi=1}^{M} \mu_{i\phi} = 1$ for all $i \in \mathcal{N}$.

Remark 1: Since the controller cannot fully obtained synchronously by the system modes in practice, thus, we use a transition probability (7) to connect the system modes and the controller, then the closed-loop system can be seen as a doubly stochastic process. The hidden Markov model as in [13] is introduced to characterize the above asynchronous phenomenon.

In this paper, we designed an asynchronous sliding surface function as

$$s_t = G_i E x_t - \int_0^t G_i(A_i + B_i K(\sigma_t)) x_s ds \tag{8}$$

where $G_i \in \mathbb{R}^{n\times n}$ is real matrix such that $G_i B_i$ is nonsingular.

Remark 2: The integral-type sliding surface designed in (8) is to restrict the dynamics onto the sliding surface and different from other sliding surface function, such as in [14]. One of our innovations is that a novel sliding surface describing the asynchronous phenomenon between the system modes and sliding controller modes is designed, which is better fit to the application since the asynchronous phenomenon is quite common in actual systems.

Consider system (4), the solution of Ex_t is given by

$$\begin{aligned} Ex_t &= Ex_0 + \int_0^t \Big(\mathbf{A}_i x_s + \mathbf{A}_{di} x_{s-d}\Big)\mathrm{d}s \\ &\quad + \int_0^t \Big(B_{\omega i}\omega_s + B_i[u_s + f(x_s,s)]\Big)\mathrm{d}s \end{aligned} \tag{9}$$

this together with (8), one obtain that

$$\begin{aligned} s_t &= G_i E x_0 + \int_0^t G_i(\Delta A_i - B_i K_\phi) x_s \mathrm{d}s \\ &\quad + \int_0^t G_i(A_{di} + \Delta A_{di}) x_{s-d}\mathrm{d}s + \int_0^t G_i B_{\omega i}\omega_s \mathrm{d}s \\ &\quad + \int_0^t G_i B_i[u_s + f(x_s,s)]\mathrm{d}s. \end{aligned} \tag{10}$$

According to the SMC theory, once the trajectories of the system reach onto the sliding surface, it satisfies that $s_t = 0$ and $\dot{s}_t = 0$. Here, we assume that $G_i B_i$ is nonsingular. Thus, by $\dot{s}_t = 0$, we can get the equivalent control u_{eq}

$$\begin{aligned} u_{\text{eq}} = &-(G_i B_i)^{-1} G_i(\Delta A_i - B_i K_\phi) x_t \\ &-(G_i B_i)^{-1} G_i(A_{di} + \Delta A_{di}) x_{t-d} \\ &-(G_i B_i)^{-1} G_i B_{\omega i}\omega_t - f(x_t,t). \end{aligned} \tag{11}$$

By substituting the controller (11) into the first formula in system (4), the sliding mode dynamics can be obtained as

$$\begin{aligned} E\dot{x}_t = &\Big(A_i + B_i K_\phi + [I - B_i(G_i B_i)^{-1} G_i]\Delta A_i\Big) x_t \\ &+ [I - B_i(G_i B_i)^{-1} G_i](A_{di} + \Delta A_{di}) x_{t-d} \\ &+ [I - B_i(G_i B_i)^{-1} G_i] B_{\omega i}\omega_t. \end{aligned} \tag{12}$$

Simply, we define $\bar{G}_i = I - B_i(G_i B_i)^{-1} G_i$, $\bar{A}_{i\phi} = A_i + B_i K_\phi$, $\bar{A}_{di} = \bar{G}_i A_{di}$, $\triangle\bar{A}_i = \bar{G}_i \triangle A_i$, $\triangle\bar{A}_{di} = \bar{G}_i \triangle A_{di}$, $\bar{B}_{\omega i} = \bar{G}_i B_{\omega i}$, $\tilde{A}_{i\phi} = \bar{A}_{i\phi} + \triangle\bar{A}_i$, and $\tilde{A}_{di} = \bar{A}_{di} + \triangle\bar{A}_{di}$; therefore, the dynamics of the sliding mode in (12) can be given as the first formula in the following equation:

$$\begin{aligned} E\dot{x}_t &= \tilde{A}_{i\phi} x_t + \tilde{A}_{di} x_{t-d} + \bar{B}_{\omega i}\omega_t \\ z_t &= \mathbf{C}_i x_t + \mathbf{C}_{di} x_{t-d} + D_{wi}\omega_t \\ x_t &= \varphi_t,\ t \in [-d,\ 0] \end{aligned} \tag{13}$$

and the uncertainties $\Delta \bar{A}_i$ and $\Delta \bar{A}_{di}$ can be written as

$$\begin{bmatrix} \Delta \bar{A}_i & \Delta \bar{A}_{di} \end{bmatrix} = \bar{M}_{1i} F_t \begin{bmatrix} N_{1i} & N_{2i} \end{bmatrix}$$

where $\bar{M}_{1i} = \bar{G}_i M_{1i}$, and M_{1i}, N_{1i}, N_{2i}, F_t, $\bar{G}_i$ are defined previously.

B. Admissibility and Passivity Analysis

Theorem 1: Given a positive scalar γ and a sufficient small positive scalar ϵ, the sliding mode dynamics (12) is admissible and robustly passive, if there exist matrices $Q > 0, Q_i > 0, R > 0, P_i \triangleq \begin{bmatrix} P_{11i} & P_{12i} \\ 0 & P_{22i} \end{bmatrix}$, $W_i \triangleq \begin{bmatrix} W_{1i} & 0_{n\times(n-r)} \end{bmatrix}$ (with $0 < P_{11i} \in \mathbb{R}^{r\times r}$ and $W_{1i} \in \mathbb{R}^{n\times r}$), such that the following conditions hold for all $i \in \mathcal{N}$ and $\phi \in \mathcal{M}$:

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} < 0 \tag{14}$$

$$\sum_{j=1}^{N} \pi_{ij} Q_j \le Q \tag{15}$$

where

$$\Theta_{11} = \begin{bmatrix} \Psi_{11i\phi} & \Psi_{12i} & \Psi_{13i} & \Psi_{14i\phi} \\ * & -Q_i & -C_{di}^T & d\bar{A}_{di}^T \\ * & * & \Psi_{33i} & d\bar{B}_{\omega i}^T \\ * & * & * & -dR^{-1} \end{bmatrix}$$

$$\Theta_{12} = \begin{bmatrix} dW_i & P_i\bar{M}_{1i} & \epsilon N_{1i}^T \\ 0 & 0 & \epsilon N_{2i}^T \\ 0 & -M_{2i} & 0 \\ 0 & dR\bar{M}_{1i} & 0 \end{bmatrix}$$

$$\Theta_{22} = -\mathrm{diag}\{dR, \epsilon I, \epsilon I\}$$

$$\Psi_{12i} = P_i\bar{A}_{di} - W_i, \quad \Psi_{14i\phi} = \sum_{\phi=1}^{M} \mu_{i\phi} d\bar{A}_{i\phi}^T$$

$$\Psi_{11i\phi} = \sum_{\phi=1}^{M} \mu_{i\phi}(P_i\bar{A}_{i\phi} + \bar{A}_{i\phi}^T P_i^T) + \sum_{j=1}^{N} \pi_{ij} P_j E + W_i + W_i^T + Q_i + dQ$$

$$\Psi_{13i} = P_i\bar{B}_{\omega i} - C_i^T, \quad \Psi_{33i} = -\gamma I - D_{\omega i} - D_{\omega i}^T.$$

Proof: First, we will prove the regularity and the impulse-free condition for system (13). Without loss of generality, we assume that the matrix E and the state vector x_t in (13) have the following forms:

$$E = \begin{bmatrix} I_r & 0_{r\times(n-r)} \\ 0_{(n-r)\times r} & 0_{(n-r)} \end{bmatrix}, \quad x_t = \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}$$

where $x_{1t} \in \mathbb{R}^r$ and $x_{2t} \in \mathbb{R}^{(n-r)}$.

It can be found from (14) that

$$\Psi_{11i\phi} = \sum_{\phi=1}^{M} \mu_{i\phi}(P_i\bar{A}_{i\phi} + \bar{A}_{i\phi}^T P_i^T) + \sum_{j=1}^{N} \pi_{ij} P_j E + W_i + W_i^T + Q_i + dQ < 0 \tag{16}$$

define

$$\bar{A}_{i\phi} = \begin{bmatrix} \bar{A}_{11i\phi} & \bar{A}_{12i\phi} \\ \bar{A}_{21i\phi} & \bar{A}_{22i\phi} \end{bmatrix}, \text{ and } Q_i = \begin{bmatrix} Q_{11i} & Q_{12i} \\ * & Q_{22i} \end{bmatrix},$$

respectively. By substituting $\bar{A}_{i\phi}$, Q_i, and P_i into (16), it yields

$$\sum_{\phi=1}^{M} \mu_{i\phi}[P_{22i}\bar{A}_{22i\phi} + \bar{A}_{22i\phi}^T P_{22i}^T] + Q_{22i} < 0$$

which implies that $\bar{A}_{22i\phi}$ is nonsingular for each $i \in \mathcal{N}$ and $\phi \in \mathcal{M}$, thus system (13) is regular and impulse-free.

Next, choose the following Lyapunov function candidate:

$$V(x_t, r_t, t) = x_t^T P_i E x_t + \int_{t-d}^{t} x_s^T Q_i x_s \mathrm{d}s + \int_{-d}^{0}\int_{t+\theta}^{t} x_s^T Q x_s \mathrm{d}s\mathrm{d}\theta + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}_s^T E^T R E\dot{x}_s \mathrm{d}s\mathrm{d}\theta \tag{17}$$

for each $i \in \mathcal{N}$, where $x_t \triangleq x_\theta, \theta \in [t-2d, t]$. Let $\mathcal{L}$ be an infinitesimal operator. For each $i \in \mathcal{N}$ and $t \ge 0$, we have

$$\begin{aligned} \mathcal{L}V(x_t, r_t, t) = {} & \sum_{\phi=1}^{M} \mu_{i\phi} 2x_t^T P_i E\dot{x}_t + x_t^T\left(\sum_{j=1}^{N} \pi_{ij} P_j\right) E x_t \\ & + \int_{t-d}^{t} x_s^T \left(\sum_{j=1}^{N} \pi_{ij} Q_j\right) x_s \mathrm{d}s + x_t^T Q_i x_t \\ & - x_{t-d}^T Q_i x_{t-d} + d x_t^T Q x_t \\ & - \int_{t-d}^{t} x_s^T Q x_s \mathrm{d}s + d\dot{x}_t^T E^T R E\dot{x}_t \\ & - \int_{t-d}^{t} \dot{x}_s^T E^T R E\dot{x}_s \mathrm{d}s. \end{aligned} \tag{18}$$

In the following, we define

$$\Psi_t = \mathcal{L}V(x_t, r_t, t) - 2z_t^T\omega_t - \gamma\omega_t^T\omega_t$$

from Newton–Leibniz formula, we have

$$2x_t^T W_i \left[x_t - x_{t-d} - \int_{t-d}^{t} \dot{x}_s \mathrm{d}s\right] = 0. \tag{19}$$

from (15), it is obvious that

$$\int_{t-d}^{t} x_s^T \left(\sum_{j=1}^{N} \pi_{ij} Q_j\right) x_s \mathrm{d}s \le \int_{t-d}^{t} x_s^T Q x_s \mathrm{d}s. \tag{20}$$

by combing (19) and (20) and noting $W_i E = W_i$, we have

$$\Psi_t \le \eta_t^T \Omega \eta_t - \Gamma_t \tag{21}$$

where

$$\eta_t = \begin{bmatrix} x_t^T & x_{t-d}^T & \omega_t^T \end{bmatrix}^T$$

$$\Gamma_t = \int_{t-d}^{t} [W_i^T x_t + RE\dot{x}_s]^T R^{-1} [W_i^T x_t + RE\dot{x}_s] \mathrm{d}s$$

$$\Omega = \begin{bmatrix} \Omega_{11i\phi} & \Omega_{12i} & \Omega_{13i} \\ * & -Q_i & -\mathbf{C}_{di}^T \\ * & * & \Psi_{33i} \end{bmatrix} + d\begin{bmatrix} \tilde{A}_{i\phi}^T \\ \tilde{A}_{di}^T \\ \bar{B}_{\omega i}^T \end{bmatrix} R \begin{bmatrix} \tilde{A}_{i\phi}^T \\ \tilde{A}_{di}^T \\ \bar{B}_{\omega i}^T \end{bmatrix}^T$$

$$\Omega_{11i\phi} = \sum_{\phi=1}^{M} \mu_{i\phi}(P_i\tilde{A}_{i\phi} + \tilde{A}_{i\phi}^T P_i^T) + \sum_{j=1}^{N} \pi_{ij} P_j E + Q_i + dQ + W_i + W_i^T + dW_i R^{-1} W_i^T$$

$$\Omega_{12i} = P_i\tilde{A}_{di} - W_i, \quad \Omega_{13i} = P_i\bar{B}_{\omega i} - \mathbf{C}_i^T$$

and Ψ_{33i} is defined in (14). Then, the inequality (14) implies

$$\Upsilon_1 + \Upsilon_2 + \Upsilon_2^T < 0 \tag{22}$$

where

$$\Upsilon_1 = \begin{bmatrix} \Psi_{11i\phi} & \Psi_{12i} & \Psi_{13i} & \Psi_{14i\phi} & dW_i \\ * & -Q_i & -C_{di}^T & d\bar{A}_{di}^T & 0 \\ * & * & \Psi_{33i} & d\bar{B}_{\omega i}^T & 0 \\ * & * & * & -dR^{-1} & 0 \\ * & * & * & * & -dR \end{bmatrix}$$

$$\Upsilon_2 = \begin{bmatrix} \bar{M}_{1i}^T P_i^T & 0 & -M_{2i}^T & d\bar{M}_{1i}^T R^T & 0 \end{bmatrix}^T F_t \begin{bmatrix} N_{1i} & N_{2i} & 0 & 0 & 0 \end{bmatrix}.$$

By using Schur complement, the inequality (22) indicates that $\Omega < 0$. Then, it notices from (21) that

$$\mathcal{L}V(x_t, r_t, t) - 2z_t^T \omega_t - \gamma\omega_t^T \omega_t \leq 0. \tag{23}$$

For any $t^* \geq 0$, integral both sides of (23) with respect to t over the time interval $[0, t^*]$ give rise to

$$\mathcal{E}\left\{\int_0^{t^*} \mathcal{L}V(x_t, r_t, t)\mathrm{d}t\right\} - \mathcal{E}\left\{2\int_0^{t^*} z_t^T \omega_t \mathrm{d}t\right\} \\ -\gamma \int_0^{t^*} \omega_t^T \omega_t \mathrm{d}t \leq 0 \tag{24}$$

under the zero initial conditions, and $V(x_{t^*}, r_{t^*}, t^*) \geq 0$, then (24) guarantees (6) in Definition 1. According to Definition 1, the system (13) is robustly passive.

Finally, we consider the asymptotic stability of system (13) under the condition of $\omega_t = 0$

$$\mathcal{L}V(x_t, r_t, t) = \bar{\eta}_t^T \bar{\Omega}\bar{\eta}_t - \Gamma_t \leq \bar{\eta}_t^T \bar{\Omega}\bar{\eta}_t$$

where

$$\bar{\Omega} \triangleq \begin{bmatrix} \Omega_{11i\phi} & \Omega_{12i} \\ * & -Q_i \end{bmatrix} + d\begin{bmatrix} \tilde{A}_{i\phi}^T \\ \tilde{A}_{di}^T \end{bmatrix} R \begin{bmatrix} \tilde{A}_{i\phi}^T \\ \tilde{A}_{di}^T \end{bmatrix}^T, \bar{\eta}_t \triangleq \begin{bmatrix} x_t \\ x_{t-d} \end{bmatrix}.$$

From (22), it implies that $\bar{\Omega} < 0$, and there exist a scalar $\lambda > 0$, such that

$$\mathcal{L}V(x_t, r_t, t) \leq -\lambda_{\min}(-\bar{\Omega})\| \bar{\eta}_t \|^2 \leq 0$$

thus, system (13) is admissible. ■

Theorem 2: Given positive scalars γ, ϵ, σ, the sliding mode dynamics (13) is admissible and robustly passive, if there exist matrices $\mathcal{Q} > 0$, $\mathcal{Q}_i > 0$, $\mathcal{R} > 0$, $\mathcal{K}_\phi$, $\mathcal{P}_i \triangleq \left[\begin{smallmatrix} \mathcal{P}_{11i} & \mathcal{P}_{12i} \\ 0 & \mathcal{P}_{22i} \end{smallmatrix}\right]$, $\mathcal{W}_i \triangleq \begin{bmatrix} \mathcal{W}_{1i} & 0_{n\times(n-r)} \end{bmatrix}$ (with $0 < \mathcal{P}_{11i} \in \mathbb{R}^{r\times r}$, $\mathcal{W}_{1i} \in \mathbb{R}^{n\times r}$), such that the following linear matrix inequalities (LMIs) hold for each $i \in \mathcal{N}$ and $\phi \in \mathcal{M}$

$$\begin{bmatrix} \widetilde{\Theta}_{11i} & \widetilde{\Theta}_{12i} \\ * & \widetilde{\Theta}_{22i} \end{bmatrix} < 0 \tag{25}$$

$$\sum_{j=1}^{N} \pi_{ij}\mathcal{Q}_j \leq \mathcal{Q} \tag{26}$$

where

$$\widetilde{\Theta}_{11i} = \begin{bmatrix} \widetilde{\Psi}_{11i\phi} & \widetilde{\Psi}_{12i} & \widetilde{\Psi}_{13i} & \widetilde{\Psi}_{14i\phi} & d\mathcal{W}_i \\ * & \widetilde{\Psi}_{22i} & -\mathcal{P}_i C_{di}^T & d\mathcal{P}_i\bar{A}_{di}^T & 0 \\ * & * & \widetilde{\Psi}_{33i} & d\bar{B}_{\omega i}^T & 0 \\ * & * & * & -d\mathcal{R} & 0 \\ * & * & * & * & \widetilde{\Psi}_{55i} \end{bmatrix}$$

$$\widetilde{\Theta}_{12i} = \begin{bmatrix} \bar{M}_{1i} & \epsilon\mathcal{P}_i N_{1i}^T & \mathcal{P}_i & d\mathcal{P}_i & \widetilde{\Psi}_{110i} \\ 0 & \epsilon\mathcal{P}_i N_{2i}^T & 0 & 0 & 0 \\ -M_{2i} & 0 & 0 & 0 & 0 \\ d\bar{M}_{1i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{\Theta}_{22i} = -\mathrm{diag}\{\epsilon I, \epsilon I, \mathcal{Q}_i, d\mathcal{Q}, \widetilde{\Psi}_{1010i}\}$$

$$\widetilde{\Psi}_{11i\phi} = \sum_{\phi=1}^{M} \mu_{i\phi}(B_i\mathcal{K}_\phi + \mathcal{K}_\phi^T B_i^T) + \mathcal{W}_i + \mathcal{W}_i^T \\ +A_i\mathcal{P}_i^T + \mathcal{P}_i A_i^T + \pi_{ii}E\mathcal{P}_i^T$$

$$\widetilde{\Psi}_{12i} = \bar{A}_{di}\mathcal{P}_i^T - \mathcal{W}_i, \quad \widetilde{\Psi}_{13i} = \bar{B}_{\omega i} - \mathcal{P}_i C_i^T$$

$$\widetilde{\Psi}_{14i\phi} = d\mathcal{P}_i A_i^T + \sum_{\phi=1}^{M} \mu_{i\phi} d\mathcal{K}_\phi^T B_i^T, \widetilde{\Psi}_{22i} = -\mathcal{P}_i - \mathcal{P}_i^T + \mathcal{Q}$$

$$\widetilde{\Psi}_{33i} = -\gamma I - D_{\omega i} - D_{\omega i}^T, \widetilde{\Psi}_{55i} = -d(\mathcal{P}_i + \mathcal{P}_i^T - \mathcal{R})$$

$$\widetilde{\Psi}_{110i} = [\sqrt{\pi_{i1}}\mathcal{P}_i \ldots \sqrt{\pi_{i(i-1)}}\mathcal{P}_i\sqrt{\pi_{i(i+1)}}\mathcal{P}_i \ldots \sqrt{\pi_{iN}}\mathcal{P}_i]$$

$$\widetilde{\Psi}_{1010i} = -\mathrm{diag}\{\Xi_1, \ldots, \Xi_{i-1}, \Xi_{i+1}, \ldots, \Xi_N\}.$$

Moreover, the matrix K_ϕ in (8) can be given as

$$K_\phi = \mathcal{K}_\phi\mathcal{P}_i^{-T}. \tag{27}$$

Proof: Let $\mathcal{P}_i = P_i^{-1}$, $\mathcal{Q}_i = Q_i^{-1}$, $\mathcal{R} = R^{-1}$, and $\mathcal{W}_i = \mathcal{P}_i^T W_i \mathcal{P}_i$, and the performance of a congruence transformation to (14) by diag $\{\mathcal{P}_i, \mathcal{P}_i, I, I, \mathcal{P}_i, I, I\}$, we have

$$\begin{bmatrix} \bar{\Theta}_{11i} & \bar{\Theta}_{12i} \\ * & \bar{\Theta}_{22i} \end{bmatrix} < 0 \tag{28}$$

where

$$\bar{\Theta}_{11i} = \begin{bmatrix} \bar{\Psi}_{11i\phi} & \widetilde{\Psi}_{12i} & \widetilde{\Psi}_{13i} & \widetilde{\Psi}_{14i\phi} \\ * & -\mathcal{P}_i Q_i \mathcal{P}_i^T & -\mathcal{P}_i C_{di}^T & d\mathcal{P}_i\bar{A}_{di}^T \\ * & * & \widetilde{\Psi}_{33i} & d\bar{B}_{\omega i}^T \\ * & * & * & -d\mathcal{R} \end{bmatrix}$$

$$\bar{\Theta}_{12i} = \begin{bmatrix} d\mathcal{W}_i & \bar{M}_{1i} & \epsilon\mathcal{P}_i N_{1i}^T \\ 0 & 0 & \epsilon\mathcal{P}_i N_{2i}^T \\ 0 & -M_{2i} & 0 \\ 0 & d\bar{M}_{1i} & 0 \end{bmatrix}$$

$$\bar{\Theta}_{22i} = -\mathrm{diag}\{d\mathcal{P}_i R\mathcal{P}_i^T, \epsilon I, \epsilon I\}$$

$$\bar{\Psi}_{11i\phi} = \sum_{\phi=1}^{M} \mu_{i\phi}(\bar{A}_{i\phi}\mathcal{P}_i^T + \mathcal{P}_i\bar{A}_{i\phi}^T) + \sum_{j=1}^{N} \pi_{ij}\mathcal{P}_i P_j E\mathcal{P}_i^T \\ +\mathcal{W}_i + \mathcal{W}_i^T + d\mathcal{P}_i Q\mathcal{P}_i^T + \mathcal{P}_i Q_i \mathcal{P}_i^T$$

and $\widetilde{\Psi}_{12i}$, $\widetilde{\Psi}_{13i}$, $\widetilde{\Psi}_{14i\phi}$, and $\widetilde{\Psi}_{33i}$ are defined in (25).

On the other hand, notice that $0 \leq (\mathcal{P}_i - \mathcal{Q}_i)Q_i(\mathcal{P}_i - \mathcal{Q}_i)^T = \mathcal{P}_i Q_i \mathcal{P}_i^T - \mathcal{P}_i - \mathcal{P}_i^T + \mathcal{Q}_i$, which implies $-\mathcal{P}_i Q_i \mathcal{P}_i^T \leq -\mathcal{P}_i - \mathcal{P}_i^T + \mathcal{Q}_i$. By the same principle as above, we have $-\mathcal{P}_i R \mathcal{P}_i^T \leq -\mathcal{P}_i - \mathcal{P}_i^T + \mathcal{R}$. Also, since $P_j E$ is singular, there exists a sufficient small scalar $\sigma > 0$ such that $P_j E \leq P_j E + \sigma\xi$, where

$$\xi \triangleq \begin{bmatrix} 0_{r\times r} & 0_{r\times(n-r)} \\ 0_{(n-r)\times r} & I_{n-r} \end{bmatrix},$$

and notices that $\Xi_j = (P_j E + \sigma\xi)^{-1} = P_j E + \sigma^{-1}\xi$.

Let $\mathcal{K}_\phi = K_\phi \mathcal{P}_i^T$ and by the Schur complement, it can be obtained that the inequality (28) would be satisfied if the inequality (25) holds. ■

C. SMC Law Design

In this section, an asynchronous SMC law will be synthesized to drive the trajectories of system (1) onto the specified sliding surface in finite time and remain there for subsequent time.

Theorem 3: Consider the system (1) and the sliding surface function is designed as (8) with K_ϕ being solvable by (27). Then, the trajectories of system (1) can be driven onto the sliding surface $s_t = 0$ in finite time by the SMC Law

$$u_t = K_\phi x_t - (G_i B_i)^{-1} G_i A_{di} x_{t-d} - \varpi_t \mathrm{sign}(s_t) \tag{29}$$

where

$$\begin{aligned} \varpi_t = & \parallel (G_i B_i)^{-1} G_i M_{1i} \parallel (\parallel N_{1i} x_t \parallel + \parallel N_{2i} x_{t-d} \parallel) \\ & + \alpha + \eta \parallel x_t \parallel + \parallel (G_i B_i)^{-1} G_i B_{\omega i} \parallel \parallel \omega_t \parallel \end{aligned} \tag{30}$$

with a positive constant α.

Proof: For simplifying calculation, select G_i as $G_i = B_i^T X_i$, where $X_i > 0$ is a positive definite matrix. So $G_i B_i$ is nonsingular. Choose the Lyapunov function candidate as

$$H_t = \frac{1}{2} s_t^T (B_i^T X_i B_i)^{-1} s_t. \tag{31}$$

From (10), we have

$$\begin{aligned} \dot{s}_t = & B_i^T X_i (\Delta A_i - B_i K_\phi) x_t + B_i^T X_i \mathbf{A}_{di} x_{t-d} \\ & + B_i^T X_i B_{\omega i} \omega_t + B_i^T X_i B_i [u_t + f(x_t, t)]. \end{aligned} \tag{32}$$

Taking the derivative of H_t and we have

$$\begin{aligned} \dot{H}_t = & s_t^T (B_i^T X_i B_i)^{-1} \dot{s}_t \\ = & s_t^T (B_i^T X_i B_i)^{-1} B_i^T X_i \triangle A_i x_t \\ & + s_t^T (B_i^T X_i B_i)^{-1} B_i^T X_i \triangle A_{di} x_{t-d} \\ & + s_t^T (B_i^T X_i B_i)^{-1} B_i^T X_i A_{di} x_{t-d} \\ & + s_t^T (B_i^T X_i B_i)^{-1} B_i^T X_i B_{\omega i} \omega_t \\ & + s_t^T [u_t + f(x_t, t)] - s_t^T K_\phi x_t \end{aligned} \tag{33}$$

then, we can get

$$\begin{aligned} \dot{H}_t \leq & \parallel s_t \parallel \parallel (B_i^T X_i B_i)^{-1} B_i^T X_i M_{1i} \parallel \\ & \times (\parallel N_{1i} x_t \parallel + \parallel N_{2i} x_{t-d} \parallel) \\ & + s_t^T (B_i^T X_i B_i)^{-1} B_i^T X_i A_{di} x_{t-d} \\ & + s_t^T (B_i^T X_i B_i)^{-1} B_i^T X_i B_{\omega i} \omega_t \\ & + s_t^T [u_t + f(x_t, t)] - s_t^T K_\phi x_t. \end{aligned} \tag{34}$$

Substituting (29) into (34) and noting $\parallel s_t \parallel \leq |s_t|$, we have

$$\dot{H}_t \leq -\alpha \parallel s_t \parallel \leq -\tilde{\alpha}\sqrt{H_t} \tag{35}$$

where $\tilde{\alpha} \triangleq \alpha\sqrt{2/\lambda_{\max}[(B_i^T X_i B_i)^{-1}]} > 0$. Thus, the trajectories of the system can be driven onto the specified sliding surface in finite time. ■

IV. Illustrative Example

Consider a delayed singular Markovian jump system (4) with two operating modes and that the controller also has two modes, that is, $\mathcal{N} = 2$ and $\mathcal{M} = 2$, and set the following parameters:

$$E = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}, \Pi = \begin{bmatrix} -0.7 & 0.7 \\ 0.4 & -0.4 \end{bmatrix}, C_1 = \begin{bmatrix} 0.9 \\ 0.4 \\ 0.7 \end{bmatrix}^T$$

$$\Omega = \begin{bmatrix} 0.4 & 0.6 \\ 0.8 & 0.2 \end{bmatrix}, \quad D_{\omega 1} = 0.2, \quad D_{\omega 2} = 0.4$$

$$A_1 = \begin{bmatrix} -0.3 & 0.5 & 1.0 \\ 0.5 & -0.4 & -1.0 \\ -0.1 & 0.3 & -0.7 \end{bmatrix}, M_{11} = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.0 \\ 0.1 & 0.0 & 0.0 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} -0.2 & 0.4 & -1.1 \\ 0.6 & -0.2 & -1.0 \\ -0.3 & 0.5 & 0.0 \end{bmatrix}, N_{11} = \begin{bmatrix} 0.1 & 0.1 & 0.0 \\ 0.1 & 0.2 & 0.1 \\ 0.2 & 0.0 & 0.1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.4 & 0.2 & 0.8 \\ 0.2 & -0.2 & -1.2 \\ -0.1 & 0.2 & -0.4 \end{bmatrix}, M_{12} = \begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.1 & 0 & 0.1 \\ 0 & 0.1 & 0.1 \end{bmatrix}$$

$$A_{d2} = \begin{bmatrix} -0.3 & 0.3 & -1.3 \\ 0.5 & -0.1 & -1.2 \\ 0.2 & -0.5 & 0.0 \end{bmatrix}, N_{12} = \begin{bmatrix} 0.3 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 \\ 0.0 & 0.2 & 0.1 \end{bmatrix}$$

$$N_{21} = \begin{bmatrix} 0.0 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 \\ 0.2 & 0.0 & 0.1 \end{bmatrix}, \quad N_{22} = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 \\ 0.0 & 0 & 0.2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1.3 \\ 1.8 \\ 0.9 \end{bmatrix}, \; B_{\omega 1} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.6 \end{bmatrix}, \; M_{21} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.0 \end{bmatrix}^T$$

$$B_2 = \begin{bmatrix} 1.1 \\ 1.3 \\ 0.8 \end{bmatrix}, \; B_{\omega 2} = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.1 \end{bmatrix}, \; M_{22} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.0 \end{bmatrix}^T$$

$$C_2 = \begin{bmatrix} -0.6 \\ 0.2 \\ 0.1 \end{bmatrix}^T, C_{d2} = \begin{bmatrix} -0.1 \\ 0.4 \\ -0.6 \end{bmatrix}^T, C_{d1} = \begin{bmatrix} 0.5 \\ 0.2 \\ -0.4 \end{bmatrix}^T.$$

In addition, $f(x_t, t) = 1.0e^{-t}\sin(t)x_t$, that is, $\eta = 1.0$. The delay time $d = 0.5$ and exogenous input $\omega_t = 1/1 + t^2$. Since quasi-sliding mode is introduced to prevent the chattering phenomenon of control signals, we replace $\mathrm{sign}(s_t)$ with $s_t/(0.01 + \parallel s_t \parallel)$. By solving the LMIs (14) and (15) in Theorem 1, and circularly optimizing parameter γ as $\gamma = 2.20$, utilizing (27), we have

$$K_1 = \begin{bmatrix} -6.56 & -7.30 & -4.87 \end{bmatrix}, K_2 = \begin{bmatrix} -3.08 & -5.37 & -1.76 \end{bmatrix}.$$

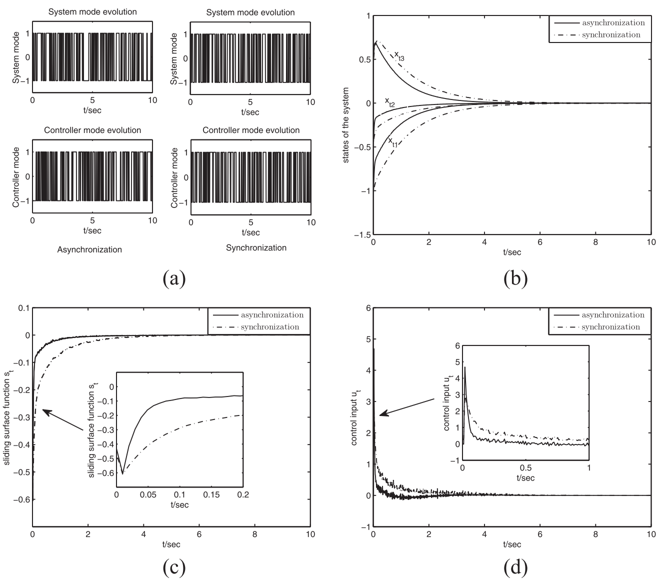

Fig. 1. Simulation results. (a) Switching signals with asynchronous and synchronous controller. (b) State responses of the closed-loop system. (c) Sliding surface function s_t. (d) SMC input u_t.

Thus, the sliding surface function in (8) is

$$s_t = \begin{cases} s_{11t} = -\int_0^t \begin{bmatrix} -19.75 & -21.39 & -15.19 \end{bmatrix} x_s \mathrm{d}s \\ \quad + \begin{bmatrix} 0.77 & 0.56 & 0 \end{bmatrix} x_t, i = 1, \phi = 1 \\ s_{12t} = -\int_0^t \begin{bmatrix} -9.30 & -15.60 & -5.85 \end{bmatrix} x_s \mathrm{d}s \\ \quad + \begin{bmatrix} 0.77 & 0.56 & 0 \end{bmatrix} x_t, i = 1, \phi = 2 \\ s_{21t} = -\int_0^t \begin{bmatrix} -20.02 & -21.61 & -15.32 \end{bmatrix} x_s \mathrm{d}s \\ \quad + \begin{bmatrix} 0.91 & 0.77 & 0 \end{bmatrix} x_t, i = 2, \phi = 1 \\ s_{22t} = -\int_0^t \begin{bmatrix} -9.57 & -15.82 & -5.98 \end{bmatrix} x_s \mathrm{d}s \\ \quad + \begin{bmatrix} 0.91 & 0.77 & 0 \end{bmatrix} x_t, i = 2, \phi = 2. \end{cases}$$

Let the adjustable parameter of α be $\alpha = 0.5$, then the asynchronous SMC law in (29) can be found as

$$u_t = \begin{cases} u_{11t} = \begin{bmatrix} -6.56 & -7.30 & -4.87 \end{bmatrix} x_t \\ \quad - \begin{bmatrix} -0.05 & 0.25 & -0.47 \end{bmatrix} x_{t-d} \\ \quad -\varpi_t \operatorname{sign}(s_{11t}), i = 1, \phi = 1 \\ u_{12t} = \begin{bmatrix} -3.08 & -5.37 & -1.76 \end{bmatrix} x_t \\ \quad - \begin{bmatrix} -0.05 & 0.25 & -0.47 \end{bmatrix} x_{t-d} \\ \quad -\varpi_t \operatorname{sign}(s_{12t}), i = 1, \phi = 2 \\ u_{21t} = \begin{bmatrix} -6.56 & -7.30 & -4.87 \end{bmatrix} x_t \\ \quad - \begin{bmatrix} 0.12 & -0.14 & -0.70 \end{bmatrix} x_{t-d} \\ \quad -\varpi_t \operatorname{sign}(s_{21t}), i = 2, \phi = 1 \\ u_{22t} = \begin{bmatrix} -3.08 & -5.37 & -1.76 \end{bmatrix} x_t \\ \quad - \begin{bmatrix} 0.12 & -0.14 & -0.70 \end{bmatrix} x_{t-d} \\ \quad -\varpi_t \operatorname{sign}(s_{22t}), i = 2, \phi = 2. \end{cases}$$

From the initial condition of $\varphi_t = \begin{bmatrix} -1.0 & 0.5 & 1.0 \end{bmatrix}^T$, $t \in [-1, 0]$, a comparison on the asynchronous and synchronous control strategy is given in Fig. 1. The simulation results shows that the asynchronous control strategy has faster response speed than synchronous control strategy. The reason is that the asynchronous control takes the delay time caused by the switching and the external disturbance in the switching process into account. The switched signal of the system modes and the controller modes is presented in Fig. 1(a), the solid line stands for the asynchronous and the dotted line expresses the synchronous control strategy. Fig. 1(b) shows the states x_t of the closed-loop system, and Fig. 1(c) and (d) depicts the sliding mode sliding surface function s_t and the SMC input u_t, respectively.

V. Conclusion

The issue of passivity-based asynchronous SMC for singular Markovian jump systems has been investigated in this paper. An asynchronous sliding surface function is proposed and sufficient conditions are given to ensure that the sliding mode dynamics are admissible and robustly passive. Furthermore, an SMC law has been synthesized to guarantee that the sliding mode dynamics satisfy the reaching condition. The effectiveness of the proposed technique has been illustrated by a numerical example. In the future, the asynchronous SMC can be extended to singular Markovian jump systems with partly known transition probabilities. Furthermore, the asynchronous observer-based and output feedback SMC problems are still open [18], which may deserve to be deeply explored in future.

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