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Stability and l_1-gain analysis for positive 2D T–S fuzzy state-delayed systems in the second FM model

Zhaoxia Duan [a], Zhengrong Xiang [a,*], Hamid Reza Karimi [b]

[a] School of Automation, Nanjing University of Science and Technology, Nanjing 210094, People's Republic of China
[b] Department of Engineering, Faculty of Engineering and Science, University of Agder, N-4898 Grimstad, Norway

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ABSTRACT

This paper considers the problems of delay-dependent stability and l_1-gain analysis for a class of positive two-dimensional (2D) Takagi–Sugeno (T–S) fuzzy linear systems with state delays described by the second FM model. Firstly, the co-positive type Lyapunov function method is applied to establish sufficient conditions of asymptotical stability for the addressed positive 2D T–S fuzzy system. Then, the l_1-gain performance analysis for the positive 2D T–S fuzzy delayed system is studied. All the obtained results are formulated in the form of linear matrix inequalities (LMIs) which are computationally tractable. Finally, an illustrative example is given to verify the effectiveness of the proposed method.

1. Introduction

2D systems have wide applications in many fields, such as circuit analysis, digital image processing, signal filtering, and thermal power engineering [1–3]. Thus the analysis and synthesis of 2D systems are interesting and challenging problems, and have received considerable attention in literature, for example, 2D state-space realization theory was researched in [4], the stability and 2D optimal control theory were studied in [5,6], and [7–9] addressed the noise attenuation and filtering problems for 2D systems.

The most popular models of two-dimensional (2D) linear systems were introduced by Roesser [10], Fornasini–Marchesini [4,11] and Kurek [12]. These models have been extended for positive systems in [13–15]. Many physical systems in the real world involve variables that have nonnegative sign, such as population levels, absolute temperature and concentrations of substances. Such systems are referred to as positive systems which mean that their states and outputs are nonnegative whenever the initial conditions and inputs are nonnegative [16–18]. This feature makes analysis and synthesis of positive systems challenging and interesting tasks. Positive 2D systems are also needed in many cases such as the wave equation in fluid dynamics, the heat equation which describes the temperature in a given region over time, and Poisson's equation. These facts stimulate the research on 2D positive discrete systems [19]. [20] investigated the choice of the forms of Lyapunov functions for positive 2D Roesser model. Stability analysis for 2D positive systems with or without delays has been investigated in [20–24].

It is well known that most of actual 2D systems are nonlinear and the aforementioned results do not work in this case. The T–S fuzzy model [25] suggests an efficient way to represent complex nonlinear positive systems by fuzzy sets and fuzzy reasoning. [26] showed that the T–S fuzzy systems can be approximated to any continuous functions in a compact set of R^n at any precision. This allows the designers to take advantages of conventional linear systems to analyze and design nonlinear systems. Therefore, T–S fuzzy control has become one of the most popular and promising research platform in the model-based fuzzy control, and the theoretic researches on the issue have been conducted actively by many fuzzy control theorists. Among these exiting stabilization conditions for T–S fuzzy systems, common quadratic Lyapunov function method (CQLF) [27–29] and piecewise quadratic Lyapunov function method [30] are two main approaches used to deal with the issue. Some works [31,32] are dealt with nonquadratic stability and stabilization with the purpose of further releasing conservatism. Authors in [33] proposed a novel non-PDC control scheme for discrete-time T–S fuzzy systems. The issues of stability and controller synthesis for 2D T–S fuzzy systems have been studied extensively [34–36,37]. On the other hand, for positive T–S fuzzy systems, co-positive type Lyapunov function method has been used to deal with the stability and stabilization [38].

* Corresponding author.
E-mail address: xiangzr@mail.njust.edu.cn (Z. Xiang).

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However, for 2D T–S fuzzy systems, unlike the usual 1D (one dimensional) T–S fuzzy systems, the system information is propagated along two directions and this fact makes the controller synthesis for 2D T–S fuzzy systems more complex and challenging.

In addition, perturbations and uncertainties widely exist in the practical systems. In some cases, the perturbations and unmodeling errors can be merged into the disturbance, which can be supposed to be bounded in the appropriate norms. Many works about disturbance attenuation performance have been done both for 2D discrete systems [7,8] and for 2D T–S fuzzy discrete systems [35]. It is important and necessary to establish a criterion evaluating the disturbance attenuation performance of positive 2D discrete-time nonlinear systems. However, to the best of our knowledge, the disturbance attenuation performance for positive 2D T–S fuzzy systems has not fully been investigated, which motivates the present work to fill such a blank.

In this paper we will study the problems of delay-dependent stability and l_1-gain analysis for positive 2D T–S fuzzy systems with delays. The main theoretical contributions of this paper are:

(1) A positive 2D fuzzy FMLSS model is built based on the combination of the non-negativity property, fuzzy system model and 2D FMLSS model. A delay-dependent stability criterion for positive 2D T–S fuzzy systems with state delays is developed.
(2) l_1-gain index is introduced for the first time to evaluate the disturbance attenuation performance of positive 2D T–S fuzzy systems.
(3) Co-positive type Lyapunov function method is firstly utilized for stability and l_1-gain performance analysis of positive 2D T–S fuzzy systems.
(4) All results are represented in form of LMIs which are conveniently computable and are of significance to the further application to practical engineering.

The paper is organized as follows. In Section 2, problem statement and some definitions concerning the positive 2D T–S fuzzy systems with delays are given. In Sections 3 and 4, the delay-dependent stability and l_1-gain analysis of positive 2D T–S fuzzy systems are developed, respectively. In Section 5, a numerical example is given to illustrate the effectiveness of the proposed approach. Finally, concluding remarks are provided in Section 6.

Notations: In this paper, the superscript "T" denotes the transpose. The notation $X>Y$ $(X\geq Y)$ means that matrix X–Y is positive definite (positive semi-definite), respectively. $A\succeq 0$ $(\preceq 0)$ means that all entries of matrix A are non-negative (non-positive). $A\succ 0$ $(\prec 0)$ means that all entries of matrix A are positive (negative). $R^{n\times m}$ denotes the set of $n\times m$ real matrices. The set of real $n\times m$ matrices with nonnegative entries will be denoted by $R_+^{n\times m}$, R_+^n denotes the set of vectors with nonnegative entries, and the set of nonnegative integers will be denoted by Z_+. The $n\times n$ identity matrix will be denoted by I_n. The l_1 norm of a 2D signal $w(k,l)=\begin{bmatrix} w_1(k,l) & w_2(k,l) & \dots & w_t(k,l)\end{bmatrix}^T$ is given by

$$\|w(k,l)\|_1=\sum_{i=1}^{t} w_i(k,l).$$

And we say $w(k,l)\in l_1\{[0,\infty),[0,\infty)\}$, if $\|w(k,l)\|_1<\infty$.

2. Problem formulation and preliminaries

In 2D systems, the system information is propagated along two independent directions. By taking this structural feature into consideration, a 2D fuzzy FMLSS model is established by connecting the local linear model from two directions. Let N be the number of fuzzy rules describing the discrete 2D nonlinear plant. The i-th rule is described by

R^i: IF $\theta_1(k,l)$ is $M_1^i,\dots,$ $\theta_\eta(k,l)$ is $M_\eta^i,\dots$ and $\theta_\alpha(k,l)$ is M_α^i,
THEN,

$$\begin{aligned}x(k+1,l+1)=&A_{1i}x(k,l+1)+A_{2i}x(k+1,l)\\&+A_{d1i}x(k-d_1,l+1)+A_{d2i}x(k+1,l-d_2)\\&+B_{1i}w(k,l+1)+B_{2i}w(k+1,l),\end{aligned}\tag{1a}$$

$$z(k,l)=C_ix(k,l)+D_iw(k,l),\tag{1b}$$

where $x(k,l)\in R^n$ is the state vector, $w(k,l)\in R^t$ is the l_1 norm bounded disturbance input, $z(k,l)\in R^r$ is the controlled output, $\theta_\eta(k,l)$, $\eta=1,2,\dots,\alpha$, are premise variables, M_η^i is a spatial fuzzy set of rule i corresponding to spatial input variable $\theta_\eta(k,l)$ $(\eta=1,2,\dots,\alpha)$. d_1 and d_2 are unknown positive integers representing delays along horizontal direction and vertical direction, respectively. A_{1i}, A_{2i}, A_{d1i}, A_{d2i}, B_{1i}, B_{2i}, C_i and D_i are real matrices with appropriate dimensions for $i\in \underline{N}=\{1,2,\dots N\}$. k and l are two integers in Z_+.

The premise variables are no longer scalar functions but vector-valued functions, that is

$$h_i=\begin{bmatrix} h_i(k,l+1) & h_i(k+1,l)\end{bmatrix},\tag{2}$$

where

$$h_i(k,l)=\frac{\mu_{M_1^i}(\theta_1(k,l))\mu_{M_2^i}(\theta_2(k,l))\dots\mu_{M_\alpha^i}(\theta_\alpha(k,l))}{\sum_{i=1}^N\left[\mu_{M_1^i}(\theta_1(k,l))\mu_{M_2^i}(\theta_2(k,l))\dots\mu_{M_\alpha^i}(\theta_\alpha(k,l))\right]}$$

is a known nonlinear function of $\theta_\eta(k,l)$ $(\eta=1,2,\dots,\alpha)$. $\mu_{M_\eta^i}(\theta_\eta(k,l))$ is the grade of membership corresponding to the fuzzy set $\mu_{M_\eta^i}$.

By definition, the fuzzy basis functions satisfy

$$\sum_{i=1}^N h_i(k,l)=1, h_i(k,l)\in[0,1],\quad \forall i\in\underline{N}.\tag{3}$$

The system behavior is described by

$$\begin{aligned}x(k+1,l+1)=&\sum_{i=1}^N h_i\left\{\begin{bmatrix}A_{1i}x(k,l+1)\\A_{2i}x(k+1,l)\end{bmatrix}\right.\\&\left.+\begin{bmatrix}A_{d1i}x(k-d_1,l+1)\\A_{d2i}x(k+1,l-d_2)\end{bmatrix}+\begin{bmatrix}B_{1i}w(k,l+1)\\B_{2i}w(k+1,l)\end{bmatrix}\right\},\end{aligned}\tag{4a}$$

$$z(k,l)=\sum_{i=1}^N h_i\begin{bmatrix}C_ix(k,l)\\D_iw(k,l)\end{bmatrix}.\tag{4b}$$

For simplicity, we take

$$\begin{bmatrix}A_1(h(k,l+1)) & A_2(h(k+1,l))\\A_{d1}(h(k,l+1)) & A_{d2}(h(k+1,l)\\B_1(h(k,l+1)) & B_2(h(k+1,l)\\C(h(k,l)) & D(h(k,l))\end{bmatrix}=\sum_{i=1}^N\begin{bmatrix}h_i(k,l+1)A_{1i} & h_i(k+1,l)A_{2i}\\h_i(k,l+1)A_{d1i} & h_i(k+1,l)A_{d2i}\\h_i(k,l+1)B_{1i} & h_i(k+1,l)B_{2i}\\h_i(k,l)C_i & h_i(k,l)D_i\end{bmatrix}.\tag{5}$$

Then systems (4a) and (4b) can be simplified as

$$\begin{aligned}x(k+1,l+1)=&A_1(h(k,l+1))x(k,l+1)+A_2(h(k+1,l))x(k+1,l)\\&+A_{d1}(h(k,l+1))x(k-d_1,l+1)\\&+A_{d2}(h(k+1,l))x(k+1,l-d_2)\\&+B_1(h(k,l+1))w(k,l+1)+B_2(h(k+1,l))w(k+1,l)\end{aligned}\tag{6a}$$

$$z(k,l)=C(h(k,l))x(k,l)+D(h(k,l))w(k,l).\tag{6b}$$

For convenience, system (4) with $w(k,l)=0$ is introduced

$$x(k+1,l+1)=\sum_{i=1}^N h_i\left\{\begin{bmatrix}A_{1i}x(k,l+1)\\A_{2i}x(k+1,l)\end{bmatrix}+\begin{bmatrix}A_{d1i}x(k-d_1,l+1)\\A_{d2i}x(k+1,l-d_2)\end{bmatrix}\right\}.\tag{7}$$

The positive boundary conditions of 2D fuzzy system (4) are defined by

$$x(k,l)=\begin{cases} h_{k,l}\geq 0, & -d_1\leq k\leq 0,\, -d_2\leq l\leq z_2;\\ 0, & -d_1\leq k\leq 0,\, l>z_2;\\ v_{k,l}\geq 0, & -d_1\leq k\leq z_1,\, -d_2\leq l\leq 0;\\ 0, & k>z_1,\, -d_2\leq l\leq 0; \end{cases} \tag{8}$$

where

$$h_{k,l}=v_{k,l}\ \text{for}\ -d_1\leq k\leq 0,\quad -d_2\leq l\leq 0, \tag{9}$$

$z_1<\infty$ and $z_2<\infty$ are positive integers.

Remark 1. When $N=1$ which means there is only one rule for system (1), 2D fuzzy system (1) will degenerate into the following general 2D FM system.

$$x(k+1,l+1)=A_1x(k,l+1)+A_2x(k+1,l)+A_{d1}x(k-d_1,l+1)$$
$$+A_{d2}x(k+1,l-d_2)+B_1w(k,l+1)+B_2w(k+1,l)$$

$$z(k,l)=Cx(k,l)+Dw(k,l)$$

Definition 1. System (4) is said to be positive if for any boundary conditions (8) and (9) and any inputs $w(k,l)\geq 0$, it satisfies $x(k,l)\geq 0$ and $z(k,l)\geq 0$, $\forall k,l\in Z_+$.

Lemma 1. *System (4) is positive if* $A_{1i}\geq 0$, $A_{2i}\geq 0$, $A_{d1i}\geq 0$ *and* $A_{d2i}\geq 0$, $\forall i\in \underline{N}$.

Proof. The matrices $A_{1i}\geq 0$, $A_{2i}\geq 0$, $A_{d1i}\geq 0$ and $A_{d2i}\geq 0$, $i\in \underline{N}$, together with the boundary conditions (8) and (9), $w(k,l)\geq 0$ and$h_i(k,l)\geq 0$, by Definition 1, system (4) is positive.

Remark 2. The positive 2D T–S fuzzy model (4) suggests an efficient way to represent some complex nonlinear positive 2D systems by fuzzy sets and fuzzy reasoning. And thus we can use some linear mathematical techniques to deal with some complex problems for positive 2D nonlinear systems.

Definition 2. System (7) is said to be asymptotically stable if $\lim_{z\to\infty} X_z=0$ for any boundary conditions (8) and (9), where $X_z=\sup\{\|x(k,l)\|_1 : k+l=z, k,l\geq 1\}$

Definition 3. Given a scalar $\gamma>0$, system (4) is said to be asymptotically stable with the l_1-gain bounded by γ, if the following conditions are satisfied:

(1) System (4) is asymptotically stable when $w(k,l)=0$;
(2) Under zero boundary condition, it holds that

$$\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\|\overline{z}(k,l)\|_1<\gamma\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\|\overline{w}(k,l)\|_1,$$
$$\forall 0\neq w(k,l)\in l_1\{[0,\infty),[0,\infty)\}, \tag{10}$$

where

$$\overline{z}(k,l)=\begin{bmatrix} z(k,l+1)\\ z(k+1,l)\end{bmatrix},\ \overline{w}(k,l)=\begin{bmatrix} w(k,l+1)\\ w(k+1,l)\end{bmatrix}.$$

3. Stability analysis

In this section, we focus on the problem of stability analysis for positive 2D T–S fuzzy system (7).

Theorem 1. *Positive 2D T–S fuzzy system (7) is asymptotically stable if there exist positive vectors* $p\succ q\in R^n_+$, $\nu_1\in R^n_+$, $\nu_2\in R^n_+$, $\varsigma_1\in R^n_+$ *and* $\varsigma_2\in R^n_+$, *such that*

$$diag\{\Phi_i^1,\Phi_i^2,\ldots,\Phi_i^n,\overline{\Phi}_i^1,\overline{\Phi}_i^2,\ldots,\overline{\Phi}_i^n,\tilde{\Phi}_i^1,\tilde{\Phi}_i^2,\ldots,\tilde{\Phi}_i^n,\widehat{\Phi}_i^1,\widehat{\Phi}_i^2,\ldots,\widehat{\Phi}_i^n\}<0,\quad \forall i\in\underline{N}, \tag{11}$$

where

$$\Phi_i^m=(a_{1i}^{mT}-E^m)p+E^mq+E^m\nu_1+d_1E^m\varsigma_1,$$
$$\overline{\Phi}_i^m=a_{2i}^{mT}p-E^mq+E^m\nu_2+d_2E^m\varsigma_2,$$
$$\tilde{\Phi}_i^m=a_{d1i}^{mT}p-E^m\nu_1-E^m\varsigma_1,$$
$$\widehat{\Phi}_i^m=a_{d2i}^{mT}p-E^m\nu_2-E^m\varsigma_2,$$

with $m=1,2,\ldots,n$, $E^m=\left[\overbrace{0,\ldots,0}^{m-1},1,\overbrace{0,\ldots,0}^{n-m}\right]$, *and* $a_{1i}^m(a_{2i}^m, a_{d1i}^m, a_{d2i}^m)$ *represents the m-th column vector of matrix* $A_{1i}A_{1i}(A_{2i}, A_{d1i}, A_{d2i})$, *respectively.*

Proof. Consider the following co-positive Lyapunov function for positive system (7)

$$V(k,l)=V_1(k,l)+V_2(k,l), \tag{12}$$

$$V_1(k,l)=x^T(k,l)(p-q)+\sum_{j=-d_1}^{-1}x^T(k+j,l)\nu_1+\sum_{\tau=-d_1+1}^{0}\sum_{j=\tau-1}^{-1}x^T(k+j,l)\varsigma_1,$$

$$V_2(k,l)=x^T(k,l)q+\sum_{j=-d_2}^{-1}x^T(k,l+j)\nu_2+\sum_{\tau=-d_2+1}^{0}\sum_{j=\tau-1}^{-1}x^T(k,l+j)\varsigma_2, \tag{13}$$

with $p\in R^n_+$, $q\in R^n_+$, $\varsigma_1\in R^n_+$, $\varsigma_2\in R^n_+$ and $p\succ q$. It is clear that $V(k,l)$ is positive definite.

The increment of function $V(k,l)$ is given by

$$\Delta V(k+1,l+1)=\Delta V_1(k+1,l+1)+\Delta V_2(k+1,l+1), \tag{14}$$

where

$$\Delta V_1(k+1,l+1)=V_1(k+1,l+1)-V_1(k,l+1),$$

$$\Delta V_2(k+1,l+1)=V_2(k+1,l+1)-V_2(k+1,l).$$

Note that the unidirectional increment can be seen as a particular case of the increment of the function $V(k,l)$ in one direction (k or l), independently of the other (respectively, l or k).

Along the trajectory of positive system (7), the increment is given by

$$\begin{aligned}
\Delta V(k+1,l+1)&\\
&=V_1(k+1,l+1)-V_1(k,l+1)+V_2(k+1,l+1)-V_2(k+1,l)\\
&=x^T(k+1,l+1)(p-q)-x^T(k,l+1)(p-q)+x^T(k+1,l+1)q-x^T(k+1,l)q\\
&\quad +x^T(k,l+1)\nu_1-x^T(k-d_1,l+1)\nu_1+x^T(k+1,l)\nu_2-x^T(k+1,l-d_2)\nu_2\\
&\quad +d_1x^T(k,l+1)\varsigma_1-\sum_{j=-d_1}^{-1}x^T(k+j,l+1)\varsigma_1\\
&\quad +d_2x^T(k+1,l)\varsigma_2-\sum_{j=-d_2}^{-1}x^T(k+1,l+j)\varsigma_2\\
&\leq x^T(k,l+1)(A_1^T(h(k,l+1))p+\nu_1+d_1\varsigma_1-(p-q))\\
&\quad +x^T(k+1,l)(A_2^T(h(k+1,l))p+\nu_2+d_2\varsigma_2-q)\\
&\quad +x^T(k-d_1,l+1)(A_{d1}^T(h(k,l+1))p-\nu_1-\varsigma_1)\\
&\quad +x^T(k+1,l-d_2)(A_{d2}^T(h(k+1,l))p-\nu_2-\varsigma_2).
\end{aligned} \tag{15}$$

From (11), we obtain

$$\begin{aligned}
&A_{1i}^Tp+\nu_1+d_1\varsigma_1-(p-q)\prec 0,\\
&A_{2i}^Tp+\nu_2+d_2\varsigma_2-q\prec 0,\\
&A_{d1i}^Tp-\nu_1-\varsigma_1\prec 0,\\
&A_{d2i}^Tp-\nu_2-\varsigma_2\prec 0,
\end{aligned} \tag{16}$$

which imply that

$$\begin{aligned}&A_1^T(h(k,l+1))p+\nu_1+d_1\varsigma_1-(p-q)\prec 0,\\&A_2^T(h(k+1,l))p+\nu_2+d_2\varsigma_2-q\prec 0,\\&A_{d1}^T(h(k,l+1))p-\nu_1-\varsigma_1\prec 0,\\&A_{d2}^T(h(k+1,l))p-\nu_2-\varsigma_2\prec 0.\end{aligned}\tag{17}$$

Then, we can get

$\Delta V(k+1,l+1)<0.$

It follows from (14) that

$$V_1(k+1,l+1)+V_2(k+1,l+1)<V_1(k,l+1)+V_2(k+1,l).\tag{18}$$

Let $D(r)$ denote the set defined by

$$D(r)\triangleq\{(k,l):k+l=r,k\ge 0,l\ge 0\}.\tag{19}$$

For any integer $r\ge\max\{z_1,z_2\}$, it follows from (18) and the boundary conditions (8) and (9) that

$$\begin{aligned}\sum_{(k,l)\in D(r)}V(k,l)&=\sum_{(k,l)\in D(r)}[V_1(k,l)+V_2(k,l)]\\&=V_1(r,0)+V_1(r-1,1)+V_1(r-2,2)+\cdots+V_1(1,r-1)+V_1(0,r)\\&\quad+V_2(r,0)+V_2(r-1,1)+V_2(r-2,2)+\cdots+V_2(1,r-1)+V_2(0,r)\\&>V_1(r+1,0)+V_1(r,1)+V_1(r-1,2)+\cdots+V_1(2,r-1)+V_1(1,r)\\&\quad+V_2(r,1)+V_2(r-1,2)+V_2(r-2,3)+\cdots+V_2(1,r)+V_2(0,r+1)\\&=\sum_{(k,l)\in D(r+1)}V(k,l).\end{aligned}$$

This implies that the sum of co-positive Lyapunov functional at the points $\{(k,l):k+l=r+1\}$ is strictly less than that at the points $\{(k,l):k+l=r\}$. Thus, we obtain

$$\lim_{r\to\infty}\sum_{(k,l)\in D(r)}V(k,l)=0,\tag{20}$$

which implies that

$$\lim_{k+l\to\infty}V(k,l)=0,\quad \lim_{k+l\to\infty}\|x(k,l)\|=0.$$

We can conclude from Definition 2 that the positive 2D T–S fuzzy system (7) is asymptotically stable.

This completes the proof.

Remark 3. It should be noted that a co-positive Lyapunov functional is constructed for the stability analysis in the derivation of Theorem 1. The motivation for using this type of Lyapunov functional is that the state of the addressed system is nonnegative and hence such a linear Lyapunov functional serves as a valid candidate. The stability result presented here is in the form of LMIs which can be conveniently verified via the LMI toolbox.

4. l_1-gain analysis

The following result establishes sufficient conditions of the asymptotical stability with l_1-gain performance for positive system (4).

Theorem 2. *For a given positive constant γ, positive 2D T–S fuzzy system (4) is asymptotically stable with the l_1-gain bounded by γ if there exist positive vectors $p\succ q\in R_+^n$, $\nu_1\in R_+^n$, $\nu_2\in R_+^n$, $\varsigma_1\in R_+^n$ and $\varsigma_2\in R_+^n$, such that*

$$\begin{aligned}&diag\left\{X_i^1,X_i^2,\ldots,X_i^n,\overline{X}_i^1,\overline{X}_i^2,\ldots,\overline{X}_i^n\right\}<0,\\&diag\left\{T_{i,g}^1,T_{i,g}^2,\ldots,T_{ig}^n,Y_{i,g}^1,Y_{i,g}^2,\ldots,Y_{i,g}^t\right\}<0,\\&diag\left\{\overline{T}_{i,f}^1,\overline{T}_{i,f}^2,\ldots,\overline{T}_{i,f}^n,\overline{Y}_{i,f}^1,\overline{Y}_{i,f}^2,\ldots,\overline{Y}_{i,f}^t\right\}<0,\ \forall i,g,f\in\underline{N},\end{aligned}\tag{21}$$

where

$$X_i^m=a_{d1i}^{mT}p-E^m\varsigma_1-E^m\nu_1,\ \overline{X}_i^m=a_{d2i}^{mT}p-E^m\varsigma_2-E^m\nu_2,$$

$$T_{i,g}^m=a_{1i}^{mT}p+E^m\nu_1-E^m(p-q)+d_1E^m\varsigma_1+\|c_g^m\|_1,$$

$$\overline{T}_{i,f}^m=a_{2i}^{mT}p+E^m\nu_2-E^mq+d_2E^m\varsigma_2+\|c_f^m\|_1$$

$$Y_{i,g}^\varepsilon=b_{1i}^{\varepsilon T}p+\|d_g^\varepsilon\|_1-\gamma,\ \overline{Y}_{i,f}^\varepsilon=b_{2i}^{\varepsilon T}p+\|d_f^\varepsilon\|_1-\gamma,$$

with $m\in\underline{n}=\{1,2,\cdots,n\}$, $\varepsilon\in\underline{t}=\{1,2,\cdots,t\}$, $a_{1i}^m(a_{2i}^m,\ a_{d1i}^m,\ a_{d2i}^m,\ c_i^m)$ represents the m-th column vector of matrices $A_{1i}(A_{2i},A_{d1i},A_{d2i},C_i)$, respectively, and $b_{1i}^\varepsilon(b_{2i}^\varepsilon,\ d_i^\varepsilon)$ represents the ε-th column vector of matrix $B_{1i}(B_{2i},D_i)$, respectively.

Proof. It is an obvious fact that (21) implies the following inequality

$$diag\{\Phi_i^1,\Phi_i^2,\ldots,\Phi_i^n,\overline{\Phi}_i^1,\overline{\Phi}_i^2,\ldots,\overline{\Phi}_i^n,\tilde{\Phi}_i^1,\tilde{\Phi}_i^2,\ldots,\tilde{\Phi}_i^n,\widehat{\Phi}_i^1,\widehat{\Phi}_i^2,\ldots,\widehat{\Phi}_i^n\}<0,\quad\forall i\in\underline{N},\tag{22}$$

where

$$\begin{aligned}\Phi_i^m&=(a_{1i}^{mT}-E^m)p+E^mq+E^m\nu_1+d_1E^m\varsigma_1,\\\overline{\Phi}_i^m&=a_{2i}^{mT}p-E^mq+E^m\nu_2+d_2E^m\varsigma_2,\\\tilde{\Phi}_i^m&=a_{d1i}^{mT}p-E^m\nu_1-E^m\varsigma_1,\\\widehat{\Phi}_i^m&=a_{d2i}^{mT}p-E^m\nu_2-E^m\varsigma_2,\end{aligned}$$

with $m\in\underline{n}=\{1,2,\cdots,n\}$, $E^m=\Big[\overbrace{0,\ldots,0}^{m-1},1,\overbrace{0,\ldots,0}^{n-m}\Big]$, and $a_{1i}^m(a_{2i}^m,\ a_{d1i}^m,\ a_{d2i}^m)$ represents the m-th column vector of matrix $A_{1i}(A_{2i},A_{d1i},A_{d2i})$, respectively.

By Theorem 1, we can obtain that positive system (4) with $w(k,l)=0$ is asymptotically stable. Now we are in a position to prove that positive system (4) has a prescribed l_1-gain index γ for any nonzero $w(k,l)\in l_1\{[0,\infty),[0,\infty)\}$. To establish the l_1-gain performance, we choose the same co-positive Lyapunov–Krasovskii functional candidate as in (13) for positive system (4). Following the proof line of Theorem 1, we can get that

$$\begin{aligned}&\Delta V(k+1,l+1)+\|\overline{z}(k,l)\|_1-\gamma\|\overline{w}(k,l)\|_1\\&\quad\le x^T(k,l+1)(A_1^T(h(k,l+1))p+\nu_1+d_1\varsigma_1-(p-q))\\&\quad+x^T(k+1,l)\left(A_2^T(h(k+1,l))p+\nu_2+d_2\varsigma_2-q\right)\\&\quad+x^T(k-d_1,l+1)(A_{d1}^T(h(k,l+1))p-\nu_1-\varsigma_1)\\&\quad+x^T(k+1,l-d_2)(A_{d2}^T(h(k+1,l))p-\nu_2-\varsigma_2)\\&\quad+w^T(k,l+1)B_1^T(h(k,l+1))p\\&\quad+w^T(k+1,l)B_2^T(h(k+1,l))p\\&\quad+\left\|\begin{bmatrix}C(h(k,l+1))x(k,l+1)+D(h(k,l+1))w(k,l+1)\\C(h(k+1,l))x(k+1,l)+D(h(k+1,l))w(k+1,l)\end{bmatrix}\right\|_1-\gamma\left\|\begin{bmatrix}w(k,l+1)\\w(k+1,l)\end{bmatrix}\right\|_1.\end{aligned}\tag{23}$$

According to the definition of 1-norm, one obtains

$$\begin{aligned}&\left\|\begin{bmatrix}C(h(k,l+1))x(k,l+1)+D(h(k,l+1))w(k,l+1)\\C(h(k+1,l))x(k+1,l)+D(h(k+1,l))w(k+1,l)\end{bmatrix}\right\|_1-\gamma\left\|\begin{bmatrix}w(k,l+1)\\w(k+1,l)\end{bmatrix}\right\|_1\\&\quad=\|C(h(k,l+1))x(k,l+1)\|_1+\|C(h(k+1,l))x(k+1,l)\|_1\\&\qquad+\|D(h(k,l+1))w(k,l+1)\|_1+\|D(h(k+1,l))w(k+1,l)\|_1\\&\qquad-\gamma\|w(k,l+1)\|_1-\gamma\|w(k+1,l)\|_1\\&\quad=\sum_{g=1}^N\sum_{f=1}^N h_g(k,l+1)h_f(k+1,l)\{\|C_gx(k,l+1)\|_1+\|C_fx(k+1,l)\|_1\\&\qquad+\|D_gw(k,l+1)\|_1+\|D_fw(k+1,l)\|_1\}-\gamma\|w(k,l+1)\|_1-\gamma\|w(k+1,l)\|_1\end{aligned}\tag{24}$$

and

$$C_gx(k,l)=\begin{bmatrix}c_g^{1,1}&c_g^{1,2}&\cdots&c_g^{1,n}\\c_g^{2,1}&c_g^{2,2}&\cdots&c_g^{2,n}\\\vdots&\vdots&\cdots&\vdots\\c_g^{r,1}&c_g^{r,2}&\cdots&c_g^{r,n}\end{bmatrix}\begin{bmatrix}x_1(k,l)\\x_2(k,l)\\\vdots\\x_n(k,l)\end{bmatrix}$$

$$= \begin{bmatrix} c_g^{1,1}x_1(k,l)+c_g^{1,2}x_2(k,l)+\cdots+c_g^{1,n}x_n(k,l) \\ c_g^{2,1}x_1(k,l)+c_g^{2,2}x_2(k,l)+\cdots+c_g^{2,n}x_n(k,l) \\ \vdots \\ c_g^{r,1}x_1(k,l)+c_g^{r,2}x_2(k,l)+\cdots+c_g^{r,n}x_n(k,l) \end{bmatrix}. \tag{25}$$

Then, we can get

$$\begin{aligned} \|C_g x(k,l)\|_1 &= \sum_{m=1}^{r}[c_g^{m,1}x_1(k,l)+c_g^{m,2}x_2(k,l)+\cdots+c_g^{m,n}x_n(k,l)] \\ &= \left(\sum_{m=1}^{r}c_g^{m,1}\right)x_1(k,l)+\left(\sum_{m=1}^{r}c_g^{m,2}\right)x_2(k,l)+\cdots+\left(\sum_{m=1}^{r}c_g^{m,n}\right)x_n(k,l) \\ &= x^T(k,l)\left[\begin{matrix}\|c_g^1\|_1 & \|c_g^2\|_1 & \cdots & \|c_g^n\|_1\end{matrix}\right]^T \end{aligned} \tag{26}$$

where c_g^j represents the j-th column vector of matrix C_g, and $c_g^{m,j}$ represents the entry located at (m,j) of matrix C_g with $g \in \underline{N}$. Similarly, we have

$$\|D_g w(k,l)\|_1 = w^T(k,l)\left[\begin{matrix}\|d_g^1\|_1 & \|d_g^2\|_1 & \cdots & \|d_g^t\|_1\end{matrix}\right]^T \tag{27}$$

$$\|\gamma w(k,l)\|_1 = w^T(k,l)[\begin{matrix}\gamma & \gamma & \cdots & \gamma\end{matrix}]^T \tag{28}$$

where d_g^j represents the j-th column vector of matrix D_g.

Substituting (26)–(28) into (24) leads to

$$\begin{aligned} &\left\|\begin{bmatrix} C(h(k,l+1))x(k,l+1)+D(h(k,l+1))w(k,l+1) \\ C(h(k+1,l))x(k+1,l)+D(h(k+1,l))w(k+1,l)\end{bmatrix}\right\|_1 - \gamma\left\|\begin{bmatrix} w(k,l+1) \\ w(k+1,l)\end{bmatrix}\right\|_1 \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N} h_i(k,l+1)h_j(k+1,l)\Big\{x^T(k,l+1)\left[\begin{matrix}\|c_i^1\|_1 & \|c_i^2\|_1 & \cdots & \|c_i^n\|_1\end{matrix}\right]^T \\ &\quad + x^T(k+1,l)\left[\begin{matrix}\|c_j^1\|_1 & \|c_j^2\|_1 & \cdots & \|c_j^n\|_1\end{matrix}\right]^T \\ &\quad + w^T(k,l+1)\left[\begin{matrix}\|d_i^1\|_1 & \|d_i^2\|_1 & \cdots & \|d_i^t\|_1\end{matrix}\right]^T \\ &\quad + w^T(k+1,l)\left[\begin{matrix}\|d_j^1\|_1 & \|d_j^2\|_1 & \cdots & \|d_j^t\|_1\end{matrix}\right]^T\Big\} \\ &\quad - w^T(k,l+1)[\begin{matrix}\gamma & \gamma & \cdots & \gamma\end{matrix}]^T - w^T(k+1,l)[\begin{matrix}\gamma & \gamma & \cdots & \gamma\end{matrix}]^T. \end{aligned} \tag{29}$$

It follows that

$$\begin{aligned} &\Delta V(k+1,l+1)+\|\overline{z}(k,l)\|_1-\gamma\|\overline{w}(k,l)\|_1 \\ &\le x^T(k,l+1)\Big\{A_1^T(h(k,l+1))p+\nu_1+d_1\varsigma_1-(p-q) \\ &\quad + \left[\begin{matrix}\|c^1(h(k,l+1))\|_1 & \|c^2(h(k,l+1))\|_1 & \cdots & \|c^n(h(k,l+1))\|_1\end{matrix}\right]^T\Big\} \\ &\quad + x^T(k+1,l)\Big\{A_2^T(h(k+1,l))p+\nu_2+d_2\varsigma_2-q \\ &\quad + \left[\begin{matrix}\|c^1h(k+1,l)\|_1 & \|c^2h(k+1,l)\|_1 & \cdots & \|c^nh(k+1,l)\|_1\end{matrix}\right]^T\Big\} \\ &\quad + x^T(k-d_1,l+1)\{A_{d1}^T(h(k,l+1))p-\nu_1-\varsigma_1\} \\ &\quad + x^T(k+1,l-d_2)\{A_{d2}^T(h(k+1,l))p-\nu_2-\varsigma_2\} \\ &\quad + w^T(k,l+1)\Big\{B_1^T(h(k,l+1))p-[\begin{matrix}\gamma & \gamma & \cdots & \gamma\end{matrix}]^T \\ &\quad + \left[\begin{matrix}\|d^1(h(k,l+1))\|_1 & \|d^2(h(k,l+1))\|_1 & \cdots & \|d^t(h(k,l+1))\|_1\end{matrix}\right]^T\Big\} \\ &\quad + w^T(k+1,l)\Big\{B_2^T(h(k+1,l))p-[\begin{matrix}\gamma & \gamma & \cdots & \gamma\end{matrix}]^T \\ &\quad + \left[\begin{matrix}\|d^1h(k+1,l)\|_1 & \|d^2h(k+1,l)\|_1 & \cdots & \|d^th(k+1,l)\|_1\end{matrix}\right]^T\Big\}, \end{aligned} \tag{30}$$

where

$$\left[\begin{matrix}c^jh(k,l) & d^jh(k,l)\end{matrix}\right] = \sum_{i=1}^{N}\left[\begin{matrix}c_i^jh(k,l) & d_i^jh(k,l)\end{matrix}\right].$$

From LMIs in (21), we get the following inequalities

$$\begin{aligned} &A_{d1i}^T p-\varsigma_1-\nu_1 \prec 0, \\ &A_{d2i}^T p-\varsigma_2-\nu_2 \prec 0, \\ &A_{1i}^T p+\nu_1+d_1\varsigma_1-(p-q)+\left[\begin{matrix}\|c_g^1\|_1 & \|c_g^2\|_1 & \cdots & \|c_g^n\|_1\end{matrix}\right]^T \prec 0, \\ &A_{2i}^T p+\nu_2+d_2\varsigma_2-q+\left[\begin{matrix}\|c_f^1\|_1 & \|c_f^2\|_1 & \cdots & \|c_f^n\|_1\end{matrix}\right]^T \prec 0, \\ &B_{1i}^T p+\left[\begin{matrix}\|d_g^1\|_1 & \|d_g^2\|_1 & \cdots & \|d_g^t\|_1\end{matrix}\right]^T-[\begin{matrix}\gamma & \gamma & \cdots & \gamma\end{matrix}]^T \prec 0, \\ &B_{2i}^T p+\left[\begin{matrix}\|d_f^1\|_1 & \|d_f^2\|_1 & \cdots & \|d_f^t\|_1\end{matrix}\right]^T-[\begin{matrix}\gamma & \gamma & \cdots & \gamma\end{matrix}]^T \prec 0. \end{aligned} \tag{31}$$

It follows from (23) and (31) that

$$\begin{aligned} &V_1(k+1,l+1)+V_2(k+1,l+1)-V_1(k,l+1)-V_2(k+1,l) \\ &\quad +\|\overline{z}(k,l)\|_1-\gamma\|\overline{w}(k,l)\|_1 < 0. \end{aligned}$$

Note that

$$\Delta V(k+1,l+1) = V_1(k+1,l+1)-V_1(k,l+1)+V_2(k+1,l+1)-V_2(k+1,l)$$

For any positive scalars $k_1, l_1 \in Z_+$, it can be verified that

$$\begin{aligned} &\sum_{k=0}^{k_1}\sum_{l=0}^{l_1}\Delta V(k+1,l+1) \\ &= \sum_{k=0}^{k_1}\sum_{l=0}^{l_1}(V_1(k+1,l+1)-V_1(k,l+1)) \\ &\quad + \sum_{k=0}^{k_1}\sum_{l=0}^{l_1}(V_2(k+1,l+1)-V_2(k+1,l)) \\ &= \sum_{l=0}^{l_1}(V_1(k_1+1,l+1)-V_1(0,l+1)) \\ &\quad + \sum_{k=0}^{k_1}(V_2(k+1,l_1+1)-V_2(k+1,0)). \end{aligned} \tag{32}$$

When $k_1, l_1 \to \infty$, we have

$$\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(\|\overline{z}(k,l)\|_1-\gamma\|\overline{w}(k,l)\|_1) < \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\Delta V(k+1,l+1). \tag{33}$$

The existence of a solution to LMIs in (21) implies that the positive 2D T–S fuzzy system (4) is asymptotically stable. Together with the zero boundary condition, one can get

$$\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\Delta V(k+1,l+1) \le 0. \tag{34}$$

Applying (34) to (33), one has

$$\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\|\overline{z}(k,l)\|_1 < \gamma\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\|\overline{w}(k,l)\|_1. \tag{35}$$

By Definition 3, positive 2D T–S fuzzy system (4) is asymptotically stable and has the l_1-gain index γ.

This completes the proof.

Remark 4. In Theorem 2, a sufficient condition of l_1-gain performance for positive 2D T–S fuzzy systems is proposed via the co-positive type Lyapunov function method, which is different from the existing results on H_∞ performance of 2D T–S fuzzy systems (see [36]), where the quadratic Lyapunov function method is utilized. In addition, the results obtained with the use of co-positive Lyapunov function are simple and are easy to compute [39,40].

Remark 5. Note that the l_1-gain performance of positive linear 2D Roesser model has been investigated in [24]. However, the major work of this paper is to analyze the l_1-gain performance for positive 2D T–S fuzzy systems in the second FM model, which is much more complex than that of [24].

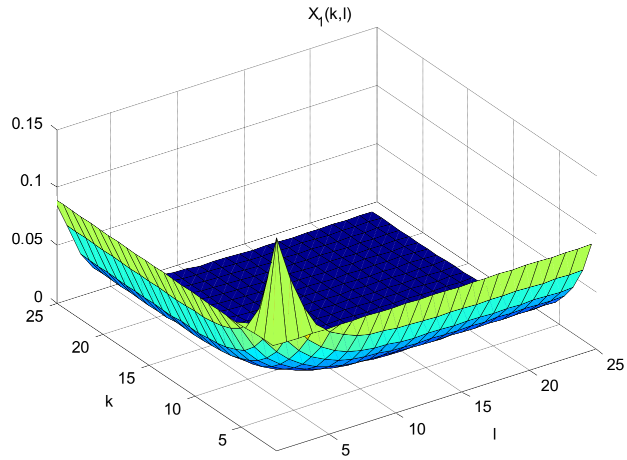

Fig. 1. State trajectory of $x_1(k,l)$.

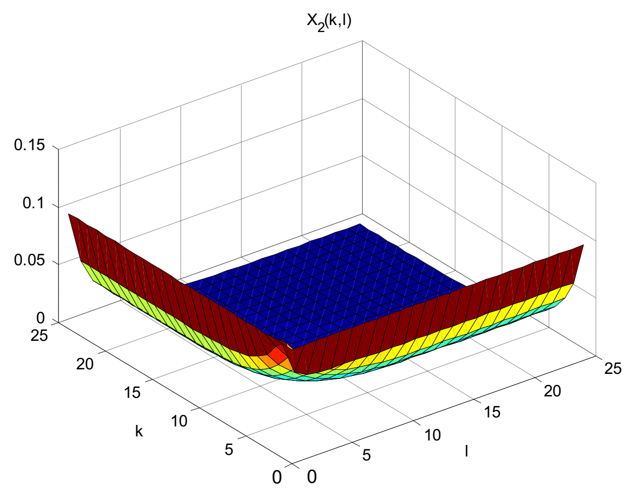

Fig. 2. State trajectory of $x_2(k,l)$.

5. Numerical example

Consider the following positive 2D nonlinear system with delays

$$x(k+1,l+1)=\begin{bmatrix}0.3 & 0.1+0.1\sin^2(x_1(k,l+1))\\ 0.3 & 0.15\end{bmatrix}x(k,l+1)$$
$$+\begin{bmatrix}0.1 & 0.3\\ 0 & 0.2+0.1\sin^2(x_1(k,l+1))\end{bmatrix}x(k+1,l)$$
$$+\begin{bmatrix}0.02 & 0.03\\ 0 & 0.02+0.01\sin^2(x_1(k,l+1))\end{bmatrix}x(k-d_1,l+1)$$
$$+\begin{bmatrix}0 & 0.02+0.01\sin^2(x_1(k,l+1))\\ 0.01 & 0.03\end{bmatrix}x(k+1,l-d_2)$$
$$+\begin{bmatrix}0.1\\ 0.1\end{bmatrix}w(k,l+1)+\begin{bmatrix}0.1\\ 0.1\end{bmatrix}w(k+1,l),$$

$$z(k,l)=\begin{bmatrix}0.1 & 0.2-0.1\sin^2(x_1(k,l+1))\end{bmatrix}x(k,l)+0.2w(k,l),$$

where $x(k,l)=\begin{bmatrix}x_1(k,l) & x_2(k,l)\end{bmatrix}^T$. We can get the positive 2D T–S fuzzy system rule according to the value of $\sin^2(x_1(k+1,l))$:

IF $\sin^2(x_1(k+1,l))$ is about 0, THEN

$$x(k+1,l+1)=A_{11}x(k,l+1)+A_{21}x(k+1,l)+A_{d11}x(k-d_1,l+1)$$
$$+A_{d21}x(k+1,l-d_2)+B_{11}w(k,l+1)+B_{21}w(k+1,l),$$

$$z(k,l)=C_1x(k,l)+D_1w(k,l).$$

IF $\sin^2(x_1(k+1,l))$ is about 1, THEN

$$x(k+1,l+1)=A_{12}x(k,l+1)+A_{22}x(k+1,l)+A_{d12}x(k-d_1,l+1)$$
$$+A_{d22}x(k+1,l-d_2)+B_{12}w(k,l+1)+B_{22}w(k+1,l),$$

$$z(k,l)=C_2x(k,l)+D_2w(k,l),$$

where

$$A_{11}=\begin{bmatrix}0.3 & 0.1\\ 0.3 & 0.15\end{bmatrix},\ A_{21}=\begin{bmatrix}0.1 & 0.3\\ 0 & 0.2\end{bmatrix},$$
$$A_{d11}=\begin{bmatrix}0.02 & 0.03\\ 0 & 0.02\end{bmatrix},\ A_{d21}=\begin{bmatrix}0 & 0.02\\ 0.01 & 0.03\end{bmatrix},$$

$$A_{12}=\begin{bmatrix}0.3 & 0.2\\ 0.3 & 0.15\end{bmatrix},\ A_{22}=\begin{bmatrix}0.1 & 0.3\\ 0 & 0.3\end{bmatrix},$$
$$A_{d12}=\begin{bmatrix}0.02 & 0.03\\ 0 & 0.03\end{bmatrix},\ A_{d22}=\begin{bmatrix}0 & 0.03\\ 0.01 & 0.03\end{bmatrix},$$

$$B_{11}=B_{21}=B_{12}=B_{22}=\begin{bmatrix}0.1\\ 0.1\end{bmatrix},\ C_1=\begin{bmatrix}0.1 & 0.2\end{bmatrix},$$
$$C_2=\begin{bmatrix}0.1 & 0.1\end{bmatrix},\ D_1=D_2=0.2.$$

Then the normalized membership functions of the attained discrete-time 2D T–S fuzzy system become

$$h_1=\begin{bmatrix}h_1(k,l+1) & h_1(k+1,l)\end{bmatrix}$$
$$=\begin{bmatrix}1-\sin^2(x_1(k,l+1)) & 1-\sin^2(x_1(k,l+1))\end{bmatrix},$$

$$h_2=\begin{bmatrix}h_2(k,l+1) & h_2(k+1,l)\end{bmatrix}=\begin{bmatrix}\sin^2(x_1(k,l+1)) & \sin^2(x_1(k,l+1))\end{bmatrix}.$$

For this example, $d_1=1$, $d_2=2$ and $\gamma=4.5$ are given. Then by using the LMI Control Toolbox [41] to solve LMIs of Theorem 2, we can get the following solution:

$$p=\begin{bmatrix}26.6966\\ 39.0618\end{bmatrix},\ q=\begin{bmatrix}3.9849\\ 23.8739\end{bmatrix},\ v_1=\begin{bmatrix}1.2606\\ 2.1802\end{bmatrix},$$

$$v_2=\begin{bmatrix}0.5193\\ 2.7029\end{bmatrix},\ \varsigma_1=\begin{bmatrix}0.2473\\ 0.4574\end{bmatrix},\ \varsigma_2=\begin{bmatrix}0.0601\\ 0.2362\end{bmatrix}.$$

The boundary conditions are given by

$$x(k,l)=\begin{bmatrix}0.1\\ 0.1\end{bmatrix},\ -d_1\le k\le 0,\quad 0\le l\le 20,\ \text{and } 0\le k\le 20,\ -d_2\le l\le 0.$$

The simulation results in Figs. 1 and 2 show the state responses of the system. It can be observed that the system is positive and asymptotically stable.

Under zero boundary conditions, by computing we get $\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\|\overline{z}(k,l)\|_1=1.2417$ and $\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\|\overline{w}(k,l)\|_1=3.9177$. It can be found that the system has a prescribed l_1-gain performance level $\gamma=4.5$.

6. Conclusions

This paper has addressed the problems of stability and l_1-gain performance analysis for discrete positive 2D T–S fuzzy systems with state delays in the second FM model. A sufficient condition for the delay-dependent asymptotic stability of positive 2D T–S fuzzy linear systems with time delays was established. Co-positive type Lyapunov function method was used to get an LMI-based sufficient criterion which ensures asymptotical stability and a prescribed l_1-gain performance level. A numerical example was finally given to illustrate the efficiency of the obtained criterion. Furthermore, future work will be devoted to the control problem for positive 2D T–S fuzzy systems.

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Zhaoxia Duan was born in Sichuan Province, China, in 1989. She received her BS degree in Automation from Nanjing University of Science and Technology (NUST), Nanjing, China, in 2011. Now she is pursuing a Ph.D. degree in Control Theory and Control Engineering from NUST, Nanjing, China. Her current research interests include robust control and filtering, switched systems, two-dimensional systems and nonlinear systems.

Zhengrong Xiang received his Ph.D. degree in Control Theory and Control Engineering at Nanjing University of Science and Technology, Nanjing, China, in 1998. Since 1998 he has been faculty member and he is currently full professor at Nanjing University of Science and Technology. He was appointed as Lecturer in 1998 and Associate Professor in 2001 at Nanjing University of Science and Technology. He is a member of the IEEE, member of the Chinese Association for Artificial Intelligence. His main research interests include switched systems, nonlinear control, robust control, and networked control systems.

Hamid Reza Karimi received his Ph.D. degree in control systems engineering from University of Tehran in 2005. He is currently a Professor in Control Systems at the Faculty of Engineering and Science of the University of Agder in Norway. His research interests are in the areas of nonlinear systems, networked control systems, robust control/filter design, time-delay systems, wavelets and vibration control of flexible structures with an emphasis on applications in engineering. He is a senior member of IEEE and serves as chairman of the IEEE chapter on control systems at IEEE Norway section. He is also serving as an editorial board member for some international journals, such as Mechatronics, Neurocomputing, Information Sciences, Asian Journal of Control, Journal of Franklin Institute, IEEE Access, Journal of Systems and Control Engineering, and International Journal of Control, Automation and Systems. He is a member of IEEE Technical Committee on Systems with Uncertainty, IFAC Technical Committee on Robust Control and IFAC Technical Committee on Automotive Control.