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An Extension of Sliding Mode Control Design for the 2-D systems in Roesser Model

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Abstract— **This letter introduces a strategy design of Sliding Mode Control (SMC) for Two-Dimensional (2-D) systems. The dynamic updating of the 2-D system is considered as a Roesser Model (RM). The proposed method consists of two stages, switching surface design and control law design. Moreover, the robustness of the proposed method for an uncertain system has been investigated. Simulation results verify the efficiency of this approach.**

I. INTRODUCTION

In nature, there are many processes, which their dynamics depend on more than one independent variable (e.g. thermal processes and long transmission lines [1]). These processes are called multi-dimensional systems. 2-D systems are mostly investigated in the literature as a multi-dimensional system. 2-D systems are often applied to theoretical aspects like filter design, image processing, and recently, Iterative Learning Control methods (see for example [2] - [6]). Over the past two decades, the stability of multi-dimensional systems in various models has been a point of high interest among researchers [7] - [12]. In [12] some new results on the stability of 2-D systems have been presented – specifically with regard to the Lyapunov stability condition which has been developed for RM. Then, robust stability problem [13] and optimal guaranteed cost control of the uncertain 2-D systems [14]-[17] came to be the area of interest. In addition, an adaptive control method for SISO 2-D systems has been presented in [18]. However, in many physical systems, the goal of control design is not only to satisfy the stability conditions but also to have a system that takes its trajectory in the predetermined hyperplane. An interesting approach to stabilize the systems and keep their states on the predetermined desired trajectory is the sliding mode control method. Generally speaking, SMC is a robust control design, which yields substantial results in invariant control systems [21]. The term invariant means that the system is robust against model uncertainties and exogenous disturbances. The behavior of the underlying SMC of systems is indeed divided into two parts. In the first part, which is called reaching mode, system states are driven to a predetermined stable switching surface. And in the second part, the system states move across or intersect the switching surface while always staying there. The latter is called sliding mode. At a glance in the literature, it is understood that there are many works in the field of SMC for 1-D continuous and discrete time systems. (see [19]-[34]) Furthermore SMC has been contributed to various control methods (see for example [33]-[34]) and several experimental works such as [30]. Recently, a SMC design for a 2-D system in RM model [23] has been presented in which the idea of a 1-D quasi-sliding mode [25] has been extended for the 2-D system. Though the sliding surfaces design problem and the conditions for the existence of an ideal quasi-sliding mode has been solved by [23] in terms of LMI.

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In this paper, using a 2-D Lyapunov function, the conditions ensuring the rest of horizontal and vertical system states on the switching surface and also the reaching condition for designing the control law are investigated. This function can also help us design the proper switching surface. Moreover, it is shown that the designed control law can be applied to some classes of 2-D uncertain systems. Simulation results of an example, which show the efficiency of the proposed SMC design, have been presented at the end of the paper. The rest of the paper is organized as follows. In Section II, some lemmas and definitions are presented. Section III and IV respectively discuss the design of switching surface and the switching control law. In Section V, the proposed control design for a case of 2-D uncertain system is investigated. Section VI includes the simulation results of the examples. Conclusions and suggestions are finally presented in Section VII.

II. PROBLEM STATEMENT

Consider a 2D system in RM model as follows

$$\begin{bmatrix} \boldsymbol{x^h(i+1,j)} \\ \boldsymbol{x^v(i,j+1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix} \begin{bmatrix} \boldsymbol{x^h(i,j)} \\ \boldsymbol{x^v(i,j)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B_1} & \boldsymbol{B_2} \\ \boldsymbol{B_3} & \boldsymbol{B_4} \end{bmatrix} \begin{bmatrix} \boldsymbol{u^h(i,j)} \\ \boldsymbol{u^v(i,j)} \end{bmatrix} \qquad (1)$$

where $x^h \in R^{n_1}$ and $x^v \in R^{n_2}$ are the horizontal and the vertical state vectors, and also, $u^h \in R^{m_1}$ and $u^v \in R^{m_2}$ are the horizontal and the vertical input controls respectively. A_l, B_l for $l = 1,2,3,4$ are constant matrices with suitable dimensions. Also, (i, j) indicates horizontal and vertical index. Furthermore, the boundary condition for (1) is

$$\begin{cases} \boldsymbol{x_0^h = x^h(0,j)} \\ \boldsymbol{x_0^v = x^v(i,0)} \end{cases} \qquad (2)$$

Definition 1: If the state vector norms $\|x^h(\boldsymbol{i},\boldsymbol{j})\|$ and $\|x^v(\boldsymbol{i},\boldsymbol{j})\|$, converge to zero when $i+j \to \infty$, then the 2D system (1) is asymptotically stable.

Here, we want to restate the Lyapunov stability for the RM that has been presented in many papers (see [7]-[9]).

Lemma 1[7]: The 2D system (1) with $u^h(i,j)=0$ and $u^v(i,j)=0$ is asymptotically stable if there exist two positive definite matrices $P_1 \in R^{n_1 \times n_1}$ and $P_2 \in R^{n_2 \times n_2}$ such that

$$\begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{P_1} & 0 \\ 0 & \boldsymbol{P_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix} - \begin{bmatrix} \boldsymbol{P_1} & 0 \\ 0 & \boldsymbol{P_2} \end{bmatrix} < 0\ . \tag{3}$$

With respect to (3), let's define two functions as follows.

$$\boldsymbol{V}(\boldsymbol{i},\boldsymbol{j}) \triangleq \frac{1}{2}\left(\begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{P_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P_2} \end{bmatrix} \begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix}\right) \tag{4}$$

$$\boldsymbol{V_{11}}(\boldsymbol{i},\boldsymbol{j}) \triangleq \frac{1}{2}\left(\begin{bmatrix} x^h(\boldsymbol{i}+\boldsymbol{1},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}+\boldsymbol{1}) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{P_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P_2} \end{bmatrix} \begin{bmatrix} x^h(\boldsymbol{i}+\boldsymbol{1},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}+\boldsymbol{1}) \end{bmatrix}\right) \tag{5}$$

The two functions (3) and (4) are called 2-D Lyapunov functions with different delays. In addition, the difference of the 2-D Lyapunov functions can be defined as

$$\Delta\boldsymbol{V}(\boldsymbol{i},\boldsymbol{j}) \triangleq \boldsymbol{V_{11}}(\boldsymbol{i},\boldsymbol{j}) - \boldsymbol{V}(\boldsymbol{i},\boldsymbol{j}) \tag{6}$$

In order to introduce the proposed SMC design, the following lemma is needed.

Lemma 2: The zero input 2-D system (1) is asymptotically stable if

$$\Delta\boldsymbol{V}(\boldsymbol{i},\boldsymbol{j}) < 0 \tag{7}$$

Proof:

Note, the zero input system (1) is

$$\begin{bmatrix} x^h(\boldsymbol{i}+\boldsymbol{1},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}+\boldsymbol{1}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix} \begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix} \tag{8}$$

By replacing the state vector (8) into (5) we can get

$$\boldsymbol{V_{11}}(\boldsymbol{i},\boldsymbol{j}) = \frac{1}{2}\left(\begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{P_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix} \begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix}\right) \tag{9}$$

Therefore, the difference equation $\Delta V(i,j)$, is

$$\begin{aligned}\Delta\boldsymbol{V}(\boldsymbol{i},\boldsymbol{j}) = {} & \frac{1}{2}\begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{P_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix} \begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix} \\ & -\frac{1}{2}\begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{P_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P_2} \end{bmatrix} \begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix} \\ = {} & \frac{1}{2}\begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix}^T Q \begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix}\end{aligned} \tag{10}$$

where

$$Q = \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{P_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} \\ \boldsymbol{A_3} & \boldsymbol{A_4} \end{bmatrix} - \begin{bmatrix} \boldsymbol{P_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P_2} \end{bmatrix}$$

Noting (10), it is clear that the expression $\Delta V(i,j) < 0$ results in the stability condition (3). In other words, both expressions (3) and (7) are equivalent. This completes the proof.

III. Switching Surface Design

In the 1-D SMC design, switching surface is commonly determined as a linear combination of the state vector. Similarly, for the 2-D system in RM, the switching surface can be determined as

$$\begin{bmatrix} s^h(\boldsymbol{i},\boldsymbol{j}) \\ s^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{C_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C_2} \end{bmatrix} \begin{bmatrix} x^h(\boldsymbol{i},\boldsymbol{j}) \\ x^v(\boldsymbol{i},\boldsymbol{j}) \end{bmatrix} \tag{11}$$

where $s^h(i,j) \in R^{m_1}$ and $s^v(i,j) \in R^{m_2}$ are the horizontal and the vertical switching surfaces respectively. C_1 and C_2 are also suitable constant matrices with proper dimensions.

Definition 3: Suppose the 2-D system (1) starts at $(i,j)=(i_0,j_0)$ with the boundary conditions (2).The boundary conditions can be out of or on the switching surface. So, if the system state trajectory moves toward the switching surface (11), it is called reaching mode. After this, if it intersects switching surface at $(i,j)=(i_1,j_1)$ and remains there for all $(i,j)>(i_1,j_1)$ then this is called sliding motion or sliding mode for 2-D systems in RM.

In 1-D system, a well-known method to design switching surface is Equivalent Control Method. In this method, first, an equivalent control law is obtained when the system (1) is set in the switching surface. Then the switching surface is designed in shadow of stability condition for augmented system with switching surface. Here we want to extend this method to the 2-D system in RM (1).

Equivalent control law: In order to obtain an equivalent input control, we should find a proper condition by which the trajectory of the 2-D system stays on the switching surface. This condition can be derived from the candidate 2-D Lyapunov function

$$V(i,j)=\frac{1}{2}\left(\begin{bmatrix}s^h(i,j)\\ s^v(i,j)\end{bmatrix}^T\begin{bmatrix}I_{m_1} & 0\\ 0 & I_{m_2}\end{bmatrix}\begin{bmatrix}s^h(i,j)\\ s^v(i,j)\end{bmatrix}\right)$$
$$=\frac{1}{2}[s^h(i,j)]^2+\frac{1}{2}[s^v(i,j)]^2 \qquad (12)$$

Also, consider

$$V_{11}(i,j)=\frac{1}{2}\left(\begin{bmatrix}s^h(i+1,j)\\ s^v(i,j+1)\end{bmatrix}^T\begin{bmatrix}I_{m_1} & 0\\ 0 & I_{m_2}\end{bmatrix}\begin{bmatrix}s^h(i+1,j)\\ s^v(i,j+1)\end{bmatrix}\right)$$
$$=\frac{1}{2}[s^h(i+1,j)]^2+\frac{1}{2}[s^v(i,j+1)]^2 \qquad (13)$$

and

$$\Delta V(i,j)=V_{11}(i,j)-V(i,j)$$
$$=\frac{1}{2}[s^h(i+1,j)]^2+\frac{1}{2}[s^v(i,j+1)]^2-\frac{1}{2}[s^h(i,j)]^2-\frac{1}{2}[s^v(i,j)]^2$$
$$=\frac{1}{2}\{[s^h(i+1,j)]^2-[s^h(i,j)]^2\}+\frac{1}{2}\{[s^v(i,j+1)]^2-[s^v(i,j)]^2\} \qquad (14)$$

Note that the 2-D Lyapunov function (12) is an energy like function, such that if the difference function (14) is decreased along $\{(i,j): i+j=d, i\geq 0, j\geq 0\}$ for some positive integer $d>0$, then the system (1) is asymptotically stable (i.e. $x^h(i,j)\to 0, x^v(i,j)\to 0$ as $d\to\infty$). As a result, the condition that sets the 2-D system (1) in the switching surface is equivalent to $\Delta V(i,j)=0$. In other words, it means that the energy variation is zero in respect to the switching surface (11).

According to (14), it is clear that a sufficient condition to have $\Delta V(i,j)=0$ is

$$\begin{bmatrix}s^h(i+1,j)\\ s^v(i,j+1)\end{bmatrix}=\begin{bmatrix}s^h(i,j)\\ s^v(i,j)\end{bmatrix} \qquad (15)$$

therefore

$$\begin{cases}s^h(i+1,j)=s^h(i,j)\\ s^v(i,j+1)=s^v(i,j)\end{cases} \qquad (16)$$

From the condition (16), the equivalent control law is derived as

$$\begin{bmatrix}C_1 & 0\\ 0 & C_2\end{bmatrix}\begin{bmatrix}x^h(i+1,j)\\ x^v(i,j+1)\end{bmatrix}$$
$$=\begin{bmatrix}C_1 & 0\\ 0 & C_2\end{bmatrix}\begin{bmatrix}A_1 & A_2\\ A_3 & A_4\end{bmatrix}\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix}$$
$$+\begin{bmatrix}C_1 & 0\\ 0 & C_2\end{bmatrix}\begin{bmatrix}B_1 & B_2\\ B_3 & B_4\end{bmatrix}\begin{bmatrix}u^h(i,j)\\ u^v(i,j)\end{bmatrix}$$
$$=\begin{bmatrix}C_1A_1 & C_1A_2\\ C_2A_3 & C_2A_4\end{bmatrix}\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix}+\begin{bmatrix}C_1B_1 & C_1B_2\\ C_2B_3 & C_2B_4\end{bmatrix}\begin{bmatrix}u^h(i,j)\\ u^v(i,j)\end{bmatrix}$$
$$=\begin{bmatrix}C_1 & 0\\ 0 & C_2\end{bmatrix}\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix} \qquad (17)$$

So, the equivalent input control is

$$\begin{bmatrix}u^h_{eq}(i,j)\\ u^v_{eq}(i,j)\end{bmatrix}=-\underbrace{\begin{bmatrix}C_1B_1 & C_1B_2\\ C_2B_3 & C_2B_4\end{bmatrix}^{-1}\begin{bmatrix}C_1(A_1-I) & C_1A_2\\ C_2A_3 & C_2(A_4-I)\end{bmatrix}}_{F}\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix}$$
$$=-F\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix} \qquad (18)$$

where we assume $\begin{bmatrix}C_1B_1 & C_1B_2\\ C_2B_3 & C_2B_4\end{bmatrix}$ is invertible. Now, the C matrix of the switching surface (11) has to be determined such that the following augmented system is stable.

$$\begin{bmatrix}x^h(i+1,j)\\ x^v(i,j+1)\end{bmatrix}=\left(\begin{bmatrix}A_1 & A_2\\ A_3 & A_4\end{bmatrix}-\begin{bmatrix}B_1 & B_2\\ B_3 & B_4\end{bmatrix}F\right)\begin{bmatrix}x^h(i,j)\\ x^v(i,j)\end{bmatrix}$$
$$\begin{bmatrix}s^h(i,j)\\ s^v(i,j)\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \qquad (19)$$

After designing the switching surface (11), the control law by which reaching condition is guaranteed, has to be designed. This condition is also guarantees the existence condition of the sliding mode.

IV. Control Law Design

, the input control is chosen as

$$\begin{bmatrix}u^h(i,j)\\ u^v(i,j)\end{bmatrix}=\begin{bmatrix}u^h_{eq}(i,j)\\ u^v_{eq}(i,j)\end{bmatrix}+\begin{bmatrix}u^h_s(i,j)\\ u^v_s(i,j)\end{bmatrix} \qquad (20)$$

where $u_S^h(i,j)$ and $u_S^v(i,j)$ are switching control actions. Also the equivalent control input in (20) can be obtained from (18). In order to design the switching control actions, we will first introduce a reaching condition by which the system (1) with the boundary condition (2) has been moved to the switching surface (11). With respect to the 2-D Lyapunov function (13), the reaching condition for the 2-D system (1) is

$$\frac{1}{2}\{[s^h(i+1,j)]^2+[s^v(i,j+1)]^2\}<\frac{1}{2}\{[s^h(i,j)]^2+[s^v(i,j)]^2\} \qquad (21)$$

Now, note that we can write

$$\Delta S\triangleq\begin{bmatrix}s^h(i+1,j)\\ s^v(i,j+1)\end{bmatrix}-\begin{bmatrix}s^h(i,j)\\ s^v(i,j)\end{bmatrix}$$

$$= \begin{bmatrix} C_1A_1 & C_1A_2 \\ C_2A_3 & C_2A_4 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} \begin{bmatrix} u^h(i,j) \\ u^v(i,j) \end{bmatrix} - \begin{bmatrix} s^h(i,j) \\ s^v(i,j) \end{bmatrix} \quad (22)$$

By replacing (20) into (22) and noting (18), we have

$$\Delta S = \begin{bmatrix} C_1A_1 & C_1A_2 \\ C_2A_3 & C_2A_4 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} - \begin{bmatrix} C_1(A_1 - I) & C_1A_2 \\ C_2A_3 & C_2(A_4 - I) \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} - \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} \begin{bmatrix} u_s^h(i,j) \\ u_s^v(i,j) \end{bmatrix} - \begin{bmatrix} s^h(i,j) \\ s^v(i,j) \end{bmatrix}$$

So,

$$\Delta S = \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} \begin{bmatrix} u_s^h(i,j) \\ u_s^v(i,j) \end{bmatrix} \quad (23)$$

By defining

$$S \triangleq \begin{bmatrix} s^{h^T}(i,j) & s^{v^T}(i,j) \end{bmatrix}^T$$

and

$$S_{11} \triangleq \begin{bmatrix} s^{h^T}(i+1,j) & s^{v^T}(i,j+1) \end{bmatrix}^T$$

and also, noting (22), the inequality (21) can be rewritten as follows

$$\tfrac{1}{2}(S + \Delta S)^2 < \tfrac{1}{2}S^2 \quad (24)$$

By replacing (23) into (24) we have

$$\frac{1}{2}\left(S + \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} \begin{bmatrix} u_s^h(i,j) \\ u_s^v(i,j) \end{bmatrix} \right)^2$$

$$= \tfrac{1}{2}S^2 - S^T \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} \begin{bmatrix} u_s^h(i,j) \\ u_s^v(i,j) \end{bmatrix} + \tfrac{1}{2}\left(\begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} \begin{bmatrix} u_s^h(i,j) \\ u_s^v(i,j) \end{bmatrix} \right)^2 < \tfrac{1}{2}S^2$$

And therefore

$$S^T \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} \begin{bmatrix} u_s^h(i,j) \\ u_s^v(i,j) \end{bmatrix} < -\tfrac{1}{2}\left(\begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} \begin{bmatrix} u_s^h(i,j) \\ u_s^v(i,j) \end{bmatrix} \right)^2 \quad (25)$$

Now, we present the following theorem.

Theorem 1: If the switching control law is chosen as

$$\begin{bmatrix} u_s^h(i,j) \\ u_s^v(i,j) \end{bmatrix} = -k \begin{bmatrix} s^h(i,j) \\ s^v(i,j) \end{bmatrix} \quad (26)$$

such that

$$\begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} + \tfrac{1}{2}k \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix}^T \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} > 0 \quad (27)$$

and $\begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} > 0$, then the reaching condition (21) or equivalently (25) is satisfied.

Proof:

The proof is easy because by substituting (26) into (25), we have

$$-S^T \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} S - \left(\frac{1}{2}\right) k S^T \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix}^T \begin{bmatrix} C_1B_1 & C_1B_2 \\ C_2B_3 & C_2B_4 \end{bmatrix} S < 0 \quad (28)$$

This completes the proof.

V. SIMULATION RESULTS

Let a 2-D uncertain system in RM be given as follows

$$\begin{bmatrix} x_1^h(i+1,j) \\ x_2^h(i+1,j) \\ x_1^v(i,j+1) \\ x_2^v(i,j+1) \end{bmatrix} = (A + \Delta A) \begin{bmatrix} x_1^h(i,j) \\ x_2^h(i,j) \\ x_1^v(i,j) \\ x_2^v(i,j) \end{bmatrix} + (B + \Delta B) \begin{bmatrix} u^h(i,j) \\ u^v(i,j) \end{bmatrix} \quad (38)$$

where

$$A = \begin{bmatrix} 0.7020 & 0.7846 & 1.1666 & 0.4806 \\ -1.6573 & -0.7190 & -1.7257 & -1.7637 \\ -1.0272 & -0.6165 & 1.6654 & -1.1104 \\ 0.1917 & -0.4467 & 1.0959 & -0.0200 \end{bmatrix}$$

$$B = \begin{bmatrix} -1.2632 & -0.3524 \\ -1.0438 & -0.2503 \\ 0.5016 & 0.8912 \\ 0.1348 & -0.0587 \end{bmatrix}$$

Suppose $\alpha = 0.5$. For this system, the switching surface is chosen as

$$\begin{bmatrix} s^h(i,j) \\ s^v(i,j) \end{bmatrix} = C \begin{bmatrix} x_1^h(i,j) \\ x_2^h(i,j) \\ x_1^v(i,j) \\ x_2^v(i,j) \end{bmatrix} \quad (39)$$

where

$$C = \begin{bmatrix} c_1^h & c_2^h & 0 & 0 \\ 0 & 0 & c_1^v & c_2^v \end{bmatrix}$$

The constant parameters c_1^h, c_2^h, c_1^v and c_2^v have to be selected such that the augmented system (19) be stable. It can be easily shown that by choosing C as

$$\boldsymbol{C}=\begin{bmatrix} c_1^h & c_2^h & 0 & 0 \\ 0 & 0 & c_1^v & c_2^v \end{bmatrix}=\begin{bmatrix} -0.3608 & -0.2825 & 0 & 0 \\ 0 & 0 & 1.3173 & 0.2140 \end{bmatrix} \quad (40)$$

the augmented system (19) is stable such that

$$\begin{bmatrix} x_1^h(i+1,j) \\ x_2^h(i+1,j) \\ x_1^v(i,j+1) \\ x_2^v(i,j+1) \end{bmatrix}=\begin{bmatrix} 1.6344 & 1.1038 & 1.2997 & 0.9881 \\ -0.8101 & -0.4095 & -1.6595 & -1.2617 \\ 0.0144 & 0.0956 & 0.8075 & 0.2061 \\ -0.0887 & -0.5883 & 1.1849 & -0.2687 \end{bmatrix}\begin{bmatrix} x_1^h(i,j) \\ x_2^h(i,j) \\ x_1^v(i,j) \\ x_2^v(i,j) \end{bmatrix}$$

$$\begin{bmatrix} s^h(i,j) \\ s^v(i,j) \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (41)$$

By simplifying (41), we have a reduced stable 2-D system as

$$\begin{bmatrix} x_1^h(i+1,j) \\ x_1^v(i,j+1) \end{bmatrix}=\begin{bmatrix} 0.2248 & -4.7821 \\ -0.1076 & -0.4612 \end{bmatrix}\begin{bmatrix} x_1^h(i,j) \\ x_1^v(i,j) \end{bmatrix} \quad (42)$$

So the control action that has been described in previous section is

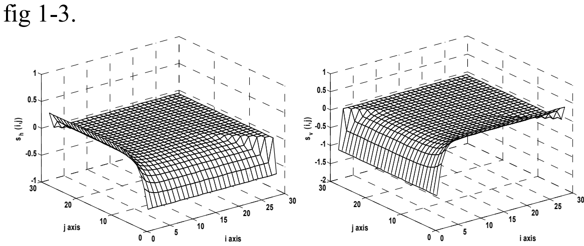
$$\begin{bmatrix} u^h(i,j) \\ u^v(i,j) \end{bmatrix}=\boldsymbol{F}\begin{bmatrix} x_1^h(i,j) \\ x_2^h(i,j) \\ x_1^v(i,j) \\ x_2^v(i,j) \end{bmatrix}-\frac{1}{2}\begin{bmatrix} s^h(i,j) \\ s^v(i,j) \end{bmatrix} \quad (43)$$

by selecting $k=\frac{1}{2}$ the condition in (34) is satisfied such that

$$(1+\alpha^2)D^TD-I=\begin{bmatrix} -0.3636 & -0.2580 \\ -0.2580 & -0.7680 \end{bmatrix} \quad (44)$$

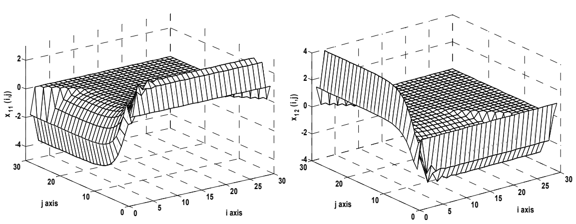
It is clear that the above matrix is a negative definite matrix.

Simulation results of this approach have been illustrated in fig 1-3.

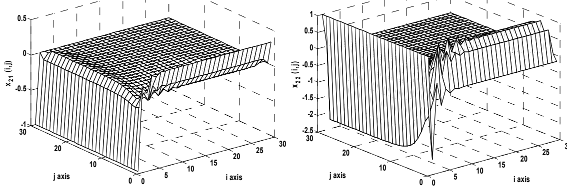


a) b)

Fig. 1. a) Horizontal sliding surface $s^h(i,j)$, b) Vertical sliding surface $s^v(i,j)$

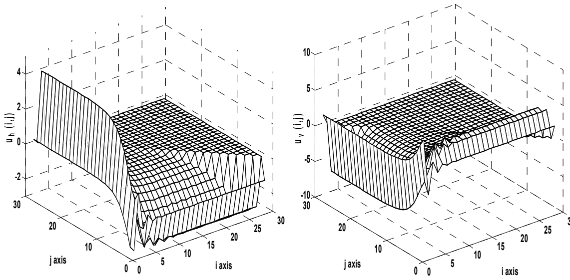


a) b)



c) d)

Fig. 2. System states a) $x_1^h(i,j)$, b) $x_2^h(i,j)$, c) $x_1^v(i,j)$ and d) $x_2^v(i,j)$



a) b)

Fig. 3. a) Horizontal input control $u^h(i,j)$, b) Vertical input control $u^v(i,j)$

VI. Conclusion

The In this paper, an extension of 1-D SMC design to the 2-D system in Roesser model has been proposed. Using a 2-D Lyapunov function, we first designed a linear switching surface, and then a feedback control law that satisfies reaching condition was obtained. This method can also be applied to 2-D uncertain systems with matching uncertainty

VII. References

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