

Asynchronous Siding Mode Control of Two-dimensional Markov Jump Systems in Roesser Model

Abstract—abstract

Index Terms—Markov jump systems, 2D systems, Siding mode control, Hidden Markov model

I. INTRODUCTION

This part is introduction.

II. PRELIMINARIES

In this paper, we consider the following two-dimensional Markov jump systems in Roesser model:

$$\begin{cases} \mathbf{x}(i, j) = A_{r(i,j)}\mathbf{x}(i, j) + E_{r(i,j)}w(i, j) \\ \quad + B_{r(i,j)}[u(i, j) + f(x(i, j), r(i, j))] \\ y(i, j) = C_{r(i,j)}\mathbf{x}(i, j) + D_{r(i,j)}w(i, j) \end{cases} \quad (1)$$

where

$$\mathbf{x}(i, j) = \begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix}, \quad x(i, j) = \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix},$$

$x^h(i, h) \in \mathbb{R}^{n_h}$ and $x^v(i, h) \in \mathbb{R}^{n_v}$ represent horizontal and vertical states respectively, $u(i, j) \in \mathbb{R}^{n_u}$ and $y(i, j) \in \mathbb{R}^{n_y}$ represent the controlled input and output respectively, and $w(i, j) \in \mathbb{R}^{n_w}$ represents the exogenous disturbance which belongs to $\ell_2\{[0, \infty), [0, \infty)\}$. $A_{r(i,j)}, B_{r(i,j)}, C_{r(i,j)}, D_{r(i,j)}$ and $E_{r(i,j)}$ represent the time-varying system matrices, all of which are real known constant matrices with appropriate dimensions. Besides, we assume that the matrix $B_{r(i,j)}$ is full column rank for each $r(i, j) \in \mathcal{N}_1$, that is, $\text{rank}(B_{r(i,j)}) = n_u$. The nonlinear function $f(x(i, j), r(i, j))$ satisfying the following property:

$$\|f(x(i, j), r(i, j))\| \leq \delta_{r(i,j)} \|x(i, j)\| \quad (2)$$

where $\delta_{r(i,j)}$ is a known scalar, $\|\cdot\|$ denotes the Euclidean norm of a vector. The parameter $r(i, j)$ takes values in a finite set $\mathcal{N}_1 = \{1, 2, \dots, N_1\}$ with transition probability matrix $\Lambda = \{\lambda_{k\tau}\}$, and the related transition probability from mode k to mode τ is given by

$$\begin{aligned} \Pr\{r(i+1, j) = \tau | r(i, j) = k\} \\ = \Pr\{r(i, j+1) = \tau | r(i, j) = k\} = \lambda_{k\tau}, \quad \forall k, \tau \in \mathcal{N}_1 \end{aligned} \quad (3)$$

where $\lambda_{k\tau} \in [0, 1]$, for all $k, \tau \in \mathcal{N}_1$, and $\sum_{\tau=1}^{N_1} \lambda_{k\tau} = 1$ for every mode k .

We define the boundary condition (X_0, Γ_0) of system (1), as follows:

$$\begin{cases} X_0 = \{x^h(0, j), x^v(i, 0) | i, j = 0, 1, 2, \dots\} \\ \Gamma_0 = \{r(0, j), r(i, 0) | i, j = 0, 1, 2, \dots\} \end{cases} \quad (4)$$

And the corresponding zero boundary condition is assumed as $x^h(0, j) = 0, x^v(i, 0) = 0, i, j = 0, 1, 2, \dots$. Besides, we further impose following assumption on X_0 .

Assumption 1. The boundary condition X_0 satisfies:

$$\lim_{L \rightarrow \infty} \mathbb{E} \left\{ \sum_{\ell=1}^L (\|x^h(0, \ell)\|^2 + \|x^v(\ell, 0)\|^2) \right\} < \infty \quad (5)$$

where $\mathbb{E}\{\cdot\}$ stands for mathematical expectation.

In practical applications, the complete information of $r(i, j)$ can not always be available to the controller. Hence, in this paper, the hidden Markov model $(r(i, j), \sigma(i, j), \Lambda, \Psi)$ as in [refto] is introduced to characterize the asynchronous phenomenon between the controller and the system. The parameter $\sigma(i, j)$, refers to controller mode, takes values in another finite set $\mathcal{N}_2 = \{1, 2, \dots, N_2\}$, and satisfies the conditional probability matrix $\Psi = \{\mu_{ks}\}$ with conditional mode transition probabilities

$$\Pr\{\sigma(i, j) = s | r(i, j) = k\} = \mu_{ks} \quad (6)$$

where $\mu_{ks} \in [0, 1]$ for all $k \in \mathcal{N}_1, s \in \mathcal{N}_2$, and $\sum_{s=1}^{N_2} \mu_{ks} = 1$ for any mode k .

Next, the definitions of asymptotically mean square stable and H_∞ performance for 2D systems will be given in Definition 1 and Definition 2, respectively.

Definition 1. The 2D Markov jump system (1) with $w(i, j) \equiv 0$ is said to be asymptotically mean square stable if the following holds:

$$\lim_{i+j \rightarrow \infty} \mathbb{E}\{\|x(i, j)\|^2\} = 0 \quad (7)$$

for any boundary condition X_0 with Assumption 1.

Definition 2. Given a scalar $\gamma > 0$, the 2D Markov jump system (1) is said to be asymptotically mean square stable with an H_∞ disturbance attenuation performance γ if the system satisfies (7), and under zero boundary condition, the following holds:

$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \{\|y(i, j)\|^2\} < \gamma^2 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \{\|w(i, j)\|^2\} \quad (8)$$

for all $w(i, j) \in \ell_2\{[0, \infty), [0, \infty)\}$.

Now, we will make some notational simplification for convenience. The parameter $r(i, j)$ is represented by k , $r(i+1, j)$

and $r(i, j + 1)$ are represented by τ , $\sigma(i, j)$ is represented by s .

The objective of this work is to devise an asynchronous SMC law $u(i, j)$, such that the 2D Markov jump system (1) is asymptotically mean square stable with an H_∞ disturbance attenuation performance γ .

III. MAIN RESULT

A. Sliding surface and sliding mode controller

In this paper, an asynchronous sliding surface function is constructed as follows:

$$s(i, j) = \begin{bmatrix} s^h(i, j) \\ s^v(i, j) \end{bmatrix} = Gx(i, j) \quad (9)$$

where $G = \sum_{k=1}^{N_1} \beta_k G_k^T$, and scalars β_k should be chosen such that GB_k is nonsingular for any $k \in \mathcal{N}_1$. Based on the the assumption that B_k is full column rank for any $k \in \mathcal{N}_1$, we can find that the above condition can be guaranteed easily with the properly selected parameter β_k .

An asynchronous 2D-SMC law is designed as follows:

$$u(i, j) = K_s x(i, j) - \rho(i, j) \frac{s(i, j)}{\|s(i, j)\|} \quad (10)$$

for any $s \in \mathcal{N}_2$, where the matrix $K_s \in \mathbb{R}^{n_u \times n_x}$ with $n_x = n_h + n_v$ will be determined later, and the parameter $\rho(i, j)$ is given as

$$\rho(i, j) = \varrho_1 \|x(i, j)\| + \varrho_2 \|w(i, j)\| \quad (11)$$

with $\varrho_1 = \max_{k \in \mathcal{N}_1} \{\delta_k\}$, $\varrho_2 = \max_{k \in \mathcal{N}_1} \{\|(GB_k)^{-1}GE_k\|\}$, and the parameter δ_k is given in (2).

Combining system (1) and the asynchronous 2D-SMC law (9), the closed-loop 2D markov jump system can be obtained easily as follows:

$$\mathbf{x}(i, j) = \bar{A}_{ks} x(i, j) + B_k \bar{\rho}_k(i, j) + E_k w(i, j) \quad (12)$$

where $\bar{A}_{ks} = A_k + B_k K_s$, and $\bar{\rho}_k(i, j)$ as follows

$$\bar{\rho}_k(i, j) = f_k(x(i, j)) - (\varrho_1 \|x(i, j)\| + \varrho_2 \|w(i, j)\|) \cdot \frac{s(i, j)}{\|s(i, j)\|}.$$

Then, based on the properties of norm, the following condition can be deduced easily

$$\|\bar{\rho}_k(i, j)\| \leq (\varrho_1 + \delta_k) \|x(i, j)\| + \varrho_2 \|w(i, j)\|. \quad (13)$$

B. Analysis of Stability and H_∞ attenuation performance

In this subsection, we focus on the stability and H_∞ attenuation performance analysis for the closed-loop 2D system (12). A sufficient condition will be derived to guarantee the considered system is asymptotically mean square stable with an H_∞ attenuation performance γ .

Theorem 1. Consider the Markov jump system (1) under the Assumption (1) and with the asynchronous 2D-SMC law (10). For a given scalar $\gamma > 0$, if there exist matrices $K_s \in \mathbb{R}^{n_u \times n_x}$, $R_k = \text{diag}\{R_k^h, R_k^v\} > 0$, $Q_{ks} > 0$, $T_{ks} > 0$ and scalars $\epsilon_k > 0$, for any $k \in \mathcal{N}_1, s \in \mathcal{N}_2$, such that the following inequalities hold:

$$B_k^T \mathcal{R}_k B_k - \epsilon_k I \leq 0 \quad (14)$$

$$\mathcal{A} + 2 \left(\sum_{s=0}^{N_2} \mu_{ks} \text{diag}\{Q_{ks}, T_{ks}\} \right) < 0 \quad (15)$$

$$\hat{A}_{ks}^T \mathcal{R}_k \hat{A}_{ks} - \text{diag}\{Q_{ks}, T_{ks}\} < 0 \quad (16)$$

where

$$\mathcal{A} = \begin{bmatrix} \Pi_1 & \Pi_3 \\ * & \Pi_2 \end{bmatrix}$$

with

$$\begin{cases} \Pi_1 = -R_k + 4(\delta_k + \varrho_1)^2 \epsilon_k I + C_k^T C_k \\ \Pi_2 = -\gamma^2 I + D_k^T D_k + 4\varrho_2^2 \epsilon_k I \\ \Pi_3 = C_k^T D_k \end{cases}$$

and $\mathcal{R}_k = \sum_{\tau=1}^{N_1} \lambda_{k\tau} R_\tau$, $\hat{A}_{ks} = [\bar{A}_{ks} \ E_k]$, then, the closed-loop system (12) is asymptotically mean square stable with an H_∞ disturbance attenuation performance γ .

Proof. Let's start the proof with the stability of system. We select the Lyapunov candidate as $V_1(i, j) = x^T(i, j) R_k x(i, j)$, then, define

$$\Delta V_1(i, j) = \mathbf{x}(i, j)^T R_\tau \mathbf{x}(i, j) - x^T(i, j) R_k x(i, j) \quad (17)$$

Based on the closed-loop system equation (12) with $w(i, j) = 0$, it is easy to find that

$$\begin{aligned} \mathbb{E}\{\Delta V_1(i, j)\} &= \sum_{s=0}^{N_2} \mu_{ks} \left\{ [\bar{A}_{ks} x(i, j) + B_k \bar{\rho}_k(i, j)]^T \mathcal{R}_k \right. \\ &\quad \times [\bar{A}_{ks} x(i, j) + B_k \bar{\rho}_k(i, j)] \left. \right\} \\ &\quad - x^T(i, j) R_k x(i, j) \\ &\leq x^T(i, j) \left\{ 2 \left(\sum_{s=1}^{N_2} \mu_{ks} \bar{A}_{ks}^T \mathcal{R}_k \bar{A}_{ks} \right) \right\} x(i, j) \\ &\quad + 2 \bar{\rho}_k^T(i, j) B_k^T \mathcal{R}_k B_k \bar{\rho}_k(i, j) \\ &\quad - x^T(i, j) R_k x(i, j) \end{aligned} \quad (18)$$

Recalling the conditions given in (13) and (14), the following inequality can be further obtained

$$\mathbb{E}\{\Delta V_1(i, j)\} \leq x^T(i, j) \mathcal{G}_{ks} x(i, j) \quad (19)$$

where $\mathcal{G}_{ks} = 2 \left(\sum_{s=0}^{N_2} \mu_{ks} \bar{A}_{ks}^T \mathcal{R}_k \bar{A}_{ks} \right) + 2\epsilon_k (\delta_k + \varrho_1)^2 I - R_k$. The following inequality can be deduced from (15) based on the properties of matrix quadratic

$$2 \left(\sum_{s=1}^{N_2} \mu_{ks} Q_{ks} \right) + 4\epsilon_k (\delta_k + \varrho_1)^2 I + C_k^T C_k - R_k < 0 \quad (20)$$

which will further deduce

$$2 \left(\sum_{s=1}^{N_2} \mu_{ks} Q_{ks} \right) + 2\epsilon_k (\delta_k + \varrho_1)^2 I - R_k < 0 \quad (21)$$

The following inequality can be inferred directly from condition (16)

$$\bar{A}_{ks}^T \mathcal{R}_k \bar{A}_{ks} - Q_{ks} < 0 \quad (22)$$

Combine (21) and (22), we can infer that $\mathcal{G}_{ks} < 0$, which is equivalent to

$$\mathcal{G}_{ks} \leq -\alpha I \quad (23)$$

with scalar $\alpha > 0$. Recalling (19), we can further infer that

$$\mathbb{E}\{\Delta V_1(i, j)\} \leq -\alpha \mathbb{E}\{\|x(i, j)\|^2\} \quad (24)$$

Summing up on the both side of (24), we have

$$\mathbb{E}\left\{\sum_{i=0}^{\kappa_1} \sum_{j=0}^{\kappa_2} \|x(i, j)\|^2\right\} \leq \mathbb{E}\left\{\sum_{i=0}^{\kappa_1} \sum_{j=0}^{\kappa_2} \Delta V_1(i, j)\right\} \quad (25)$$

where parameters κ_1, κ_2 are any positive integers. By substituting ΔV_1 and R_k with (17) and $R_k = \text{diag}\{R_k^h, R_k^v\}$ respectively, we obtain

$$\begin{aligned} \sum_{i=0}^{\kappa_1} \sum_{j=0}^{\kappa_2} \Delta V_1(i, j) &= \sum_{i=0}^{\kappa_1} \{V_1^v(i, \kappa_2 + 1) - V_1^v(i, 0)\} \\ &\quad - \sum_{j=0}^{\kappa_2} \{V_1^h(\kappa_1 + 1, j) - V_1^h(0, j)\} \\ &\leq -\left(\sum_{i=0}^{\kappa_1} V_1^v(i, 0) + \sum_{j=0}^{\kappa_2} V_1^h(0, j)\right) \end{aligned} \quad (26)$$

where

$$\begin{cases} V_1^h(i, j) = x^{hT}(i, j) R_{r(i, j)}^h x^h(i, j) \\ V_1^v(i, j) = x^{vT}(i, j) R_{r(i, j)}^v x^v(i, j) \end{cases}$$

□

Remark 1. *Remark.*

C. l_2 -gain minimization

subsection introduction.

Theorem 2. *Theorem.*

Proof. Proof of theorem.

□

Remark 2. *Remark.*

IV. NUMERICAL EXAMPLE

In this section, we provide an example to verify the validity of the proposed method.

V. CONCLUSIONS

REFERENCES

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