Two-Dimensional Dissipative Control and Filtering for Roesser Model

Choon Ki Ahn, *Senior Member, IEEE*, Peng Shi, *Fellow, IEEE*, and Michael V. Basin, *Senior Member, IEEE*

***Abstract*—This paper investigates the problems of two-dimensional (2-D) dissipative control and filtering for a linear discrete-time Roesser model. First, a novel sufficient condition is proposed such that the discrete-time Roesser system is asymptotically stable and 2-D (Q, S, R)-α-dissipative. Special cases, such as 2-D passivity performance and 2-D H_∞ performance, and feedback interconnected systems are also discussed. Based on this condition, new 2-D (Q, S, R)-α-dissipative state-feedback and output-feedback control problems are defined and solved for a discrete-time Roesser model. The design problems of 2-D (Q, S, R)-α-dissipative filters of observer form and general form are also considered using a linear matrix inequality (LMI) approach. Two examples are given to illustrate the effectiveness and potential of the proposed design techniques.**

***Index Terms*—Control and filtering, dissipativity, Roesser model, two-dimensional (2-D) system.**

I. Introduction

Two-dimensional (2-D) systems and their practical applications in many areas (e.g., circuit analysis, image processing, signal filtering, water stream heating, and thermal power engineering) have received much attention from researchers over the past few decades [1]–[3]. This has led to wide acceptance in recent years of 2-D systems theory as an important tool in solving control and filtering problems. Roesser [4] and Fornasini [5] developed 2-D realization theory in a state-space framework, Hinamoto [6] dealt with 2-D system stability, and Bisiacco [7] proposed the 2-D optimal control theory. Disturbance attenuation problems were considered by Du and Xie, who established H_∞ control theory for 2-D systems based on the 2-D bounded real lemma [8]. This result was extended to filtering problems for 2-D systems with parametric uncertainties [8], polytopic uncertainties [9], state-dependent noise [10], and time delays [11]. Wu *et al.* designed a full-order H_∞ filter for 2-D systems with Markovian jump parameters [12] and also investigated the H_∞ model reduction problem for 2-D state-delayed discrete systems [13]. Recently, results on $l_2 - l_\infty$ stability analysis were established for a class of 2-D nonlinear disturbed systems [14], [15]. Note that the Roesser model has been a widely used model for the study of 2-D systems among some existing models of 2-D systems [16]–[23].

The theory of dissipative systems, first introduced by Willems [24], [25], is applied in a wide range of areas, such as circuits, systems, networks, and controls. The dissipativity theory provides a unified framework for the analysis and design of control and signal processing systems with an input-output energy-related characterization. It generalizes many independent theorems or lemmas, including the stability theorem, the bounded real lemma, the passivity theorem, the Kalman-Yakubovich-Popov (KYP) lemma, and the circle criterion [26]–[28]. Thus, in recent years, dissipativity and its application in control and filtering problems have been extensively studied, and many significant research results have been reported in the literature. Dissipative controller design methods were proposed for linear continuous-time [29] and discrete-time [30] systems. Dissipative state-feedback and output-feedback controllers were designed for linear time-delayed systems in [31]. Sufficient conditions for the state-feedback dissipative control of stochastic impulsive systems were obtained in [32]. Wu et al. defined a novel dissipativity for stochastic nonlinear systems by introducing the escape time in [33]. Wu et al. also analyzed dissipativity for discrete-time stochastic delayed neural networks in [34]. The dissipative static output-feedback control problems of linear systems were investigated in [35], [36]. Recently, a reliable dissipative filter [37] was designed for a class of fuzzy systems with time-varying delays and sensor failures. Unfortunately, these studies were all restricted to one-dimensional (1-D) systems due to the dynamical and structural complexity of 2-D systems, which differ significantly from 1-D systems. To the best of the authors' knowledge, no results on dissipative control and filtering for 2-D systems have been presented in the literature, even though their 1-D results have been known for approximately three decades. This fact motivates our present research.

Based on the above considerations, the purpose of this paper is to solve the problems of dissipative control and filtering for 2-D systems in the discrete-time Roesser model for the first time. First, we establish a new sufficient condition, such that 2-D systems in the Roesser model are asymptotically stable

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and 2-D (Q, S, R)-α-dissipative. Some special cases (2-D passivity performance, 2-D H_∞ performance, and mixed 2-D H_∞/passivity performance) and interconnected 2-D systems are also discussed. Based on this result, we then focus on defining and solving new 2-D (Q, S, R)-α-dissipative state-feedback and output-feedback control problems. Next, we develop systematic methods for the design of 2-D (Q, S, R)-α-dissipative filters of observer form and general form. The problems addressed in this paper can be cast into convex optimization problems in terms of linear matrix inequalities (LMIs) [38], [39], which can be efficiently solved using existing software packages. The main contributions of this paper are summarized as follows.

1) The 2-D (Q, S, R)-α-dissipativity is defined for the discrete-time Roesser model and, based on this definition, a sufficient LMI condition is derived, which unifies the conditions of 2-D passivity performance and 2-D H_∞ performance. The feedback interconnection of 2-D (Q, S, R)-α-dissipative systems is also discussed (Section II).
2) 2-D (Q, S, R)-α-dissipative control problems are defined and solved via state feedback and output feedback controllers based on an LMI approach (Section III).
3) 2-D (Q, S, R)-α-dissipative filter design methods based on observer form and general form are proposed for 2-D state-space systems in the Roesser form using an LMI approach (Section IV).

This paper is organized as follows. Section II proposes a new criterion for the 2-D (Q, S, R)-α-dissipativity of 2-D systems in the Roesser model. In Section III, 2-D (Q, S, R)-α-dissipative state-feedback and output-feedback control problems are defined and solved. In Section IV, 2-D (Q, S, R)-α-dissipative filters of observer form and general form are designed. Section V presents two numerical examples. Finally, conclusions are drawn in Section VI.

II. 2-D Dissipativity Performance Bound

Consider the following 2-D discrete-time linear system in the Roesser model:

$$x^h(i+1, j) = A_{11}x^h(i, j) + A_{12}x^v(i, j) + B_1u(i, j) \quad (1)$$

$$x^v(i, j+1) = A_{21}x^h(i, j) + A_{22}x^v(i, j) + B_2u(i, j) \quad (2)$$

$$y(i, j) = C_1x^h(i, j) + C_2x^v(i, j) + Du(i, j) \quad (3)$$

where $x^h(i, j) \in R^m$ is the horizontal state, $x^v(i, j) \in R^n$ is the vertical state, $u(i, j) \in R^p$ is the input, and $y(i, j) \in R^q$ is the output. $A_{11} \in R^{m\times m}$, $A_{12} \in R^{m\times n}$, $A_{21} \in R^{n\times m}$, $A_{22} \in R^{n\times n}$, $B_1 \in R^{m\times p}$, $B_2 \in R^{n\times p}$, $C_1 \in R^{q\times m}$, $C_2 \in R^{q\times n}$, and $D \in R^{q\times p}$ are the system matrices.

We now define the 2-D (Q, S, R)-α-dissipativity based on the 1-D dissipativity in [30], [34], [40], [41].

Definition 1 (2-D (Q, S, R)-α-Dissipativity): Given some scalar $\alpha > 0$, matrices Q, S, and R with Q and R real symmetric, (1)–(3) is strictly 2-D (Q, S, R)-α-dissipative, for any $T_i \geq 0$ and $T_j \geq 0$, if the following condition is satisfied under zero boundary conditions:

$$\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} y^T(i, j)Qy(i, j) + 2\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} y^T(i, j)Su(i, j) + \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} u^T(i, j)Ru(i, j) \geq \alpha \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} u^T(i, j)u(i, j). \quad (4)$$

Without a loss of generality, we assume that $Q \leq 0$ and that $-Q = Q_*^TQ_*$ for some Q_*.

Throughout the paper, the notation $\star$ is used as an ellipsis for terms that are induced by symmetry, $diag\{\cdot\}$ denotes a block-diagonal matrix, and $sym\{A\}$ indicates $A + A^T$.

In this section, we will focus our study on the dissipativity performance bound of the 2-D linear system (1)–(3) in the Roesser model.

Theorem 1: Given some scalar $\alpha > 0$, matrices Q, S, and R with Q and R real symmetric, the 2-D system (1)–(3) is strictly 2-D (Q, S, R)-α-dissipative if there exist symmetric positive definite matrices P_h and P_v such that

$$\Gamma = \begin{bmatrix} \Gamma_{1,1} & \Gamma_{1,2} & \Gamma_{1,3} \\ \star & \Gamma_{2,2} & \Gamma_{2,3} \\ \star & \star & \Gamma_{3,3} \end{bmatrix} < 0 \quad (5)$$

where

$$\begin{aligned}
\Gamma_{1,1} &= A_{11}^TP_hA_{11} + A_{21}^TP_vA_{21} - P_h - C_1^TQC_1 \\
\Gamma_{2,2} &= A_{12}^TP_hA_{12} + A_{22}^TP_vA_{22} - P_v - C_2^TQC_2 \\
\Gamma_{3,3} &= B_1^TP_hB_1 + B_2^TP_vB_2 - R + \alpha I \\
&\quad - D^TQD - sym\{S^TD\} \\
\Gamma_{1,2} &= A_{11}^TP_hA_{12} + A_{21}^TP_vA_{22} - C_1^TQC_2 \\
\Gamma_{1,3} &= A_{11}^TP_hB_1 + A_{21}^TP_vB_2 - C_1^TQD - C_1^TS \\
\Gamma_{2,3} &= A_{12}^TP_hB_1 + A_{22}^TP_vB_2 - C_2^TQD - C_2^TS.
\end{aligned}$$

Furthermore, condition (5) guarantees that the unforced 2-D system (1)–(3) with $u(i, j) = 0$ is asymptotically stable.

Proof: First, we study the dissipativity of the 2-D system (1)–(3) with $u(i, j) \neq 0$. By considering $V(x^h(i, j), x^v(i, j)) = {x^h}^T(i, j)P_hx^h(i, j) + {x^v}^T(i, j)P_vx^v(i, j)$ and defining $\Psi(i, j) = [{x^h}^T(i, j) \quad {x^v}^T(i, j) \quad u^T(i, j)]^T$, we obtain

$$\begin{aligned}
&\Delta V\left(x^h(i, j), x^v(i, j)\right) - y^T(i, j)Qy(i, j) \\
&\quad - 2y^T(i, j)Su(i, j) - u^T(i, j)[R - \alpha I]u(i, j) \\
&= \Psi^T(i, j)\Gamma\Psi(i, j)
\end{aligned} \quad (6)$$

where $\Delta V(x^h(i, j), x^v(i, j)) = V(x^h(i+1, j), x^v(i, j+1)) - V(x^h(i, j), x^v(i, j))$. $\Gamma < 0$ implies

$$\begin{aligned}
&\Delta V\left(x^h(i, j), x^v(i, j)\right) - y^T(i, j)Qy(i, j) \\
&\quad - 2y^T(i, j)Su(i, j) - u^T(i, j)[R - \alpha I]u(i, j) < 0.
\end{aligned} \quad (7)$$

Note that

$$\begin{aligned}\sum_{i=0}^{T_i}\sum_{j=0}^{T_j}&\left\{\Delta V\left(x^h(i,j),x^v(i,j)\right)-y^T(i,j)Qy(i,j)\right.\\&\left.-2y^T(i,j)Su(i,j)-u^T(i,j)[R-\alpha I]u(i,j)\right\}\\=&\sum_{i=0}^{T_i}\Big[x^{vT}(i,T_j+1)P_vx^v(i,T_j+1)-x^{vT}(i,0)P_vx^v(i,0)\Big]\\&+\sum_{j=0}^{T_j}\Big[x^{hT}(T_i+1,j)P_hx^h(T_i+1,j)-x^{hT}(0,j)P_hx^h(0,j)\Big]\\&-\sum_{i=0}^{T_i}\sum_{j=0}^{T_j}\left\{y^T(i,j)Qy(i,j)+2y^T(i,j)Su(i,j)\right.\\&\left.+u^T(i,j)[R-\alpha I]u(i,j)\right\}\\\geq&-\sum_{i=0}^{T_i}\sum_{j=0}^{T_j}\left\{y^T(i,j)Qy(i,j)+2y^T(i,j)Su(i,j)\right.\\&\left.+u^T(i,j)[R-\alpha I]u(i,j)\right\}\end{aligned}\tag{8}$$

under zero boundary conditions. From (7) and (8), we have

$$\sum_{i=0}^{T_i}\sum_{j=0}^{T_j}\left\{y^T(i,j)Qy(i,j)+2y^T(i,j)Su(i,j)+u^T(i,j)[R-\alpha I]u(i,j)\right\}>0\tag{9}$$

which leads to (4). Therefore, according to Definition 1, the 2-D system (1)–(3) is strictly 2-D (Q,S,R)-α-dissipative.

Next, we prove the stability of the 2-D system (1)–(3) with $u(i,j)=0$. To this end, we consider $V(x^h(i,j),x^v(i,j))=x^{hT}(i,j)P_hx^h(i,j)+x^{vT}(i,j)P_vx^v(i,j)$. We then have

$$\begin{aligned}\Delta V&\left(x^h(i,j),x^v(i,j)\right)\\&=\begin{bmatrix}x^h(i,j)\\x^v(i,j)\\0\end{bmatrix}^T\left\{\Gamma-[C_1\quad C_2\quad 0]^TQ_*^TQ_*\right.\\&\quad\left.\times[C_1\quad C_2\quad 0]\right\}\begin{bmatrix}x^h(i,j)\\x^v(i,j)\\0\end{bmatrix}.\end{aligned}\tag{10}$$

$\Gamma<0$ implies $\Delta V(x^h(i,j),x^v(i,j))<0$ for $x^h(i,j)\neq 0$ and $x^v(i,j)\neq 0$. According to Theorem 1 in [17], we have

$$\lim_{i,j\to\infty}\begin{bmatrix}x^h(i,j)\\x^v(i,j)\end{bmatrix}=0.\tag{11}$$

This completes the proof. ■

Remark 1: The LMI (5) is not only over the matrix variables P_h and P_v, but also over the scalar α. The scalar α was introduced in the right-hand side of the 2-D (Q,S,R)-α-dissipativity definition (4), because it makes the 2-D system (1)–(3) satisfy the strict dissipativity inequality: $\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} y^T(i,j)Qy(i,j)+2\sum_{i=0}^{T_i}\sum_{j=0}^{T_j}y^T(i,j)Su(i,j)+\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} u^T(i,j)Ru(i,j)>0$ when $u(i,j)\neq 0$. In this case, the scalar α can be regarded as a tunable parameter for determining the strictness of the dissipativity in Definition 1. If there exists a scalar $\alpha>0$ satisfying (4), then the 2-D system (1)–(3) is strictly dissipative. If α is increased, (4) becomes more difficult to satisfy. Thus, we can obtain the maximum value of α, which is the optimal dissipativity performance bound α^*, such that the 2-D system (1)–(3) is strictly 2-D (Q,S,R)-α-dissipative. The value of α^* can be obtained by maximizing α subject to $P_h>0$, $P_v>0$, and (5) via MATLAB LMI Control Toolbox [39] or Scilab [42] (developed by INRIA, France).

Remark 2: Recently, a set of stability conditions was proposed for linear repetitive processes based on a dissipative setting [43]. The reference [43] focused on stability and did not address the explicit design of the control law. However, the present paper presents the concept on 2-D (Q,S,R)-α-dissipativity for the Roesser model and proposes solutions to control and filtering problems for the Roesser model based on 2-D (Q,S,R)-α-dissipativity. Thus, in this paper, we consider and solve different problems from those addressed in [43].

Remark 3: For the first time, this paper proposes a 2-D (Q,S,R)-α-dissipativity concept for the Roesser model. Thus, this paper presents a new possibility for application of 2-D (Q,S,R)-α-dissipativity to create new control and filtering methods for 2-D systems. For example, the 2-D (Q,S,R)-α-dissipativity proposed in this paper can be applied to the stability analysis of the 2-D Fornasini-Marchesini model [44]–[47]. It is expected to be applied to the development of 2-D dissipative sampled-data control, 2-D dissipative stochastic filtering, and 2-D fault-distribution dependent dissipative control based on the results in [48]–[50], respectively.

Theorem 2: The LMI condition (5) is equivalent to the following LMI condition:

$$\begin{bmatrix}-P & PA & PB & 0\\ \star & -P & -C^TS & C^TQ_*^T\\ \star & \star & -R+\alpha I-sym\{S^TD\} & D^TQ_*^T\\ \star & \star & \star & -I\end{bmatrix}<0\tag{12}$$

where

$$\begin{aligned}A&\triangleq\begin{bmatrix}A_{11} & A_{12}\\A_{21} & A_{22}\end{bmatrix},\quad B\triangleq\begin{bmatrix}B_1\\B_2\end{bmatrix},\\C&\triangleq[C_1\quad C_2],\quad P\triangleq\begin{bmatrix}P_h & 0\\ \star & P_v\end{bmatrix}.\end{aligned}\tag{13}$$

Proof: Based on the definitions (13), the LMI condition (5) is equivalently represented by

$$\begin{bmatrix}A^TPA-P-C^TQC & A^TPB-C^TQD-C^TS\\ \star & B^TPB-R+\alpha I-D^TQD-sym\{S^TD\}\end{bmatrix}<0\tag{14}$$

which is rewritten as

$$\begin{bmatrix}-P & -C^TS\\ \star & -R+\alpha I-sym\{S^TD\}\end{bmatrix}+\begin{bmatrix}A & B\\Q_*C & Q_*D\end{bmatrix}^T\begin{bmatrix}P & 0\\ \star & I\end{bmatrix}\begin{bmatrix}A & B\\Q_*C & Q_*D\end{bmatrix}<0.$$

By the Schur complement, we have

$$\begin{bmatrix} -P & -C^TS & A^T & C^TQ_*^T \\ \star & -R+\alpha I - sym\{S^TD\} & B^T & D^TQ_*^T \\ \star & \star & -P^{-1} & 0 \\ \star & \star & \star & -I \end{bmatrix} < 0. \tag{15}$$

Define

$$\Pi \triangleq \begin{bmatrix} 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ P & 0 & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}. \tag{16}$$

Pre-multiplying and post-multiplying (15) by Π^T and Π, respectively, yields (12). This completes the proof. ∎

The LMI condition (12) plays an important role in the design of 2-D dissipative controllers and filters. We apply Theorem 2 to derive 2-D (Q, S, R)-α-dissipative controllers and filters in Sections III and IV, respectively.

Remark 4: Focus on the horizontal state, fix j, and let A_{12}, A_{21}, A_{22}, B_2, C_2, and P_v be null matrices. Define $\mathbf{x}(i) \triangleq x^h(i,j)$, $\mathbf{u}(i) \triangleq u(i,j)$, and $\mathbf{y}(i) \triangleq y(i,j)$. From (1), (3), and (4), we obtain the following 1-D system:

$$\mathbf{x}(i+1) = A_{11}\mathbf{x}(i) + B_1\mathbf{u}(i) \tag{17}$$

$$\mathbf{y}(i) = C_1\mathbf{x}(i) + D\mathbf{u}(i) \tag{18}$$

and the following 1-D dissipativity definition:

$$\sum_{i=0}^{T_i} \mathbf{y}^T(i)Q\mathbf{y}(i) + 2\sum_{i=0}^{T_i} \mathbf{y}^T(i)S\mathbf{u}(i) + \sum_{i=0}^{T_i} \mathbf{u}^T(i)R\mathbf{u}(i) \geq \alpha \sum_{i=0}^{T_i} \mathbf{u}^T(i)\mathbf{u}(i). \tag{19}$$

Define

$$\bar{\Pi} \triangleq \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{bmatrix}. \tag{20}$$

The condition (14) then becomes

$$\begin{bmatrix} A_{11}^TP_hA_{11} - P_h - C_1^TQC_1 & A_{11}^TP_hB_1 - C_1^TQD - C_1^TS \\ \star & B_1^TP_hB_1 - R + \alpha I - D^TQD - sym\{S^TD\} \end{bmatrix} < 0 \tag{21}$$

through a congruence transformation with $\bar{\Pi}$ and the Schur complement. (21) is a matrix inequality condition for the dissipativity of the 1-D system (17) and (18).

We now provide a characterization of dissipativity; in other words, passivity of the 2-D system (1)–(3), by choosing $Q = 0$, $S = I$, and $R = 2\alpha I$, as follows:

Corollary 1: Given some scalar $\alpha > 0$, the 2-D system (1)–(3) is asymptotically stable and passive if there exists a positive definite block-diagonal matrix $P = diag\{P_h, P_v\}$, where $P_h = {P_h}^T > 0$ and $P_v = {P_v}^T > 0$, such that

$$\begin{bmatrix} -P & PA & PB \\ \star & -P & -C^T \\ \star & \star & -\alpha I - sym\{D\} \end{bmatrix} < 0. \tag{22}$$

Remark 5: A sufficient matrix inequality condition is given in Corollary 1 to guarantee the passivity of the 2-D system (1)–(3). Clearly, by minimizing α subject to $P > 0$ and (22), the optimal passivity performance bound α^* is obtained.

Remark 6: Note that although only a delay-free 2-D system is investigated in Theorems 1 and 2 and Corollary 1, the presented results can be easily extended to 2-D systems with time-delays by using similar techniques.

Remark 7: By choosing $Q = -I$, $S = 0$, and $R = (\alpha^2 + \alpha)I$, we can obtain the following matrix inequality condition for the H_∞ performance of the 2-D system (1)–(3) from Theorems 1 and 2:

$$\begin{bmatrix} -P & PA & PB & 0 \\ \star & -P & 0 & C^T \\ \star & \star & -\alpha^2 I & D^T \\ \star & \star & \star & -I \end{bmatrix} < 0 \tag{23}$$

where α is an H_∞ performance bound. By minimizing α subject to $P > 0$ and (23), the optimal H_∞ performance bound α^* can be obtained.

Remark 8: Let $\mu \in [0, 1]$ be a weighting parameter that defines the trade-off between 2-D passive and H_∞ performances. By choosing $Q = -\mu I$, $S = (1-\mu)I$, and $R = [(\alpha^2 - \alpha)\mu + 2\alpha]I$, we can obtain a condition for the mixed 2-D H_∞/passivity performance as follows:

$$\begin{bmatrix} -P & PA & PB & 0 \\ \star & -P & -(1-\mu)C^T & \sqrt{\mu}C^T \\ \star & \star & -\left((\alpha^2-\alpha)\mu+\alpha\right)I-(1-\mu)sym\{D\} & \sqrt{\mu}D^T \\ \star & \star & \star & -I \end{bmatrix} < 0. \tag{24}$$

The 2-D (Q, S, R)-α-dissipativity concept proposed in this paper can be a useful tool for analyzing large-scale 2-D systems when they are represented by a collection of interconnected 2-D subsystems. We restrict our attention to the case in which only two 2-D systems are interconnected. Generalizations of these results to interconnections of more than two 2-D systems can be obtained easily from these results. Consider the feedback interconnection of two 2-D (Q, S, R)-α-dissipative systems, S_A and S_B, where S_A and S_B are given by

$$S_k: \begin{cases} \begin{bmatrix} x_k^h(i+1,j) \\ x_k^v(i,j+1) \end{bmatrix} = A_k \begin{bmatrix} x_k^h(i,j) \\ x_k^v(i,j) \end{bmatrix} + B_k u_k(i,j) \\ y_k(i,j) = C_k \begin{bmatrix} x_k^h(i,j) \\ x_k^v(i,j) \end{bmatrix} + D_k u_k(i,j), \ k = A, B \end{cases}$$

where A_k, B_k, C_k, and D_k are the system matrices. The feedback interconnection of S_A and S_B is depicted in Fig. 1.

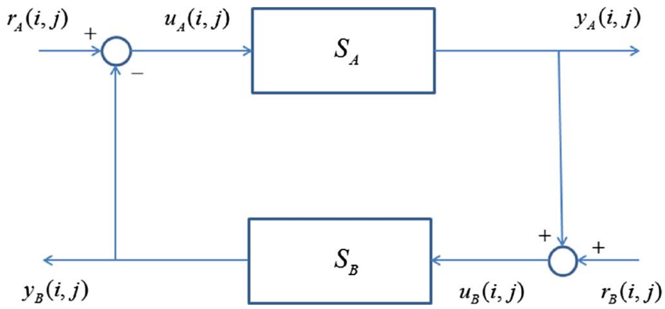

Fig. 1. Feedback interconnection of S_A and S_B.

This interconnected system forms a new 2-D system, which has a mapping from the new input $r_{IC}(i,j)$ to the new output $y_{IC}(i,j)$, where

$$r_{IC}(i,j) \triangleq \begin{bmatrix} r_A(i,j) \\ r_B(i,j) \end{bmatrix},\ y_{IC}(i,j) \triangleq \begin{bmatrix} y_A(i,j) \\ y_B(i,j) \end{bmatrix}. \tag{25}$$

In the following theorem, we find a new condition for the dissipativity performance of the interconnected 2-D system.

Theorem 3: Given scalars $\alpha_k > 0$, matrices Q_k, S_k, and R_k with Q_k and R_k real symmetric, assume that there exist symmetric positive definite matrices $P_{h,k}$ and $P_{v,k}$ satisfying the LMI condition (5) for the 2-D systems S_k, where $k = A, B$. The interconnected system is then 2-D (Q_{IC}, S_{IC}, R_{IC})-α_{IC}-dissipative, where

$$Q_{IC} = \begin{bmatrix} Q_A + R_B - \alpha_B I & -2S_A \\ 2S_B & Q_B + R_A - \alpha_A I \end{bmatrix}, \tag{26}$$

$$S_{IC} = \begin{bmatrix} S_A & R_B - \alpha_B I \\ -R_A + \alpha_A I & S_B \end{bmatrix}, \tag{27}$$

$$R_{IC} = \begin{bmatrix} R_A & 0 \\ \star & R_B \end{bmatrix}, \tag{28}$$

$$\alpha_{IC} = \alpha_A + \alpha_B. \tag{29}$$

Proof: From the interconnection in Fig. 1, we have

$$u_A(i,j) = r_A(i,j) - y_B(i,j) \tag{30}$$

$$u_B(i,j) = r_B(i,j) + y_A(i,j). \tag{31}$$

According to Theorem 1, the 2-D systems S_k are 2-D (Q_k, S_k, R_k)-α_k-dissipative, which guarantees that

$$\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} y_A^T(i,j)Q_A y_A(i,j) + 2\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} y_A^T(i,j)S_A u_A(i,j) + \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} u_A^T(i,j)R_A u_A(i,j) \ge \alpha_A \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} u_A^T(i,j)u_A(i,j),$$

$$\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} y_B^T(i,j)Q_B y_B(i,j) + 2\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} y_B^T(i,j)S_B u_B(i,j) + \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} u_B^T(i,j)R_B u_B(i,j) \ge \alpha_B \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} u_B^T(i,j)u_B(i,j).$$

By adding these two inequalities and considering (30) and (31), we obtain the following relation:

$$\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} y_{IC}^T(i,j)Q_{IC}y_{IC}(i,j) + 2\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} y_{IC}^T(i,j)S_{IC}r_{IC}(i,j) + \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} r_{IC}^T(i,j)R_{IC}r_{IC}(i,j) \ge \alpha_{IC}\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} r_{IC}^T(i,j)r_{IC}(i,j).$$

This completes the proof. ∎

In the next result, a new condition is proposed for the asymptotic stability of the unforced interconnected system.

Corollary 2: Assume that the conditions in Theorem 3 are satisfied. Then, $Q_{IC} < 0$ implies the asymptotic stability of the unforced interconnected system, where Q_{IC} is defined in (26).

Proof: Define

$$x_{IC}^h(i,j) \triangleq \begin{bmatrix} x_A^h(i,j) \\ x_B^h(i,j) \end{bmatrix},\ x_{IC}^v(i,j) \triangleq \begin{bmatrix} x_A^v(i,j) \\ x_B^v(i,j) \end{bmatrix}. \tag{32}$$

When $r_{IC}(i,j) = 0$, (30) and (31) become

$$u_A(i,j) = -y_B(i,j), \tag{33}$$

$$u_B(i,j) = y_A(i,j). \tag{34}$$

Define $\Delta V_{IC}(x_{IC}^h(i,j), x_{IC}^v(i,j)) \triangleq V_{IC}(x_{IC}^h(i+1,j), x_{IC}^v(i,j+1)) - V_{IC}(x_{IC}^h(i,j), x_{IC}^v(i,j))$, where $V_{IC}(x_{IC}^h(i,j), x_{IC}^v(i,j)) = x_{IC}^{h\,T}(i,j)P_{h,IC}x_{IC}^h(i,j) + x_{IC}^{v\,T}(i,j)P_{v,IC}x_{IC}^v(i,j)$,

$$P_{h,IC} = \begin{bmatrix} P_{h,A} & 0 \\ \star & P_{h,B} \end{bmatrix},$$

$$P_{v,IC} = \begin{bmatrix} P_{v,A} & 0 \\ \star & P_{v,B} \end{bmatrix}.$$

Note that $P_{h,IC}$ and $P_{v,IC}$ are symmetric positive definite matrices. Following the proof of Theorem 1, we have

$$\begin{aligned} &\Delta V_{IC}(x_{IC}^h(i,j), x_{IC}^v(i,j)) \\ &\quad < y_A^T(i,j)Q_A y_A(i,j) + 2y_A^T(i,j)S_A u_A(i,j) \\ &\qquad + u_A^T(i,j)[R_A - \alpha_A I]u_A(i,j) \\ &\qquad + y_B^T(i,j)Q_B y_B(i,j) + 2y_B^T(i,j)S_B u_B(i,j) \\ &\qquad + u_B^T(i,j)[R_B - \alpha_B I]u_B(i,j) \\ &\quad = y_A^T(i,j)Q_A y_A(i,j) - 2y_A^T(i,j)S_A y_B(i,j) \\ &\qquad + y_B^T(i,j)[R_A - \alpha_A I]y_B(i,j) \\ &\qquad + y_B^T(i,j)Q_B y_B(i,j) + 2y_B^T(i,j)S_B y_A(i,j) \\ &\qquad + y_A^T(i,j)[R_B - \alpha_B I]y_A(i,j) \\ &\quad = y_{IC}^T(i,j)Q_{IC}y_{IC}(i,j) \end{aligned} \tag{35}$$

using (33) and (34), where $y_{IC}(i,j)$ is defined in (25). $Q_{IC} < 0$ implies $\Delta V_{IC}(x_{IC}^h(i,j), x_{IC}^v(i,j)) < 0$. According to Theorem 1 in [17], we can guarantee $x_{IC}^h(i,j) \to 0$ and $x_{IC}^v(i,j) \to 0$ as $i,j \to \infty$. Thus, we have $x_A^h(i,j) \to 0$, $x_B^h(i,j) \to 0$,

$x_A^v(i,j) \to 0$, and $x_B^v(i,j) \to 0$ as $i,j \to \infty$. This completes the proof. ■

Remark 9: Theorem 3 and Corollary 2 can be easily generalized to interconnections of any finite number of 2-D systems and then utilized to analyze the dissipativity and stability of large-scale 2-D systems.

III. 2-D Dissipative Control

In this section, we consider the 2-D dissipative control problems for the Roesser model via state-feedback and output-feedback controllers. An LMI approach to solve these problems is presented. Consider the following 2-D model in the Roesser form:

$$x^h(i+1,j) = A_{11}x^h(i,j) + A_{12}x^v(i,j) + B_1u(i,j) + G_1w(i,j) \tag{36}$$

$$x^v(i,j+1) = A_{21}x^h(i,j) + A_{22}x^v(i,j) + B_2u(i,j) + G_2w(i,j) \tag{37}$$

$$y(i,j) = C_1x^h(i,j) + C_2x^v(i,j) + G_3w(i,j) \tag{38}$$

$$z(i,j) = E_1x^h(i,j) + E_2x^v(i,j) + Fu(i,j) + G_4w(i,j) \tag{39}$$

where $x^h(i,j) \in R^m$ is the horizontal state, $x^v(i,j) \in R^n$ is the vertical state, $u(i,j) \in R^p$ is the control input, $w(i,j) \in R^l$ is the external disturbance, $y(i,j) \in R^q$ is the output, and $z(i,j) \in R^r$ is the controlled output. A_{11}, A_{12}, A_{21}, A_{22}, B_1, B_2, C_1, C_2, E_1, E_2, F, G_1, G_2, G_3, and G_4 are the system matrices.

A. State-Feedback Control

Let us design the following state-feedback controller:

$$u(i,j) = K\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \tag{40}$$

where $K = [K_1\ K_2]$ is a controller gain. With this controller, using the definitions in (13), we obtain the following closed-loop system:

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = (A+BK)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Gw(i,j) \tag{41}$$

$$z(i,j) = (E+FK)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + G_4w(i,j) \tag{42}$$

where

$$G \triangleq \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \quad E \triangleq [E_1\ E_2]. \tag{43}$$

Definition 2 (2-D (Q,S,R)-α-Dissipative State-Feedback Control): Given some scalar $\alpha > 0$, matrices Q, S, and R with Q and R real symmetric, the controller (40) is a 2-D (Q,S,R)-α-dissipative state-feedback controller for any $T_i \geq 0$ and $T_j \geq 0$ if the following condition is satisfied under zero boundary conditions:

$$\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} z^T(i,j)Qz(i,j) + 2\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} z^T(i,j)Sw(i,j) + \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} w^T(i,j)Rw(i,j) \geq \alpha\sum_{i=0}^{T_i}\sum_{j=0}^{T_j} w^T(i,j)w(i,j). \tag{44}$$

In the following theorem, we can obtain a new 2-D (Q,S,R)-α-dissipative state-feedback control by solving the LMI problem.

Theorem 4: Given some scalar $\alpha > 0$, matrices Q, S, and R with Q and R real symmetric, where $Q = -Q_*^TQ_* \leq 0$ for some Q_*, the controller (40) is a 2-D (Q,S,R)-α-dissipative state-feedback controller and guarantees the asymptotic stability of the 2-D system (36)–(39) if there exist a positive definite block-diagonal matrix $X_c = diag\{X_h^c, X_v^c\}$ and a matrix Y_c, where $X_h^c = X_h^{cT} > 0$ and $X_v^c = X_v^{cT} > 0$, such that

$$\begin{bmatrix} -X_c & AX_c + BY_c & G & 0 \\ \star & -X_c & -(EX_c + FY_c)^TS & (EX_c + FY_c)^TQ_*^T \\ \star & \star & -R + \alpha I - sym\{S^TG_4\} & G_4^TQ_*^T \\ \star & \star & \star & -I \end{bmatrix} < 0. \tag{45}$$

The gain matrix of the controller (40) is then given by $K = Y_c\, diag\{X_h^{c-1}, X_v^{c-1}\}$.

Proof: From the result of Theorem 2, (12), (13), and (43), according to the following correspondences:

$$\begin{aligned} A &\longleftarrow A+BK, & B &\longleftarrow G, \\ C &\longleftarrow E+FK, & D &\longleftarrow G_4 \end{aligned}$$

we obtain $\Omega_{sc} < 0$, where

$$\Omega_{sc} = \begin{bmatrix} -P & P(A+BK) & PG & 0 \\ \star & -P & -(E+FK)^TS & (E+FK)^TQ_*^T \\ \star & \star & -R + \alpha I - sym\{S^TG_4\} & G_4^TQ_*^T \\ \star & \star & \star & -I \end{bmatrix}. \tag{46}$$

Define $X_c = P^{-1} = diag\{P_h^{-1}, P_v^{-1}\}$, $Y_c = KP^{-1} = K\, diag\{P_h^{-1}, P_v^{-1}\}$. Pre-multiplying and post-multiplying $\Omega_{sc} < 0$ by $diag\{P^{-1}, P^{-1}, I, I\}$ yield the LMI (45). The controller gain is then given by $K = Y_cX_c^{-1} = Y_c\ diag\{X_h^{c-1}, X_v^{c-1}\}$. This completes the proof. ■

B. Output-Feedback Control

Assume that F is a null matrix and B is a full column rank matrix. Design the following output-feedback controller for the 2-D system (36)–(39):

$$x_c^h(i+1,j)=A_{c,11}x_c^h(i,j)+A_{c,12}x_c^v(i,j)+B_{c,1}y(i,j) \quad (47)$$

$$x_c^v(i,j+1)=A_{c,21}x_c^h(i,j)+A_{c,22}x_c^v(i,j)+B_{c,2}y(i,j) \quad (48)$$

$$u(i,j)=C_{c,1}x_c^h(i,j)+C_{c,2}x_c^v(i,j)+D_cy(i,j) \quad (49)$$

where $A_{c,11}$, $A_{c,12}$, $A_{c,21}$, $A_{c,22}$, $B_{c,1}$, $B_{c,2}$, $C_{c,1}$, $C_{c,2}$, and D_c are controller gains. Using the definitions in (13) and (43), the augmented system of the 2-D system (36)–(39) and this controller is then of the form

$$\begin{bmatrix}\tilde{x}^h(i+1,j)\\ \tilde{x}^v(i,j+1)\end{bmatrix} = \Xi[\bar{\mathbb{A}}+\mathbb{A}\Theta_c\mathbb{C}]\Xi^T\begin{bmatrix}\tilde{x}^h(i,j)\\ \tilde{x}^v(i,j)\end{bmatrix} + \Xi[\bar{\mathbb{G}}+\mathbb{A}\Theta_c\mathbb{G}_3]w(i,j) \quad (50)$$

$$z(i,j)=\bar{\mathbb{E}}\Xi^T\begin{bmatrix}\tilde{x}^h(i,j)\\ \tilde{x}^v(i,j)\end{bmatrix}+G_4w(i,j) \quad (51)$$

where

$$\begin{aligned}
&\Xi=\begin{bmatrix}I&0&0&0\\0&0&I&0\\0&I&0&0\\0&0&0&I\end{bmatrix},\ \bar{\mathbb{A}}=\begin{bmatrix}A&0\\ \star&0\end{bmatrix},\ \bar{\mathbb{G}}=\begin{bmatrix}G\\0\end{bmatrix}\\
&\mathbb{A}=\begin{bmatrix}B&0\\ \star&I\end{bmatrix},\ \mathbb{C}=\begin{bmatrix}C&0\\ \star&I\end{bmatrix},\ \mathbb{G}_3=\begin{bmatrix}G_3\\0\end{bmatrix}\\
&\Theta_c=\begin{bmatrix}D_c&C_c\\B_c&A_c\end{bmatrix},\ \tilde{x}^h(i+1,j)=\begin{bmatrix}x^h(i+1,j)\\x_c^h(i+1,j)\end{bmatrix}\\
&\bar{\mathbb{E}}=[E\quad 0],\ \tilde{x}^v(i,j+1)=\begin{bmatrix}x^v(i,j+1)\\x_c^v(i,j+1)\end{bmatrix}\\
&A_c=\begin{bmatrix}A_{c,11}&A_{c,12}\\A_{c,21}&A_{c,22}\end{bmatrix},\ B_c=\begin{bmatrix}B_{c,1}\\B_{c,2}\end{bmatrix}\\
&C_c=[\,C_{c,1}\quad C_{c,2}\,].
\end{aligned} \quad (52)$$

Definition 3 (2-D (Q,S,R)-α-Dissipative Output-Feedback Control): Given some scalar $\alpha>0$, matrices Q, S, and R with Q and R real symmetric, the controller (47)–(49) is a 2-D (Q,S,R)-α-dissipative output-feedback controller for any $T_i\geq 0$ and $T_j\geq 0$ if (44) is satisfied under zero boundary conditions.

The following result gives the existence of a 2-D (Q,S,R)-α-dissipative output-feedback controller.

Theorem 5: Assume that F is a null matrix and B is a full column rank matrix. Given a scalar $\alpha>0$, matrices Q, S, and R with Q and R real symmetric, where $Q=-Q_*^TQ_*\leq 0$ for some Q_*, the controller (47)–(49) is a 2-D (Q,S,R)-α-dissipative output-feedback controller and guarantees the asymptotic stability of the 2-D system (36)–(39) if there exist a positive definite block-diagonal matrix $P=diag\{P_h,P_v\}$ and matrices M_c, N_c, where $P_h={P_h}^T>0$ and $P_v={P_v}^T>0$, such that $\Xi\mathbb{A}M_c=P\Xi\mathbb{A}$ and

$$\begin{bmatrix}-P & P\Xi\bar{\mathbb{A}}+\Xi\mathbb{A}N_c\mathbb{C} & P\Xi\bar{\mathbb{G}}+\Xi\mathbb{A}N_c\mathbb{G}_3 & 0\\ \star & -\Xi^TP\Xi & -\bar{\mathbb{E}}^TS & \bar{\mathbb{E}}^TQ_*^T\\ \star & \star & -R+\alpha I-sym\{S^TG_4\} & G_4^TQ_*^T\\ \star&\star&\star&-I\end{bmatrix}<0. \quad (53)$$

The controller gain is then given by $\Theta_c=M_c^{-1}N_c$.

Proof: From the result of Theorem 2 and (12), according to the following correspondences:

$$A\longleftarrow \Xi(\bar{\mathbb{A}}+\mathbb{A}\Theta_c\mathbb{C})\Xi^T,\quad B\longleftarrow \Xi(\bar{\mathbb{G}}+\mathbb{A}\Theta_c\mathbb{G}_3),\quad C\longleftarrow \bar{\mathbb{E}}\Xi^T,\quad D\longleftarrow G_4$$

we obtain $\Omega_{oc}<0$, where

$$\Omega_{oc}=\begin{bmatrix}-P & P\Xi(\bar{\mathbb{A}}+\mathbb{A}\Theta_c\mathbb{C})\Xi^T & P\Xi(\bar{\mathbb{G}}+\mathbb{A}\Theta_c\mathbb{G}_3) & 0\\ \star & -P & -\Xi\bar{\mathbb{E}}^TS & \Xi\bar{\mathbb{E}}^TQ_*^T\\ \star&\star&-R+\alpha I-sym\{S^TG_4\} & G_4^TQ_*^T\\ \star&\star&\star&-I\end{bmatrix}. \quad (54)$$

Pre-multiplying and post-multiplying $\Omega_{oc}<0$ by $diag\{I,\Xi^T,I,I\}$ and $diag\{I,\Xi,I,I\}$, respectively, yield

$$\begin{bmatrix}-P & P\Xi(\bar{\mathbb{A}}+\mathbb{A}\Theta_c\mathbb{C}) & P\Xi(\bar{\mathbb{G}}+\mathbb{A}\Theta_c\mathbb{G}_3) & 0\\ \star & -\Xi^TP\Xi & -\bar{\mathbb{E}}^TS & \bar{\mathbb{E}}^TQ_*^T\\ \star&\star&-R+\alpha I-sym\{S^TG_4\} & G_4^TQ_*^T\\ \star&\star&\star&-I\end{bmatrix}<0. \quad (55)$$

Since B is a full column rank matrix, $\mathbb{A}$ has full column rank. Then, $\Xi\mathbb{A}$ is also a full column rank matrix because Ξ is an invertible matrix. It follows from $\Xi\mathbb{A}M_c=P\Xi\mathbb{A}$ that M_c is also full rank, and thus invertible, yielding $\Xi\mathbb{A}=P\Xi\mathbb{A}M_c^{-1}$. By using this fact and defining $\Theta_c=M_c^{-1}N_c$, we obtain (53) from (54). This completes the proof. ■

Remark 10: Theorem 5 provides a sufficient solvability condition for the 2-D (Q,S,R)-α-dissipative output-feedback control problem of 2-D systems. A desired output feedback controller can be obtained by solving the LMI (53) with the linear matrix equality (LME) $\Xi\mathbb{A}M_c=P\Xi\mathbb{A}$ in Theorem 5. The problem of finding M_c, N_c, and P that satisfy the LMI (53) with the LME $\Xi\mathbb{A}M_c=P\Xi\mathbb{A}$ is convex; hence, it can be solved with efficient and reliable algorithms [38]. For example, Scilab [42] provides a solution to the optimization problem subject to LMIs and LMEs. First, construct the evaluation function, where the term contents of LMIs and LMEs are specified. We can then obtain a solution by calling the "lmisolver" command with the evaluation function.

In Theorem 5, a sufficient condition for the 2-D (Q,S,R)-α-dissipative output-feedback control is presented by inserting an LME on a Lyapunov variable. However, it suffers from potential conservatism. In many cases, it can even lead to infeasibility of the optimization, even if a solution exists. A common source of conservatism is a structural restriction imposed on a Lyapunov variable. Next, we propose a new sufficient condition, which is less conservative, for the 2-D (Q, S, R)-α-dissipative output-feedback control. The structural restriction imposed on a Lyapunov variable is bypassed by using auxiliary structured slack variables. The extra degree of freedom supplied by the introduction of slack variables provides additional flexibility to obtain a solution to the 2-D (Q, S, R)-α-dissipative output-feedback control problem.

Theorem 6: Assume that F is a null matrix and B is a full column rank matrix. Given a scalar $\alpha > 0$, matrices Q, S, and R with Q and R real symmetric, where $Q = -Q_*^T Q_* \leq 0$ for some Q_*, the controller (47)–(49) is a 2-D (Q, S, R)-α-dissipative output-feedback controller and guarantees asymptotic stability for the 2-D system (36)–(39) if there exist a positive definite block-diagonal matrix $P = diag\{P_h, P_v\}$, two positive definite matrices $\bar{\mathbb{S}}_1$, $\bar{\mathbb{S}}_2$, and a matrix $\mathbb{L}_1$, where $P_h = {P_h}^T > 0$ and $P_v = {P_v}^T > 0$, such that

$$\begin{bmatrix} -\bar{\mathbb{S}} - \bar{\mathbb{S}}^T + \mathbb{T}^{-T} P \mathbb{T}^{-1} & \bar{\mathbb{S}}\mathbb{T}\Xi\bar{\mathbb{A}}\Xi^T + \mathbb{L}\mathbb{C}\Xi^T & \bar{\mathbb{S}}\mathbb{T}\Xi\bar{\mathbb{G}} + \mathbb{L}\mathbb{G}_3 & 0 \\ \star & -P & -\Xi\bar{\mathbb{E}}^T S & \Xi\bar{\mathbb{E}}^T Q_*^T \\ \star & \star & -R + \alpha I - sym\{S^T G_4\} & G_4^T Q_*^T \\ \star & \star & \star & -I \end{bmatrix} < 0 \quad (56)$$

where

$$\bar{\mathbb{S}} \triangleq \begin{bmatrix} \bar{\mathbb{S}}_1 & 0 \\ \star & \bar{\mathbb{S}}_2 \end{bmatrix}, \quad \mathbb{L} \triangleq \begin{bmatrix} \mathbb{L}_1 \\ 0 \end{bmatrix}$$

and $\mathbb{T}$ is an invertible transformation matrix satisfying $\mathbb{T}\Xi\mathbb{A} = \begin{bmatrix} I \\ 0 \end{bmatrix}$. The controller gain is then given by $\Theta_c = \bar{\mathbb{S}}_1^{-1}\mathbb{L}_1$.

Proof: By defining $\mathbb{S} = \mathbb{T}^T\bar{\mathbb{S}}\mathbb{T}$ and using the definitions in Theorem 6, we obtain

$$\mathbb{T}^T\mathbb{L} = \mathbb{T}^T\bar{\mathbb{S}}\begin{bmatrix} I \\ 0 \end{bmatrix}\Theta_c = \mathbb{T}^T\bar{\mathbb{S}}\mathbb{T}\Xi\mathbb{A}\Theta_c = \mathbb{S}\Xi\mathbb{A}\Theta_c. \quad (57)$$

Pre-multiplying and post-multiplying (56) by $diag\{\mathbb{T}^T, I, I, I\}$ and $diag\{\mathbb{T}, I, I, I\}$, respectively, give

$$\begin{bmatrix} -\mathbb{T}^T\bar{\mathbb{S}}\mathbb{T} - \mathbb{T}^T\bar{\mathbb{S}}^T\mathbb{T} + P & \mathbb{T}^T\bar{\mathbb{S}}\mathbb{T}\Xi\bar{\mathbb{A}}\Xi^T + \mathbb{T}^T\mathbb{L}\mathbb{C}\Xi^T & \mathbb{T}^T\bar{\mathbb{S}}\mathbb{T}\Xi\bar{\mathbb{G}} + \mathbb{T}^T\mathbb{L}\mathbb{G}_3 & 0 \\ \star & -P & -\Xi\bar{\mathbb{E}}^T S & \Xi\bar{\mathbb{E}}^T Q_*^T \\ \star & \star & -R + \alpha I - sym\{S^T G_4\} & G_4^T Q_*^T \\ \star & \star & \star & -I \end{bmatrix} < 0 \quad (58)$$

which can be represented by

$$\begin{bmatrix} -\mathbb{S} - \mathbb{S}^T + P & \mathbb{S}\Xi(\bar{\mathbb{A}} + \mathbb{A}\Theta_c\mathbb{C})\Xi^T & \mathbb{S}\Xi(\bar{\mathbb{G}} + \mathbb{A}\Theta_c\mathbb{G}_3) & 0 \\ \star & -P & -\Xi\bar{\mathbb{E}}^T S & \Xi\bar{\mathbb{E}}^T Q_*^T \\ \star & \star & -R + \alpha I - sym\{S^T G_4\} & G_4^T Q_*^T \\ \star & \star & \star & -I \end{bmatrix} < 0 \quad (59)$$

using the relation (57) and the definition $\mathbb{S} = \mathbb{T}^T\bar{\mathbb{S}}\mathbb{T}$. By pre-multiplying and post-multiplying (59) by $diag\{\mathbb{S}^{-1}, I, I, I\}$ and $diag\{\mathbb{S}^{-T}, I, I, I\}$, respectively, the inequality (59) can be rewritten as

$$\begin{bmatrix} -\mathbb{S}^{-1} - \mathbb{S}^{-T} + \mathbb{S}^{-1}P\mathbb{S}^{-T} & \Xi(\bar{\mathbb{A}} + \mathbb{A}\Theta_c\mathbb{C})\Xi^T & \Xi(\bar{\mathbb{G}} + \mathbb{A}\Theta_c\mathbb{G}_3) & 0 \\ \star & -P & -\Xi\bar{\mathbb{E}}^T S & \Xi\bar{\mathbb{E}}^T Q_*^T \\ \star & \star & -R + \alpha I - sym\{S^T G_4\} & G_4^T Q_*^T \\ \star & \star & \star & -I \end{bmatrix} < 0. \quad (60)$$

In view of the inequality $P^{-1} \geq \mathbb{S}^{-1} + \mathbb{S}^{-T} - \mathbb{S}^{-1}P\mathbb{S}^{-T}$ resulting from $(\mathbb{S}^{-1} - P^{-1})P(\mathbb{S}^{-1} - P^{-1})^T \geq 0$, (60) implies

$$\begin{bmatrix} -P^{-1} & \Xi(\bar{\mathbb{A}} + \mathbb{A}\Theta_c\mathbb{C})\Xi^T & \Xi(\bar{\mathbb{G}} + \mathbb{A}\Theta_c\mathbb{G}_3) & 0 \\ \star & -P & -\Xi\bar{\mathbb{E}}^T S & \Xi\bar{\mathbb{E}}^T Q_*^T \\ \star & \star & -R + \alpha I - sym\{S^T G_4\} & G_4^T Q_*^T \\ \star & \star & \star & -I \end{bmatrix} < 0. \quad (61)$$

Finally, by pre-multiplying and post-multiplying (61) by $diag\{P, I, I, I\}$ and $diag\{P, I, I, I\}$, respectively, we obtain $\Omega_{oc} < 0$, where Ω_{oc} is defined in (54). This completes the proof. ■

Remark 11: The projection lemma technique [51] is a typical method for solving H_∞ output-feedback control problems. If we use this technique to solve the 2-D (Q, S, R)-α-dissipative output-feedback control problem, we obtain two matrix inequalities, which are not LMIs. Solving these matrix inequalities is difficult; thus, we require additional procedures, such as the cone complementarity linearization (CCL) approach [52] or the partitioned matrix approach [53], to change the nonconvex feasibility problem into a convex one. In Theorem 5, the LME insertion approach [54], [55] was employed to change the nonconvex feasibility problem to a convex one. The main advantages of the approach are as follows: 1) The sufficient solvability condition for the 2-D (Q, S, R)-α-dissipative output-feedback control can be obtained by adding a new LME; and 2) the obtained problem is convex, and thus, it can be solved easily using existing software, such as Scilab [42]. However, it may suffer from conservatism due to the LME. In Theorem 6, a new less conservative LMI condition for the 2-D (Q,S,R)-α-dissipative output-feedback control was proposed, based on state coordinate transformation and slack matrices.

IV. 2-D Dissipative Filtering

In this section, attention is focused on the design of 2-D dissipative filters of the observer form and the general form for a Roesser model. The following 2-D model in the Roesser form is considered:

$$x^h(i+1,j) = A_{11}x^h(i,j) + A_{12}x^v(i,j) + G_1w(i,j) \quad (62)$$

$$x^v(i,j+1) = A_{21}x^h(i,j) + A_{22}x^v(i,j) + G_2w(i,j) \quad (63)$$

$$y(i,j) = C_1x^h(i,j) + C_2x^v(i,j) + G_3w(i,j) \quad (64)$$

$$z(i,j) = E_1x^h(i,j) + E_2x^v(i,j) \quad (65)$$

where $x^h(i,j) \in R^m$ is the horizontal state, $x^v(i,j) \in R^n$ is the vertical state, $w(i,j) \in R^l$ is the external disturbance, $y(i,j) \in R^q$ is the measured output, and $z(i,j) \in R^r$ is the signal to be estimated. A_{11}, A_{12}, A_{21}, A_{22}, C_1, C_2, G_1, G_2, G_3, E_1, and E_2 are the system matrices.

A. 2-D Filter of Observer Form

Design the following 2-D observer based filter:

$$\hat{x}^h(i+1,j) = A_{11}\hat{x}^h(i,j) + A_{12}\hat{x}^v(i,j) + L_1\{y(i,j) - C_1\hat{x}^h(i,j) - C_2\hat{x}^v(i,j)\} \quad (66)$$

$$\hat{x}^v(i,j+1) = A_{21}\hat{x}^h(i,j) + A_{22}\hat{x}^v(i,j) + L_2\{y(i,j) - C_1\hat{x}^h(i,j) - C_2\hat{x}^v(i,j)\} \quad (67)$$

$$\hat{z}(i,j) = E_1\hat{x}^h(i,j) + E_2\hat{x}^v(i,j) \quad (68)$$

where $\hat{x}^h(i,j)$, $\hat{x}^v(i,j)$, and $\hat{z}(i,j)$ are the estimates of $x^h(i,j)$, $x^v(i,j)$, and $z(i,j)$, respectively, and $L = [L_1^T \; L_2^T]^T$ is a filter gain. Define the filtering errors $e^h(i,j) = x^h(i,j) - \hat{x}^h(i,j)$, $e^v(i,j) = x^v(i,j) - \hat{x}^v(i,j)$, and $\tilde{z}(i,j) = z(i,j) - \hat{z}(i,j)$. We then obtain the following 2-D filtering error system:

$$\begin{bmatrix} e^h(i+1,j) \\ e^v(i,j+1) \end{bmatrix} = (A - LC)\begin{bmatrix} e^h(i,j) \\ e^v(i,j) \end{bmatrix} + (G - LG_3)w(i,j) \quad (69)$$

$$\tilde{z}(i,j) = E\begin{bmatrix} e^h(i,j) \\ e^v(i,j) \end{bmatrix} \quad (70)$$

using the definitions in (13) and (43).

Definition 4 (2-D (Q,S,R)-α-Dissipative Filtering: Observer Form): Given some scalar $\alpha > 0$, matrices Q, S, and R with Q and R real symmetric, the filter (66)–(68) is a 2-D (Q,S,R)-α-dissipative filter of observer form for any $T_i \geq 0$ and $T_j \geq 0$ if the following relation is satisfied under zero boundary conditions:

$$\sum_{i=0}^{T_i}\sum_{j=0}^{T_j}\tilde{z}^T(i,j)Q\tilde{z}(i,j) + 2\sum_{i=0}^{T_i}\sum_{j=0}^{T_j}\tilde{z}^T(i,j)Sw(i,j) + \sum_{i=0}^{T_i}\sum_{j=0}^{T_j}w^T(i,j)Rw(i,j) \geq \alpha\sum_{i=0}^{T_i}\sum_{j=0}^{T_j}w^T(i,j)w(i,j). \quad (71)$$

The following theorem shows that there exists a new 2-D (Q,S,R)-α-dissipative filter of observer form for the 2-D system (62)–(65).

Theorem 7: Given some scalar $\alpha > 0$, matrices Q, S, and R with Q and R real symmetric, where $Q = -Q_*^TQ_* \leq 0$ for some Q_*, the filter (66)–(68) is a 2-D (Q,S,R)-α-dissipative filter of observer form for the 2-D system (62)–(65) if there exist a positive definite block-diagonal matrix $P = diag\{P_h, P_v\}$ and a matrix Y_f, where $P_h = P_h^T > 0$ and $P_v = P_v^T > 0$, such that

$$\begin{bmatrix} -P & PA - Y_fC & PG - Y_fG_3 & 0 \\ \star & -P & -E^TS & E^TQ_*^T \\ \star & \star & -R + \alpha I & 0 \\ \star & \star & \star & -I \end{bmatrix} < 0. \quad (72)$$

The gain matrix of the filter (66)–(68) is then given by $L = diag\{P_h^{-1}, P_v^{-1}\}Y_f$.

Proof: By using Theorem 2, (12), (13), and (43), according to the following correspondences:

$$A \longleftarrow A - LC, \quad B \longleftarrow G - LG_3, \quad C \longleftarrow E, \quad D \longleftarrow 0$$

we obtain

$$\begin{bmatrix} -P & P(A - LC) & P(G - LG_3) & 0 \\ \star & -P & -E^TS & E^TQ_*^T \\ \star & \star & -R + \alpha I & 0 \\ \star & \star & \star & -I \end{bmatrix} < 0. \quad (73)$$

Defining $Y_f = PL$ yields the LMI (72). The filter gain is then given by $L = P^{-1}Y_f = diag\{P_h^{-1}, P_v^{-1}\}Y_f$. This completes the proof. ■

B. 2-D Filter of General Form

Assume that G_3 is a null matrix and C is a full row rank matrix. Consider the design problem of a 2-D filter of general form for (62)–(65)

$$\hat{x}^h(i+1,j) = A_{f,11}\hat{x}^h(i,j) + A_{f,12}\hat{x}^v(i,j) + G_{f,1}y(i,j), \quad (74)$$

$$\hat{x}^v(i,j+1) = A_{f,21}\hat{x}^h(i,j) + A_{f,22}\hat{x}^v(i,j) + G_{f,2}y(i,j), \quad (75)$$

$$\hat{z}(i,j) = E_{f,1}\hat{x}^h(i,j) + E_{f,2}\hat{x}^v(i,j) + F_fy(i,j) \quad (76)$$

where $A_{f,11}$, $A_{f,12}$, $A_{f,21}$, $A_{f,22}$, $G_{f,1}$, $G_{f,2}$, $E_{f,1}$, $E_{f,2}$, and F_f are filter gains. Using the definitions in (13) and (43), the augmented system of the 2-D system (62)–(65) and the filter (74)–(76) is then of the form

$$\begin{bmatrix}\bar{x}^h(i+1,j)\\ \bar{x}^v(i,j+1)\end{bmatrix} = \Xi[\bar{\mathcal{A}}+\mathcal{A}\Theta_f\mathcal{C}]\Xi^T\begin{bmatrix}\bar{x}^h(i,j)\\ \bar{x}^v(i,j)\end{bmatrix} + \Xi\bar{\mathcal{G}}w(i,j) \tag{77}$$

$$\tilde{z}(i,j) = [\bar{\mathcal{E}}+\mathcal{I}\Theta_f\mathcal{C}]\Xi^T\begin{bmatrix}\bar{x}^h(i,j)\\ \bar{x}^v(i,j)\end{bmatrix} \tag{78}$$

where

$$\Xi = \begin{bmatrix} I & 0 & 0 & 0\\ 0 & 0 & I & 0\\ 0 & I & 0 & 0\\ 0 & 0 & 0 & I\end{bmatrix},\ \bar{\mathcal{A}} = \begin{bmatrix} A & 0\\ \star & 0\end{bmatrix},\ \bar{\mathcal{G}} = \begin{bmatrix} G\\ 0\end{bmatrix}$$

$$\bar{\mathcal{E}} = [\,E \quad 0\,],\ \mathcal{A} = \begin{bmatrix} 0 & 0\\ \star & I\end{bmatrix},\ \mathcal{C} = \begin{bmatrix} C & 0\\ \star & I\end{bmatrix}$$

$$\mathcal{I} = [\,-I \quad 0\,],\ \bar{x}^h(i+1,j) = \begin{bmatrix} x^h(i+1,j)\\ \hat{x}^h(i+1,j)\end{bmatrix}$$

$$\Theta_f = \begin{bmatrix} F_f & E_f\\ G_f & A_f\end{bmatrix},\ \bar{x}^v(i,j+1) = \begin{bmatrix} x^v(i,j+1)\\ \hat{x}^v(i,j+1)\end{bmatrix}$$

$$A_f = \begin{bmatrix} A_{f,11} & A_{f,12}\\ A_{f,21} & A_{f,22}\end{bmatrix},\ E_f = [\,E_{f,1} \quad E_{f,2}\,]$$

$$G_f = \begin{bmatrix} G_{f,1}\\ G_{f,2}\end{bmatrix}. \tag{79}$$

Definition 5 (2-D (Q,S,R)-α-Dissipative Filtering: General Form): Given some scalar $\alpha > 0$, matrices Q, S, and R with Q and R real symmetric, the filter (74)–(76) is a 2-D (Q,S,R)-α-dissipative filter of general form for any $T_i \geq 0$ and $T_j \geq 0$ if (71) is satisfied under zero boundary conditions.

In the following theorem, we present a new 2-D (Q,S,R)-α-dissipative filter of general form for the 2-D system (62)–(65).

Theorem 8: Assume that G_3 is a null matrix and C is a full row rank matrix. Given a scalar $\alpha > 0$, matrices Q, S, and R with Q and R real symmetric, where $Q = -Q_*^T Q_* \leq 0$ for some Q_*, the filter (74)–(76) is a 2-D (Q,S,R)-α-dissipative filter of general form for the 2-D system (62)–(65) if there exist a positive definite block-diagonal matrix $X = diag\{X_h, X_v\}$ and matrices M_f, N_f, where $X_h = X_h^T > 0$ and $X_v = X_v^T > 0$, such that $M_f\mathcal{C}\Xi^T = \mathcal{C}\Xi^T X$ and

$$\begin{bmatrix} -\Xi^T X\Xi & \star & \star & \star\\ (\bar{\mathcal{A}}\Xi^T X + \mathcal{A}N_f\mathcal{C}\Xi^T)^T & -X & \star & \star\\ \bar{\mathcal{G}}^T & -S^T\bar{\mathcal{E}}\Xi^T X - S^T\mathcal{I}N_f\mathcal{C}\Xi^T & -R+\alpha I & \star\\ 0 & Q_*\bar{\mathcal{E}}\Xi^T X + Q_*\mathcal{I}N_f\mathcal{C}\Xi^T & 0 & -I\end{bmatrix} < 0. \tag{80}$$

The filter gain is then given by $\Theta_f = N_f M_f^{-1}$.

Proof: Using Theorem 2, (12), (13), and (43), according to the following correspondences:

$$A \longleftarrow \Xi(\bar{\mathcal{A}}+\mathcal{A}\Theta_f\mathcal{C})\Xi^T,\quad B \longleftarrow \Xi\bar{\mathcal{G}}$$
$$C \longleftarrow (\bar{\mathcal{E}}+\mathcal{I}\Theta_f\mathcal{C})\Xi^T,\quad D \longleftarrow 0$$

we obtain $\Omega_{fg} < 0$, where

$$\Omega_{fg} = \begin{bmatrix} -P & \star & \star & \star\\ \Xi(\bar{\mathcal{A}}^T + \mathcal{C}^T\Theta_f^T\mathcal{A}^T)\Xi^T P & -P & \star & \star\\ \bar{\mathcal{G}}^T\Xi^T P & -S^T(\bar{\mathcal{E}}+\mathcal{I}\Theta_f\mathcal{C})\Xi^T & -R+\alpha I & \star\\ 0 & Q_*(\bar{\mathcal{E}}+\mathcal{I}\Theta_f\mathcal{C})\Xi^T & 0 & -I\end{bmatrix}. \tag{81}$$

Let $X = P^{-1}$. Pre-multiplying and post-multiplying $\Omega_{fg} < 0$ by $diag\{\Xi^T X, X, I, I\}$ and $diag\{X\Xi, X, I, I\}$, respectively, yield

$$\begin{bmatrix} -\Xi^T X\Xi & \star & \star & \star\\ X\Xi(\bar{\mathcal{A}}^T + \mathcal{C}^T\Theta_f^T\mathcal{A}^T) & -X & \star & \star\\ \bar{\mathcal{G}}^T & -S^T(\bar{\mathcal{E}}+\mathcal{I}\Theta_f\mathcal{C})\Xi^T X & -R+\alpha I & \star\\ 0 & Q_*(\bar{\mathcal{E}}+\mathcal{I}\Theta_f\mathcal{C})\Xi^T X & 0 & -I\end{bmatrix} < 0. \tag{82}$$

Note that $\mathcal{C}$ has full row rank because C is a full row rank matrix. Thus, the matrix $\mathcal{C}\Xi^T$ is also a full row rank matrix because Ξ^T is an invertible matrix. It follows from $M_f\mathcal{C}\Xi^T = \mathcal{C}\Xi^T X$ that M_f is also full rank, and thus invertible, yielding $\mathcal{C}\Xi^T = M_f^{-1}\mathcal{C}\Xi^T X$. By using this fact and defining $\Theta_f = N_f M_f^{-1}$, we obtain (80) from (82). This completes the proof. ■

Remark 12: Theorem 8 gives a sufficient condition for the solvability of the 2-D (Q,S,R)-α-dissipative filtering problem of general form. The problem for solving the LMI (80) with the LME $M_f\mathcal{C}\Xi^T = \mathcal{C}\Xi^T X$ in Theorem 8 is convex; hence, it can be solved with efficient and reliable algorithms [38].

The insertion of the LME $M_f\mathcal{C}\Xi^T = \mathcal{C}\Xi^T X$ on the Lyapunov variable X in Theorem 8 may lead to potential conservatism. A new less conservative LMI condition for the 2-D (Q,S,R)-α-dissipative filtering of general form is proposed with the introduction of slack variables and a state transformation matrix. The structural restriction imposed on the Lyapunov variable X is bypassed by employing auxiliary slack variables with structure.

Theorem 9: Assume that G_3 is a null matrix and C is a full row rank matrix. Given a scalar $\alpha > 0$, matrices Q, S, and R with Q and R real symmetric, where $Q = -Q_*^T Q_* \leq 0$ for some Q_*, the filter (74)–(76) is a 2-D (Q,S,R)-α-dissipative filter of general form for the 2-D system (62)–(65) if there exist a positive definite block-diagonal matrix $X = diag\{X_h, X_v\}$, two positive definite matrices $\bar{\mathcal{Q}}_1$, $\bar{\mathcal{Q}}_2$, and a matrix $\mathcal{Y}_1$, where $X_h = {X_h}^T > 0$ and $X_v = {X_v}^T > 0$, such that

$$\begin{bmatrix} -X & \star & \star & \star \\ \bar{\mathcal{Q}}^T\mathcal{T}^{-T}\Xi\bar{\mathcal{A}}^T\Xi^T + \mathcal{Y}^T\mathcal{A}^T\Xi^T & -\bar{\mathcal{Q}} - \bar{\mathcal{Q}}^T + \mathcal{T}X\mathcal{T}^T & \star & \star \\ \bar{\mathcal{G}}^T\Xi^T & -S^T\bar{\mathcal{E}}\Xi^T\mathcal{T}^{-1}\bar{\mathcal{Q}} - S^T\mathcal{I}\mathcal{Y} & -R+\alpha I & \star \\ 0 & Q_*\bar{\mathcal{E}}\Xi^T\mathcal{T}^{-1}\bar{\mathcal{Q}} + Q_*\mathcal{I}\mathcal{Y} & 0 & -I \end{bmatrix} < 0 \tag{83}$$

where

$$\bar{\mathcal{Q}} \triangleq \begin{bmatrix} \bar{\mathcal{Q}}_1 & 0 \\ \star & \bar{\mathcal{Q}}_2 \end{bmatrix}, \quad \mathcal{Y} \triangleq [\,\mathcal{Y}_1 \quad 0\,]$$

and $\mathcal{T}$ is an invertible transformation matrix satisfying $\mathcal{C}\Xi^T\mathcal{T}^{-1} = [I \quad 0]$. The filter gain is then given by $\Theta_f = \mathcal{Y}_1\bar{\mathcal{Q}}_1^{-1}$.

Proof: Define $\mathcal{Q} = \mathcal{T}^{-1}\bar{\mathcal{Q}}\mathcal{T}^{-T}$. Based on the definitions of $\mathcal{T}$, $\bar{\mathcal{Q}}$, $\mathcal{Y}$, and $\mathcal{Q}$, the following relation is obtained:

$$\begin{aligned} \mathcal{Y}\mathcal{T}^{-T} &= \Theta_f[I \quad 0]\bar{\mathcal{Q}}\mathcal{T}^{-T} = \Theta_f\mathcal{C}\Xi^T\mathcal{T}^{-1}\bar{\mathcal{Q}}\mathcal{T}^{-T} \\ &= \Theta_f\mathcal{C}\Xi^T\mathcal{Q}. \end{aligned} \tag{84}$$

By pre-multiplying and post-multiplying (83) by $diag\{I, \mathcal{T}^{-1}, I, I\}$ and $diag\{I, \mathcal{T}^{-T}, I, I\}$, respectively, we obtain

$$\begin{bmatrix} -X & \star & \star & \star \\ \mathcal{T}^{-1}\bar{\mathcal{Q}}^T\mathcal{T}^{-T}\Xi\bar{\mathcal{A}}^T\Xi^T + \mathcal{T}^{-1}\mathcal{Y}^T\mathcal{A}^T\Xi^T & -\mathcal{T}^{-1}\bar{\mathcal{Q}}\mathcal{T}^{-T} - \mathcal{T}^{-1}\bar{\mathcal{Q}}^T\mathcal{T}^{-T} + X & \star & \star \\ \bar{\mathcal{G}}^T\Xi^T & -S^T\bar{\mathcal{E}}\Xi^T\mathcal{T}^{-1}\bar{\mathcal{Q}}\mathcal{T}^{-T} - S^T\mathcal{I}\mathcal{Y}\mathcal{T}^{-T} & -R+\alpha I & \star \\ 0 & Q_*\bar{\mathcal{E}}\Xi^T\mathcal{T}^{-1}\bar{\mathcal{Q}}\mathcal{T}^{-T} + Q_*\mathcal{I}\mathcal{Y}\mathcal{T}^{-T} & 0 & -I \end{bmatrix} < 0 \tag{85}$$

which is equivalently represented by

$$\begin{bmatrix} -X & \star & \star & \star \\ \mathcal{Q}^T\Xi(\bar{\mathcal{A}}^T + \mathcal{C}^T\Theta_f^T\mathcal{A}^T)\Xi^T & -\mathcal{Q} - \mathcal{Q}^T + X & \star & \star \\ \bar{\mathcal{G}}^T\Xi^T & -S^T(\bar{\mathcal{E}} + \mathcal{I}\Theta_f\mathcal{C})\Xi^T\mathcal{Q} & -R+\alpha I & \star \\ 0 & Q_*(\bar{\mathcal{E}} + \mathcal{I}\Theta_f\mathcal{C})\Xi^T\mathcal{Q} & 0 & -I \end{bmatrix} < 0 \tag{86}$$

using (84) and the definition $\mathcal{Q} = \mathcal{T}^{-1}\bar{\mathcal{Q}}\mathcal{T}^{-T}$. Pre-multiplying and post-multiplying (86) by $diag\{I, \mathcal{Q}^{-T}, I, I\}$ and $diag\{I, \mathcal{Q}^{-1}, I, I\}$, respectively, yield

$$\begin{bmatrix} -X & \star & \star & \star \\ \Xi(\bar{\mathcal{A}}^T + \mathcal{C}^T\Theta_f^T\mathcal{A}^T)\Xi^T & -\mathcal{Q}^{-1} - \mathcal{Q}^{-T} + \mathcal{Q}^{-T}X\mathcal{Q}^{-1} & \star & \star \\ \bar{\mathcal{G}}^T\Xi^T & -S^T(\bar{\mathcal{E}} + \mathcal{I}\Theta_f\mathcal{C})\Xi^T & -R+\alpha I & \star \\ 0 & Q_*(\bar{\mathcal{E}} + \mathcal{I}\Theta_f\mathcal{C})\Xi^T & 0 & -I \end{bmatrix} < 0. \tag{87}$$

From $(\mathcal{Q}^{-1} - X^{-1})X(\mathcal{Q}^{-1} - X^{-1})^T \geq 0$, it is clear that $X^{-1} \geq \mathcal{Q}^{-1} + \mathcal{Q}^{-T} - \mathcal{Q}^{-T}X\mathcal{Q}^{-1}$. Hence, the condition (87) implies

$$\begin{bmatrix} -X & \star & \star & \star \\ \Xi(\bar{\mathcal{A}}^T + \mathcal{C}^T\Theta_f^T\mathcal{A}^T)\Xi^T & -X^{-1} & \star & \star \\ \bar{\mathcal{G}}^T\Xi^T & -S^T(\bar{\mathcal{E}} + \mathcal{I}\Theta_f\mathcal{C})\Xi^T & -R+\alpha I & \star \\ 0 & Q_*(\bar{\mathcal{E}} + \mathcal{I}\Theta_f\mathcal{C})\Xi^T & 0 & -I \end{bmatrix} < 0. \tag{88}$$

If we pre-multiply and post-multiply (88) by $diag\{X^{-1}, I, I, I\}$ and $diag\{X^{-1}, I, I, I\}$, respectively, we obtain $\Omega_{fg} < 0$ with $X = P^{-1}$, where Ω_{fg} is defined in (81). This completes the proof. ■

Remark 13: As mentioned in Remark 11, we used the LME insertion approach [54], [55] to obtain the solvability condition for the 2-D (Q, S, R)-α-dissipative filtering problem in Theorem 8. However, the LME $M_f\mathcal{C}\Xi^T = \mathcal{C}\Xi^T X$ in Theorem 8 may lead to conservatism. This problem is improved by Theorem 9, which proposes a new less conservative condition using state coordinate transformation and slack matrices.

V. Application Examples

A. 2-D Dissipative Control for Thermal Process

Consider the thermal process represented by the following partial differential equation [1], [8]:

$$\frac{\partial T(x,t)}{\partial x} = -\frac{\partial T(x,t)}{\partial t} - T(x,t) + U(t) \tag{89}$$

where $T(x,t)$ is usually the temperature at x(space)$\in [0, x_f]$ and t(time) $\in [0, \infty)$, and $U(t)$ is a given force function. Note that thermal processes in heat exchangers, chemical reactors, and pipe furnaces are expressed by (89). Let $x^h(i,j) = T(i-1, j)$ and $x^v(i,j) = T(i,j)$, where $T(i,j) = T(i\Delta x, j\Delta t)$. Equation (89) can then be represented by the Roesser model (36) and (37) with $A_{11} = 0$, $A_{12} = 1$, $A_{21} = \Delta t/\Delta x$, $A_{22} = 1 - (\Delta t/\Delta x) - \Delta t$, $B_1 = 0$, and $B_2 = \Delta t$, where $\Delta x = 0.2$ and $\Delta t = 0.1$. Let $C_1 = 1$, $C_2 = 10$, $E_1 = 0.4$, $E_2 = 0.3$, $F = 0$, $G_1 = 0.1$, $G_2 = 0.08$, $G_3 = 0.05$, and $G_4 = 0$. $w(i,j)$ is given by white noise with a mean of 0 and a variance of 1. The boundary conditions are given by $x^h(0,j) = 7$ and $x^v(i,0) = -4.7$ $(0 \leq i \leq 90, 0 \leq j \leq 100)$. Using Theorem 5 with $Q = -1$, $S = 1$, and $R = 3$, the optimal dissipativity performance bound is obtained as $\alpha^* = 2.9108$. Figs. 2 and 3 show the horizontal and vertical state trajectories, respectively. The horizontal and vertical states ($x^h(i,j)$ and $x^h(i,j)$) converge from the boundary conditions to a small region around zero as i and j increase. Figs. 4 and 5 show the horizontal and vertical state trajectories, respectively, for $w(i,j) = 0$. These two figures demonstrate the asymptotic stability of the closed-loop system.

Next, we check the 2-D (Q, S, R)-α-dissipativity performance of the proposed controller. Define the following function:

$$D_c(T_i, T_j) = \left\{ \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} \left[z^T(i,j)Qz(i,j) + 2z^T(i,j)Sw(i,j) + w^T(i,j)Rw(i,j) \right] \right\} \Bigg/ \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} w^T(i,j)w(i,j)$$

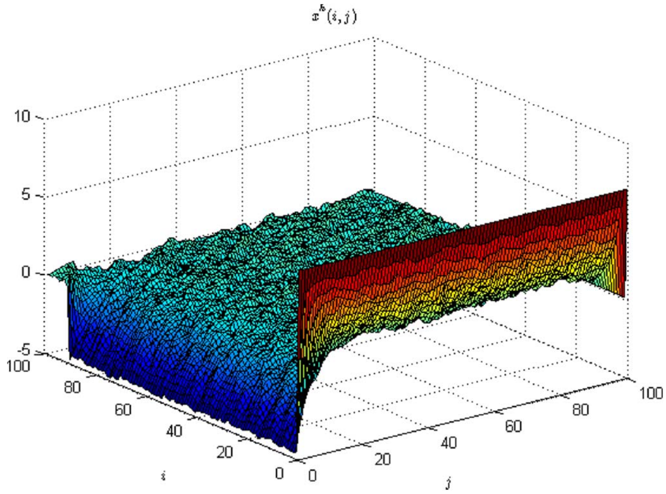

Fig. 2. Horizontal state trajectory $x^h(i,j)$.

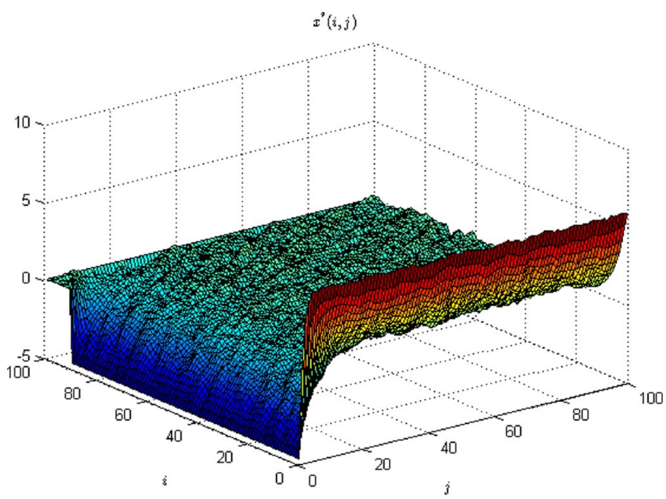

Fig. 3. Vertical state trajectory $x^v(i,j)$.

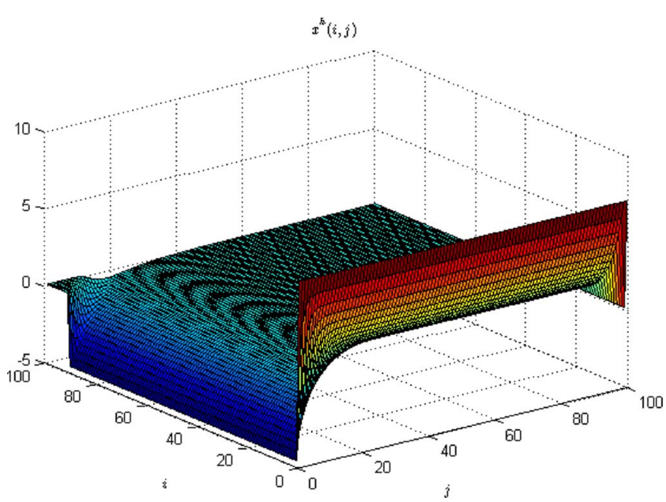

Fig. 4. Horizontal state trajectory $x^h(i,j)$ for $w(i,j)=0$.

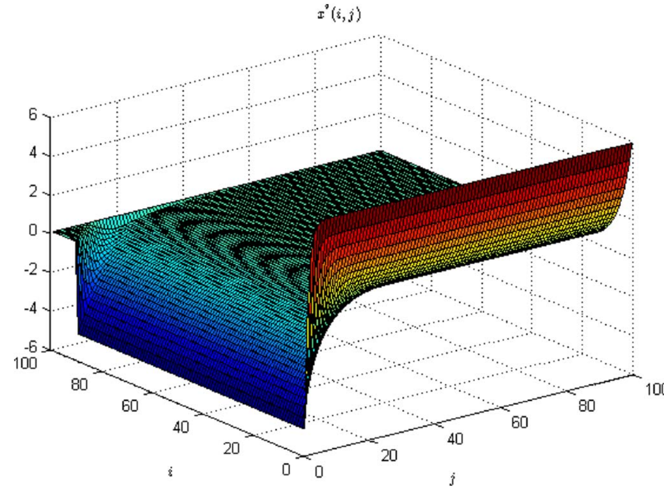

Fig. 5. Vertical state trajectory $x^v(i,j)$ for $w(i,j)=0$.

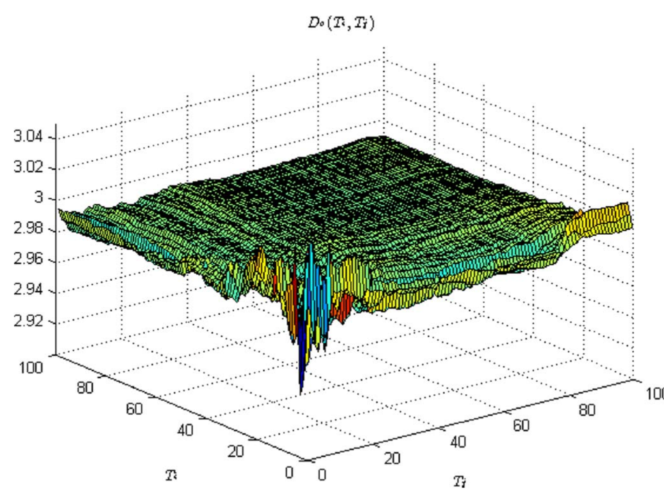

Fig. 6. Plot of $D_c(T_i,T_j)$.

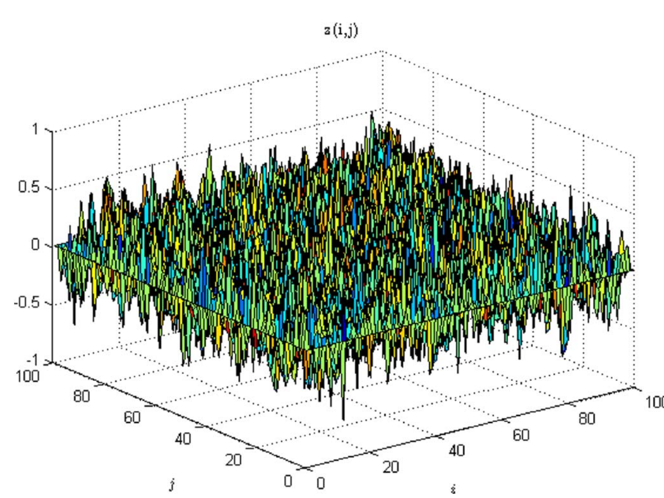

Fig. 7. State of random field $z(i,j)$.

where $w(i,j)\neq 0$ for $i=0,\ldots,T_i$ and $j=0,\ldots,T_j$. Note that the 2-D (Q,S,R)-α-dissipativity performance (44) for control is equivalently represented by $D_c(T_i,T_j)\geq\alpha$. Fig. 6 shows $D_c(T_i,T_j)$ for $0\leq T_i\leq 100$ and $0\leq T_j\leq 100$. The minimum value of $D_c(T_i,T_j)$ is obtained as 2.9394, which is above the optimal dissipativity performance bound $\alpha^*=2.9108$. Thus, the 2-D (Q,S,R)-α-dissipativity performance (44) is achieved.

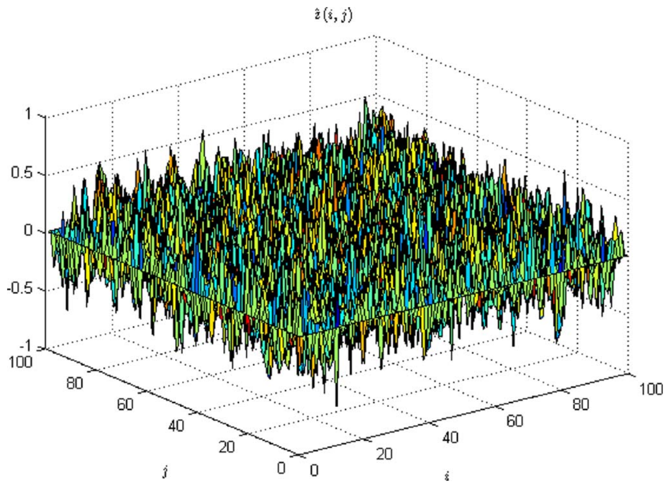

Fig. 8. Estimate $\hat{z}(i,j)$.

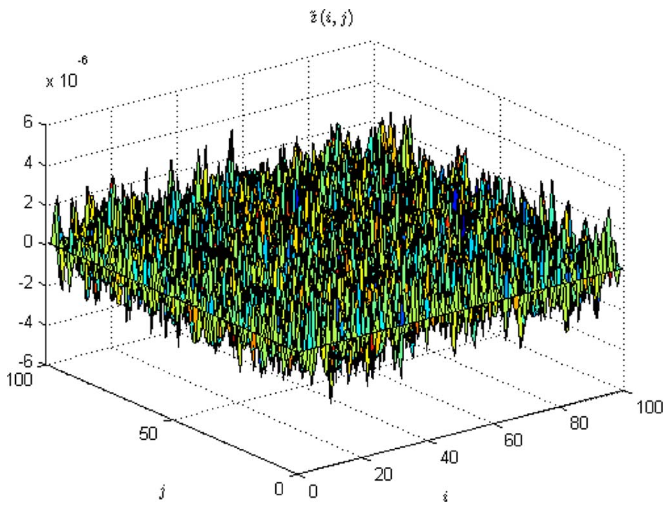

Fig. 9. Filtering error $\tilde{z}(i,j)$.

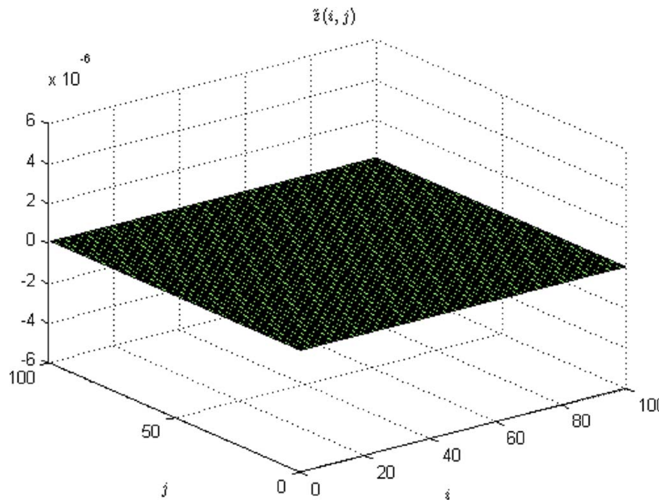

Fig. 10. Filtering error $\tilde{z}(i,j)$ for $w(i,j)=0$.

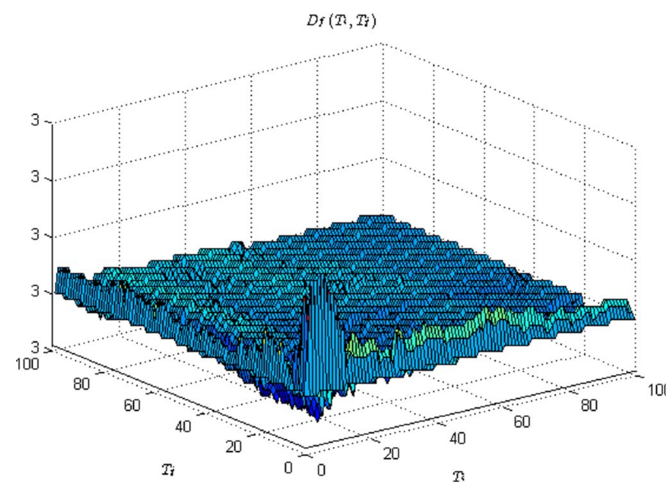

Fig. 11. Plot of $D_f(T_i,T_j)$.

B. 2-D Dissipative Filtering for a Stationary Random Field

Consider the following stationary random field model in image processing [1], [56], [57]:

$$\eta(i+1,j+1) = a_1\eta(i,j+1) + a_2\eta(i+1,j) - a_1a_2\eta(i,j) + v_1(i,j) \quad (90)$$

where $i=0,1,\cdots$ and $j=0,1,\cdots$ are the vertical and horizontal position variables, $\eta(i,j)$ is the state of the random field at special coordinates (i,j), $v_1(i,j)$ is a noise input, and $a_1^2<1$ and $a_2^2<1$ as a_1 and a_2 represent the vertical and horizontal correlations of the random field, respectively. Let $x^h(i,j)=\eta(i,j+1)-a_2\eta(i,j)$ and $x^v(i,j)=\eta(i,j)$. The stationary random field model (90) can then be represented by the Roesser model (62) and (63) with $A_{11}=a_1$, $A_{12}=0$, $A_{21}=1$, $A_{22}=a_2$, $G_1=1$, and $G_2=0$, where $a_1=0.3$ and $a_2=0.2$. Let $C_1=0$, $C_2=1$, $D=0$, $E_1=0$, and $E_2=1$. By using Theorem 8 with $Q=-1$, $S=1$, and $R=3$, we can obtain the optimal dissipativity performance bound $\alpha^*=2.9997$. Figs. 7 and 8 show the state of the random field $z(i,j)=\eta(i,j)$ and its estimate $\hat{z}(i,j)$, respectively, when the system has zero boundary conditions and $w(i,j)$ is given by white noise with a mean of 0 and a variance of 0.2. Fig. 9 shows that the filtering error $\tilde{z}(i,j)$ is kept within a very small interval $[-4\times10^{-6}, 4\times10^{-6}]$ around zero. Fig. 10 demonstrates that the filtering error $\tilde{z}(i,j)$ is zero for $w(i,j)=0$ under zero boundary conditions.

Next, we check the 2-D (Q,S,R)-α-dissipativity performance. Define the following function:

$$D_f(T_i,T_j) = \left\{ \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} \left[\tilde{z}^T(i,j)Q\tilde{z}(i,j) + 2\tilde{z}^T(i,j)Sw(i,j) + w^T(i,j)Rw(i,j)\right] \right\} \Big/ \sum_{i=0}^{T_i}\sum_{j=0}^{T_j} w^T(i,j)w(i,j)$$

where $w(i,j)\neq 0$ for $i=0,\ldots,T_i$ and $j=0,\ldots,T_j$. The 2-D (Q,S,R)-α-dissipativity performance (71) for filtering is equal to $D_f(T_i,T_j)\geq\alpha$. Fig. 11 shows the plot of $D_f(T_i,T_j)$ for $0\leq T_i\leq 100$ and $0\leq T_j\leq 100$. The minimum value of $D_f(T_i,T_j)$ for $0\leq T_i\leq 100$ and $0\leq T_j\leq 100$ is obtained as 3, which is higher than the optimal dissipativity performance bound $\alpha^*=2.9997$.

VI. Conclusion

In this paper, we have presented systematic methods for the design of dissipative controllers and filters for 2-D systems in the discrete-time Roesser model. First, a new sufficient condition was established for evaluation of the (Q, S, R)-α-dissipativity performance of 2-D systems in the Roesser model. By using this condition, 2-D (Q, S, R)-α-dissipative state-feedback and output-feedback controllers were designed via an LMI approach. Furthermore, we proposed solutions to design problems of 2-D (Q, S, R)-α-dissipative filters of observer form and general form. Two application examples for 2-D dissipative control and filtering were provided to illustrate the theoretical results obtained. The results presented in this paper can be broadly extended to the cases of dissipative control and filtering for 2-D nonlinear systems, 2-D time-delayed systems, 2-D Takagi-Sugeno fuzzy systems, and 2-D systems with polytopic or parametric uncertainties.

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