Sliding Mode Control

THEORY AND APPLICATIONS

Christopher Edwards
and Sarah K. Spurgeon

Sliding Mode Control
Theory and Applications

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Sliding Mode Control: Theory and Applications

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We dedicate this book to Chris's parents and Sarah's children.

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Series Introduction

Control systems has a long distinguished tradition stretching back to nineteenth-century dynamics and stability theory. Its establishment as a major engineering discipline in the 1950s arose, essentially, from Second World War driven work on frequency response methods by, amongst others, Nyquist, Bode and Wiener. The intervening 40 years has seen quite unparalleled developments in the underlying theory with applications ranging from the ubiquitous PID controller widely encountered in the process industries through to high-performance/fidelity controllers typical of aerospace applications. This development has been increasingly underpinned by the rapid developments in the, essentially enabling, technology of computing software and hardware.

This view of mathematically model-based systems and control as a mature discipline masks relatively new and rapid developments in the general area of robust control. Here intense research effort is being directed to the development of high-performance controllers which (at least) are robust to specified classes of plant uncertainty. One measure of this effort is the fact that, after a relatively short period of work, 'near world' tests of classes of robust controllers have been undertaken in the aerospace industry. Again, this work is supported by computing hardware and software developments, such as the toolboxes available within numerous commercially marketed controller design/simulation packages.

Recently, there has been increasing interest in the use of so-called intelligent control techniques such as fuzzy logic and neural networks. Basically, these rely on learning (in a prescribed manner) the input-output behaviour of the plant to be controlled. Already, it is clear that there is little to be gained by applying these techniques to cases where mature mathematical model-based approaches yield high-performance control. Instead, their role is (in general terms) almost certainly going to lie in areas where the processes encountered are ill-defined, complex, nonlinear, time-varying and stochastic. A detailed evaluation of their (relative) potential awaits the appearance of a rigorous supporting base (underlying theory and implementation architectures for example) the essential elements of which are beginning to appear in learned journals and conferences.

Elements of control and systems theory/engineering are increasingly finding use outside traditional numerical processing environments. One such general area in which there is increasing interest is intelligent command and control systems which are central, for example, to innovative manufacturing and management of advanced transportation systems. Another is discrete event systems which mix numeric and logic decision making.

It was in response to these exciting new developments that the present book series of System and Control was conceived. It publishes high-quality research texts and reference works in the diverse areas which systems and control now includes. In addition to basic theory, experimental and/or application studies are welcome, as are expository texts where theory, verification and applications come together to provide a unifying coverage of a particular topic or topics.

E. Rogers
J. O'Reilly

Preface

In the formulation of any control problem there will typically be discrepancies between the actual plant and the mathematical model developed for controller design. This mismatch may be due to unmodelled dynamics, variation in system parameters or the approximation of complex plant behaviour by a straightforward model. The engineer must ensure that the resulting controller has the ability to produce the required performance levels in practice despite such plant/model mismatches. This has led to an intense interest in the development of so-called *robust* control methods which seek to solve this problem. One particular approach to robust controller design is the so-called *sliding mode* control methodology.

Sliding mode control is a particular type of *Variable structure control.* Variable structure control systems (VSCS) are characterised by a suite of feedback control laws and a decision rule. The decision rule, termed the switching function, has as its input some measure of the current system behaviour and produces as an output the particular feedback controller which should be used at that instant in time. The result is a variable structure system, which may be regarded as a combination of subsystems where each subsystem has a fixed control structure and is valid for specified regions of system behaviour. One of the advantages of introducing this additional complexity into the system is the ability to combine useful properties of each of the composite structures of the system. Furthermore, the system may be designed to possess new properties not present in any of the composite structures alone. Utilisation of these natural ideas began in the Soviet Union in the late 1950s.

In sliding mode control, VSCS are designed to drive and then constrain the system state to lie within a neighbourhood of the switching function. There are two main advantages to this approach. Firstly, the dynamic behaviour of the system may be tailored by the particular choice of switching function. Secondly, the closed-loop response becomes totally insensitive to a particular class of uncertainty. The latter invariance property clearly makes the methodology an appropriate candidate for robust control. In addition, the ability to specify performance directly makes sliding mode control attractive from the design perspective.

The sliding mode design approach consists of two components. The first involves the design of a switching function so that the sliding motion satisfies design specifications. The second is concerned with the selection of a control law which will make the switching function attractive to the system state. Note that this control law is not necessarily discontinuous.

This text provides the reader with a thorough grounding in the sliding mode control area and as such is appropriate for the graduate with a basic knowledge of classical control theory and some knowledge of state-space methods. From this basis, more advanced theoretical results are developed. Resulting design procedures are emphasised using MATLAB mfiles.[1]

[1] MATLAB is a registered trademark of Mathworks, Inc.

Fully worked design examples are an additional tutorial feature. Industrial case studies, which present the results of sliding mode controller implementations, are used to illustrate the successful practical application of the theory.

The book is structured as follows. Chapter 1 introduces the concept of sliding mode control and illustrates the attendant features of robustness and performance specification using a straightforward example and graphical exposition. The necessary background for more formal development is presented in Chapter 2 to enhance readability.

Chapter 3 formulates the general multivariable sliding mode control problem. The interpretation of the sliding surface design problem as a straightforward linear state feedback problem for a subsystem is emphasised. Possible control strategies to enforce sliding motion are identified and the problem of smoothing undesirable discontinuous signals addressed. Controller design issues are considered in Chapter 4. A number of methods for sliding surface design are described. The so-called unit vector control structure is exploited as this lends itself to the development of simple numeric design algorithms. Frameworks for the solution of both regulation and tracking problems are presented. A design study from the flight control area is used to illustrate the application of the proposed design methods.

As developed thus far, the controllers require full state information. In practice, it may be impossible or impractical to measure all of the system states. The next three chapters address this problem. Chapter 5 investigates the possibilities of obtaining sliding mode schemes where the control law and decision rule are a function of the measured outputs. The associated limitations relating to both nominal system structure and attendant design freedoms are demonstrated. A design algorithm is presented and illustrated using numerical examples. Chapters 6 and 7 consider the possibilities of both state reconstruction and control using sliding mode techniques. Again design frameworks are emphasised.

The book concludes with industrial studies describing both sliding mode controller design and implementation. Chapter 8 considers two automotive studies. The automotive environment is harsh; components are subject to large temperature variations and shock, and parameters can be expected to vary significantly during the lifetime of the vehicle. Robustness is thus a key requirement. Further, there is the need to produce high levels of performance and safety from inexpensive components. Sliding mode design will be seen to fulfil these requirements.

Chapter 9 considers sliding mode control of a gas-fired furnace. The furnace dynamics are highly nonlinear and any control strategy is required to yield robust performance over a large operating range. Tracking requirements are essential; when firing ceramics, for example, it is essential that the furnace temperature follows a prespecified profile exactly if the end product is to be of good quality. Efficiency of combustion is also a priority; large magnitude interacting control signals result in significant gas wastage. It will be established that sliding mode control can achieve the desired specifications.

In terms of the interdependence of the individual chapters, Chapters 1 and 2 are not formally connected to the rest of the text. Chapter 1 only seeks to motivate some of the ideas that are expressed rigorously in Chapter 3. Chapter 2 is meant to indicate the topics in systems theory that are required in subsequent chapters.

Readers who have not encountered the state-space and Lyapunov theory presented there are encouraged to refer to more specialist texts in these particular areas which develop the ideas in a more measured and expert way. Chapter 3 is the cornerstone of the book; in particular Section 3.6 very much sets the tone of the control laws which are discussed in Chapters 4, 5 and 7. Chapter 4 is quite strongly linked with Chapter 3. Although its main thrust is to describe different hyperplane design strategies, it does introduce in a tutorial way the integral action and model-following approaches for reference tracking, which subsequently appear in Chapters 5 and 7. Although Chapter 5 (by necessity) considers and develops different control strategies because of the removal of the state availability assumption, parallels are still drawn with the results of Chapter 3. Chapter 6, which is primarily concerned with the design of observers, does utilise some of the ideas and results from Chapter 5 because a similar canonical form is used as the fundamental design framework. As a result, some of the proofs are more sketched but suitable cross-referencing has been made to the original development. (This is true of the run of chapters from 3 through to 7.) Chapter 7 essentially reworks some of the ideas of Chapter 4 (the reference tracking methodologies) and considers the state feedback controllers in conjunction with the state observers developed in Chapter 6. The first actuator case study in Chapter 8 could (almost) be tackled once the concepts from Chapter 1 have been understood. The idle speed control problem in Section 8.3 and the furnace case study in Chapter 9 rely heavily on the integral action controller exposition from Chapter 7.

Finally it is our pleasure to thank our families, friends and colleagues who have helped in the preparation of this book. Several deserve a special mention. First and foremost we would like to thank Aamer Bhatti for his assistance in the preparation of Chapter 8. We are especially grateful to him for allowing us to use, at the time of writing, unpublished work pertaining to the control of engine speed. We also wish to thank Russell Jones of Lucas Varity for his help with the automotive actuator problem, which also appears in Chapter 8. The furnace application described in Chapter 9 would not have taken place without the help and technical assistance of Haydn Porch, Sean Goodhart, Ruth Davies and Patrick Holmes. We thank them for their cooperation and patience during the trials initially at the Midlands Research Centre at Solihull and subsequently at the Gas Research Centre in Loughborough.

In terms of the preparation of the document we would like to thank Doug Pratt for drawing many of the figures which appear throughout the text. We are also very grateful to the brave souls who helped proofread and provide valuable feedback: we would like to thank Robert Cortez who suffered the earliest draft and also our graduate students Ashu Akoachere and Guido Hermman. We are particularly grateful to Xinghuo Yu for his thorough reading of the first draft and for providing many pertinent and insightful comments which we have tried to incorporate. Last but not least, we would like to thank Xiao-Yun Lu for helping us read and correct the final proofs. Of course, despite our best efforts, we are sure that many typographical errors still remain; for these we take full responsibility.

Chris Edwards
Sarah Spurgeon

March 1998

Chapter 1

An Introduction to Sliding Mode Control

1.1 INTRODUCTION

Variable structure control systems (VSCS) evolved from the pioneering work in Russia of Emel'yanov and Barbashin in the early 1960s. The ideas did not appear outside of Russia until the mid 1970s when a book by Itkis (1976) and a survey paper by Utkin (1977) were published in English. VSCS concepts have subsequently been utilised in the design of robust regulators, model-reference systems, adaptive schemes, tracking systems, state observers and fault detection schemes. The ideas have successfully been applied to problems as diverse as automatic flight control, control of electric motors, chemical processes, helicopter stability augmentation systems, space systems and robots. This chapter seeks to motivate and introduce the concepts which will be considered more formally later in the book.

Variable structure control systems, as the name suggests, are a class of systems whereby the 'control law' is deliberately changed during the control process according to some defined rules which depend on the state of the system. For the purpose of illustration consider the double integrator given by

$$\ddot{y}(t) = u(t) \tag{1.1}$$

Initially consider the effect of using the feedback control law

$$u(t) = -ky(t) \tag{1.2}$$

where k is a strictly positive scalar. One way of analysing the resulting closed-loop motion is by means of a phase portrait, which essentially is a plot of velocity against position. Substituting for the control action in equation (1.1), and multiplying the resulting equation throughout by $\dot{y}$, yields

$$\dot{y}\ddot{y} = -k\dot{y}y \tag{1.3}$$

Integrating this expression gives the following relationship between velocity and position

$$\dot{y}^2 + ky^2 = c \tag{1.4}$$

where c represents a constant of integration resulting from the initial conditions and is strictly positive. Importantly, time does not appear explicitly in the expression in (1.4). In the special case when $k = 1$, equation (1.4) represents a circle with centre at the origin and radius $\sqrt{c}$. More generally, a plot of $\dot{y}$ against y is an ellipse which depends on the initial conditions as shown in Figure 1.1. In terms of regulation – i.e. steering arbitrary initial conditions to the origin – a control law of the form given in (1.2) is not appropriate since, as shown in Figure 1.1, the y and $\dot{y}$ variables do not move towards the origin.[1]

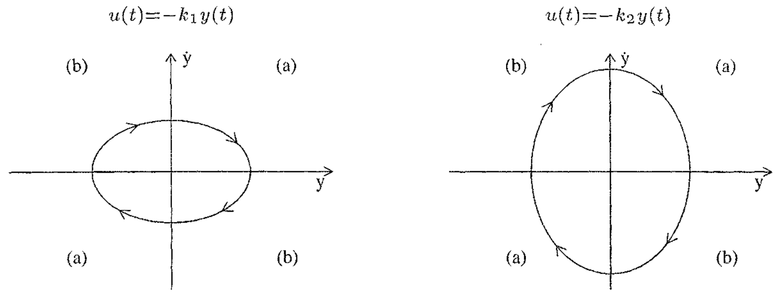

Figure 1.1: Phase portraits of simple harmonic motion

Consider instead the control law

$$u(t) = \begin{cases} -k_1 y(t) & \text{if } y\dot{y} < 0 \\ -k_2 y(t) & \text{otherwise} \end{cases} \tag{1.5}$$

where $0 < k_1 < 1 < k_2$. The phase plane $(y, \dot{y})$ is partitioned by the switching rule into four quadrants separated by the axes as shown in Figure 1.1. The control law $u = -k_2 y$ will be in effect in the quadrants of the phase plane labelled (a). In this region, the distance from the origin of the points in the phase portrait decreases along the system trajectory. Likewise, in region (b) when the control law $u = -k_1 y$ is in operation, the distance from the origin of the points in the phase portrait also decreases. This system clearly fits the description of a VSCS given earlier. The phase portrait for the closed-loop system under the variable structure control law u is obtained by splicing together the appropriate regions from the two phase portraits in Figure 1.1. In this way the phase portrait must spiral in towards the origin and an asymptotically stable motion results, as shown in Figure 1.2. This can be verified more formally by considering the function

$$V(y, \dot{y}) = y^2 + \dot{y}^2 \tag{1.6}$$

which, from Pythagoras' theorem, represents the square of the distance from the point $(y, \dot{y})$ to the origin in the phase plane and may be viewed as representing the energy of the system. The time derivative of $V(y, \dot{y})$ along the closed-loop trajectories is given by

$$\begin{aligned} \dot{V} &= 2\dot{y}y + 2\ddot{y}\dot{y} \\ &= 2\dot{y}(y + u) = \begin{cases} 2y\dot{y}(1 - k_1) & \text{if } y\dot{y} < 0 \\ 2y\dot{y}(1 - k_2) & \text{if } y\dot{y} > 0 \end{cases} \end{aligned}$$

[1] Since y and $\dot{y}$ remain bounded for all time the closed-loop is stable; it is not, however, asymptotically stable – see Section 2.2 for details.

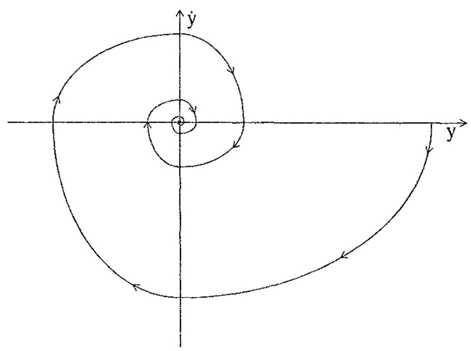

Figure 1.2: Phase portrait of the system under VSCS

This is always negative by the way the gains are constructed. Thus the distance from the origin is always decreasing, which agrees with the intuitive observation made earlier. By introducing a rule for switching between two control structures, which independently do not provide stability, a stable closed-loop system has been obtained.

A more significant example results from using the variable structure law given by

$$u(t) = \begin{cases} -1 & \text{if } s(y,\dot{y}) > 0 \\ 1 & \text{if } s(y,\dot{y}) < 0 \end{cases} \tag{1.7}$$

where the *switching function* is defined by

$$s(y,\dot{y}) = my + \dot{y} \tag{1.8}$$

where m is a positive design scalar. The reason for the use of the term 'switching function' is clear, since the function given in equation (1.8) is used to decide which control structure is in use at any point $(y, \dot{y})$ in the phase plane. The expression in equation (1.7) is usually written more concisely as

$$u(t) = -\text{sgn}\,(s(t)) \tag{1.9}$$

where $\text{sgn}(\cdot)$ is the *signum*, or more colloquially, the sign function. The signum function exhibits the property that

$$s\,\text{sgn}(s) = |s| \tag{1.10}$$

This simple result will be exploited often in the analysis which follows.

The expression given in (1.7) is used to control the double integrator. For large values of $\dot{y}$ the phase portrait, obtained from joining the parabolic components of the constituent laws, is shown in Figure 1.3. The dotted line in the figure represents the set of points for which $s(y,\dot{y}) = 0$; in this case a straight line through the origin of gradient $-m$. However, for values of $\dot{y}$ satisfying the inequality $m|\dot{y}| < 1$ then

$$s\dot{s} = s\,(m\dot{y} + \ddot{y}) = s\,(m\dot{y} - \text{sgn}(s)) < |s|\,(m|\dot{y}| - 1) < 0$$

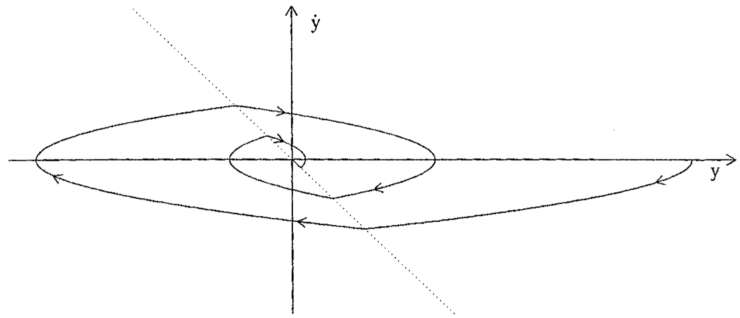

Figure 1.3: Phase portrait of the system for large $\dot{y}$

or equivalently

$$\lim_{s \to 0^+} \dot{s} < 0 \qquad \text{and} \qquad \lim_{s \to 0^-} \dot{s} > 0 \tag{1.11}$$

Consequently, when $m|\dot{y}| < 1$ the system trajectories on either side of the line

$$\mathcal{L}_s = \{(y, \dot{y}) \ : \ s(y, \dot{y}) = 0\} \tag{1.12}$$

point towards the line. This is demonstrated in Figure 1.4, which shows different phase portraits intercepting the same point on the line $\mathcal{L}$ from different initial conditions.

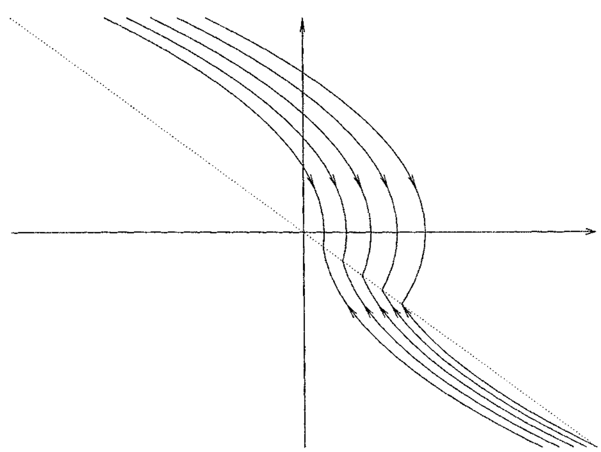

Figure 1.4: Phase portrait of the system under VSC near the origin

Intuitively, high frequency switching between the two different control structures will take place as the system trajectories repeatedly cross the line $\mathcal{L}_s$. This high frequency motion is described as *chattering*. If infinite frequency switching were possible, the motion would be trapped or constrained to remain on the line $\mathcal{L}_s$. The motion when confined to the line $\mathcal{L}_s$ satisfies the differential equation obtained from rearranging $s(y, \dot{y}) = 0$, namely

$$\dot{y}(t) = -my(t) \tag{1.13}$$

This represents a first-order decay and the trajectories will 'slide' along the line $\mathcal{L}_s$ to the origin (Figure 1.5).

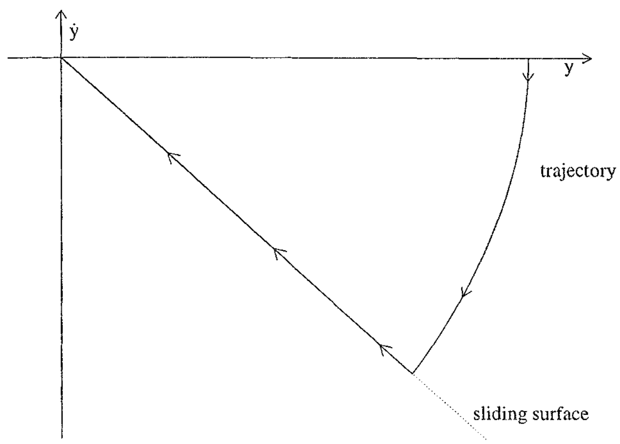

Figure 1.5: Phase portrait of a sliding motion

Such dynamical behaviour is described as an *ideal sliding mode* or an *ideal sliding motion* and the line $\mathcal{L}_s$ is termed the *sliding surface*. During sliding motion, the system behaves as a reduced-order system which is apparently independent of the control. The control action, rather than prescribing the dynamic performance, ensures instead that the conditions given in (1.11) are satisfied; this guarantees that $s(y, \dot{y}) = 0$. The conditions in (1.11) are usually written more conveniently as

$$s\dot{s} < 0 \tag{1.14}$$

which is referred to as the *reachability condition*. Thus, in terms of VSCS design, the choice of the switching function, represented in this situation by the parameter m, governs the performance response; whilst the control law itself is designed to guarantee that the reachability condition (1.14) is satisfied. In this case, as argued earlier, the reachability condition is only satisfied in a domain of the phase plane

$$\Omega = \{(y, \dot{y}) \ : \ m|\dot{y}| < 1\}$$

It should be noted that the control action required to bring about such a motion is discontinuous, and on the sliding surface is not even defined. The discontinuous requirement will be discussed in later chapters.

1.2 PROPERTIES OF THE SLIDING MOTION

This section explores in more detail the properties of the ideal sliding motion and the control action necessary to maintain such a motion. Consider once again the double integrator in equation (1.1) and the control law given in (1.7). The phase portrait in Figure 1.6 is from a simulation of the closed-loop behaviour when $m = 1$, and the initial conditions are given by $y = 1$ and $\dot{y} = 0$. The two-stage nature of the dynamics is readily observed: the initial (parabolic) motion towards the sliding surface, followed by a motion along the line $\dot{y} = -y$ towards the origin.

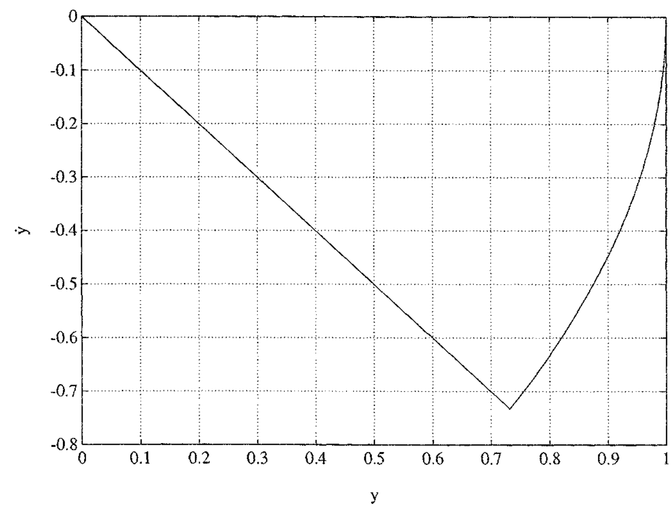

Figure 1.6: Phase portrait of a sliding motion

The control action associated with this simulation is given in Figure 1.7. It can be seen that sliding takes place after 0.732 second when high frequency switching takes place.

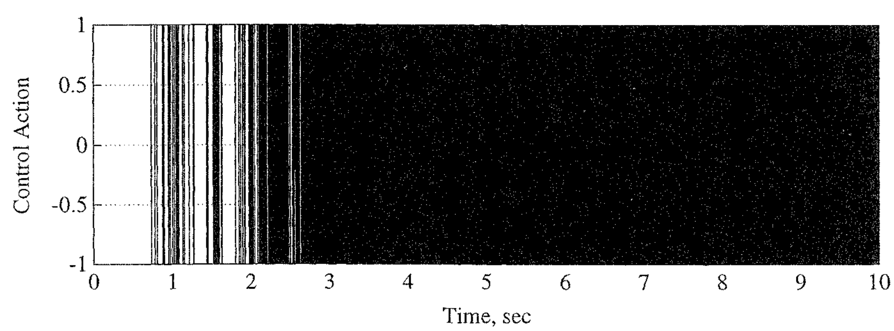

Figure 1.7: Discontinuous control action

Before considering the properties of the sliding motion, an interpretation of the control signal given in Figure 1.7 will be given in terms of its 'average' or low frequency behaviour. As argued earlier, the purpose of the control action is to ensure that the trajectories are driven towards and forced to remain on $\mathcal{L}_s$ to guarantee a sliding motion. It is natural therefore to explore the relationship between the control action and the switching function rather than between the control action and the plant output. Suppose at time t_s the switching surface is reached and an ideal sliding motion takes place. It follows that the switching function satisfies $s(t) = 0$ for all $t > t_s$, which in turn implies that $\dot{s}(t) = 0$ for all $t \geq t_s$. However, from equations (1.1) and (1.8)

$$\dot{s}(t) = m\dot{y}(t) + u(t) \tag{1.15}$$

and thus since $\dot{s}(t) = 0$ for all $t \geq t_s$, it follows from (1.15) that a control law which maintains the motion on $\mathcal{L}_s$ is

$$u(t) = -m\dot{y}(t) \qquad (t \geq t_s) \tag{1.16}$$

This control law is referred to as the *equivalent control* action. This is not the control signal which is actually applied to the plant but may be thought of as the control signal which is applied 'on average'. This can be demonstrated by passing the discontinuous control signal (in Figure 1.7) through the low pass filter

$$\tau\dot{u}_a(t) + u_a(t) = u(t) \tag{1.17}$$

to obtain the low frequency component $u_a(t)$. Figure 1.8 shows $u_a(t)$, together with the associated equivalent control when $\tau = 0.04$. It can be seen that the filtered (or averaged) control signal agrees with the equivalent control action defined in equation (1.16). Of course agreement can only take place once sliding is established – in this case after 0.732 second. Consequently, the dotted line, representing the equivalent control, is only drawn from this time onwards, as shown in Figure 1.8.

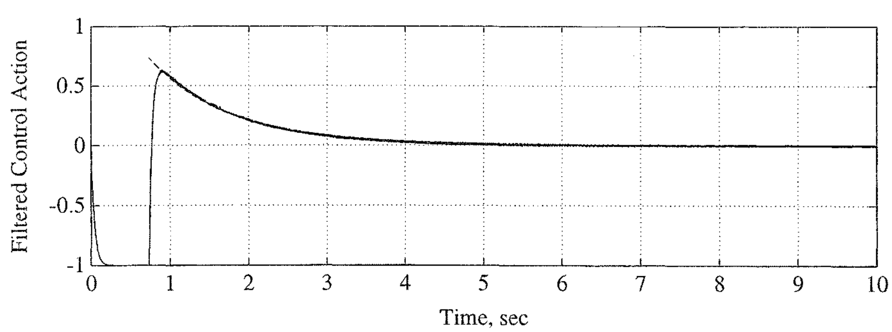

Figure 1.8: Equivalent control

The control signal applied to the plant may be thought of as comprising 'low' and 'high' frequency components so that

$$u(t) = \underbrace{u_a(t)}_{low\ frequency} + \underbrace{(u(t) - u_a(t))}_{high\ frequency}$$

For systems which may be modelled as strictly proper transfer functions,[2] for example, the high frequency component may be considered to be beyond the bandwidth; hence the control action that affects the dynamic response is $u_a(t)$, i.e. the equivalent control. A more dramatic demonstration of the strength of the notion of an equivalent control will be given later when the effects of plant/model mismatches will be examined.

Suppose the double integrator in equation (1.1) is a linear approximation of the real system on which the control law is to be implemented. Suppose that the real system is in fact a pendulum, formed from a light rod and a heavy mass, as shown in Figure 1.9.

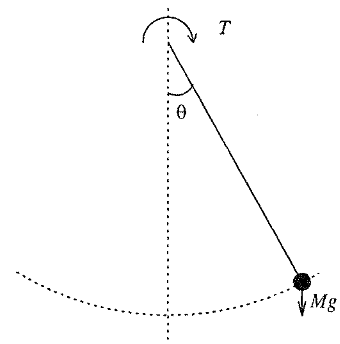

Figure 1.9: Schematic of a pendulum

The variable θ represents the angular displacement from the vertical and T represents a torque applied at the point of suspension, which will be considered to be the control input to the system. Ignoring the effects of friction, the system can be represented mathematically as

$$\ddot{\theta}(t) = -\frac{l}{g}\sin\theta(t) + \frac{1}{Ml^2}u(t) \tag{1.18}$$

where M represents the mass of the bob, l the length of the rod and g the acceleration due to gravity. By appropriate scaling, the essential dynamics of the system are captured by

$$\ddot{y}(t) = -a_1 \sin y(t) + u(t) \tag{1.19}$$

where a_1 is a positive scalar. Equation (1.19) will subsequently be referred to as the *normalised pendulum equation* or *pendulum system.* The double integrator of (1.1) may therefore be considered to be a linear approximation of the normalised pendulum equation which is obtained from ignoring the nonlinear sine term. The phase portrait of the closed-loop system obtained from using the control law (1.7) in the normalised pendulum equation (1.19), when $a_1 = 0.25$ and the initial conditions are $\dot{y} = 0$ and $y = 1$, is shown in Figure 1.10. (For comparison the dotted line represents the phase portrait of the nominal closed-loop system when $a_1 = 0$).

[2]A transfer function $G(s)$, where s is the Laplace variable, is strictly proper if the order of the numerator polynomial is strictly less than the denominator polynomial and thus $G(jw) \to 0$ as $w \to \infty$.

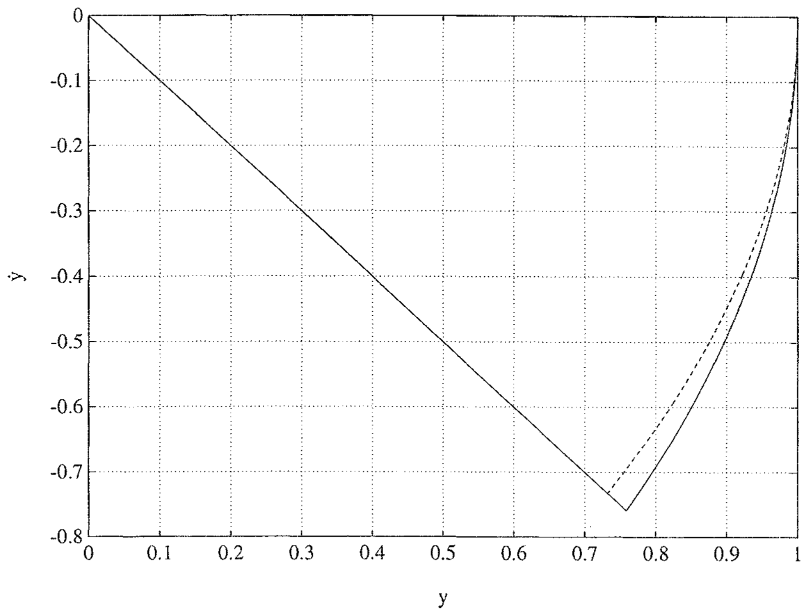

Figure 1.10: Controlled pendulum

The key result is that, in finite time, the phase portrait intercepts the sliding surface $\mathcal{L}_s$ and is forced to remain there. The significance of this is that, once ideal sliding is established, the double-integrator system and the normalised pendulum behave in an identical fashion, namely

$$\dot{y}(t) = -my(t) \tag{1.20}$$

An alternative interpretation is that the effect of the nonlinear term $a_1 \sin y(t)$, which may be construed as a disturbance or uncertainty in the nominal double-integrator system, has been completely rejected. As such, the closed-loop system is said to be *robust*, i.e. it is insensitive to mismatches between the model used for control law design and the plant on which it will be implemented.

The idea of completely rejecting a disturbance or uncertainty is decidedly different from other control approaches such as $\mathcal{H}_2$ and $\mathcal{H}_\infty$. These linear methodologies attempt to *minimise* in some sense the transfer functions relating the disturbances to the outputs of interest. Indeed, at first, it is difficult to imagine how the effect of an unknown disturbance can be *cancelled*. It is at this point that the concept of the equivalent control can be used to provide insight. Arguing as before, once an ideal sliding motion has been attained, $s(t) = 0$ and $\dot{s}(t) = 0$ for all subsequent time. From equations (1.8) and (1.19) it follows that

$$\dot{s}(t) = m\dot{y}(t) - a_1 \sin y(t) + u(t) \tag{1.21}$$

The equivalent control is obtained by equating the expression in (1.21) to zero, resulting in

$$u_{eq}(t) = -m\dot{y}(t) + a_1 \sin y(t) \tag{1.22}$$

This expression captures precisely the uncertainty or disturbance in the closed-loop system, in this case the $a_1 \sin y(t)$ term, and cancels its effect. Of course the control action applied to the plant *does not* utilise any knowledge of the uncertainty. Indeed, as shown in Figure 1.11, the control action appears no different from that used in the nominal double-integrator plant (Figure 1.7).

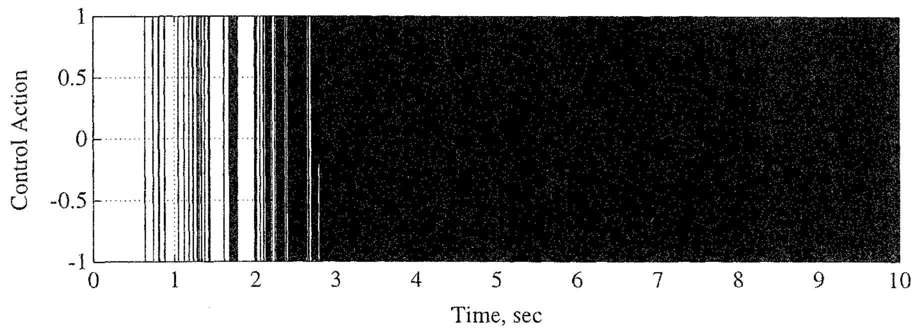

Figure 1.11: Applied control action

However, passing the control action in Figure 1.11 through the low pass filter given in (1.17) provides the average applied control signal shown in Figure 1.12. This is quite different to the average control signal obtained previously and, as expected, is visually identical to the equivalent control from (1.22), shown as a dotted line.

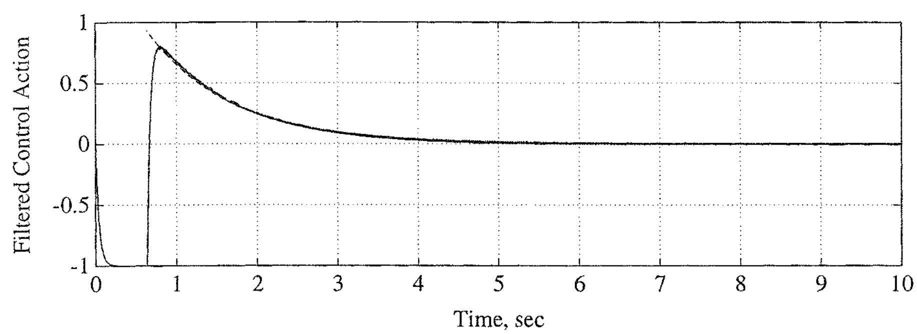

Figure 1.12: Filtered control action compared to the equivalent control

Apart from the robustness properties exhibited by the sliding motion, another benefit accruing from this situation is that the pendulum is forced to behave as a first-order system. This guarantees that no overshoot will occur when attempting to regulate the pendulum from an arbitrary initial displacement to the origin (provided sliding can be attained quickly enough). This can be seen from Figure 1.13, which displays the exponential decay characteristic typical of a first-order system. The two properties of an ideal sliding motion described previously, namely disturbance rejection and order reduction, are the key properties that have motivated the study of controllers which induce sliding motions.

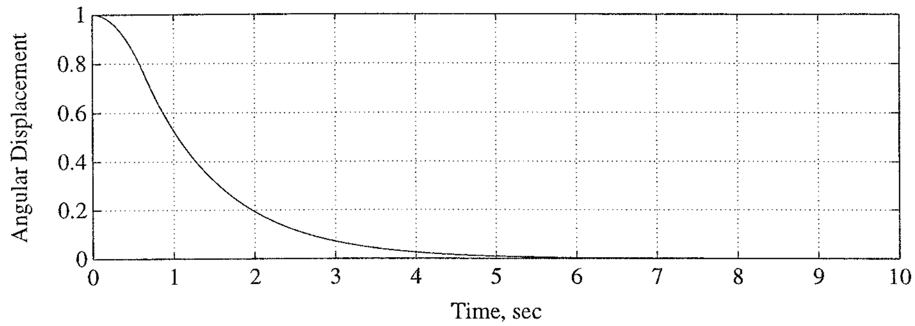

Figure 1.13: Evolution of displacement with respect to time

1.3 DIFFERENT CONTROLLER DESIGNS

Consider the pendulum example from the previous section together with the control law in (1.7) and consider the response of the system to large initial displacements. Figure 1.14 represents a plot of the switching function with initial conditions $y = 3$ and $\dot{y} = 0$.

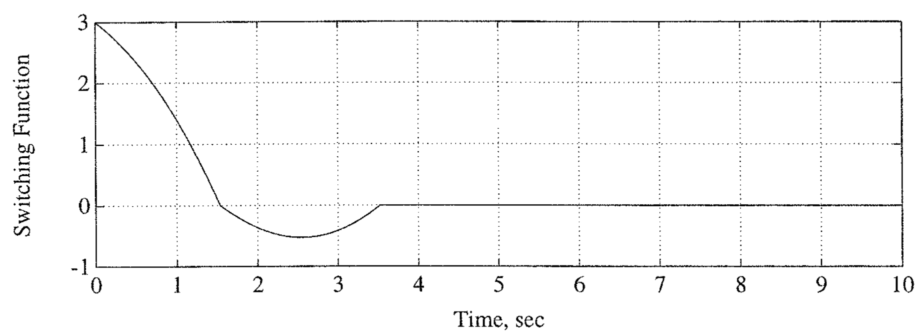

Figure 1.14: Switching function with respect to time

It can be seen that at approximately 1.5 seconds the trajectory intercepts the switching line $\mathcal{L}_s$ but insufficient control energy is available to maintain a sliding motion. The sign of the control law switches, the trajectory pierces the switching line and moves away before intercepting the line again at approximately 3.5 seconds, at which point sliding takes place. This result is not perhaps surprising since the initial conditions correspond to releasing the pendulum from near the upward vertical which will intuitively generate larger angular velocities. This also agrees with the theory presented earlier since even for the nominal double integrator, a sliding motion could only be guaranteed in the region of the phase plane for which the angular velocity is less than $1/m$. The result in this case is that the pendulum crosses the downward vertical before a sliding motion can be established and thus overshoot occurs.

Since the key properties of robustness and order reduction are only obtained once sliding is induced, from the perspective of control law design, the time taken to induce sliding should be minimised, and the region in which sliding takes place maximised. If the magnitude of the control signal is limited to the range ± 1 then the bang-bang controller given in (1.7) may well represent a legitimate control scheme. Otherwise it is instructive to consider modifying the control law given in (1.7), through the inclusion of a linear feedback component, to attempt to ensure that once the trajectories reach the sliding surface they are forced to remain there.

A candidate control structure is given by

$$u(t) = l_1 y(t) + l_2 \dot{y}(t) - \rho \operatorname{sgn}(s(t)) \tag{1.23}$$

where l_1, l_2 and ρ represent scalars yet to be designed. The intention is to choose the three parameters in (1.23) so that the inequality $s\dot{s} < 0$ is always satisfied for the normalised pendulum equation. From the definition of the switching function given in (1.8) it follows that

$$s\dot{s} = s(m\dot{y} + \ddot{y}) = s(m\dot{y} - a_1 \sin(y) + u) \tag{1.24}$$

Substituting for the control action in equation (1.24) gives

$$s\dot{s} = s(m\dot{y} - a_1 \sin(y) + l_1 y + l_2 \dot{y} - \rho \operatorname{sgn}(s)) \tag{1.25}$$

By choosing $l_1 = 0$ and $l_2 = -m$ it follows that

$$s\dot{s} = -s a_1 \sin(y) - \rho |s| < |s|(a_1 - \rho)$$

Thus finally, by choosing $\rho > a_1 + \eta$ where η is a positive design scalar, the inequality

$$s\dot{s} < -\eta |s| \tag{1.26}$$

is established. In the literature this is referred to as the *η-reachability condition.* The previous control law only satisfied the reachability condition in a region of the phase plane; the control law in (1.23) guarantees that, whenever the sliding surface is reached, an ideal sliding motion takes place. A motion such as the one obtained in Figure 1.14, with the trajectory piercing the switching surface, is now no longer possible. It should be noted that the linear component corresponds exactly to the nominal equivalent control expression given in (1.16). This is by no means accidental since, ignoring the nonlinear terms in the preceding analysis, the linear component has been chosen to ensure that $\dot{s}(t) = 0$.

Using the control law

$$u(t) = -m\dot{y}(t) - \rho \operatorname{sgn}(s(t)) \tag{1.27}$$

with $m = 1$ and $\rho = 1$, and the same initial conditions $y = 3$ and $\dot{y} = 0$, the switching function associated with the simulated closed-loop response is given in Figure 1.15. It reveals that no piercing of the sliding surface occurs, although it could be argued that the time taken to reach the sliding surface is a little slow. This is reflected in the angular response given in Figure 1.16. Sliding is achieved early enough so that the approach to the downward vertical is governed by the reduced-order motion (and so no overshoot can occur), but the closed-loop performance is unduly sluggish.

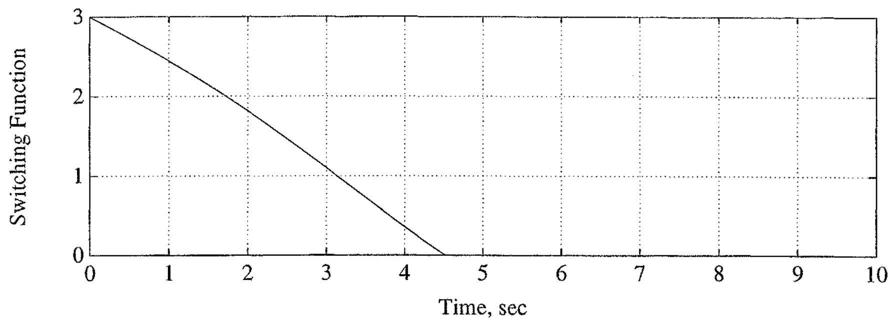

Figure 1.15: Switching function with respect to time

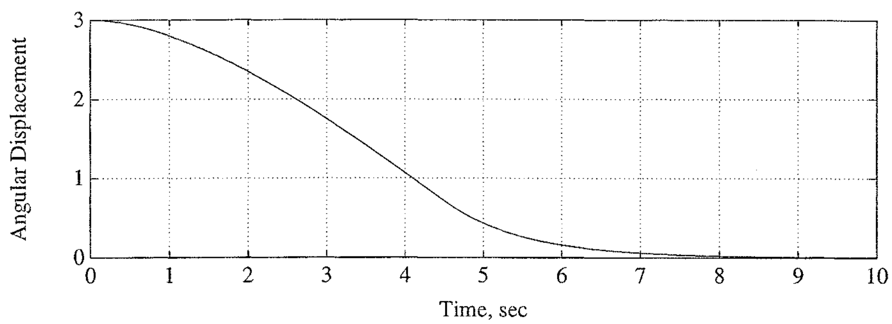

Figure 1.16: Angular displacement with respect to time

The most serious objection to the control law given earlier is perhaps that choosing $\rho = 1$ is rather conservative in the sense that it need only be greater than 0.25 for the η-reachability condition to be satisfied. A lower value of ρ would reduce the amplitude of the high frequency switching, which is advantageous from the point of view of limiting wear and tear on the actuators. Unfortunately, retaining the same linear component and reducing the scalar ρ results in even slower attainment of the sliding surface. To overcome this difficulty, a judicious modification to (1.27) is to add the term $-\Phi s$, where Φ is a positive design scalar, so that $l_1 = -\Phi m$ and $l_2 = -(m + \Phi)$ making

$$u(t) = -(m+\Phi)\,\dot{y}(t) - \Phi m\,y(t) - \rho\,\mathrm{sgn}(s(t)) \tag{1.28}$$

Arguing as before, it can be established that with this control law the inequality

$$s\dot{s} \le -\Phi s^2 - \eta|s| \tag{1.29}$$

is satisfied. Consequently, since $\Phi s^2 \ge 0$, an η-reachability condition has been established and a sliding motion will take place. By ignoring the nonlinear term in (1.29) it follows that

$$\frac{d}{dt}|s(t)| \le -\Phi|s(t)|$$

which implies

$$|s(t)| \le |s(0)|e^{-\Phi t}$$

where $|s(0)|$ represents the initial distance away from the sliding surface. The parameter Φ can thus be seen to affect the rate at which the sliding surface is attained. As a result of this modification, ρ can be chosen as small as possible (to reduce the amplitude of the switching) and Φ can be chosen to determine the time taken to attain sliding.

The following simulation results are obtained from using the control law in (1.28) with $\Phi = 1$ and $\rho = 0.3$ and assuming, as before, the initial conditions $\dot{y} = 0$ and $y = 3$. Figure 1.17 represents the switching function as a function of time and demonstrates that the time to reach the sliding surface has been greatly reduced.

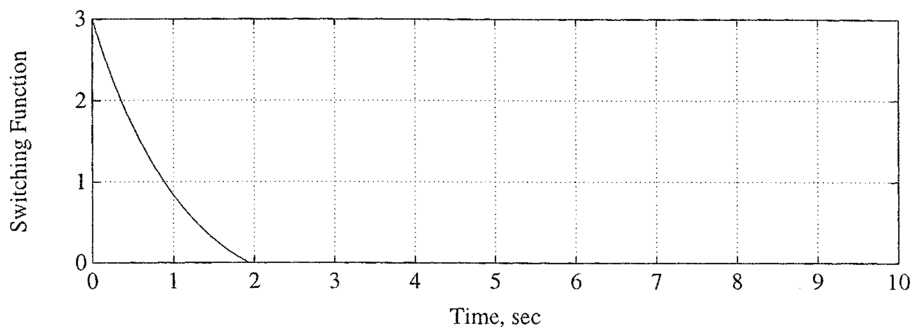

Figure 1.17: Switching function with respect to time

To obtain a faster response, the value of Φ can be increased assuming that sufficient control energy is available. As argued earlier, the reaching-time behaviour displayed in Figure 1.17 is preferable to Figure 1.15 since all the robustness and order reduction properties only occur once the surface has been attained. The corresponding angular displacement is given in Figure 1.18.

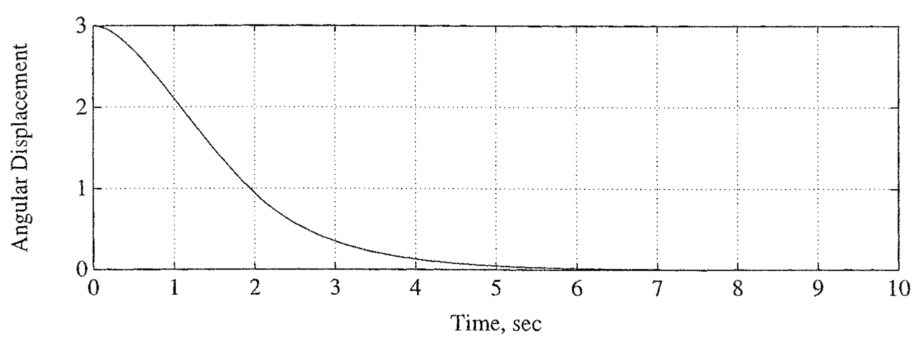

Figure 1.18: Angular displacement with respect to time

Here again the settling time is much improved when compared with the one obtained in Figure 1.16. The big advantage of the controller given in (1.28) over (1.27) is that the control signal is much less aggressive in the sense that the amplitude of the switching is now ± 0.3 as shown in Figure 1.19.

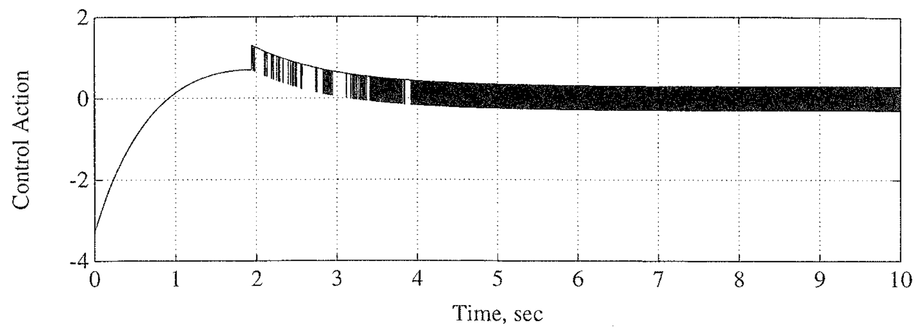

Figure 1.19: Evolution of control action with respect to time

In the nominal double-integrator case with $\rho = 0$ in equation (1.28), it follows that the closed-loop system satisfies

$$\ddot{y}(t) + (m + \Phi)\,\dot{y}(t) + \Phi m\, y(t) = 0 \tag{1.30}$$

This represents a stable motion with poles at $\{-\Phi, -m\}$. It was established earlier that the pole at $-\Phi$ corresponds to the rate at which the sliding surface is attained. The other pole, located at $-m$, corresponds to the pole of the sliding motion. It can thus be argued that the linear part of the control action establishes a sliding mode for the nominal system whilst the discontinuous component counteracts the effects of the uncertainty or nonlinearity.

1.4 PSEUDO-SLIDING WITH A SMOOTH CONTROL ACTION

In certain problems, such as control of electric motors and power converters, the control action is naturally discontinuous and sliding mode ideas can be used to obtain extremely high performance.[3] Although the control signal obtained from Figure 1.19 is preferable to the earlier designs in terms of its chattering behaviour, in many situations such a control signal would still not be considered acceptable. A natural solution is to attempt to smooth the discontinuity in the signum function to obtain an arbitrarily close but continuous approximation. One possible approximation is the sigmoid-like function

$$\nu_\delta(s) = \frac{s}{(|s| + \delta)} \tag{1.31}$$

where δ is a small positive scalar, which is shown in Figure 1.20. It can be visualised that as $\delta \to 0$, the function $\nu_\delta(\cdot)$ tends pointwise to the signum function. The variable δ can be used to trade off the requirement of maintaining ideal performance with that of ensuring a smooth control action.

Assuming the same initial conditions as in the previous section and using the control law given in (1.28) with $\nu_\delta(s)$ replacing $\mathrm{sgn}(s)$ with $\delta = 0.005$, the closed-loop response given below is obtained.

[3] For details see Utkin (1992).

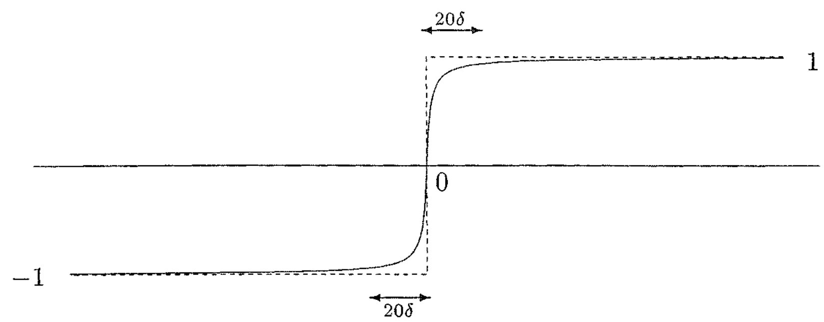

Figure 1.20: A differentiable approximation of the signum function

The angular displacement (Figure 1.21) is indistinguishable from Figure 1.18. The key effect, however, is that the control action is smooth, as shown in Figure 1.22.

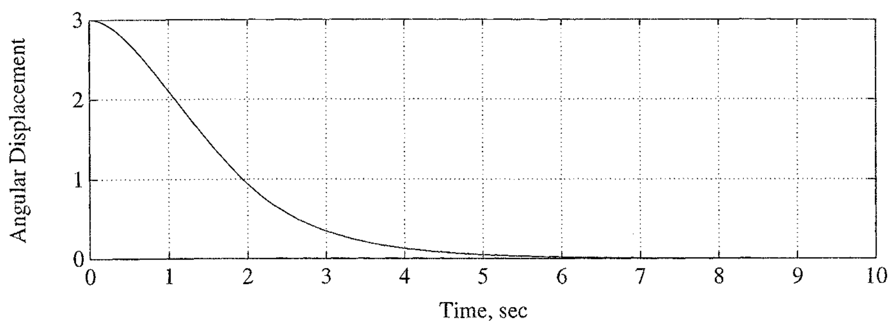

Figure 1.21: Angular displacement with respect to time

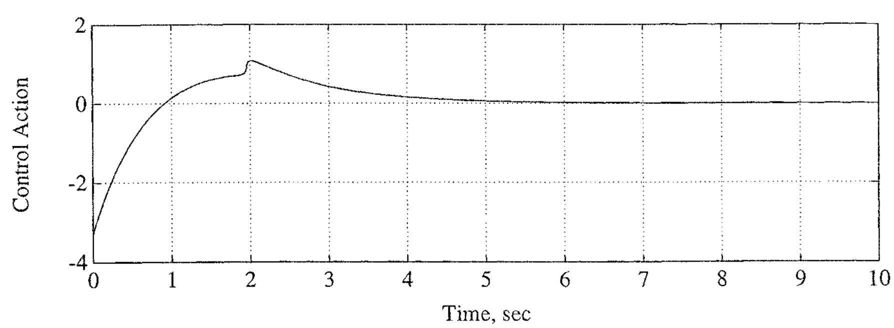

Figure 1.22: Evolution of control action with respect to time

Such continuous approximations enable 'sliding mode' controllers to be utilised in situations where high frequency chattering effects would be unacceptable. It should be stressed that ideal sliding no longer takes place: the continuous control action only drives the states to a neighbourhood of the switching surface. However, arbitrarily close approximation to ideal sliding can be obtained by making δ small. In the literature this is often referred to as *pseudo-sliding.*

1.5 A STATE-SPACE APPROACH

Phase plane analysis is an effective way of analysing the second-order systems considered so far. In future, however, multivariable systems of high order will need to be analysed and therefore a more general framework must be established. The state-space approach pioneered in the 1960s will provide an excellent means of accomplishing this objective.

For the purposes of illustration, consider once again the double integrator in (1.1). If a new vector variable

$$x \triangleq \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$$

is introduced then equation (1.1) can be written in *state-space* form as

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{1.32}$$

The switching function from (1.8) can also be conveniently expressed in matrix terms as

$$s(y, \dot{y}) = Sx(t) \tag{1.33}$$

where

$$S = \begin{bmatrix} m & 1 \end{bmatrix}$$

The normalised pendulum can also be written in state-space form as

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) - \begin{bmatrix} 0 \\ 1 \end{bmatrix} a_1 \sin x_1(t) \tag{1.34}$$

In this representation it can be seen that the nonlinearity or uncertainty represented by $\sin(x_1)$ acts in the 'channel' of the input – i.e. in the second of the coupled pair of differential equations. It will be proved in Chapter 3 that VSCS with a sliding mode have the ability to completely reject the effect of bounded uncertainty acting in the input channels – which is referred to as *matched uncertainty.* This effect was of course noted earlier for the normalised pendulum. Uncertainty which does not act in the input channels is referred to as *unmatched uncertainty.*

In the remainder of the book, the uncertain linear time invariant system with m inputs given by

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t, x, u) \tag{1.35}$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $f(\cdot)$ is an unknown bounded function, will be studied. Switching functions of the form

$$s(x) = Sx \tag{1.36}$$

where $S \in \mathbb{R}^{m \times n}$ will be used to induce a sliding motion on the hyperplane

$$\mathcal{S} = \{x \in \mathbb{R}^n \ : \ s(x) = 0\} \tag{1.37}$$

Before formally exploring in a general multivariable context the properties of sliding modes, it is necessary to review some properties of linear systems and ideas of stability for nonlinear systems; these aspects are fundamental to the analysis in the remainder of the book.

1.6 NOTES AND REFERENCES

The earliest work in English on the properties of sliding modes appeared in the late 1970s in the form of two books; Itkis (1976) and Utkin (1992). References to the early work, pioneered mainly in the former USSR, are given in Utkin (1992). VSCS concepts have subsequently been utilised in the design of robust regulators, model-reference systems, adaptive schemes, tracking systems, state observers and fault detection strategies. Many of these ideas will be examined later in the book, where more extensive references will be given.

A recent survey paper which gives many references to the various application areas in which sliding mode ideas have been utilised is Hung *et al.* (1993).

The concept of equivalent control is attributed to Utkin (1977). A rigorous interpretation of the relationship between the equivalent control law and the low frequency components of a discontinuous control action maintaining a sliding motion is described in Utkin (1992). The η-reachability condition stems from the work of Slotine (1984).

The control law described in equation (1.28) is essentially the single-input simplification of the control law proposed by Ryan & Corless (1984). This will be discussed at length in Chapter 3.

All the simulations in the chapter have been performed using the Runge–Kutta integration routines in the SIMULINK libraries of MATLAB.

The example of the VSCS for the double integrator in Section 1.1 is taken from Utkin (1977). The pendulum example has been used by many authors to demonstrate various nonlinear phenomenon; see for example Khalil (1992).

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