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Dissipativity based fault detection for 2D Markov jump systems with asynchronous modes[☆]

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ABSTRACT

The problem of fault detection for two-dimensional (2D) Markov jump systems characterized in the form of Roesser model is considered in this paper. An asynchronous fault detection filter is designed to produce a residual signal. The fault detection filter changes from one mode to another asynchronously with the plant's transitions. More specifically, the filter's mode transitions depend on the plant's mode through some conditional probabilities. Moreover, the transition probabilities of the plant and the conditional probabilities are only partially accessible, which appears more practical in real application systems. Under such a framework, sufficient conditions are developed to ensure the asymptotic mean square stability and $(\mathcal{Q}, \mathcal{R}, \mathcal{S})$-$\mu$-dissipativity of the overall fault detection system. The parameters in the fault detection filter are given in terms of solutions of an optimization problem. Finally, some simulations are carried out to demonstrate the validity of the presented design techniques.

1. Introduction

Markov jump system (MJS) is well-known as an important member in the domain of stochastic systems, which has a multi-mode feature, changing from one mode to another according to some transition probabilities. MJS appears to be quite effective to model complicated dynamical plants, such as systems with abrupt changes in its parameters or structure, e.g., the Markovian jumping genetic regulatory networks (Cui, Liu, & Wang, 2017). Consequently, MJSs have been given intense attention and enjoyed a great prosperity during the last few decades. To mention a few, the main contribution of Li and Ugrinovskii (2007) is a sufficient and necessary condition for the existence of an output feedback controller for MJSs. The study in Costa and de Saporta (2017) has led to a new linear filter with optimal mean square estimation error for Markov jump linear systems. Several theoretical results concerning optimal H_2 sampled-data control have been obtained in Geromel and Gabriel (2015). Note that, in Geromel and Gabriel (2015) and Li and Ugrinovskii (2007), the transition probabilities or transition rates of the Markov chain are fully accessible, which are sometimes not attainable in practical situations. In Luan, Zhao, and Liu (2013), uncertain transition probabilities described by a Gaussian stochastic process have been taken into account when investigating H_∞ control problem for MJSs. And in Zhang, Boukas, and Lam (2008), the considered MJSs include only partially known transition probabilities.

On the other hand, there is increasing concern for two-dimensional (2D) systems over these years since 2D systems can represent a good number of practical processes in the fields such as modeling of partial differential equations (Marszalek, 1984) and image processing (Roesser, 1975). To describe 2D systems, several mathematical models have been proposed, including the Roesser model (Roesser, 1975) and Fornasini–Marchesini model (Fornasini & Marchesini, 1976). The Fornasini–Marchesini model was proposed to design 2D digital filter, which uses a single vector to represent a function of two independent variables. The Roesser model was firstly developed for studying image processing. Different from the Fornasini–Marchesini model, the state in Roesser model is composed of two sub-vectors denoting horizontal state and vertical state, respectively. Since 2D system is not a straightforward generation of 1D system, the existing achievements for 1D system cannot be extended to 2D systems directly. Hence, it is not trivial to restudy the analysis and synthesis for 2D system. A fair number of works have been reported in literature. Some results on stability of 2D systems can be found

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in El-Agizi and Fahmy (1979), Kurek (1995) and Knorn and Middleton (2013). The work (Ahn, Shi, & Basin, 2015) has carried out studies on the problems of dissipative control and filtering for 2D systems based on Roesser model. Using linear matrix inequality (LMI) method, Du, Xie, and Soh (2001) have investigated model reduction problem for 2D systems. Recently, 2D MJSs, a kind of 2D systems with a Markov jumping attribute, are entering researchers' horizon. For example, control and filtering problems have been considered for 2D MJSs in Hien and Trinh (2017), Wu, Shi, Gao, and Wang (2008), and Wu, Shen, Shi, Shu, and Su (2019). However, overall, the research on 2D systems is not sufficient yet, and calls for further development.

It is possible that unexpected changes and abnormalities will take place in engineering systems. Once abnormalities happen, they will inevitably undermine the system performance and even result in catastrophic situations. Therefore, fault detection is an emergent and challenging problem to be settled, especially for security-critical applications, such as power networks, vehicle systems and aero engines. Generally speaking, fault detection helps check whether there is abnormality or fault and find out when the fault happens by generating a residual signal and evaluating it by means of a residual function. There are quite a lot of contributions concerning the fault detection problem. In 1976, the authors of Willsky (1976) conducted a survey on fault detection and proposed the concept of analytical redundancy which is critical for model-based fault detection. In Li, Gao, Shi, and Lam (2016), an observer-based fault detection scheme has been proposed for fuzzy systems with sensor fault. Authors of He, Wang, and Zhou (2009) have investigated the robust fault detection problem for networked systems, where the network status varies in a Markovian style. A fault detection filter has been designed for 2D MJSs in Wu, Yao, and Zheng (2012), with which the overall system is asymptotically mean square stable with a generalized H_2 performance.

It is worth noting that, in the aforementioned works such as Costa and de Saporta (2017), Li and Ugrinovskii (2007), Wu et al. (2008), Wu et al. (2012), Zhang et al. (2008), the filter or controller therein is synchronous with the plant, i.e., their transitions are under the control of the same Markov parameter. Then, a question arises that how the controller or filter obtains the precise mode information of the plant. As a matter of fact, it is rather difficult to realize synchronization in real applications. One simple method to circumvent this difficulty is to project a mode-independent filter or controller, such as in Dolgov and Hanebeck (2017) and de Souza, Trofino, and Barbosa (2006). However, the disadvantage of mode-independent methods is obvious, namely, the mode of the plant will be completely ignored even if part of the mode information is available. Taking these into consideration, researchers begin to focus more on asynchronous issues. For example, in Costa, Fragoso, and Marques (2005), Costa, Fragoso, and Todorov (2015), Oliveira and Costa (2018), Shen, Wu, Shi, Shu, and Karimi (2019), Todorov, Fragoso, and Costa (2018), Wu, Shi, Su, and Chu (2014) Wu, Shi, Shu, Su, and Lu (2017), and Zhang, Zhu, Shi, and Zhao (2015), there are some works that investigate asynchronization issues. In Wu et al. (2014), the asynchronous filter proposed is under the control of a piecewise homogeneous Markov chain, whose transition probabilities keep time-invariant for each mode of the original system. Using the same description method as in Wu et al. (2014), Zhang et al. (2015) have investigated the resilient H_∞ filtering issue in asynchronous situations. Deserved to be mentioned, Shen et al. (2019) and Wu et al. (2017) have introduced hidden Markov model to describe the mismatching or asynchronization between the modes of the controller and the plant, where the asynchronization is induced by defective network transmission. In fact, the hidden Markov model $(\theta_k, \hat{\theta}_k)$ has been discussed early in the work (Costa et al., 2005), where the controller's mode $\hat{\theta}_k$ was seen as an estimate of the plant's mode θ_k. In Costa et al. (2015), Oliveira and Costa (2018) and Todorov et al. (2018), this model has been employed to address H_2, H_∞ and mixed H_2/H_∞ control problems from the detector-based perspective. However, as far as we know, there is no report on asynchronous fault detection filtering for 2D MJSs yet, which inspires the present work.

In this paper, we concentrate on designing an asynchronous fault detection filter for 2D MJSs. The considered 2D system is in the form of Roesser model. In our framework, the key point appears in a hidden Markov model describing the asynchronous phenomenon between the plant and the fault detection filter. In this asynchronization model, a conditional probabilistic characterization is used to describe the relationship of the plant's mode and the filter's mode. Besides, part of the transition probabilities and conditional probabilities is unknown. It is worth noting that our framework is a very unified one since it covers synchronous, mode-independent and asynchronous cases simultaneously, and is also applicable to extreme cases with transition probabilities and conditional probabilities completely known or completely unknown. Based on Lyapunov function method and Projection Lemma, the fault detection filter is given by solutions of a set of LMIs, and the overall fault detection system is asymptotic mean square stable and $(\mathcal{Q}, \mathcal{R}, \mathcal{S})$-$\mu$-dissipative. Finally, the influence of asynchronization level and the amount of available probabilities on the dissipativity performance are further discussed through an example related to Darboux equation. The simulation provides strong evidence for the validity of the presented design techniques.

2. Preliminaries

In this paper, our attention is focused on the following 2D MJS characterized by Roesser model:

$$\begin{cases} x_{i,j}^{\dagger} = A_{\alpha_{i,j}} x_{i,j} + B_{\alpha_{i,j}} u_{i,j} + D_{\alpha_{i,j}} w_{i,j} + E_{\alpha_{i,j}} f_{i,j} \\ y_{i,j} = C_{\alpha_{i,j}} x_{i,j} + F_{\alpha_{i,j}} u_{i,j} + G_{\alpha_{i,j}} w_{i,j} + H_{\alpha_{i,j}} f_{i,j} \end{cases} \tag{1}$$

where

$$x_{i,j}^{\dagger} = \begin{bmatrix} x_{i+1,j}^{h} \\ x_{i,j+1}^{v} \end{bmatrix}, \quad x_{i,j} = \begin{bmatrix} x_{i,j}^{h} \\ x_{i,j}^{v} \end{bmatrix}.$$

The state of system (1) evolves in two directions. $x_{i,j}^h \in \mathbb{R}^{n_h}$ and $x_{i,j}^v \in \mathbb{R}^{n_v}$ denote the states in horizontal and vertical directions, respectively. $y_{i,j} \in \mathbb{R}^{n_y}$ is system's measurement. $w_{i,j} \in \mathbb{R}^{n_w}$ denotes an exogenous disturbance input. $u_{i,j} \in \mathbb{R}^{n_u}$ and $f_{i,j} \in \mathbb{R}^{n_f}$ respectively represent an input vector and a fault vector, which are both deterministic. $w_{i,j}$, $u_{i,j}$ and $f_{i,j}$ all belong to $l_2\{[0,\infty), [0,\infty)\}$. $A_{\alpha_{i,j}}, B_{\alpha_{i,j}}, C_{\alpha_{i,j}}, D_{\alpha_{i,j}}, E_{\alpha_{i,j}}, F_{\alpha_{i,j}}, G_{\alpha_{i,j}}, H_{\alpha_{i,j}}$ are real-valued matrices which are appropriately dimensioned. Note that these system matrices are related to the Markov parameter $\alpha_{i,j}$ taking values in the limited set $\mathscr{L}_1 = \{1, 2, \ldots, L_1\}$. This Markov process follows a mode transition probability matrix $\Psi = [\psi_{pq}]$ with ψ_{pq} defined as

$$\Pr\{\alpha_{i+1,j} = q | \alpha_{i,j} = p\} = \Pr\{\alpha_{i,j+1} = q | \alpha_{i,j} = p\} = \psi_{pq}, \tag{2}$$

where ψ_{pq} has restrictions $\psi_{pq} \geq 0$ and $\sum_{q=1}^{L_1} \psi_{pq} = 1$, $\forall p, q \in \mathscr{L}_1$.

When performing fault detection, the first step and also the most important step is to generate a residual signal which is used to indicate whether a fault takes place or not in this system. Hence, the residual signal must be closely correlated with the fault signal. In this paper, we are dedicated to designing a following fault detection filter:

$$\begin{cases} \hat{x}_{i,j}^{\dagger} = \hat{A}_{\beta_{i,j}} \hat{x}_{i,j} + \hat{B}_{\beta_{i,j}} y_{i,j} \\ z_{i,j} = \hat{C}_{\beta_{i,j}} \hat{x}_{i,j} \end{cases} \tag{3}$$

where

$$\hat{x}^{\dagger}_{i,j} = \begin{bmatrix} \hat{x}^h_{i+1,j} \\ \hat{x}^v_{i,j+1} \end{bmatrix}, \ \hat{x}_{i,j} = \begin{bmatrix} \hat{x}^h_{i,j} \\ \hat{x}^v_{i,j} \end{bmatrix}.$$

In this fault detection filter, $\hat{x}^h_{i,j} \in \mathbb{R}^{n_h}$ and $\hat{x}^v_{i,j} \in \mathbb{R}^{n_v}$ are its horizontal state and vertical state, respectively. $z_{i,j} \in \mathbb{R}^{n_f}$ is a residual signal that we are interested in. $\hat{A}_{\beta_{i,j}}$, $\hat{B}_{\beta_{i,j}}$ and $\hat{C}_{\beta_{i,j}}$ are some matrices to be figured out. We can note that the fault detection filter (3) depends directly on mode $\beta_{i,j}$ rather than $\alpha_{i,j}$. In other words, the mode of fault detection filter (3) is asynchronous with that of system (1). The parameter $\beta_{i,j}$ takes values in the limited set $\mathscr{L}_2 = \{1, 2, \ldots, L_2\}$. Its transitions depend on system mode $\alpha_{i,j}$ according to the following conditional probability

$$\Pr\{\beta_{i,j} = t | \alpha_{i,j} = p\} = \theta_{pt}. \tag{4}$$

It is obvious that $\theta_{pt} \in [0, 1]$, $\sum_{t=1}^{L_2} \theta_{pt} = 1$, $\forall p \in \mathscr{L}_1$, $t \in \mathscr{L}_2$. We refer to $\Theta = [\theta_{pt}]$ as a conditional probability matrix.

Remark 1. Note that an asynchronous fault detection filter is considered in this work. The asynchronization finds expression in the hidden Markov model $(\alpha_{i,j}, \beta_{i,j}, \Psi, \Theta)$ which fully reflects the relationship of system (1)'s mode and the fault detection filter (3)'s mode through conditional probability (4).

In this work, we consider the case that the entries of transition probability matrix Ψ and conditional probability matrix Θ are only partially accessible, i.e., some entries of Ψ and Θ are unknown. For example, Ψ and Θ may look like

$$\Psi = \begin{bmatrix} ? & ? & ? & \psi_{14} \\ \psi_{21} & \psi_{22} & ? & ? \\ \psi_{31} & ? & \psi_{33} & ? \\ \psi_{41} & ? & ? & ? \end{bmatrix}, \ \Theta = \begin{bmatrix} \theta_{11} & ? & \theta_{13} & ? \\ ? & ? & \theta_{23} & \theta_{24} \\ ? & \theta_{32} & \theta_{33} & ? \\ \theta_{41} & ? & ? & \theta_{44} \end{bmatrix} \tag{5}$$

if system (1) and fault detection filter (3) have four modes, where "?" denotes that the corresponding probability is unavailable. For $p \in \mathscr{L}_1$, we denote $\mathscr{L}_1 = \mathscr{L}^p_{1\mathcal{K}} \cup \mathscr{L}^p_{1\mathcal{U}}$ and $\mathscr{L}_2 = \mathscr{L}^p_{2\mathcal{K}} \cup \mathscr{L}^p_{2\mathcal{U}}$, where

$$\begin{cases} \mathscr{L}^p_{1\mathcal{K}} = \{q : \psi_{pq} \text{ is known}\} \\ \mathscr{L}^p_{1\mathcal{U}} = \{q : \psi_{pq} \text{ is unknown}\} \end{cases} \tag{6}$$

and

$$\begin{cases} \mathscr{L}^p_{2\mathcal{K}} = \{t : \theta_{pt} \text{ is known}\} \\ \mathscr{L}^p_{2\mathcal{U}} = \{t : \theta_{pt} \text{ is unknown}\} \end{cases} \tag{7}$$

In the sequel, for convenience purpose, we will let p, t denote $\alpha_{i,j}$ and $\beta_{i,j}$, respectively, and let q denote $\alpha_{i+1,j}$ or $\alpha_{i,j+1}$. Then, we can denote these matrices in (1) and (3) briefly, for example, $A_{\alpha_{i,j}} \triangleq A_p$, $\hat{A}_{\beta_{i,j}} \triangleq \hat{A}_t$.

In order to enhance the fault detection performance, we will introduce frequency weighting to the spectrum of the fault signal $f_{i,j}$, which can be realized by the system below:

$$\begin{cases} \bar{x}^{\dagger}_{i,j} = \bar{A}\bar{x}_{i,j} + \bar{B} f_{i,j} \\ \bar{f}_{i,j} = \bar{C}\bar{x}_{i,j} \end{cases} \tag{8}$$

where

$$\bar{x}^{\dagger}_{i,j} = \begin{bmatrix} \bar{x}^h_{i+1,j} \\ \bar{x}^v_{i,j+1} \end{bmatrix}, \ \bar{x}_{i,j} = \begin{bmatrix} \bar{x}^h_{i,j} \\ \bar{x}^v_{i,j} \end{bmatrix},$$

$\bar{x}^h_{i,j} \in \mathbb{R}^{n_{\bar{h}}}$ and $\bar{x}^v_{i,j} \in \mathbb{R}^{n_{\bar{v}}}$ denote horizontal and vertical states, respectively. $\bar{f}_{i,j}$ is the result of frequency weighting of $f_{i,j}$. $\bar{A}$, $\bar{B}$ and $\bar{C}$ are some pre-given and appropriately dimensioned matrices.

Next, we can obtain an augmented model including all the states of (1), (3) and (8). The dynamics of the overall fault detection system is as follows:

$$\begin{cases} \tilde{x}^{\dagger}_{i,j} = \tilde{A}_{pt}\tilde{x}_{i,j} + \tilde{B}_{pt}\tilde{u}_{i,j} \\ \tilde{z}_{i,j} = \tilde{C}_t\tilde{x}_{i,j} \end{cases} \tag{9}$$

where

$$\tilde{x}^{\dagger}_{i,j} = \begin{bmatrix} \tilde{x}^h_{i+1,j} \\ \tilde{x}^v_{i,j+1} \end{bmatrix}, \ \tilde{x}_{i,j} = \begin{bmatrix} \tilde{x}^h_{i,j} \\ \tilde{x}^v_{i,j} \end{bmatrix}, \ \tilde{z}_{i,j} = z_{i,j} - \bar{f}_{i,j}$$

$$\tilde{x}^h_{i,j} = \begin{bmatrix} x^h_{i,j} \\ \bar{x}^h_{i,j} \\ \hat{x}^h_{i,j} \end{bmatrix}, \ \tilde{x}^v_{i,j} = \begin{bmatrix} x^v_{i,j} \\ \bar{x}^v_{i,j} \\ \hat{x}^v_{i,j} \end{bmatrix}, \ \tilde{u}_{i,j} = \begin{bmatrix} u_{i,j} \\ w_{i,j} \\ f_{i,j} \end{bmatrix}$$

$$\tilde{A}_{pt} = \Xi\mathring{A}_{pt}\Xi^{\top}, \ \tilde{B}_{pt} = \Xi\mathring{B}_{pt}, \ \tilde{C}_t = \mathring{C}_t\Xi^{\top}$$

$$\mathring{A}_{pt} = \begin{bmatrix} A_p & 0 & 0 \\ 0 & \bar{A} & 0 \\ \hat{B}_t C_p & 0 & \hat{A}_t \end{bmatrix}, \mathring{C}_t = \begin{bmatrix} 0 & -\bar{C} & \hat{C}_t \end{bmatrix}$$

$$\mathring{B}_{pt} = \begin{bmatrix} B_p & D_p & E_p \\ 0 & 0 & \bar{B} \\ \hat{B}_t F_p & \hat{B}_t G_p & \hat{B}_t H_p \end{bmatrix}$$

$$\Xi = \begin{bmatrix} I_{n_h} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{n_{\bar{h}}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{n_h} & 0 \\ 0 & I_{n_v} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{n_{\bar{v}}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{n_v} \end{bmatrix}$$

Note that matrix Ξ satisfies $\Xi\Xi^{\top} = \Xi^{\top}\Xi = I$. We define the boundary condition $(\mathcal{X}_0, \Lambda_0, \Omega_0)$ of system (9) in the following:

$$\begin{cases} \mathcal{X}_0 = \{\tilde{x}^h_{0,j}, \tilde{x}^v_{i,0} | i, j = 0, 1, 2, \ldots\} \\ \Lambda_0 = \{\alpha_{0,j}, \alpha_{i,0} | i, j = 0, 1, 2, \ldots\} \\ \Omega_0 = \{\beta_{0,j}, \beta_{i,0} | i, j = 0, 1, 2, \ldots\} \end{cases} \tag{10}$$

And the zero boundary condition is defined as $\{\tilde{x}^h_{0,j} = 0, \tilde{x}^v_{i,0} = 0 | i, j = 0, 1, 2, \ldots\}$. Furthermore, we have the following assumption concerning $\mathcal{X}_0$.

Assumption 1 (Wu et al. (2019)). Assume that $\mathcal{X}_0$ satisfies:

$$\lim_{K\to\infty} \mathbb{E}\left\{\sum_{k=0}^{K} (\|\tilde{x}^h_{0,k}\|^2 + \|\tilde{x}^v_{k,0}\|^2)\right\} < \infty, \tag{11}$$

where $\|\cdot\|$ denotes Euclidean vector norm.

The last step of fault detection is to perform a residual evaluation process. We define an evaluation function $\Gamma_{a,b}(z)$ and a threshold $\bar{\Gamma}$ as follows:

$$\Gamma_{a,b}(z) = \sqrt{\sum_{i=i_0}^{i_0+a} \sum_{j=j_0}^{j_0+b} z^{\top}_{i,j} z_{i,j}}, \ \bar{\Gamma} = \sup_{w\neq 0, u\neq 0, f=0} \Gamma_{a,b}(z), \tag{12}$$

where i_0 and j_0 are initial evaluation time. We judge whether faults occur or not by comparing $\Gamma_{a,b}(z)$ and $\bar{\Gamma}$. $\Gamma_{a,b}(z) > \bar{\Gamma}$ implies a fault occurs, and then, an alarm will be started. Otherwise, no fault takes place. In the following, we will recall some definitions which are essential for the derivation of our main results.

Definition 1 (Wu et al. (2019)). The overall fault detection system (9) is said to be asymptotically mean square stable if, with $\tilde{u}_{i,j} \equiv 0$, the condition

$$\lim_{i+j\to\infty} \mathbb{E}\{\|\tilde{x}_{i,j}\|^2\} = 0 \tag{13}$$

is satisfied for any boundary condition $(\mathcal{X}_0, \Lambda_0, \Omega_0)$.

The dissipativity theory has been elaborated in Willems (1972). By saying a system is dissipative, we mean that the consumed energy in this system is no more than the supplied energy. Dissipativity is fairly important as it has strong implication on systems' stability. To define $(\mathcal{Q},\ \mathcal{R},\ \mathcal{S})$-$\mu$-dissipativity, we firstly give the following quadratic polynomial as the energy supply rate:

$$W(\tilde{u}_{i,j},\tilde{z}_{i,j}) \triangleq \tilde{z}_{i,j}^{\top}\mathcal{Q}\tilde{z}_{i,j} + 2\tilde{z}_{i,j}^{\top}\mathcal{R}\tilde{u}_{i,j} + \tilde{u}_{i,j}^{\top}\mathcal{S}\tilde{u}_{i,j}, \tag{14}$$

where $\mathcal{Q}$, $\mathcal{R}$ and $\mathcal{S}$ are real matrices, $\mathcal{S}$ is symmetric, $\mathcal{Q}$ is negative semi-definite and can be denoted as $-\mathcal{Q} \triangleq Q^{\top}Q \geq 0$.

Definition 2 (*Ahn et al. (2015)*). Assume that the overall fault detection system (9) is asymptotically mean square stable. Then (9) with $\tilde{u}_{i,j} \in l_2\{[0,\infty),[0,\infty)\}$ is said to be $(\mathcal{Q},\ \mathcal{R},\ \mathcal{S})$-$\mu$-dissipative, if under zero boundary condition, the following condition

$$\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\mathbb{E}\{W(\tilde{u}_{i,j},\tilde{z}_{i,j})\} \geq \mu\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\tilde{u}_{i,j}^{\top}\tilde{u}_{i,j} \tag{15}$$

holds for any positive integers N_1 and N_2, where the scalar $\mu > 0$.

In conclusion, the key point of the current paper is to put forward an asynchronous fault detection method for 2D MJS (1), such that the overall fault detection system (9) is stochastically mean square stable and $(\mathcal{Q},\ \mathcal{R},\ \mathcal{S})$-$\mu$-dissipative.

Remark 2. It should be noted that the framework in this work is rather unified. This can be explained from three aspects:

(1) As mentioned above, the asynchronization expresses itself by virtue of hidden Markov model, which is a unified model covering mode-independent case ($\mathscr{L}_2 = \{1\}$), synchronous case ($\mathscr{L}_1 = \mathscr{L}_2$ and $\theta_{pt} = 1$ for $p = t$) and asynchronous case simultaneously (Wu et al., 2017).
(2) In our work, the transition probability matrix and the conditional probability matrix are both partially accessible, which may look like (5). As a matter of fact, completely known and completely unknown probability matrices are two extreme cases of (5).
(3) The results based on dissipativity theory can be extended to H_∞ ($\mathcal{Q} = -I$, $\mathcal{R} = 0$, $\mathcal{S} = (\mu^2+\mu)I$) or passivity ($\mathcal{Q} = 0$, $\mathcal{R} = I$, $\mathcal{S} = 2\mu I$) based cases by appropriately adjusting $\mathcal{Q}$, $\mathcal{R}$ and $\mathcal{S}$ (Wu, Dong, Su, & Li, 2018).

Therefore, the results derived in this work are quite general.

3. Main results

The main purpose of this section is to propose a design method of the fault detection filter (3) for 2D MJS (1). To begin with, we will propose a sufficient condition ensuring the overall fault detection system (9) is stochastically mean square stable and $(\mathcal{Q},\ \mathcal{R},\ \mathcal{S})$-$\mu$-dissipative.

Theorem 1. *Consider the fault detection system (9) with Assumption 1. System (9) is asymptotically mean square stable and $(\mathcal{Q},\ \mathcal{R},\ \mathcal{S})$-$\mu$-dissipative, if there exist matrices $\Upsilon_p = diag\{\Upsilon_p^h, \Upsilon_p^v\} > 0$, $\Pi_{pt} > 0$, and matrices $\hat{A}_t$, $\hat{B}_t$, $\hat{C}_t$, such that, $\forall p \in \mathscr{L}_1$, $t \in \mathscr{L}_{2\mathcal{U}}^p$,*

$$\Pi_p^{\mathcal{K}} + (1-\theta_p^{\mathcal{K}})\Pi_{pt} - \Upsilon_p < 0 \tag{16}$$

holds, and $\forall p \in \mathscr{L}_1$, $t \in \mathscr{L}_2$, $q \in \mathscr{L}_{1\mathcal{U}}^p$,

$$\begin{bmatrix} -(\Upsilon_p^{\mathcal{K}} + (1-\psi_p^{\mathcal{K}})\Upsilon_q)^{-1} & 0 & \tilde{A}_{pt} & \tilde{B}_{pt} \\ * & -I & Q\tilde{C}_t & 0 \\ * & * & -\Pi_{pt} & -\tilde{C}_t^{\top}\mathcal{R} \\ * & * & * & \mu I - \mathcal{S} \end{bmatrix} < 0 \tag{17}$$

holds, where

$$\Pi_p^{\mathcal{K}} = \sum_{t\in\mathscr{L}_{2\mathcal{K}}^p}\theta_{pt}\Pi_{pt},\ \theta_p^{\mathcal{K}} = \sum_{t\in\mathscr{L}_{2\mathcal{K}}^p}\theta_{pt},$$

$$\Upsilon_p^{\mathcal{K}} = \sum_{q\in\mathscr{L}_{1\mathcal{K}}^p}\psi_{pq}\Upsilon_q,\ \psi_p^{\mathcal{K}} = \sum_{q\in\mathscr{L}_{1\mathcal{K}}^p}\psi_{pq}.$$

Proof. To begin with, we will have some derivative results from (16) and (17) to facilitate the verification of asymptotic mean square stability and $(\mathcal{Q},\ \mathcal{R},\ \mathcal{S})$-$\mu$-dissipativity. We firstly consider the case with $\theta_p^{\mathcal{K}} < 1$ and $\psi_p^{\mathcal{K}} < 1$. We can know that (16) implies

$$\sum_{t\in\mathscr{L}_{2\mathcal{U}}^p}\frac{\theta_{pt}}{1-\theta_p^{\mathcal{K}}}\left\{\Pi_p^{\mathcal{K}} + (1-\theta_p^{\mathcal{K}})\Pi_{pt} - \Upsilon_p\right\} < 0 \tag{18}$$

holds. Then,

$$\sum_{t=1}^{L_2}\theta_{pt}\Pi_{pt} - \Upsilon_p < 0 \tag{19}$$

holds. On the other hand, (17) implies

$$\begin{bmatrix} -(\Upsilon_p^{\mathcal{K}} + (1-\psi_p^{\mathcal{K}})\Upsilon_q)^{-1} & \tilde{A}_{pt} \\ * & -\Pi_{pt} \end{bmatrix} < 0. \tag{20}$$

According to Schur Complement, (20) and (17) are equivalent to

$$\tilde{A}_{pt}^{\top}(\Upsilon_p^{\mathcal{K}} + (1-\psi_p^{\mathcal{K}})\Upsilon_q)\tilde{A}_{pt} - \Pi_{pt} < 0 \tag{21}$$

and

$$\begin{aligned} &\begin{bmatrix} -\Pi_{pt} & -\tilde{C}_t^{\top}\mathcal{R} \\ * & \mu I - \mathcal{S} \end{bmatrix} + \begin{bmatrix} \tilde{C}_t^{\top}Q^{\top} \\ 0 \end{bmatrix}\begin{bmatrix} Q\tilde{C}_t & 0 \end{bmatrix} \\ &+ \begin{bmatrix} \tilde{A}_{pt}^{\top} \\ \tilde{B}_{pt}^{\top} \end{bmatrix}(\Upsilon_p^{\mathcal{K}} + (1-\psi_p^{\mathcal{K}})\Upsilon_q)\begin{bmatrix} \tilde{A}_{pt} & \tilde{B}_{pt} \end{bmatrix} < 0 \end{aligned} \tag{22}$$

respectively. In a similar line as (18), the inequalities (21) and (22) ensure

$$\tilde{A}_{pt}^{\top}\bar{\Upsilon}_p\tilde{A}_{pt} - \Pi_{pt} < 0 \tag{23}$$

and

$$\begin{aligned} &\begin{bmatrix} -\Pi_{pt} & -\tilde{C}_t^{\top}\mathcal{R} \\ * & \mu I - \mathcal{S} \end{bmatrix} + \begin{bmatrix} \tilde{C}_t^{\top}Q^{\top} \\ 0 \end{bmatrix}\begin{bmatrix} Q\tilde{C}_t & 0 \end{bmatrix} \\ &+ \begin{bmatrix} \tilde{A}_{pt}^{\top} \\ \tilde{B}_{pt}^{\top} \end{bmatrix}\bar{\Upsilon}_p\begin{bmatrix} \tilde{A}_{pt} & \tilde{B}_{pt} \end{bmatrix} < 0 \end{aligned} \tag{24}$$

hold, respectively, where $\bar{\Upsilon}_p = \sum_{q=1}^{L_1}\psi_{pq}\Upsilon_q$. Besides, when $\theta_p^{\mathcal{K}} = 1$ and $\psi_p^{\mathcal{K}} = 1$, i.e., Ψ and Θ are completely known, (19), (23), and (24) undoubtedly hold by (16) and (17). We rewrite (24) as

$$\mathscr{M}_{pt} < \begin{bmatrix} \Pi_{pt} & 0 \\ 0 & 0 \end{bmatrix}, \tag{25}$$

where

$$\mathscr{M}_{pt} = \begin{bmatrix} \tilde{C}_t^{\top}Q^{\top}Q\tilde{C}_t & -\tilde{C}_t^{\top}\mathcal{R} \\ * & \mu I - \mathcal{S} \end{bmatrix} + \begin{bmatrix} \tilde{A}_{pt}^{\top} \\ \tilde{B}_{pt}^{\top} \end{bmatrix}\bar{\Upsilon}_p\begin{bmatrix} \tilde{A}_{pt} & \tilde{B}_{pt} \end{bmatrix}.$$

In the sequel, the inequalities (19), (23) and (25) will be useful to carry forward the proof.

Next, we will analyze the asymptotic mean square stability for system (9) with $\tilde{u}_{i,j} \equiv 0$. The Lyapunov function is chosen as $V(i,j) = \tilde{x}_{i,j}^{\top}\Upsilon_p\tilde{x}_{i,j}$. Then, the forward difference of $V(i,j)$ is defined as

$$\Delta V(i,j) = \tilde{x}_{i,j}^{\dagger\top}\Upsilon_q\tilde{x}_{i,j}^{\dagger} - \tilde{x}_{i,j}^{\top}\Upsilon_p\tilde{x}_{i,j}, \tag{26}$$

where $\Upsilon_p = diag\{\Upsilon^h_{\alpha_{i,j}}, \Upsilon^v_{\alpha_{i,j}}\}$ and $\Upsilon_q = diag\{\Upsilon^h_{\alpha_{i+1,j}}, \Upsilon^v_{\alpha_{i,j+1}}\}$. Then, we can obtain that

$$\begin{aligned}&\sum_{i=0}^{i_1}\sum_{j=0}^{j_1}\Delta V(i,j)\\&=\sum_{j=0}^{j_1}\left\{\tilde{x}^{h\top}_{i_1+1,j}\Upsilon^h_{\alpha_{i_1+1,j}}\tilde{x}^h_{i_1+1,j}-\tilde{x}^{h\top}_{0,j}\Upsilon^h_{\alpha_{0,j}}\tilde{x}^h_{0,j}\right\}\\&\quad+\sum_{i=0}^{i_1}\left\{\tilde{x}^{v\top}_{i,j_1+1}\Upsilon^v_{\alpha_{i,j_1+1}}\tilde{x}^v_{i,j_1+1}-\tilde{x}^{v\top}_{i,0}\Upsilon^v_{\alpha_{i,0}}\tilde{x}^v_{i,0}\right\}\\&\geq-\sum_{j=0}^{j_1}\tilde{x}^{h\top}_{0,j}\Upsilon^h_{\alpha_{0,j}}\tilde{x}^h_{0,j}-\sum_{i=0}^{i_1}\tilde{x}^{v\top}_{i,0}\Upsilon^v_{\alpha_{i,0}}\tilde{x}^v_{i,0}\end{aligned}\tag{27}$$

where i_1 and j_1 are arbitrary positive integers. According to the dynamics of system (9) with $\tilde{u}_{i,j}\equiv 0$, $\Delta V(i,j)$ can be computed as

$$\Delta V(i,j)=\tilde{x}^\top_{i,j}(\tilde{A}^\top_{pt}\Upsilon_q\tilde{A}_{pt}-\Upsilon_p)\tilde{x}_{i,j}.\tag{28}$$

We further figure out its expectation as

$$\begin{aligned}\mathbb{E}\{\Delta V(i,j)\}&=\mathbb{E}\{\tilde{x}^\top_{i,j}(\tilde{A}^\top_{pt}\Upsilon_q\tilde{A}_{pt}-\Upsilon_p)\tilde{x}_{i,j}|\tilde{x}_{i,j},\alpha_{i,j}=p\}\\&=\mathbb{E}\left\{\tilde{x}^\top_{i,j}\left(\sum_{t=1}^{L_2}\theta_{pt}\tilde{A}^\top_{pt}\bar{\Upsilon}_p\tilde{A}_{pt}-\Upsilon_p\right)\tilde{x}_{i,j}\right\}\end{aligned}\tag{29}$$

Let $-\eta_1$ be the maximal eigenvalue of $\left(\sum_{t=1}^{L_2}\theta_{pt}\Pi_{pt}-\Upsilon_p\right)$ for all $p\in\mathscr{L}_1$. Note that (19) implies $-\eta_1<0$. It follows from (23) that

$$\begin{aligned}\mathbb{E}\{\Delta V(i,j)\}&<\mathbb{E}\left\{\tilde{x}^\top_{i,j}\left(\sum_{t=1}^{L_2}\theta_{pt}\Pi_{pt}-\Upsilon_p\right)\tilde{x}_{i,j}\right\}\\&\leq-\eta_1\mathbb{E}\{\|\tilde{x}_{i,j}\|^2\}\end{aligned}\tag{30}$$

We add both sides of (30), then,

$$\begin{aligned}&\mathbb{E}\left\{\sum_{i=0}^{i_1}\sum_{j=0}^{j_1}\|\tilde{x}_{i,j}\|^2\right\}\\&\leq-\frac{1}{\eta_1}\mathbb{E}\left\{\sum_{i=0}^{i_1}\sum_{j=0}^{j_1}\Delta V(i,j)\right\}\\&\leq\frac{1}{\eta_1}\mathbb{E}\left\{\sum_{j=0}^{j_1}\tilde{x}^{h\top}_{0,j}\Upsilon^h_{\alpha_{0,j}}\tilde{x}^h_{0,j}+\sum_{i=0}^{i_1}\tilde{x}^{v\top}_{i,0}\Upsilon^v_{\alpha_{i,0}}\tilde{x}^v_{i,0}\right\}\end{aligned}\tag{31}$$

where the second "$\leq$" comes from (27). Let $i_1\to\infty$ and $j_1\to\infty$, then (31) yields that

$$\begin{aligned}&\mathbb{E}\left\{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\|\tilde{x}_{i,j}\|^2\right\}\\&\leq\frac{1}{\eta_1}\mathbb{E}\left\{\sum_{i=0}^{\infty}\tilde{x}^{h\top}_{0,i}\Upsilon^h_{\alpha_{0,i}}\tilde{x}^h_{0,i}+\tilde{x}^{v\top}_{i,0}\Upsilon^v_{\alpha_{i,0}}\tilde{x}^v_{i,0}\right\}\\&\leq\frac{\eta_2}{\eta_1}\mathbb{E}\left\{\sum_{i=0}^{\infty}\|\tilde{x}^h_{0,i}\|^2+\|\tilde{x}^v_{i,0}\|^2\right\}\end{aligned}\tag{32}$$

where η_2 is the maximal eigenvalue of $diag\{\Upsilon^h_{\alpha_{0,i}}, \Upsilon^v_{\alpha_{i,0}}\}$. With Assumption 1, we get that

$$\mathbb{E}\left\{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\|\tilde{x}_{i,j}\|^2\right\}<\infty.\tag{33}$$

Thus, (13) holds. Consequently, the asymptotic mean square stability of system (9) is proved.

We investigate ($\mathcal{Q}$, $\mathcal{R}$, $\mathcal{S}$)-μ-dissipativity by considering the following performance index:

$$J(N_1,N_2)=\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\mathbb{E}\left\{\mu\tilde{u}^\top_{i,j}\tilde{u}_{i,j}-W(\tilde{u}_{i,j},\tilde{z}_{i,j})\right\},\tag{34}$$

where $W(\tilde{u}_{i,j},\tilde{z}_{i,j})$ is defined in (14). We can know from (27) that, under zero boundary condition, $\sum_{i=0}^{i_1}\sum_{j=0}^{j_1}\Delta V(i,j)\geq 0$ holds. And, when $\tilde{u}_{i,j}\not\equiv 0$,

$$\begin{aligned}\Delta V(i,j)=&\begin{bmatrix}\tilde{x}^\top_{i,j} & \tilde{u}^\top_{i,j}\end{bmatrix}\begin{bmatrix}\tilde{A}^\top_{pt}\\ \tilde{B}^\top_{pt}\end{bmatrix}\Upsilon_q\begin{bmatrix}\tilde{A}_{pt} & \tilde{B}_{pt}\end{bmatrix}\begin{bmatrix}\tilde{x}_{i,j}\\ \tilde{u}_{i,j}\end{bmatrix}\\&-\tilde{x}^\top_{i,j}\Upsilon_p\tilde{x}_{i,j}\end{aligned}\tag{35}$$

Thus, we can deduce as follows:

$$\begin{aligned}&J(N_1,N_2)\\&\leq\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\mathbb{E}\left\{\mu\tilde{u}^\top_{i,j}\tilde{u}_{i,j}-W(\tilde{u}_{i,j},\tilde{z}_{i,j})+\Delta V(i,j)\right\}\\&=\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\mathbb{E}\left\{\begin{bmatrix}\tilde{x}^\top_{i,j} & \tilde{u}^\top_{i,j}\end{bmatrix}\sum_{t=1}^{L_2}\theta_{pt}\mathscr{M}_{pt}\begin{bmatrix}\tilde{x}_{i,j}\\ \tilde{u}_{i,j}\end{bmatrix}-\tilde{x}^\top_{i,j}\Upsilon_p\tilde{x}_{i,j}\right\}\end{aligned}\tag{36}$$

Using inequalities (25) and (19), we can further obtain

$$J(N_1,N_2)<\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\mathbb{E}\left\{\tilde{x}^\top_{i,j}\left(\sum_{t=1}^{L_2}\theta_{pt}\Pi_{pt}-\Upsilon_p\right)\tilde{x}_{i,j}\right\}<0,\tag{37}$$

which means (15) is satisfied, i.e., system (9) is ($\mathcal{Q}$, $\mathcal{R}$, $\mathcal{S}$)-μ-dissipative. Thus, we finish the proof. □

Remark 3. It is imaginable that the matrix inequalities in the sufficient condition will be rather complex since two mode sequences are involved in this fault detection system. The dimensions of these matrix inequalities will increase along with the growth of L_2. This will bring about inconvenience when solving the filter parameters in (3). To deal with this problem, we bring in a matrix Π_{pt}. As a result, the conditional probabilities θ_{pt} appear only in (16) and intertwine only with Π_{pt}. Thus, all other decision variables are separated from θ_{pt} which simplifies the sufficient condition.

Despite all this, there is still a problem to be solved, i.e. the nonlinearity in (17), before we can present a design method for the fault detection filter. The nonlinearity will be dealt with in Theorem 2 by virtue of slack matrix technique as well as Projection Lemma (Feng & Han, 2015; Gahinet & Apkarian, 1994).

We denote $n\triangleq n_h+n_v$ and $\bar{n}\triangleq n_{\bar{h}}+n_{\bar{v}}$, and introduce a matrix $Z_{pt}=\begin{bmatrix}\mathcal{Z}_{pt} & 0\end{bmatrix}+\mathcal{Z}_t$ with $\mathcal{Z}_{pt}$ and $\mathcal{Z}_t$ defined as follows:

$$\mathcal{Z}_{pt}=\begin{bmatrix}Z_{11,pt} & Z_{12,pt}\\ Z_{21,pt} & Z_{22,pt}\\ Z_{31,pt} & Z_{32,pt}\end{bmatrix},\quad \mathcal{Z}_t=\begin{bmatrix}0 & 0 & \breve{Z}_t\\ 0 & 0 & T\breve{Z}_t\\ 0 & 0 & \breve{Z}_t\end{bmatrix},\tag{38}$$

where $Z_{11,pt}\in\mathbb{R}^{n\times n}$, $Z_{22,pt}\in\mathbb{R}^{\bar{n}\times\bar{n}}$, $\breve{Z}_t\in\mathbb{R}^{n\times n}$, and

$$T=\begin{cases}\begin{bmatrix}I_{\bar{n}} & 0_{n\times(n-\bar{n})}\end{bmatrix} & \text{if } \bar{n}<n\\ I_n & \text{if } \bar{n}=n\\ \begin{bmatrix}I_n\\ 0_{(\bar{n}-n)\times n}\end{bmatrix} & \text{if } \bar{n}>n\end{cases}$$

We describe the positive definite matrix $\Upsilon_p = diag\{\Upsilon_p^h, \Upsilon_p^v\}$ as follows:

$$\Upsilon_p^h = \begin{bmatrix} \Upsilon_{11,p}^h & \Upsilon_{12,p}^h & \Upsilon_{13,p}^h \\ * & \Upsilon_{22,p}^h & \Upsilon_{23,p}^h \\ * & * & \Upsilon_{33,p}^h \end{bmatrix}, \quad \Upsilon_p^v = \begin{bmatrix} \Upsilon_{11,p}^v & \Upsilon_{12,p}^v & \Upsilon_{13,p}^v \\ * & \Upsilon_{22,p}^v & \Upsilon_{23,p}^v \\ * & * & \Upsilon_{33,p}^v \end{bmatrix}. \tag{39}$$

Theorem 2. *Consider the fault detection system (9) with Assumption 1. System (9) is asymptotically mean square stable and $(\mathcal{Q}, \mathcal{R}, \mathcal{S})$-$\mu$-dissipative, if there exist matrices $\breve{\Upsilon}_p > 0$ with $\breve{\Upsilon}_p$ being in the following form:*

$$\breve{\Upsilon}_p = \begin{bmatrix} \Upsilon_{11,p}^h & 0 & \Upsilon_{12,p}^h & 0 & \Upsilon_{13,p}^h & 0 \\ 0 & \Upsilon_{11,p}^v & 0 & \Upsilon_{12,p}^v & 0 & \Upsilon_{13,p}^v \\ \Upsilon_{12,p}^{h\top} & 0 & \Upsilon_{22,p}^h & 0 & \Upsilon_{23,p}^h & 0 \\ 0 & \Upsilon_{12,p}^{v\top} & 0 & \Upsilon_{22,p}^v & 0 & \Upsilon_{23,p}^v \\ \Upsilon_{13,p}^{h\top} & 0 & \Upsilon_{23,p}^{h\top} & 0 & \Upsilon_{33,p}^h & 0 \\ 0 & \Upsilon_{13,p}^{v\top} & 0 & \Upsilon_{23,p}^{v\top} & 0 & \Upsilon_{33,p}^v \end{bmatrix}$$

and $\breve{\Pi}_{pt} > 0$, $\breve{A}_t$, $\breve{B}_t$, $\breve{C}_t$ and $\breve{Z}_t$, such that, $\forall p \in \mathscr{L}_1$, $t \in \mathscr{L}_{2\mathcal{U}}^p$,

$$\breve{\Pi}_p^{\mathcal{K}} + (1 - \theta_p^{\mathcal{K}})\breve{\Pi}_{pt} - \breve{\Upsilon}_p < 0 \tag{40}$$

holds, and $\forall p \in \mathscr{L}_1$, $t \in \mathscr{L}_2$, $q \in \mathscr{L}_{1\mathcal{U}}^p$,

$$\tilde{\mathcal{U}}_p^\top X_{pt} \tilde{\mathcal{U}}_p < 0, \ \bar{X}_{pt} < 0 \tag{41}$$

holds, where

$$\tilde{\mathcal{U}}_p = \begin{bmatrix} 0 & 0 & \mathscr{A}_p & \mathscr{B}_p \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \ X_{pt} = \begin{bmatrix} X_{1,pt} & X_{2,pt} \\ * & \bar{X}_{pt} \end{bmatrix}$$

$$\mathscr{A}_p = \begin{bmatrix} A_p & 0 & 0 \\ 0 & \bar{A} & 0 \end{bmatrix}, \ \mathscr{B}_p = \begin{bmatrix} B_p & D_p & E_p \\ 0 & 0 & \bar{B} \end{bmatrix}$$

$$X_{1,pt} = \breve{\Upsilon}_p^{\mathcal{K}} + (1 - \psi_p^{\mathcal{K}})\breve{\Upsilon}_q - \mathcal{Z}_t - \mathcal{Z}_t^\top$$

$$X_{2,pt} = \begin{bmatrix} 0 & M_{1,pt} & M_{2,pt} \end{bmatrix}$$

$$M_{1,pt} = \begin{bmatrix} \breve{B}_t C_p & 0 & \breve{A}_t \\ T\breve{B}_t C_p & 0 & T\breve{A}_t \\ \breve{B}_t C_p & 0 & \breve{A}_t \end{bmatrix}$$

$$M_{2,pt} = \begin{bmatrix} \breve{B}_t F_p & \breve{B}_t G_p & \breve{B}_t H_p \\ T\breve{B}_t F_p & T\breve{B}_t G_p & T\breve{B}_t H_p \\ \breve{B}_t F_p & \breve{B}_t G_p & \breve{B}_t H_p \end{bmatrix}$$

$$\bar{X}_{pt} = \begin{bmatrix} -I & Q\breve{\mathscr{C}}_t & 0 \\ * & -\breve{\Pi}_{pt} & -\breve{\mathscr{C}}_t^\top \mathcal{R} \\ * & * & \mu I - \mathcal{S} \end{bmatrix}, \ \breve{\mathscr{C}}_t = \begin{bmatrix} 0 & -\bar{C} & \breve{C}_t \end{bmatrix}$$

$$\breve{\Pi}_p^{\mathcal{K}} = \sum_{t \in \mathscr{L}_{2\mathcal{K}}^p} \theta_{pt} \breve{\Pi}_{pt}, \ \breve{\Upsilon}_p^{\mathcal{K}} = \sum_{q \in \mathscr{L}_{1\mathcal{K}}^p} \psi_{pq} \breve{\Upsilon}_q$$

Furthermore, if inequalities (40) and (41) are solvable, then the filter parameters in (3) can be figured out as follows:

$$\hat{A}_t = \breve{Z}_t^{-1}\breve{A}_t, \ \hat{B}_t = \breve{Z}_t^{-1}\breve{B}_t, \hat{C}_t = \breve{C}_t. \tag{42}$$

Proof. Firstly, with the help of following notations:

$$\breve{\Pi}_{pt} = \Xi^\top \Pi_{pt} \Xi, \ \breve{\Upsilon}_p = \Xi^\top \Upsilon_p \Xi, \tag{43}$$

we can readily find that (16) and (40) are equivalent by performing a congruence transformation to (40) with matrix Ξ.

On the other hand, condition (41) can be rewritten as

$$\tilde{\mathcal{U}}_p^\top X_{pt} \tilde{\mathcal{U}}_p < 0, \ \tilde{\mathcal{V}}^\top X_{pt} \tilde{\mathcal{V}} < 0, \tag{44}$$

where

$$\tilde{\mathcal{V}} = \begin{bmatrix} 0_{(2n+\bar{n}+n_u+n_w+2n_f)\times(2n+\bar{n})} & I_{2n+\bar{n}+n_u+n_w+2n_f} \end{bmatrix}^\top.$$

We are able to find matrices $\mathcal{U}_p$ and $\mathcal{V}$ that satisfy $\mathcal{U}_p\tilde{\mathcal{U}}_p = 0$ and $\mathcal{V}\tilde{\mathcal{V}} = 0$:

$$\mathcal{U}_p = \begin{bmatrix} -I & 0 & 0 & \mathscr{A}_p & \mathscr{B}_p \end{bmatrix}, \quad \mathcal{V} = \begin{bmatrix} I_{2n+\bar{n}} & 0_{(2n+\bar{n})\times(2n+\bar{n}+n_u+n_w+2n_f)} \end{bmatrix}. \tag{45}$$

Then, according to Projection Lemma, (41) is equivalent to the existence of matrix $\mathcal{Z}_{pt}$ such that

$$X_{pt} + \mathcal{V}^\top \mathcal{Z}_{pt} \mathcal{U}_p + \mathcal{U}_p^\top \mathcal{Z}_{pt}^\top \mathcal{V} < 0 \tag{46}$$

holds, where $\mathcal{Z}_{pt}$ is defined in (38). Substituting X_{pt}, $\mathcal{Z}_{pt}$, $\mathcal{U}_p$ and $\mathcal{V}$ into (46), and using the notations:

$$\breve{A}_t = \breve{Z}_t\hat{A}_t, \ \breve{B}_t = \breve{Z}_t\hat{B}_t, \ \breve{C}_t = \hat{C}_t, \tag{47}$$

we get

$$\begin{bmatrix} \Upsilon_{1,pt}^z & 0 & Z_{pt}\mathring{A}_{pt} & Z_{pt}\mathring{B}_{pt} \\ * & -I & Q\mathring{C}_t & 0 \\ * & * & -\breve{\Pi}_{pt} & -\mathring{C}_t^\top \mathcal{R} \\ * & * & * & \mu I - \mathcal{S} \end{bmatrix} < 0, \tag{48}$$

where $\Upsilon_{1,pt}^z = \breve{\Upsilon}_p^{\mathcal{K}} + (1-\psi_p^{\mathcal{K}})\breve{\Upsilon}_q - Z_{pt} - Z_{pt}^\top$. Note that (48) implies Z_{pt} and $\breve{Z}_t$ are both invertible. A well-known fact is that

$$-Z_{pt}(\breve{\Upsilon}_p^{\mathcal{K}} + (1-\psi_p^{\mathcal{K}})\breve{\Upsilon}_q)^{-1} Z_{pt}^\top \le \breve{\Upsilon}_p^{\mathcal{K}} + (1-\psi_p^{\mathcal{K}})\breve{\Upsilon}_q - Z_{pt} - Z_{pt}^\top \tag{49}$$

Therefore, (48) is sufficient to guarantee

$$\begin{bmatrix} -\Upsilon_{2,pt}^z & 0 & Z_{pt}\mathring{A}_{pt} & Z_{pt}\mathring{B}_{pt} \\ * & -I & Q\mathring{C}_t & 0 \\ * & * & -\breve{\Pi}_{pt} & -\mathring{C}_t^\top \mathcal{R} \\ * & * & * & \mu I - \mathcal{S} \end{bmatrix} < 0 \tag{50}$$

holds, where $\Upsilon_{2,pt}^z = Z_{pt}(\breve{\Upsilon}_p^{\mathcal{K}} + (1 - \psi_p^{\mathcal{K}})\breve{\Upsilon}_q)^{-1} Z_{pt}^\top$. Performing a congruence transformation to (50) with matrix $diag\{Z_{pt}^{-1}, I, I, I\}$ yields

$$\begin{bmatrix} -(\breve{\Upsilon}_p^{\mathcal{K}} + (1-\psi_p^{\mathcal{K}})\breve{\Upsilon}_q)^{-1} & 0 & \mathring{A}_{pt} & \mathring{B}_{pt} \\ * & -I & Q\mathring{C}_t & 0 \\ * & * & -\breve{\Pi}_{pt} & -\mathring{C}_t^\top \mathcal{R} \\ * & * & * & \mu I - \mathcal{S} \end{bmatrix} < 0. \tag{51}$$

Furthermore, by Schur Complement, (51) is equivalent to

$$\begin{bmatrix} -\breve{\Pi}_{pt} & -\mathring{C}_t^\top \mathcal{R} \\ * & \mu I - \mathcal{S} \end{bmatrix} + \begin{bmatrix} \mathring{C}_t^\top Q^\top \\ 0 \end{bmatrix} \begin{bmatrix} Q\mathring{C}_t & 0 \end{bmatrix} + \begin{bmatrix} \mathring{A}_{pt}^\top \\ \mathring{B}_{pt}^\top \end{bmatrix} (\breve{\Upsilon}_p^{\mathcal{K}} + (1-\psi_p^{\mathcal{K}})\breve{\Upsilon}_q) \begin{bmatrix} \mathring{A}_{pt} & \mathring{B}_{pt} \end{bmatrix} < 0. \tag{52}$$

Pre- and post-multiplying (52) by $diag\{\Xi, I\}$ and its transpose, we can discover the equivalence between (22) and (52). As has been discussed in the proof of Theorem 1, (22) and (17) are equivalent. Given all the discussion above, we can see that (41) ensures (17) holds. In conclusion, LMIs (40) and (41) are sufficient to guarantee the asymptotic mean square stability and $(\mathcal{Q}, \mathcal{R}, \mathcal{S})$-$\mu$-dissipativity of the filter detection system (9). Thus, we complete the proof. □

Remark 4. In the proof of Theorem 2, a slack matrix Z_{pt} is used to handle the nonlinear term in (17). Note that matrix Z_{pt} contains a considerable number of variables, which will undoubtedly aggravate computational burden. In the proposed design method, we improve this drawback by virtue of Projection Lemma, which is inspired by Li, Lam, Gao, and Li (2014). As a result, most of the variables are removed from Z_{pt}, and only $\check{Z}_t$ is retained in the final result.

Remark 5. Note that, in Theorem 2, the fault detection filter is devised given a predefined dissipativity parameter μ. The parameter μ shows the level of dissipativity performance. A larger μ means the system has a better dissipativity performance. In fact, based on the result in Theorem 2, μ can be optimized (the optimized μ is denoted by μ^*), i.e., the fault detection filter can be designed by settling the following convex optimization problem:

$$\begin{aligned} &\min \quad -\mu \\ &\text{s.t.} \quad (40)(41) \end{aligned} \tag{53}$$

Remark 6. It can be observed from (40) and (41) that the number of LMIs used to solve the fault detection problem grows along with the increase of the number of unknown entries in Ψ and Θ. More LMIs mean more computational resources are required. As a result, there should be a tradeoff between the computational efficiency and the amount of resource devoted to acquiring Ψ and Θ.

4. Numerical example

We will take an example to show the correctness and validity of the proposed fault detection filter. We consider a partial differential equation called Darboux equation (Marszalek, 1984), which is closely related to some practical processes, such as in water stream heating, air drying, and etc. In the present work, it is assumed that the parameters in this equation have a Markov jumping feature. Then, the equation is presented as follows:

$$\begin{aligned} \frac{\partial^2 \mathscr{F}(x,\tau)}{\partial x \partial \tau} =& a_0(\alpha_{x,\tau})\mathscr{F}(x,\tau) + a_1(\alpha_{x,\tau})\frac{\partial \mathscr{F}(x,\tau)}{\partial \tau} \\ &+ a_2(\alpha_{x,\tau})\frac{\partial \mathscr{F}(x,\tau)}{\partial x} + b(\alpha_{x,\tau})\mathscr{G}(x,\tau), \end{aligned} \tag{54}$$

where the vector function $\mathscr{F}(x,\tau)$ is at the state space $x \in [0, x_f]$ and time $\tau \in [0,\infty]$; $\mathscr{G}(x,\tau)$ is an input function probably including control input $u(x,\tau)$, disturbance $w(x,\tau)$ and fault input $f(x,\tau)$. $\alpha_{x,\tau}$ denotes a Markov process taking values in $\mathscr{L}_1$. We are interested in estimating $r(x,\tau)$ based on the measurable output $y(x,\tau)$, where

$$\begin{aligned} y(x,\tau) =& c_1(\alpha_{x,\tau})\mathscr{F}(x,\tau) + d(\alpha_{x,\tau})\mathscr{G}(x,\tau) \\ &+ c_2(\alpha_{x,\tau})\left\{\frac{\partial \mathscr{F}(x,\tau)}{\partial \tau} - a_2(\alpha_{x,\tau})\mathscr{F}(x,\tau)\right\} \end{aligned} \tag{55}$$

$$r(x,\tau) = g(\alpha_{x,\tau})f(x,\tau) \tag{56}$$

In a similar way as in Wu et al. (2012), we convert the partial differential model described by (54)–(56) to a 2D difference model in the form of (1) by making following definitions:

$$x^h_{i,j} \triangleq \mathscr{S}(i\Delta x, j\Delta\tau), \quad x^v_{i,j} \triangleq \mathscr{F}(i\Delta x, j\Delta\tau), \tag{57}$$

where $\mathscr{S}(x,\tau) \triangleq \frac{\partial \mathscr{F}(x,\tau)}{\partial \tau} - a_2(\alpha_{x,\tau})\mathscr{F}(x,\tau)$. Δx and $\Delta\tau$ denote step sizes.

In (54)–(56), $a_0(\alpha_{x,\tau})$, $a_1(\alpha_{x,\tau})$, $a_2(\alpha_{x,\tau})$, $b(\alpha_{x,\tau})$, $c_1(\alpha_{x,\tau})$, $c_2(\alpha_{x,\tau})$, $d(\alpha_{x,\tau})$ and $g(\alpha_{x,\tau})$ are some real parameters. By assigning values to them, system matrices in (1) can be determined accordingly. Arranging the system matrices of (1) as

$$\left(\begin{array}{c|c|c|c} A_p & B_p & D_p & E_p \\ \hline C_p & F_p & G_p & H_p \end{array}\right) \qquad p = 1, 2, 3,$$

Table 1
Probability matrices Ψ and Θ with different unknown entries.

	Ψ	Θ
1	$\begin{bmatrix} 0.6 & 0.13 & 0.27 \\ 0.1 & 0.1 & 0.8 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.4 & 0.5 & 0.1 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$
2	$\begin{bmatrix} 0.6 & ? & ? \\ 0.1 & 0.1 & 0.8 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.2 & ? & ? \\ 0.4 & 0.5 & 0.1 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$
3	$\begin{bmatrix} 0.6 & ? & ? \\ ? & ? & 0.8 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.2 & ? & ? \\ ? & 0.5 & ? \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$
4	$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$	$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$

we present their values as follows:
Mode 1:

$$\left(\begin{array}{cc|c|c|c} 0.8 & -0.1 & 0.8 & 0.5 & 1.2 \\ 0.2 & -0.5 & 1 & -0.2 & 1 \\ \hline 1.5 & 2.1 & 1.2 & 0.8 & 1.5 \end{array}\right)$$

Mode 2:

$$\left(\begin{array}{cc|c|c|c} -0.5 & 0.4 & 0.8 & 0.4 & 1.2 \\ 1 & -0.1 & 0.1 & -1.2 & 1 \\ \hline 1 & 2 & 1 & 0.4 & 0.8 \end{array}\right)$$

Mode 3:

$$\left(\begin{array}{cc|c|c|c} 0.2 & 0 & 0.1 & 0.2 & 0.6 \\ -0.1 & -0.4 & 0.1 & -0.2 & 0.5 \\ \hline 1 & 0.2 & 0.8 & 0.2 & 0.6 \end{array}\right)$$

Using the proposed method, we plan to design a three-mode fault detection filter with dissipativity parameters given by

$$\mathcal{Q} = -1, \; \mathcal{R} = \begin{bmatrix} 2 & 5 & 5 \end{bmatrix}, \; \mathcal{S} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

And the frequency weighting system (8) is given by

$$\bar{A} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \; \bar{B} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, \; \bar{C} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}.$$

A number of cases with different transition probability matrices Ψ and conditional probability matrices Θ are simulated. Firstly, we study the cases with different Ψ and Θ given in Table 1. We can see that the unknown entries in Ψ and Θ increase progressively from the 1st row to the 4th row. Then, we solve an optimization problem (53) for all the combinations of Ψ and Θ in Table 1, and present the optimized dissipativity performance μ^* in Table 2. By observing the values presented in Table 2, we can conclude that the fault detection system will have a better dissipativity performance if more information about Ψ and Θ is available. Secondly, we study the influence of asynchronous level on dissipativity performance using the transition probability matrix Ψ_1 in Table 1 and conditional probability matrices Θ presented in Table 3. Table 4 clearly shows that the dissipativity performance deteriorates when the asynchronous level between the plant and the fault detection filter increases. All these observed facts strongly indicate the correctness of the obtained results in Theorems 1 and 2.

Next, we will present more detailed simulations with Ψ_3 and Θ_3 given in Table 1. By solving the optimization problem (53), we get the following parameters in fault detection filter (3):

Table 2
Optimized dissipativity performance μ^* for Ψ and Θ with different unknown entries.

Θ	Ψ			
	1	2	3	4
1	3.6482	3.6034	3.5946	3.4569
2	3.6429	3.5995	3.5890	3.4564
3	3.6395	3.5973	3.5863	3.4558
4	3.6367	3.5957	3.5848	3.4552

Table 3
Conditional probability matrices Θ of different asynchronous level.

Synchronous (Case I)	Partly synchronous (Case II)
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Partly synchronous (Case III)	Completely asynchronous (Case IV)
$\begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.4 & 0.5 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.4 & 0.5 & 0.1 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$

Table 4
Optimized dissipativity performance μ^* for Θ of different asynchronous level.

	Case I	Case II	Case III	Case IV
μ^*	3.6933	3.6615	3.6507	3.6482

Mode 1:

$$\left(\begin{array}{c|c} \hat{A}_1 & \hat{B}_1 \\ \hline \hat{C}_1 & \end{array}\right) = \left(\begin{array}{cc|c} -0.0650 & -0.1635 & -0.6407 \\ 0.0670 & -0.5291 & -0.1878 \\ \hline -0.0112 & -0.0061 & \end{array}\right)$$

Mode 2:

$$\left(\begin{array}{c|c} \hat{A}_2 & \hat{B}_2 \\ \hline \hat{C}_2 & \end{array}\right) = \left(\begin{array}{cc|c} 0.0564 & -0.1167 & -0.6289 \\ 0.0935 & -0.5921 & -0.2584 \\ \hline -0.0107 & -0.0061 & \end{array}\right)$$

Mode 3:

$$\left(\begin{array}{c|c} \hat{A}_3 & \hat{B}_3 \\ \hline \hat{C}_3 & \end{array}\right) = \left(\begin{array}{cc|c} 0.0558 & -0.1626 & -0.6216 \\ 0.0858 & -0.5378 & -0.2658 \\ \hline -0.0107 & -0.0060 & \end{array}\right)$$

We further present some necessary parameters: $u_{i,j} = 0.5$; the disturbance input $w_{i,j} = 0.9^{i+j}\tilde{w}_{i,j}$, where $\tilde{w}_{i,j}$ is a random signal between -1 and 1; the boundary condition and fault signal are given as follows:

$$x^h(0,j) = \begin{cases} 0.2, & 0 \le j \le 14 \\ 0, & else \end{cases}$$

$$x^v(i,0) = \begin{cases} 0.3, & 0 \le i \le 14 \\ 0, & else \end{cases}$$

$$f(i,j) = \begin{cases} 2, & 6 \le i,j \le 14 \\ 0, & else \end{cases}$$

The weighted fault signal $\bar{f}_{i,j}$ is displayed in Fig. 1.

Based on the parameters given above, the state evolution of the fault detection filter as well as the residual signal are obtained, which are exhibited in Figs. 2–4. The simulation result demonstrates that the filter is stable. And, with the generated residual signal, we are able to conduct the fault detection process using the evaluation function $\Gamma_{a,b}(z)$ and threshold $\bar{\Gamma}$ defined in (12). To be more specific, Fig. 5 displays the evaluation values of $\Gamma_{a,b}(z)$ for both fault free and faulty cases. For the example we discuss, we choose the threshold

$$\bar{\Gamma} = \sup_{w\neq 0, u\neq 0, f=0} \sqrt{\sum_{i=0}^{30}\sum_{j=0}^{30} z_{i,j}^{\top} z_{i,j}} = 0.0849.$$

Then, through simulation, we find that

$$\sqrt{\sum_{i=0}^{9}\sum_{j=0}^{12} z_{i,j}^{\top} z_{i,j}} = 0.0875 > 0.0849.$$

Thus, the fault can be detected.

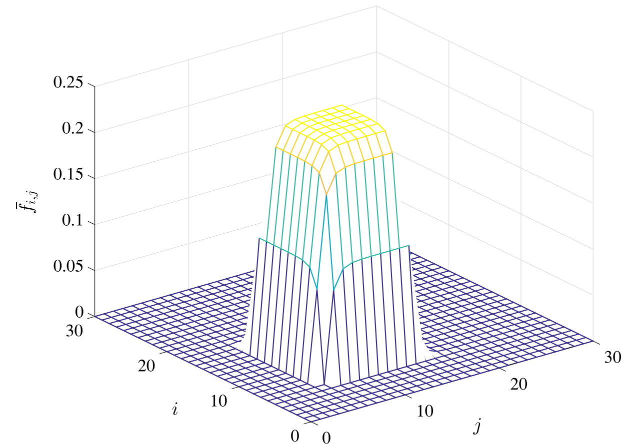

Fig. 1. Weighted fault signal $\bar{f}_{i,j}$.

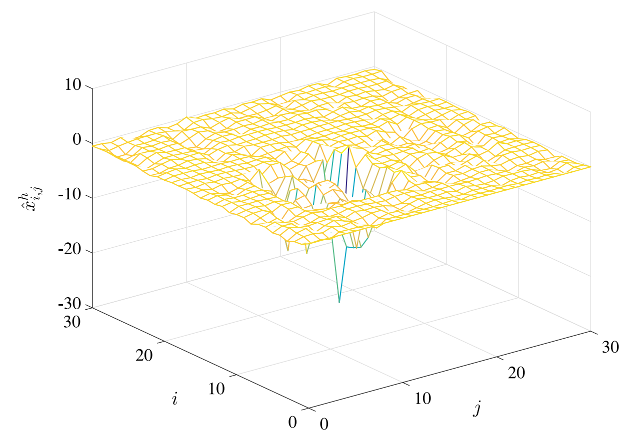

Fig. 2. Horizontal state $\hat{x}^h_{i,j}$ of fault detection filter.

5. Conclusion

In this work, we have worked on the fault detection problem for 2D MJSs established based on the Roesser model. In our framework, some more practical situations have been taken into consideration. On the one hand, we have considered the asynchronization phenomenon between the plant and the fault detection filter, i.e., their modes are not always consistent. Specially, the asynchronization is described by hidden Markov model. On the other hand, the transition probabilities of the Markov process and the conditional probabilities involved in the hidden Markov model are assumed to be only partially known. By virtue of Lyapunov function technique, we have obtained a sufficient condition such that the overall fault detection system is asymptotically mean square stable and $(\mathcal{Q},\mathcal{R},\mathcal{S})$-$\mu$-dissipative.

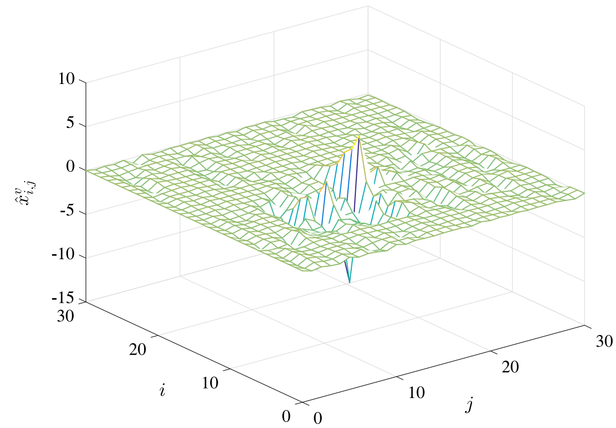

Fig. 3. Vertical state $\hat{x}^v_{i,j}$ of fault detection filter.

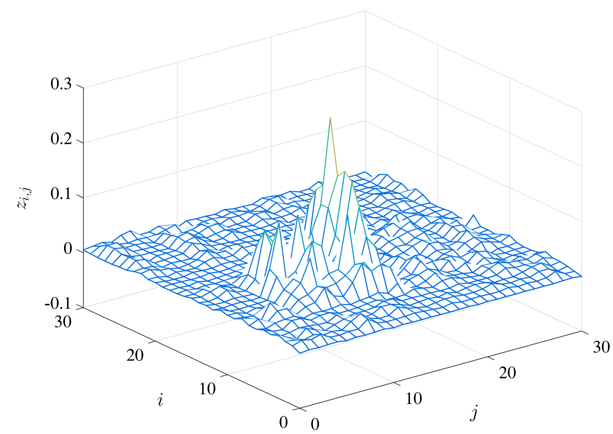

Fig. 4. Residual signal $z_{i,j}$.

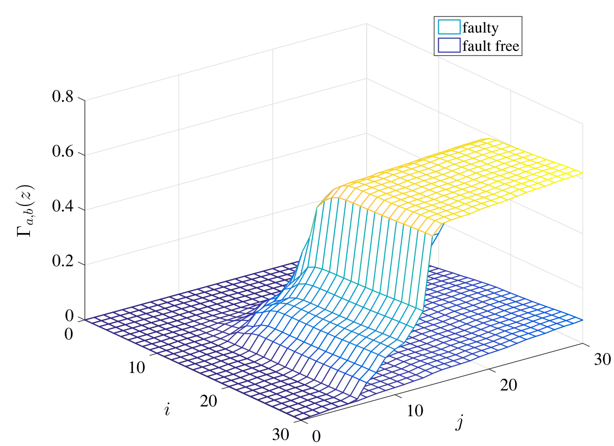

Fig. 5. Evaluation values $\Gamma_{a,b}(z)$ for both fault free and faulty cases.

Moreover, Projection Lemma has facilitated the design of a cost efficient fault detection filter. Finally, we have verified our design method through an example related to the Darboux equation.

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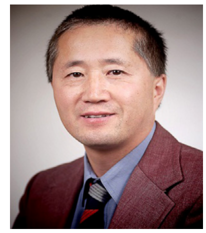

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