An Event-Triggered Approach to Sliding Mode Control of Markovian Jump Lur'e Systems Under Hidden Mode Detections

Jun Song, Yugang Niu, and Jing Xu

***Abstract*—The asynchronous sliding mode control (SMC) problem is investigated for networked Markovian jump Lur'e systems, in which the information of system modes is unavailable to the sliding mode controller but could be estimated by a mode detector via a hidden Markov model (HMM). In order to mitigate the burden of data communication, an event-triggered protocol is proposed to determine whether the system state should be released to the controller at certain time-point according to a specific triggering condition. By constructing a novel *common* sliding surface, this paper designs an event-triggered asynchronous SMC law, which just depends on the hidden mode information. A combination of the stochastic Lur'e-type Lyapunov functional and the HMM approach is exploited to establish the sufficient conditions of the mean square stability with a prescribed H_∞ performance and the reachability of a sliding region around the specified sliding surface. Moreover, the solving algorithm for the control gain matrices is given via a convex optimization problem. Finally, an example from the dc motor device system is provided.**

***Index Terms*—Event-triggered strategy, hidden Markov model (HMM), Markovian jump Lur'e systems, sliding mode control (SMC).**

I. Introduction

MARKOVIAN jump systems (MJSs) are an important kind of hybrid systems in which the *mode signal*, responsible for controlling the switch among dynamic modes, is modeled by a time-homogeneous Markov chain. The key feature for this class of systems is that the Markov chain could model some abrupt changes in the dynamics of the system due to, for instance, environmental disturbances, component failures or repairs, changes in subsystems interconnections, etc. Examples of the MJSs can be found in economics, wind turbine, and networked control [17], [31], [33]. Over the past few decades, the controller/filter synthesis for MJSs have been attracting a wonderful of research for continuous-time case in [3], [14], [15], and [18] and for discrete-time case in [27] and [29].

It is well-known that the nonlinearities are ubiquitous in practice and the control problem for nonlinear MJSs has been the hot topic in control field, especially, the so-called Markovian jump Lur'e systems composed of a linear part and a nonlinearity for each mode. Recently, by employing the stochastic Lur'e-type Lyapunov function approach which renders less conservative than the stochastic quadratic Lyapunov function approach, the stochastic stabilization and the ℓ_2 gain minimization problems for the discrete-time Markovian jump Lur'e systems were studied in [7]. Following this excellent result, some preliminary results have been reported in the literature concerning the relative control/filtering issues for the Markovian jump Lur'e systems, see [22], [38], [40] and the references therein.

It should be noted that all of these aforementioned works concerning the Markovian jump Lur'e systems are based on the implicit assumption that the controller/filter has full knowledge about the *mode signal* at any time instant. Unfortunately, this assumption is not realistic in many practical scenarios. For example, in networked control systems, the *mode signal* of the plant cannot be completely accessible because of communication delays and inevitable packet dropout, which may result in the asynchronization phenomenon between controller/filter modes and system modes. To deal with this problem, two typical approaches have been recently adopted to cater for the effect of limited and/or uncertain knowledge about the mode signal: the one is to consider the case that the mode signal can be only observed at particular sampling times [24] and the other is to suppose that the mode signal can be detected at any time instant via a hidden Markov model (HMM) [5], [30], which can cover some well-studied situations as special cases, such as complete observation, cluster observation, and no information. To date, the asynchronous control/filter of *linear* MJSs has gained some initial research interest and the corresponding research on *nonlinear* MJSs is still ongoing. Especially, the HMM-based asynchronous H_∞ control issue for the discrete-time Markovian jump Lur'e systems has not been adequately investigated, which constitutes one of the motivations for the present research.

As an effective robust control scheme to tackle parameter variations and external disturbances, sliding mode control (SMC) has been widely used in various applications [10], [21], [28], [37]. Under the assumption that

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the controller has full knowledge about the mode signal at any time instant, solutions to standard SMC synthesis problems for various kinds of the uncertain MJSs were reported in [2], [19], and [36] for continuous-time case and in [8] and [39] for discrete-time case. When only partial mode information is available to the controller, the above-mentioned results cannot be employed to solve the asynchronous SMC problem for Markovian jump Lur'e systems under the hidden mode detections, which remains as an open research issue till now.

In network environments, an important issue is how to transmit signals more effectively by utilizing the available limited network bandwidth. To alleviate the unnecessary waste of communication/computation resources, a recently popular communication schedule called event-triggered strategy has been proposed in [6] and [16]. The main idea of an event-triggered scheme is to transmit certain information only when a predesigned event happens. In comparison with the conventional time-triggered communication, a notable advantage of the event-triggering scheme is its capability of saving communication resource while preserving the guaranteed system performance. In recent years, some particular research interesting has been paid to the event-triggered control/estimation of MJSs. For example, the problem of event-triggered control for a class of fuzzy MJSs with general switching policies was studied in [4]. The event-triggered H_∞ state estimation problem was investigated in [13] for a class of stochastic genetic regulatory networks with both Markovian jumping parameters and time-varying delays. On another research front, the event-triggered SMC techniques have drawn increasing attention in the past few years [1], [34], [35]. For instance, the event-triggered SMC problem was studied in [35] for uncertain stochastic systems based on output feedback over limited communication networks. However, the event-triggered SMC problem of the MJSs has not received proper research attention, especially when the mode-dependent Lur'e nonlinearities and the hidden mode detections become another research focus.

Summarizing the discussion made so far, it is of both theoretical significance and practical importance to co-design event-triggered strategy and the asynchronous SMC scheme for the networked Markovian jump Lur'e systems under the hidden mode detections. This appears to be a challenging task with three essential difficulties identified as follows: 1) how to design a suitable sliding surface and an SMC law just by employing the estimated modes via an HMM detector? 2) how to tackle the event-triggered strategy and the Lur'e-type nonlinearities in designing asynchronous SMC scheme? and 3) how to guarantee the mean-square H_∞ stability of the controlled Markovian jump Lur'e systems and the reachability of the specified sliding surface under the hidden mode detections and the event-triggered updated mechanism? It is, therefore, the main motivation of this paper is to shorten such a gap by launching a systematic investigation.

In this paper, we endeavor to study the event-triggered asynchronous SMC problem for the Markovian jump Lur'e systems under hidden mode detections. Based on an HMM, a detector is employed to observe the mode signal of the plant and emit an estimated mode to the controller. An event-triggered strategy is introduced to decrease the frequency of data transmission between the plant and the controller. The main contributions of this paper are highlighted as follows.

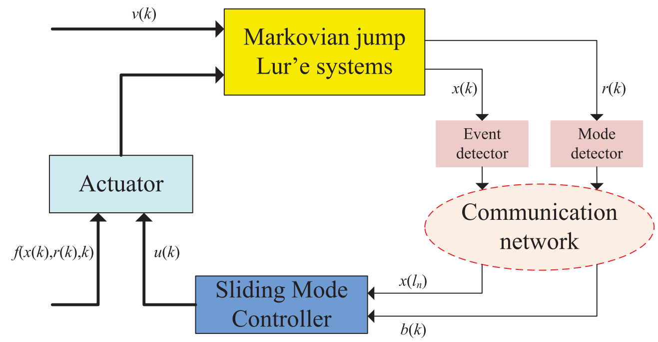

Fig. 1. Event-triggered asynchronous SMC for the networked Markovian jump Lur'e systems.

1) For the concerned Markovian jump Lur'e systems, a novel common sliding surface is developed and a novel event-triggered asynchronous SMC law is constructed appropriately by only using the detected modes.
2) The mean-square stability with H_∞ disturbance rejection performance and the reachability of the sliding mode dynamics are analyzed, respectively, by combining the stochastic Lur'e-type Lyapunov function and the HMM approach.
3) The solving algorithm for the proposed event-triggered asynchronous SMC scheme is established in terms of a convex optimization problem, which can be computed directly *offline* via standard software package.

Notations: Throughout this paper, all matrices are supposed to have compatible dimensions. $\mathbb{N}_+$ denotes the set of natural numbers. $\mathbb{E}\{\cdot\}$ denotes the expectation operator with respect to probability measure. For a real symmetric matrix M, $M > 0$ ($M < 0$) represents that M is a positive-definite (negative-definite) matrix. The shorthand "diag$\{\cdot\}$" denotes a block diagonal matrix. In symmetric block matrices, the symbol "$\star$" is used as an ellipsis for terms induced for symmetry. For a probability space $(\Omega, \mathcal{F}, \Pr\{\cdot\})$, Ω is the sample space, $\mathcal{F}$ is the σ-algebra of events, and $\Pr\{\cdot\}$ is the probability measure defined on $\mathcal{F}$. The function $\mathrm{sgn}(\cdot) : \mathbb{R}^m \to \mathbb{R}^m$ is defined as $\mathrm{sgn}(s) = [\,\mathrm{sgn}(s_1)\ \mathrm{sgn}(s_2)\ \cdots\ \mathrm{sgn}(s_m)\,]^T$ with

$$\mathrm{sgn}(s_i) = \begin{cases} 1, & s_i > 0 \\ 0, & s_i = 0 \\ -1, & s_i < 0 \end{cases}$$

for $i = 1, 2, \ldots, m$.

II. Problem Formulation

This paper discusses a networked Markovian jump Lur'e system under a remote sliding mode controller as shown in Fig. 1, in which an event detector is utilized to determine whether or not the current state should be transmitted to the controller and a mode detector is employed to emit an estimated mode signal to the controller.

A. System Description

Given the probability space $(\Omega, \mathcal{F}, \Pr\{\cdot\})$, we consider the following Markovian jump Lur'e systems:

$$\begin{cases} x(k+1) = [A(r(k)) + \Delta A(r(k), k)]x(k) \\ \qquad\quad + D(r(k))\varphi(r(k), H(r(k))x(k)) \\ \qquad\quad + B(r(k))\big[u(k) + f(x(k), r(k), k)\big] \\ \qquad\quad + G(r(k))v(k) \\ z(k) = C(r(k))x(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system state, $u(k) \in \mathbb{R}^m$ is the control input, $z(k) \in \mathbb{R}^p$ is the controlled output, and $v(k) \in \mathbb{R}^{n_v}$ is the external disturbance belonging to $\ell_2[0, +\infty)$. $\varphi(r(k), H(r(k))x(k)) \in \mathbb{R}^q$ is a mode-dependent memoryless nonlinearity. The stochastic process $\{r(k) = i,\ k \geq 0\}$ is described via a homogeneous Markov chain which taking values on a finite set $\mathbb{S} \triangleq \{1, 2, \ldots, N\}$, and having the following transition probability from mode i at sample time k to mode j at sample time $k+1$:

$$\pi_{ij} = \Pr\{r(k+1) = j \mid r(k) = i\} \quad \forall i, j \in \mathbb{S} \tag{2}$$

where $\pi_{ij} \in [0, 1]$, and $\sum_{j=1}^{N} \pi_{ij} = 1$. The transition probability matrix is defined as $\Pi = \big[\pi_{ij}\big]_{i,j\in\mathbb{S}}$.

For each possible $r(k) = i \in \mathbb{S}$, define the known constant matrices as $A_i \triangleq A(r(k))$, $B_i \triangleq B(r(k))$, $C_i \triangleq C(r(k))$, $D_i \triangleq D(r(k))$, $G_i \triangleq G(r(k))$, $H_i \triangleq H(r(k))$, and the known Lur'e nonlinearity as $\varphi_i(H_i x(k)) \triangleq \varphi(r(k), H(r(k))x(k))$. The input matrix B_i is full column rank, that is, $\mathrm{rank}(B_i) = m$. The uncertainties $\Delta A_i(k) \triangleq \Delta A(r(k), k)$ satisfy $\Delta A_i(k) = M_i \Phi_i(k) E_i$, with M_i and E_i known constant matrices, and $\Phi_i(k)$ an unknown time-varying matrix satisfying $\Phi_i^T(k)\Phi_i(k) \leq I$. Besides, for any $r(k) = i \in \mathbb{S}$, the external disturbance $f_i(x(k), k) \triangleq f(x(k), r(k), k)$ possesses

$$\|f_i(x(k), k)\| \leq \mu_i \|x(k)\| \tag{3}$$

where $\mu_i \geq 0$ is a known scalar.

Here, we make the following assumption as in [7].

Assumption 1: The nonlinear function $\varphi_i(\cdot) : \mathbb{R}^q \to \mathbb{R}^q$ verifies a cone-bounded sector condition and is decentralized being associated with each mode $r(k) = i \in \mathbb{S}$, which satisfies the conditions: 1) $\varphi(0) = 0$ and 2) there exists N *diagonal* positive definite matrices $\Omega_i \in \mathbb{R}^{q\times q}$, such that $\forall y \in \mathbb{R}^q$ $\forall h = 1, 2, \ldots, q$,

$$\varphi_{i,(h)}(y)\big[\varphi_i(y) - \Omega_i y\big]_{(h)} \leq 0.$$

From Assumption 1, the following inequality holds $\forall y \in \mathbb{R}^q$ $\forall i \in \mathbb{S}$:

$$\mathbf{SC}(i, y, \Lambda_i) \triangleq \varphi_i^T(y)\Lambda_i\big[\varphi_i(y) - \Omega_i y\big] \leq 0 \tag{4}$$

where Λ_i is any *diagonal* positive semidefinite matrices. Clearly, the condition (4) implies the following relation for any $y \in \mathbb{R}^q$:

$$0 \leq \varphi_i^T(y)\Lambda_i\varphi_i(y) \leq \varphi_i^T(y)\Lambda_i\Omega_i y \leq y^T\Omega_i\Lambda_i\Omega_i y. \tag{5}$$

B. Communication Under Event-Triggered Protocol and Hidden Mode Detections

In this paper, the event-triggered strategy will be proposed to save computer resources. Denote by $\{l_0, l_1, \ldots\}$ the sequence of the time instants at which the current state of the plant (1) is transmitted to the controller. Each time instant l_n is defined according to the following event-triggered condition:

$$\varepsilon^T(k)T\varepsilon(k) \geq \alpha x^T(k)Tx(k) \tag{6}$$

where $\alpha \in [0, 1]$ is the prescribed event-triggering parameter, and $\varepsilon(k) \triangleq x(k) - x(l_n)$ is the error for $k \in [l_n, l_{n+1})$, $n = 0,\ 1,\ 2,\ \ldots, \infty$. $T > 0$ is the weighting matrix to be designed later.

In practical applications, it is not always possible to directly measure the information of system modes $r(k)$. Instead, a detector $b(k)$ may be utilized to obtain the estimation of $r(k)$ with some probability [5]. In this case, the emitted signal $b(k)$ from detector to controller may not synchronize with the system mode $r(k)$. The HMM $\big(r(k), b(k)\big)$ as in [5] and [30] is introduced to characterize the above asynchronous phenomenon as follows:

$$\begin{aligned} \xi_{i\chi} &= \Pr\{b(k) = \chi \mid r(k) = i\} \\ &\quad i \in \mathbb{S},\ \ \chi \in \mathbb{D},\ \ \mathbb{D} \triangleq \{1, 2, \ldots, J\} \end{aligned} \tag{7}$$

where $\xi_{i\chi}$ is the mode detection probability belonging to $[0, 1]$. For any $i \in \mathbb{S}$, it has $\sum_{\chi=1}^{J} \xi_{i\chi} = 1$. The MDPM Θ is defined as $\Theta \triangleq \big[\xi_{i\chi}\big]$. It is clear that the HMM (7) covers the mode-dependent ($\mathbb{D} = \mathbb{S}$, $\xi_{i\chi} = 1$ for $\chi = i$) and mode-independent ($\mathbb{D} = \{1\}$) cases.

C. Problem of Interest

Definition 1 [5], [23]: The system (1) with $v(k) = 0$ is said to be mean square stable (MSS) if, for any initial condition $\{x(0) \neq 0,\ r(0) \in \mathbb{S}\}$, the condition $\lim_{k\to\infty} \mathbb{E}\{\|x(k)\|^2\} \mid_{x(0),r(0)} = 0$ holds.

Definition 2: Given a scalar $\gamma > 0$, the system (1) is said to be MSS with an H_∞ disturbance attenuation level γ if the system is MSS, and under the zero initial condition $x(0) = 0$, the condition $\mathbb{E}\{\sum_{k=0}^{\infty} \|z(k)\|^2\} \leq \gamma^2 \sum_{k=0}^{\infty} \|v(k)\|^2$ holds for all $v(k) \in \ell_2[0, \infty)$.

Now, our objective is to design an asynchronous SMC law $u(k)$, depending on only the estimated mode $b(k)$, such that the resultant closed-loop system is MMS with a prescribed H_∞ disturbance attention level under the event-triggered strategy (6) and the hidden mode detections (7).

III. Event-Triggered Asynchronous Sliding Mode Control

A. Designing of Sliding Surface and Sliding Mode Controller

In this paper, a novel *common* sliding function is constructed as follows:

$$s(k) = Zx(k) \tag{8}$$

where $Z \triangleq \sum_{i=1}^{N} \beta_i B_i^T$, and scalars $\beta_i \in [0, 1]$ ($i \in \mathbb{S}$) will be chosen such that $X_i \triangleq ZB_i$ is nonsingular for any $i \in \mathbb{S}$. As discussed in [20], based on the assumption that $\mathrm{rank}(B_i) = m$

for any $i \in \mathbb{S}$, the above nonsingularity can be guaranteed easily by selecting the parameters β_i properly.

Remark 1: In the recent years, the SMC problems of discrete-time MJSs have gained some particular research attenuations, see [8], [39]. It is noted that all of them employed the mode-dependent sliding surfaces, which render a shortcoming that the reachability of sliding surfaces may not always be attained due to the switching frequently from one mode to another. The above drawback may be overcome by the proposed *common* sliding surface (8).

Notice that the system mode $r(k)$ can be estimated via a detector $b(k)$. That means that the controller design for the system (1) can just utilize the mode information emitting from the detector. To this end, we design an appropriate event-triggered asynchronous SMC law as follows:

$$u(k) = K_\chi x(l_n) + L_\chi \varphi_\chi\big(H_\chi x(l_n)\big) - \varrho\|x(l_n)\| \cdot \mathrm{sgn}(s(l_n)) \tag{9}$$

for $\chi \in \mathbb{D}$, $k \in [l_n, l_{n+1})$, $n = 0, 1, 2, \ldots, \infty$, where the matrices $K_\chi \in \mathbb{R}^{m\times n_x}$, $L_\chi \in \mathbb{R}^{m\times q}$ ($\chi \in \mathbb{D}$) will be determined later, and the scalar ϱ is given by

$$\varrho \triangleq \max_{i\in\mathcal{N}}\{\delta_i\},\ \ \delta_i \triangleq \|X_i^{-1}ZM_i\| \cdot \|E_i\| + \mu_i \tag{10}$$

with μ_i defined in (3).

Remark 2: Actually, the effect of stochastic jumping from one detected mode to another is just reflected in the asynchronous SMC law (9) via matrices K_χ and L_χ, which are the solutions of some coupled matrix inequalities concerning the transition probabilities π_{ij} and the mode detection probabilities $\xi_{i\chi}$ (see Theorem 3 later). On the other hand, the asynchronous SMC law (9) takes the effects of the event-triggered mechanism (6) and the Lur'e-type nonlinearities into account appropriately. The introduction of the term $L_\chi\varphi_\chi(H_\chi x(l_n))$ in (9) is inspired by [7].

Remark 3: Very recently, the problem of passivity-based asynchronous SMC for a class of uncertain singular MJSs with time-delay and nonlinear perturbations was addressed in [11]. However, there are three essential differences between this paper and [11]: 1) the considered system (1) is *discrete-time* but the one in [11] is *continuous-time*; 2) this paper proposes the *event*-triggered asynchronous SMC approach but [11] developed the *time*-triggered one; and 3) the proposed asynchronous SMC law (9) only depends on the detected mode $d(k)$ not the system mode $r(k)$, however, the SMC law (29) in [11] still used the system mode signal so that it cannot be practically implemented under the asynchronous phenomenon.

By substituting (9) into (1), the closed-loop system is obtained as

$$\begin{aligned} x(k+1) = {} & \tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) + D_i\varphi_i(H_ix(k)) \\ & + B_iL_\chi\varphi_\chi\big(H_\chi x(l_n)\big) + G_iv(k) + B_i\rho_i(k) \end{aligned} \tag{11}$$

where $\tilde{A}_{i\chi} \triangleq A_i + \tilde{Z}_i\Delta A_i(k) + B_iK_\chi$, $\tilde{Z}_i \triangleq I - B_iX_i^{-1}Z$, $\rho_i(k) \triangleq U_i(k) - \varrho\|x(l_n)\|\mathrm{sgn}(s(l_n))$, and $U_i(k) \triangleq X_i^{-1}Z\Delta A_i(k)x(k) + f_i(x(k), k)$.

Noticing that $\|\mathrm{sgn}(s(l_n))\| \le \sqrt{m}$, we have

$$\|\rho_i(k)\| \le \big(\delta_i + \varrho\sqrt{m}\big)\|x(k)\| + \varrho\sqrt{m}\|\varepsilon(k)\|. \tag{12}$$

In the sequel, the mean square stability with a prescribed H_∞ disturbance rejection performance of the sliding mode dynamics (11) and the reachability for the specified sliding surface (8) will be analyzed, respectively.

B. Analysis of Sliding Mode Dynamics

In the following theorem, by referring to the work [7], the stochastic Lur'e-type Lyapunov functional approach is utilized for reducing the conservatism in tackling the mode-dependent Lur'e-type nonlinearities $\varphi_i(H_ix(k))$, $i \in \mathbb{S}$.

Theorem 1: Consider the Markovian jump Lur'e system (1) with given event-triggered parameter $\alpha \in [0, 1]$ and the event-triggered asynchronous SMC law (9). The closed-loop system (11) is MSS with a prescribed disturbance attenuation level $\gamma > 0$ if for any $i \in \mathbb{S}$ and $\chi \in \mathbb{D}$, there exist positive definite matrices P_i, $T \in \mathbb{R}^{n_x\times n_x}$, matrices $K_\chi \in \mathbb{R}^{m\times n_x}$, $L_\chi \in \mathbb{R}^{m\times q}$, diagonal positive semidefinite matrices $\Psi_{i\chi}$, $\Xi_{i\chi}$, $\Delta_i \in \mathbb{R}^{q\times q}$, and positive scalars ς_i, ω_i such that the following inequalities hold:

$$B_i^T\mathbb{P}_iB_i - \varsigma_iI \le 0 \tag{13}$$

$$L_\chi^TB_i^T\mathbb{P}_iB_iL_\chi - \Psi_{i\chi} \le 0 \tag{14}$$

$$B_i^T\mathbb{H}_iB_i - \omega_iI \le 0 \tag{15}$$

$$L_\chi^TB_i^T\mathbb{H}_iB_iL_\chi - \Xi_{i\chi} \le 0 \tag{16}$$

$$\mathbb{X}_i + \sum_{\chi=1}^{J}\xi_{i\chi}\Big[\mathbb{Y}_{i\chi}^T(\mathbb{P}_i + \mathbb{H}_i)\mathbb{Y}_{i\chi} + \mathbb{I}_\chi^T\big(\Psi_{i\chi} + \Xi_{i\chi}\big)\mathbb{I}_\chi\Big] < 0 \tag{17}$$

where $\mathbb{P}_i \triangleq \sum_{j=1}^{N}\pi_{ij}P_j$, $\mathbb{H}_i \triangleq \sum_{j=1}^{N}\pi_{ij}H_j^T\Omega_j\Delta_j\Omega_jH_j$, and

$$\begin{aligned} \mathbb{X}_i &\triangleq \begin{bmatrix} \mathbb{C}_i & H_i^T\Omega_i\Delta_i & 0 & 0 \\ \star & -3\Delta_i & 0 & 0 \\ \star & \star & \mathbb{T}_i & 0 \\ \star & \star & \star & -\gamma^2I \end{bmatrix} \\ \mathbb{C}_i &\triangleq -P_i + 6(\delta_i + \varrho\sqrt{m})^2(\varsigma_i + \omega_i)I + \alpha T + C_i^TC_i, \\ \mathbb{T}_i &\triangleq -T + 6m\varrho^2(\varsigma_i + \omega_i)I, \\ \mathbb{Y}_{i\chi} &\triangleq \big[\sqrt{3}\tilde{A}_{i\chi}\quad \sqrt{3}D_i\quad -\sqrt{3}B_iK_\chi\quad \sqrt{3}G_i\big] \\ \mathbb{I}_\chi &\triangleq \big[\sqrt{3}\Omega_\chi H_\chi\quad 0\quad -\sqrt{3}\Omega_\chi H_\chi\quad 0\big]. \end{aligned}$$

Proof: For any system mode $r(k) = i \in \mathbb{S}$, we consider the following Lur'e-type Lyapunov functional candidate for sliding mode dynamics (11):

$$V(x(k), i) \triangleq V_1(x(k), i) + V_2(x(k), i) \tag{18}$$

where $V_1(x(k), i) \triangleq x^T(k)P_ix(k)$, and $V_2(x(k), i) \triangleq \varphi_i^T(H_ix(k))\Delta_i\Omega_iH_ix(k)$.

Along the state trajectories of (11), one has

$$\begin{aligned} &\mathbb{E}\{\Delta V_1(x(k), i)\} \\ &\quad \triangleq \mathbb{E}\big\{V_1(x(k+1), r(k+1)) \mid x(k), i\big\} - V_1(x(k), i) \\ &\quad \le \sum_{\chi=1}^{J}\xi_{i\chi}\Big\{3\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) + D_i\varphi_i(H_ix(k)) \\ &\qquad + G_iv(k)\Big]^T\mathbb{P}_i\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) \end{aligned}$$

$$+ D_i\varphi_i(H_ix(k)) + G_iv(k)\Big] + 3\varphi_\chi^T\big(H_\chi x(l_n)\big)L_\chi^TB_i^T\mathbb{P}_iB_iL_\chi\varphi_\chi\big(H_\chi x(l_n)\big)\Big\} + 3\rho_i^T(k)B_i^T\mathbb{P}_iB_i\rho_i(k) - x^T(k)P_ix(k). \quad (19)$$

Notice that from (12) and (13), we have

$$\rho_i^T(k)B_i^T\mathbb{P}_iB_i\rho_i(k) \le 2\varsigma_i\Big[\big(\delta_i+\varrho\sqrt{m}\big)^2x^T(k)x(k) + m\varrho^2\varepsilon^T(k)\varepsilon(k)\Big]. \quad (20)$$

Besides, by exploiting the condition (14) and the inequality (5), we obtain for any $\chi \in \mathbb{D}$

$$\varphi_\chi^T\big(H_\chi x(l_n)\big)L_\chi^TB_i^T\mathbb{P}_iB_iL_\chi\varphi_\chi\big(H_\chi x(l_n)\big) \le (x(k)-\varepsilon(k))^TH_\chi^T\Omega_\chi\Psi_{i\chi}\Omega_\chi H_\chi(x(k)-\varepsilon(k)). \quad (21)$$

Thus, substituting (20) and (21) into (19) yields

$$\begin{aligned}\mathbb{E}\{\Delta V_1(x(k),i)\} \le \sum_{\chi=1}^{J}\xi_{i\chi}\Big\{3\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) + D_i\varphi_i(H_ix(k)) \\ + G_iv(k)\Big]^T\mathbb{P}_i\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) \\ + D_i\varphi_i(H_ix(k)) + G_iv(k)\Big] \\ + 3(x(k)-\varepsilon(k))^TH_\chi^T\Omega_\chi\Psi_{i\chi}\Omega_\chi H_\chi(x(k)-\varepsilon(k))\Big\} \\ + 6\varsigma_i\Big[\big(\delta_i+\varrho\sqrt{m}\big)^2x^T(k)x(k) + m\varrho^2\varepsilon^T(k)\varepsilon(k)\Big] \\ - x^T(k)P_ix(k).\end{aligned} \quad (22)$$

On the other hand, recalling the relationships in (5) gives

$$\begin{aligned}\mathbb{E}\{\Delta V_2(x(k))\} \triangleq \mathbb{E}\big\{V_2(x(k+1), r(k+1)) \mid x(k), i\big\} - V_2(x(k), i) \\ \le \sum_{\chi=1}^{J}\xi_{i\chi}\Big\{3\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) + D_i\varphi_i(H_ix(k)) \\ + G_iv(k)\Big]^T\mathbb{H}_i\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) \\ + D_i\varphi_i(H_ix(k)) + G_iv(k)\Big] \\ + 3\varphi_\chi^T\big(H_\chi x(l_n)\big)L_\chi^TB_i^T\mathbb{H}_iB_iL_\chi\varphi_\chi\big(H_\chi x(l_n)\big)\Big\} \\ + 3\rho_i^T(k)B_i^T\mathbb{H}_iB_i\rho_i(k) \\ - \varphi_i^T(H_ix(k))\Delta_i\Omega_iH_ix(k).\end{aligned} \quad (23)$$

By adopting the similar operations to (20) and (21), we have from the conditions (15) and (16)

$$\rho_i^T(k)B_i^T\mathbb{H}_iB_i\rho_i(k) \le 2\omega_i\Big[\big(\delta_i+\varrho\sqrt{m}\big)^2x^T(k)x(k) + m\varrho^2\varepsilon^T(k)\varepsilon(k)\Big] \quad (24)$$

$$\begin{aligned}\varphi_\chi^T\big(H_\chi x(l_n)\big)L_\chi^TB_i^T\mathbb{H}_iB_iL_\chi\varphi_\chi\big(H_\chi x(l_n)\big) \\ \le \varphi_\chi^T\big(H_\chi x(l_n)\big)\Xi_{i\chi}\varphi_\chi\big(H_\chi x(l_n)\big) \\ \le (x(k)-\varepsilon(k))^TH_\chi^T\Omega_\chi\Xi_{i\chi}\Omega_\chi H_\chi(x(k)-\varepsilon(k))\end{aligned} \quad (25)$$

which implies that

$$\begin{aligned}\mathbb{E}\{\Delta V_2(x(k))\} \\ \le \sum_{\chi=1}^{J}\xi_{i\chi}\Big\{3\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) + D_i\varphi_i(H_ix(k)) \\ + G_iv(k)\Big]^T\mathbb{H}_i\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) \\ + D_i\varphi_i(H_ix(k)) + G_iv(k)\Big] \\ + 3(x(k)-\varepsilon(k))^TH_\chi^T\Omega_\chi\Xi_{i\chi}\Omega_\chi H_\chi(x(k)-\varepsilon(k))\Big\} \\ + 6\omega_i\Big[\big(\delta_i+\varrho\sqrt{m}\big)^2x^T(k)x(k) + m\varrho^2\varepsilon^T(k)\varepsilon(k)\Big] \\ - \varphi_i^T(H_ix(k))\Delta_i\Omega_iH_ix(k).\end{aligned} \quad (26)$$

Now, define an extended state vector as

$$\eta(k) \triangleq \begin{bmatrix} x^T(k) & \varphi_i^T(H_ix(k)) & \varepsilon^T(k) & v^T(k)\end{bmatrix}^T.$$

Then, combining the event-triggered scheme in (6) and the inequality in (4), we have

$$\mathbb{E}\big\{\Delta V(x(k),i)\big\} \le \mathbb{E}\big\{\Delta V_1(x(k),i)\big\} + \mathbb{E}\big\{\Delta V_2(x(k),i)\big\} + \alpha x^T(k)Tx(k) - \varepsilon^T(k)T\varepsilon(k) - 3\mathbf{SC}(i, H_ix(k), \Delta_i). \quad (27)$$

When $v(k) = 0$, it is easily shown from (22), (26), and (27) that the condition (17) guarantees $\mathbb{E}\big\{\Delta V(x(k),i)\big\} < 0$ for any $i \in \mathbb{S}$, which means that the sliding mode dynamics in (11) with $v(k) = 0$ is MSS.

In what follows, we consider the H_∞ disturbance attenuation performance of the sliding mode dynamics (11) under the zero initial condition. To this end, we exploit the following index for $k \in [l_n, l_{n+1})$:

$$J_i(k) \triangleq \mathbb{E}\big\{\Delta V(x(k),i)\big\} + \mathbb{E}\Big\{\|z(k)\|^2\Big\} - \gamma^2\|v(k)\|^2.$$

Following a similar reasoning as above, it is shown that the condition (17) implies $J_i(k) \le 0$. Thus, it has $\sum_{k=0}^{\infty}J_i(k) \le 0$, which renders $\mathbb{E}\big\{\sum_{k=0}^{\infty}\|z(k)\|^2\big\} \le \gamma^2\sum_{k=0}^{\infty}\|v(k)\|^2$ under the zero initial condition. ■

C. Analysis of Reachability

This section carries out the reachability analysis by using an extended stochastic Lur'e-type Lyapunov functional. Around the specified sliding surface (8), i.e., $s(k) = 0$, we define a *time-varying* sliding region as follows:

$$\mathbf{O} \triangleq \big\{s(k) \;\big|\; \|s(k)\| \le \tilde{\rho}(k)\big\} \quad (28)$$

where $\tilde{\rho}(k) \triangleq \max_{i\in\mathbb{S}}\Big\{\sqrt{[(\hat{\rho}_i(k))/(\lambda_{\min}(Q_i))]}\Big\}$ with

$$\begin{aligned}\hat{\rho}_i(k) \triangleq 6\big(\|G_i^T\mathbb{P}_iG_i\| + \|G_i^T\mathbb{H}_iG_i\| + \|G_i^TZ^T\mathbb{Q}_iZG_i\|\big)\cdot\|v(k)\|^2 \\ + 12\big(\|B_i^T\mathbb{P}B_i\| + \|B_i^T\mathbb{H}_iB_i\| + \|X_i^T\mathbb{Q}_iX_i\|\big) \\ \times\left[\big(\delta_i+\varrho\sqrt{m}\big)^2 + m\varrho^2\alpha\frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}\right]\cdot\|x(k)\|^2\end{aligned}$$

and the matrices $\mathbb{Q}_i$ and T defined in Theorem 2.

Assuming that Theorem 1 hold and noticing the fact that $v(k) \in \ell_2[0,+\infty)$, it is easily shown that $\tilde{\rho}(k)$ is just a *vicinity* of the sliding surface $s(k) = 0$, that is, the quasi-sliding mode (QSM). The following theorem guarantees the QSM.

Theorem 2: Consider the Markovian jump Lur'e system (1) with given event-triggered parameter $\alpha \geq 0$ and the event-triggered asynchronous SMC law (9). For any $i \in \mathbb{S}$ and $\chi \in \mathbb{D}$, there exist positive definite matrices P_i, $T \in \mathbb{R}^{n_x \times n_x}$, $Q_i \in \mathbb{R}^{m\times m}$, matrices $K_\chi \in \mathbb{R}^{m\times n_x}$, $L_\chi \in \mathbb{R}^{m\times q}$, and diagonal positive semidefinite matrices $\Psi_{i\chi}$, $\Xi_{i\chi}$, $\Gamma_{i\chi}$, $\Delta_i \in \mathbb{R}^{q\times q}$ satisfying the conditions (14), (16) and the following matrix inequalities:

$$L_\chi^T X_i^T \mathbb{Q}_i X_i L_\chi - \Gamma_{i\chi} \leq 0 \tag{29}$$

$$\vec{\mathbb{X}}_i + \sum_{\chi=1}^{J} \xi_{i\chi}\Big[\vec{\mathbb{Y}}_{i\chi}^T(\mathbb{P}_i + \mathbb{H}_i)\vec{\mathbb{Y}}_{i\chi} + \mathbb{Z}_{i\chi}^T \mathbb{Q}_i \mathbb{Z}_{i\chi} + \vec{\mathbb{I}}_\chi^T\big(\Psi_{i\chi} + \Xi_{i\chi} + \Gamma_{i\chi}\big)\vec{\mathbb{I}}_\chi\Big] < 0 \tag{30}$$

where

$$\vec{\mathbb{X}}_i \triangleq \begin{bmatrix} -P_i + \alpha T & H_i^T \Omega_i \Delta_i & 0 \\ \star & -3\Delta_i & 0 \\ \star & \star & -T \end{bmatrix}$$
$$\vec{\mathbb{Y}}_{i\chi} \triangleq \begin{bmatrix} \sqrt{3}\tilde{A}_{i\chi} & \sqrt{3}D_i & -\sqrt{3}B_i K_\chi \end{bmatrix}$$
$$\mathbb{Z}_{i\chi} \triangleq \begin{bmatrix} \sqrt{3}\vec{A}_{i\chi} & \sqrt{3}ZD_i & -\sqrt{3}X_i K_\chi \end{bmatrix}$$
$$\vec{\mathbb{I}}_\chi \triangleq \begin{bmatrix} \sqrt{3}\Omega_\chi H_\chi & 0 & -\sqrt{3}\Omega_\chi H_\chi \end{bmatrix}$$
$$\mathbb{Q}_i \triangleq \sum_{j=1}^{N} \pi_{ij} Q_j,\ \vec{A}_{i\chi} \triangleq ZA_i + X_i K_\chi$$

and other matrices are defined in Theorem 1, then the state trajectories of the closed-loop system (11) will be driven into the sliding region **O** *in mean square* under the event-triggered asynchronous SMC law (9).

Proof: Combining (8) and (11), we have

$$\begin{aligned} s(k+1) = {} & \vec{A}_{i\chi}x(k) - X_i K_\chi \varepsilon(k) + ZD_i\varphi_i(H_i x(k)) \\ & + X_i L_\chi \varphi_\chi\big(H_\chi x(l_n)\big) + ZG_i v(k) + X_i \rho_i(k). \end{aligned} \tag{31}$$

Now, we consider the following extended Lur'e-type Lyapunov functional:

$$\vec{V}(k,i) \triangleq V(x(k),i) + s^T(k) Q_i s(k). \tag{32}$$

where the Lyapunov functional $V(x(k),i)$ is defined in (18).

Along with the sliding function in (31), we obtain

$$\begin{aligned} &\mathbb{E}\big\{s^T(k+1)Q(r(k+1))s(k+1) \mid x(k), r(k)=i\big\} \\ &\leq \sum_{\chi=1}^{J}\xi_{i\chi}\bigg\{3\Big[\vec{A}_{i\chi}x(k) - X_iK_\chi\varepsilon(k) + ZD_i\varphi_i(H_ix(k))\Big]^T \\ &\quad \times \mathbb{Q}_i\Big[\vec{A}_{i\chi}x(k) - X_iK_\chi\varepsilon(k) + ZD_i\varphi_i(H_ix(k))\Big] \\ &\quad + 3\varphi_\chi^T\big(H_\chi x(l_n)\big)L_\chi^T X_i^T \mathbb{Q}_i X_i L_\chi \varphi_\chi\big(H_\chi x(l_n)\big)\bigg\} \\ &\quad + 3(ZG_iv(k) + X_i\rho_i(k))^T \mathbb{Q}_i(ZG_iv(k) + X_i\rho_i(k)). \end{aligned} \tag{33}$$

By resorting to the condition (29), it follows from (5) that:

$$\begin{aligned} &\varphi_\chi^T\big(H_\chi x(l_n)\big)L_\chi^T X_i^T \mathbb{Q}_i X_i L_\chi \varphi_\chi\big(H_\chi x(l_n)\big) \\ &\quad \leq \varphi_\chi^T\big(H_\chi x(l_n)\big)\Gamma_{i\chi}\varphi_\chi\big(H_\chi x(l_n)\big) \\ &\quad \leq (x(k) - \varepsilon(k))^T H_\chi^T \Omega_\chi \Gamma_{i\chi}\Omega_\chi H_\chi(x(k) - \varepsilon(k)). \end{aligned} \tag{34}$$

Substituting (34) into (33) renders

$$\begin{aligned} &\mathbb{E}\big\{s^T(k+1)Q(r(k+1))s(k+1) \mid x(k), r(k)=i\big\} \\ &\leq \sum_{\chi=1}^{J}\xi_{i\chi}\bigg\{3\Big[\vec{A}_{i\chi}x(k) - X_iK_\chi\varepsilon(k) + ZD_i\varphi_i(H_ix(k))\Big]^T \\ &\quad \times \mathbb{Q}_i\Big[\vec{A}_{i\chi}x(k) - X_iK_\chi\varepsilon(k) + ZD_i\varphi_i(H_ix(k))\Big] \\ &\quad + 3(x(k) - \varepsilon(k))^T H_\chi^T \Omega_\chi \Gamma_{i\chi}\Omega_\chi H_\chi(x(k) - \varepsilon(k))\bigg\} \\ &\quad + 6\big(\|G_i^T Z^T \mathbb{Q}_i ZG_i\| \cdot \|v(k)\|^2 + \|X_i^T \mathbb{Q}_i X_i\| \cdot \|\rho_i(k)\|^2\big). \end{aligned} \tag{35}$$

Following a similar argument as above and bearing (4), (6), (19), and (23) in mind, we obtain from the conditions (14) and (16) that:

$$\begin{aligned} &\mathbb{E}\big\{\Delta\vec{V}(k,i)\big\} \\ &\leq \sum_{\chi=1}^{J}\xi_{i\chi}\bigg\{3\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) + D_i\varphi_i(H_ix(k))\Big]^T \\ &\quad \times \mathbb{P}_i\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) + D_i\varphi_i(H_ix(k))\Big] \\ &\quad + 3(x(k) - \varepsilon(k))^T H_\chi^T \Omega_\chi \Psi_{i\chi}\Omega_\chi H_\chi(x(k) - \varepsilon(k))\bigg\} \\ &\quad + 6\Big(\|G_i^T \mathbb{P}_i G_i\| \cdot \|v(k)\|^2 + \|B_i^T \mathbb{P}_i B_i\| \cdot \|\rho_i(k)\|^2\Big) \\ &\quad + \sum_{\chi=1}^{J}\xi_{i\chi}\bigg\{3\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) + D_i\varphi_i(H_ix(k))\Big]^T \\ &\quad \times \mathbb{H}_i\Big[\tilde{A}_{i\chi}x(k) - B_iK_\chi\varepsilon(k) + D_i\varphi_i(H_ix(k))\Big] \\ &\quad + 3(x(k) - \varepsilon(k))^T H_\chi^T \Omega_\chi \Xi_{i\chi}\Omega_\chi H_\chi(x(k) - \varepsilon(k))\bigg\} \\ &\quad + 6\Big(\|G_i^T \mathbb{H}_i G_i\| \cdot \|v(k)\|^2 + \|B_i^T \mathbb{H}_i B_i\| \cdot \|\rho_i(k)\|^2\Big) \\ &\quad + \sum_{\chi=1}^{J}\xi_{i\chi}\bigg\{3\Big[\vec{A}_{i\chi}x(k) - X_iK_\chi\varepsilon(k) + ZD_i\varphi_i(H_ix(k))\Big]^T \\ &\quad \times \mathbb{Q}_i\Big[\vec{A}_{i\chi}x(k) - X_iK_\chi\varepsilon(k) + ZD_i\varphi_i(H_ix(k))\Big] \\ &\quad + 3(x(k) - \varepsilon(k))^T H_\chi^T \Omega_\chi \Gamma_{i\chi}\Omega_\chi H_\chi(x(k) - \varepsilon(k))\bigg\} \\ &\quad + 6\Big(\|G_i^T Z^T \mathbb{Q}_i ZG_i\| \cdot \|v(k)\|^2 + \|X_i^T \mathbb{Q}_i X_i\| \cdot \|\rho_i(k)\|^2\Big) \\ &\quad - x^T(k)P_ix(k) - \varphi_i^T(H_ix(k))\Delta_i\Omega_iH_ix(k) + \alpha x^T(k)Tx(k) \\ &\quad -\varepsilon^T(k)T\varepsilon(k) - 3\mathbf{SC}(i, H_ix(k), \Delta_i) - s^T(k)Q_is(k) \\ &\leq \vec{\eta}^T(k)\bigg\{\vec{\mathbb{X}}_i + \sum_{\chi=1}^{J}\xi_{i\chi}\Big[\vec{\mathbb{Y}}_{i\chi}^T(\mathbb{P}_i + \mathbb{H}_i)\vec{\mathbb{Y}}_{i\chi} \\ &\quad + \mathbb{Z}_{i\chi}^T\mathbb{Q}_i\mathbb{Z}_{i\chi} + \vec{\mathbb{I}}_\chi^T\big(\Psi_{i\chi} + \Xi_{i\chi} + \Gamma_{i\chi}\big)\vec{\mathbb{I}}_\chi\Big]\bigg\}\vec{\eta}(k) \\ &\quad - \lambda_{\min}(Q_i)\|s(k)\|^2 + \vec{\rho}_i(k) \end{aligned} \tag{36}$$

where $\eta(k) \triangleq \begin{bmatrix} x^T(k) & \varphi_i^T(H_ix(k)) & \varepsilon^T(k)\end{bmatrix}^T$ and

$$\begin{aligned} \vec{\rho}_i(k) \triangleq {} & 6\big(\|G_i^T\mathbb{P}_iG_i\| + \|G_i^T\mathbb{H}_iG_i\| \\ & + \|G_i^TZ^T\mathbb{Q}_iZG_i\|\big)\cdot\|v(k)\|^2 + 6\big(\|B_i^T\mathbb{P}B_i\| \\ & + \|B_i^T\mathbb{H}_iB_i\| + \|X_i^T\mathbb{Q}_iX_i\|\big)\cdot\|\rho_i(k)\|^2. \end{aligned}$$

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Notice that during any time interval $k \in [l_n, l_{n+1})$, one has from (6) and (12) that

$$\|\rho_i(k)\|^2 \le 2\left[\left(\delta_i + \varrho\sqrt{m}\right)^2 + m\varrho^2\alpha\frac{\lambda_{\max}(T)}{\lambda_{\min}(T)}\right]\|x(k)\|^2. \tag{37}$$

Thus, when the state trajectories escape from the region $\mathbf{O}$ around the specified sliding surface (8), that is,

$$\|s(k)\| > \tilde{\rho}(k) \ge \sqrt{\frac{\hat{\rho}_i(k)}{\lambda_{\min}(Q_i)}} \ge \sqrt{\frac{\vec{\rho}_i(k)}{\lambda_{\min}(Q_i)}} \quad \forall i \in \mathbb{S}$$

it yields from (30) and (36) that

$$\begin{aligned}
&\mathbb{E}\{\Delta\tilde{V}(k,i)\} \\
&\le \vec{\eta}^T(k)\Bigg\{\vec{\mathbb{X}}_i + \sum_{\chi=1}^{J}\xi_{i\chi}\Big[\vec{\mathbb{Y}}_{i\chi}^T(\mathbb{P}_i + \mathbb{H}_i)\vec{\mathbb{Y}}_{i\chi} + \mathbb{Z}_{i\chi}^T\mathbb{Q}_i\mathbb{Z}_{i\chi} \\
&\quad + \vec{\mathbb{I}}_\chi^T\big(\Psi_{i\chi} + \Xi_{i\chi} + \Gamma_{i\chi}\big)\vec{\mathbb{I}}_\chi\Big]\Bigg\}\vec{\eta}(k) \\
&< 0.
\end{aligned} \tag{38}$$

This implies that the state trajectories of the closed-loop system (11) are strictly decreasing (*with mean square*) outside the region $\mathbf{O}$ defined in (28). ■

Remark 4: Since the adopted event-triggered strategy schedules the system state $x(k)$ to the controller until the event-triggered condition (6) is attained, the communication burden will be greatly reduced. Nevertheless, it should be also noted that the introduction of event-triggered protocol may sacrifice certain SMC performance. Actually, it is seen from (28) that the sliding region $\mathbf{O}$ with event-triggered protocol is larger than the one without event-triggered protocol [i.e., $\alpha = 0$ in (6)] due to the additional term $m\varrho^2\alpha[(\lambda_{\max}(T))/(\lambda_{\min}(T))]\cdot \|x(k)\|^2$.

D. Synthesis of SMC Law

In order to achieve the mean square stability with a prescribed H_∞ performance of the sliding mode dynamics (11) and the reachability of the specified sliding function (8) simultaneously, the gain matrices K_χ and L_χ in the designed SMC scheme (9) should be determined by Theorems 1 and 2 simultaneously. In what follows, we derive some sufficient conditions to synthesize the event-triggered asynchronous SMC law (9).

Theorem 3: Consider the following assertions.

1) Given the parameters α and γ, there exist positive definite matrices P_i, $T \in \mathbb{R}^{n\times n}$, $Q_i \in \mathbb{R}^{m\times m}$, matrices $K_\chi \in \mathbb{R}^{m\times n}$, $L_\chi \in \mathbb{R}^{m\times q}$, diagonal positive semidefinite matrices $\Psi_{i\chi}$, $\Xi_{i\chi}$, $\Gamma_{i\chi}$, $\Delta_i \in \mathbb{R}^{q\times q}$, and positive scalars ς_i, ω_i such that the conditions (13)–(17) and (29) and (30) hold for any $i \in \mathbb{S}$ and $\chi \in \mathbb{D}$.
2) Given the parameters α and γ, there exist positive definite matrices P_i, T, $X_{i\chi}$, $Z_{i\chi} \in \mathbb{R}^{n_x\times n_x}$, $Q_i \in \mathbb{R}^{m\times m}$, $Y_{i\chi} \in \mathbb{R}^{q\times q}$, $W_{i\chi} \in \mathbb{R}^{n_v\times n_v}$, matrices $K_\chi \in \mathbb{R}^{m\times n_x}$, $L_\chi \in \mathbb{R}^{m\times q}$, diagonal positive semidefinite matrices $\Psi_{i\chi}$, $\Xi_{i\chi}$, $\Gamma_{i\chi}$, $\Delta_i \in \mathbb{R}^{q\times q}$, and positive scalars ς_i, ω_i such that the conditions (13)–(16), (29) and the following matrix inequalities hold for any $i \in \mathbb{S}$ and $\chi \in \mathbb{D}$:

$$\begin{aligned}
&\mathbb{Y}_{i\chi}^T(\mathbb{P}_i + \mathbb{H}_i)\mathbb{Y}_{i\chi} \\
&\quad + \mathbb{I}_\chi^T\big(\Psi_{i\chi} + \Xi_{i\chi} + \Gamma_{i\chi}\big)\mathbb{I}_\chi + \tilde{\mathbb{Z}}_{i\chi}^T\mathbb{Q}_i\tilde{\mathbb{Z}}_{i\chi} \le \mathbb{W}_{i\chi}
\end{aligned} \tag{39}$$

$$\mathbb{X}_i + \sum_{\chi=1}^{J}\xi_{i\chi}\mathbb{W}_{i\chi} < 0 \tag{40}$$

where $\tilde{\mathbb{Z}}_{i\chi} \triangleq [\sqrt{3}\bar{A}_{i\chi} \quad \sqrt{3}ZD_i \quad -\sqrt{3}X_iK_\chi \quad 0]$ and $\mathbb{W}_{i\chi} \triangleq \mathrm{diag}\{X_{i\chi}, Y_{i\chi}, Z_{i\chi}, W_{i\chi}\}$.
3) Given the parameters α and γ, there exist positive definite matrices $\tilde{P}_i$, $\tilde{T}$, $\tilde{X}_{i\chi}$, $\tilde{Z}_{i\chi} \in \mathbb{R}^{n_x\times n_x}$, $\tilde{Q}_i \in \mathbb{R}^{m\times m}$, $\tilde{Y}_{i\chi} \in \mathbb{R}^{q\times q}$, $\tilde{W}_{i\chi} \in \mathbb{R}^{n_v\times n_v}$, matrices $\mathcal{K}_\chi \in \mathbb{R}^{m\times n_x}$, $\mathcal{L}_\chi \in \mathbb{R}^{m\times q}$, $\mathcal{J}_\chi \in \mathbb{R}^{n_x\times n_x}$, $\mathcal{V}_\chi \in \mathbb{R}^{q\times q}$, diagonal positive semidefinite matrices $\tilde{\Psi}_{i\chi}$, $\tilde{\Xi}_{i\chi}$, $\tilde{\Gamma}_{i\chi}$, $\tilde{\Delta}_i \in \mathbb{R}^{q\times q}$, and positive scalars $\tilde{\varsigma}_i$, $\tilde{\omega}_i$, ϑ_i such that the following coupled LMIs hold for any $i \in \mathbb{S}$ and $\chi \in \mathbb{D}$:

$$\begin{bmatrix} -\tilde{\varsigma}_iI & \mathbb{B}_i \\ \mathbb{B}_i^T & -\vec{\mathbb{P}} \end{bmatrix} \le 0 \tag{41}$$

$$\begin{bmatrix} \tilde{\Psi}_{i\chi} - \mathcal{V}_\chi^T - \mathcal{V}_\chi & \mathbb{L}_{i\chi} \\ \mathbb{L}_{i\chi}^T & -\vec{\mathbb{P}} \end{bmatrix} \le 0 \tag{42}$$

$$\begin{bmatrix} -\tilde{\omega}_iI & \tilde{\mathbb{B}}_i \\ \tilde{\mathbb{B}}_i & -\vec{\Delta} \end{bmatrix} \le 0 \tag{43}$$

$$\begin{bmatrix} \tilde{\Xi}_{i\chi} - \mathcal{V}_\chi^T - \mathcal{V}_\chi & \vec{\mathbb{L}}_{i\chi} \\ \vec{\mathbb{L}}_{i\chi}^T & -\vec{\Delta} \end{bmatrix} \le 0 \tag{44}$$

$$\begin{bmatrix} \tilde{\Gamma}_{i\chi} - \mathcal{V}_\chi^T - \mathcal{V}_\chi & \hat{\mathbb{L}}_{i\chi} \\ \hat{\mathbb{L}}_{i\chi}^T & -\vec{\mathbb{Q}} \end{bmatrix} \le 0 \tag{45}$$

$$\begin{bmatrix} \tilde{\mathbb{W}}_{i\chi} & \vec{\mathbb{R}}_{i\chi} & \tilde{\mathbb{E}}_i \\ \vec{\mathbb{R}}_{i\chi}^T & \Sigma_{i\chi} & \tilde{\mathbb{M}}_i \\ \tilde{\mathbb{E}}_i^T & \tilde{\mathbb{M}}_i^T & -\mathrm{diag}\{\vartheta_iI, \vartheta_iI\} \end{bmatrix} \le 0 \tag{46}$$

$$\begin{bmatrix} \tilde{\mathbb{X}}_i & \tilde{\mathbb{T}}_i \\ \tilde{\mathbb{T}}_i^T & \Upsilon_i \end{bmatrix} < 0 \tag{47}$$

where $\vec{\mathbb{P}} \triangleq \mathrm{diag}\{\tilde{P}_1, \ldots, \tilde{P}_N\}$, $\vec{\mathbb{Q}} \triangleq \mathrm{diag}\{\tilde{Q}_1, \ldots, \tilde{Q}_N\}$, $\vec{\Delta} \triangleq \mathrm{diag}\{\tilde{\Delta}_1, \ldots, \tilde{\Delta}_N\}$, and

$$\begin{aligned}
\mathbb{B}_i &\triangleq \left[\sqrt{\pi_{i1}}B_i^T\tilde{\varsigma}_i \quad \cdots \quad \sqrt{\pi_{iN}}B_i^T\tilde{\varsigma}_i\right] \\
\mathbb{L}_{i\chi} &\triangleq \left[\sqrt{\pi_{i1}}\mathcal{L}_\chi^TB_i^T \quad \cdots \quad \sqrt{\pi_{iN}}\mathcal{L}_\chi^TB_i^T\right] \\
\tilde{\mathbb{B}}_i &\triangleq \left[\sqrt{\pi_{i1}}B_i^TH_1^T\Omega_1\tilde{\omega}_i \cdots \sqrt{\pi_{iN}}B_i^TH_N^T\Omega_N\tilde{\omega}_i\right] \\
\vec{\mathbb{L}}_{i\chi} &\triangleq \left[\sqrt{\pi_{i1}}\mathcal{L}_\chi^TB_i^TH_1^T\Omega_1 \quad \cdots \quad \sqrt{\pi_{iN}}\mathcal{L}_\chi^TB_i^TH_N^T\Omega_N\right] \\
\hat{\mathbb{L}}_{i\chi} &\triangleq \left[\sqrt{\pi_{i1}}\mathcal{L}_\chi^TX_i^T \quad \cdots \quad \sqrt{\pi_{iN}}\mathcal{L}_\chi^TX_i^T\right] \\
\tilde{\mathbb{W}}_{i\chi} &\triangleq \mathrm{diag}\Big\{\tilde{X}_{i\chi} - \mathcal{J}_\chi^T - \mathcal{J}_\chi, -\tilde{Y}_{i\chi} \\
&\qquad \tilde{Z}_{i\chi} - \mathcal{J}_\chi^T - \mathcal{J}_\chi, -\tilde{W}_{i\chi}\Big\} \\
\Sigma_{i\chi} &\triangleq -\mathrm{diag}\Big\{\vec{\mathbb{P}}, \vec{\Delta}, \tilde{\Psi}_{i\chi}, \tilde{\Xi}_{i\chi}, \tilde{\Gamma}_{i\chi}, \vec{\mathbb{Q}}\Big\}
\end{aligned}$$

$$\vec{\mathbb{R}}_{i\chi} \triangleq \begin{bmatrix} \tilde{\mathbb{A}}_{i\chi} & \hat{\mathbb{A}}_{i\chi} & \sqrt{3}\mathcal{J}_\chi^T H_\chi^T \Omega_\chi & \sqrt{3}\mathcal{J}_\chi^T H_\chi^T \Omega_\chi & \sqrt{3}\mathcal{J}_\chi^T H_\chi^T \Omega_\chi & \bar{\mathbb{A}}_{i\chi} \\ \tilde{\mathbb{D}}_{i\chi} & \hat{\mathbb{D}}_{i\chi} & 0 & 0 & 0 & \bar{\mathbb{D}}_{i\chi} \\ \tilde{\mathbb{K}}_{i\chi} & \hat{\mathbb{K}}_{i\chi} & -\sqrt{3}\mathcal{J}_\chi^T H_\chi^T \Omega_\chi & -\sqrt{3}\mathcal{J}_\chi^T H_\chi^T \Omega_\chi & -\sqrt{3}\mathcal{J}_\chi^T H_\chi^T \Omega_\chi & \bar{\mathbb{K}}_{i\chi} \\ \tilde{\mathbb{G}}_{i\chi} & \hat{\mathbb{G}}_{i\chi} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\mathbb{A}}_{i\chi} \triangleq \left[\sqrt{3\pi_{i1}}\tilde{\mathbf{A}}_{i\chi}^T \quad \cdots \quad \sqrt{3\pi_{iN}}\tilde{\mathbf{A}}_{i\chi}^T \right]$$
$$\hat{\mathbb{A}}_{i\chi} \triangleq \left[\sqrt{3\pi_{i1}}\tilde{\mathbf{A}}_{i\chi}^T H_1^T \Omega_1 \quad \cdots \quad \sqrt{3\pi_{iN}}\tilde{\mathbf{A}}_{i\chi}^T H_N^T \Omega_N \right]$$
$$\bar{\mathbb{A}}_{i\chi} \triangleq \left[\sqrt{3\pi_{i1}}\vec{\mathbf{A}}_{i\chi}^T \quad \cdots \quad \sqrt{3\pi_{iN}}\vec{\mathbf{A}}_{i\chi}^T \right]$$
$$\tilde{\mathbb{D}}_{i\chi} \triangleq \left[\sqrt{3\pi_{i1}}\tilde{Y}_{i\chi} D_i^T \quad \cdots \quad \sqrt{3\pi_{iN}}\tilde{Y}_{i\chi} D_i^T \right]$$
$$\hat{\mathbb{D}}_{i\chi} \triangleq \left[\sqrt{3\pi_{i1}}\tilde{Y}_{i\chi} D_i^T H_1^T \Omega_1 \quad \cdots \quad \sqrt{3\pi_{iN}}\tilde{Y}_{i\chi} D_i^T H_N^T \Omega_N \right]$$
$$\bar{\mathbb{D}}_{i\chi} \triangleq \left[\sqrt{3\pi_{i1}}\tilde{Y}_{i\chi} D_i^T Z^T \quad \cdots \quad \sqrt{3\pi_{iN}}\tilde{Y}_{i\chi} D_i^T Z^T \right]$$
$$\tilde{\mathbb{K}}_{i\chi} \triangleq \left[-\sqrt{3\pi_{i1}}\mathcal{K}_\chi^T B_i^T \quad \cdots \quad -\sqrt{3\pi_{iN}}\mathcal{K}_\chi^T B_i^T \right]$$
$$\hat{\mathbb{K}}_{i\chi} \triangleq \left[-\sqrt{3\pi_{i1}}\mathcal{K}_\chi^T B_i^T H_1^T \Omega_1 \quad \cdots \quad -\sqrt{3\pi_{iN}}\mathcal{K}_\chi^T B_i^T H_N^T \Omega_N \right]$$
$$\bar{\mathbb{K}}_{i\chi} \triangleq \left[-\sqrt{3\pi_{i1}}\mathcal{K}_\chi^T X_i^T \quad \cdots \quad -\sqrt{3\pi_{iN}}\mathcal{K}_\chi^T X_i^T \right]$$
$$\tilde{\mathbb{G}}_{i\chi} \triangleq \left[\sqrt{3\pi_{i1}}\tilde{W}_{i\chi} G_i^T \quad \cdots \quad \sqrt{3\pi_{iN}}\tilde{W}_{i\chi} G_i^T \right],$$
$$\hat{\mathbb{G}}_{i\chi} \triangleq \left[\sqrt{3\pi_{i1}}\tilde{W}_{i\chi} G_i^T H_1^T \Omega_1 \quad \cdots \quad \sqrt{3\pi_{iN}}\tilde{W}_{i\chi} G_i^T H_N^T \Omega_N \right]$$
$$\tilde{\mathbf{A}}_{i\chi} \triangleq A_i \mathcal{J}_\chi + B_i \mathcal{K}_\chi,\ \vec{\mathbf{A}}_{i\chi} \triangleq Z A_i \mathcal{J}_\chi + X_i \mathcal{K}_\chi$$
$$\tilde{\mathbb{E}}_i \triangleq \begin{bmatrix} \mathcal{J}_\chi^T E_i^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\tilde{\mathbb{M}}_i \triangleq \begin{bmatrix} \underbrace{0 \ \cdots \ 0}_{N} & \underbrace{0 \ \cdots \ 0}_{N} & 0 & 0 & 0 & \underbrace{0 \ \cdots \ 0}_{N} \\ \tilde{\mathbf{M}}_i & \hat{\mathbf{M}}_i & 0 & 0 & 0 & \underbrace{0 \ \cdots \ 0}_{N} \end{bmatrix}^T$$
$$\tilde{\mathbf{M}}_i \triangleq \left[\sqrt{3\pi_{i1}} M_i^T \tilde{Z}_i^T \vartheta_i \quad \cdots \quad \sqrt{3\pi_{iN}} M_i^T \tilde{Z}_i^T \vartheta_i \right],$$
$$\hat{\mathbf{M}}_i \triangleq \left[\sqrt{3\pi_{i1}} M_i^T \tilde{Z}_i^T H_1^T \Omega_1 \vartheta_i \quad \cdots \quad \sqrt{3\pi_{iN}} M_i^T \tilde{Z}_i^T H_N^T \Omega_N \vartheta_i \right]$$
$$\tilde{\mathbb{X}}_i \triangleq \begin{bmatrix} -\tilde{P}_i & \tilde{P}_i H_i^T \Omega_i & 0 & 0 \\ \star & -3\tilde{\Delta}_i & 0 & 0 \\ \star & \star & -\tilde{T} & 0 \\ \star & \star & \star & -\gamma^2 I \end{bmatrix}$$
$$\Upsilon_i \triangleq -\mathrm{diag}\{\tilde{\varsigma}_i I, \tilde{\omega}_i I, \tilde{T}, I, \tilde{\varsigma}_i I, \tilde{\omega}_i I, \mathbf{X}_i, \mathbf{Y}_i, \mathbf{Z}_i, \mathbf{W}_i\},$$
$$\mathbf{X}_i \triangleq \mathrm{diag}\{\tilde{X}_{i1}, \ldots, \tilde{X}_{iJ}\},\ \mathbf{Y}_i \triangleq \mathrm{diag}\{\tilde{Y}_{i1}, \ldots, \tilde{Y}_{iJ}\}$$
$$\mathbf{Z}_i \triangleq \mathrm{diag}\{\tilde{Z}_{i1}, \ldots, \tilde{Z}_{iJ}\},\ \mathbf{W}_i \triangleq \mathrm{diag}\{\tilde{W}_{i1}, \ldots, \tilde{W}_{iJ}\}$$
$$\vec{\mathbb{T}}_i \triangleq \begin{bmatrix} \sqrt{6}\vec{\delta}_i \tilde{P}_i & \sqrt{6}\vec{\delta}_i \tilde{P}_i & \sqrt{\alpha}\tilde{P}_i & \tilde{P}_i C_i^T & 0 & 0 & \hat{\mathbf{P}}_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{\Delta}_i & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{6m\varrho}\tilde{T} & \sqrt{6m\varrho}\tilde{T} & 0 & 0 & \hat{\mathbf{T}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{\mathbf{I}}_i \end{bmatrix}$$
$$\vec{\delta}_i \triangleq \delta_i + \varrho\sqrt{m},\ \hat{\mathbf{P}}_i \triangleq \left[\sqrt{\xi_{i1}}\tilde{P}_i \quad \cdots \quad \sqrt{\xi_{iJ}}\tilde{P}_i \right]$$
$$\hat{\Delta}_i \triangleq \left[\sqrt{\xi_{i1}}\tilde{\Delta}_i \quad \cdots \quad \sqrt{\xi_{iJ}}\tilde{\Delta}_i \right]$$
$$\hat{\mathbf{T}} \triangleq \left[\sqrt{\xi_{i1}}\tilde{T} \quad \cdots \quad \sqrt{\xi_{iJ}}\tilde{T} \right]$$
$$\hat{\mathbf{I}}_i \triangleq \left[\sqrt{\xi_{i1}} I \quad \cdots \quad \sqrt{\xi_{iJ}} I \right].$$

We have that 3) $\Rightarrow$ 2) $\Rightarrow$ 1). Moreover, if the coupled LMIs (41)–(47) hold, the mean square stability with a desired H_∞ disturbance attenuation level γ of the closed-loop system (11) and the reachability of the sliding region $\mathbf{O}$ defined in (28) around the sliding surface (8) can be guaranteed simultaneously by the event-triggered asynchronous SMC law (9) with $K_\chi = \mathcal{K}_\chi \mathcal{J}_\chi^{-1}$ and $L_\chi = \mathcal{L}_\chi \mathcal{V}_\chi^{-1}$.

Proof 2) $\Rightarrow$ *1):* Notice that $\Psi_{i\chi} + \Xi_{i\chi} + \Gamma_{i\chi} > \Psi_{i\chi} + \Xi_{i\chi}$. It is readily shown that the conditions (17) and (30) are ensured by the following matrix inequality:

$$\mathbb{X}_i + \sum_{\chi=1}^{J} \xi_{i\chi} \Big[\mathbb{Y}_{i\chi}^T (\mathbb{P}_i + \mathbb{H}_i) \mathbb{Y}_{i\chi} + \mathbb{I}_\chi^T (\Psi_{i\chi} + \Xi_{i\chi} + \Gamma_{i\chi}) \mathbb{I}_\chi + \tilde{\mathbb{Z}}_{i\chi}^T \mathbb{Q}_i \tilde{\mathbb{Z}}_{i\chi} \Big] < 0$$

which is guaranteed by the conditions (39) and (40).

3) $\Rightarrow$ *2):* Denote $\tilde{P}_i \triangleq P_i^{-1}$, $\tilde{Q}_i \triangleq Q_i^{-1}$, $\tilde{T} \triangleq T^{-1}$, $\tilde{X}_{i\chi} \triangleq X_{i\chi}^{-1}$, $\tilde{Y}_{i\chi} \triangleq Y_{i\chi}^{-1}$, $\tilde{Z}_{i\chi} \triangleq Z_{i\chi}^{-1}$, $\tilde{W}_{i\chi} \triangleq W_{i\chi}^{-1}$, $\tilde{\Phi}_{i\chi} \triangleq \Phi_{i\chi}^{-1}$, $\tilde{\Xi}_{i\chi} \triangleq \Xi_{i\chi}^{-1}$, $\tilde{\Gamma}_{i\chi} \triangleq \Gamma_{i\chi}^{-1}$, $\tilde{\Delta}_i \triangleq \Delta_i^{-1}$, $\tilde{\varsigma}_i \triangleq \varsigma_i^{-1}$, $\tilde{\omega}_i \triangleq \omega_i^{-1}$, $\mathcal{K}_\chi \triangleq K_\chi \mathcal{J}_\chi$, and $\mathcal{L}_i \triangleq L_\chi \mathcal{V}_\chi$. By resorting to the following matrix inequalities:

$$\left(\Phi_{i\chi} - \mathcal{V}_\chi^T\right) \Phi_{i\chi}^{-1} \left(\Phi_{i\chi} - \mathcal{V}_\chi\right) \geq 0$$
$$\left(\Xi_{i\chi} - \mathcal{V}_\chi^T\right) \Xi_{i\chi}^{-1} \left(\Xi_{i\chi} - \mathcal{V}_\chi\right) \geq 0$$
$$\left(\Gamma_{i\chi} - \mathcal{V}_\chi^T\right) \Gamma_{i\chi}^{-1} \left(\Gamma_{i\chi} - \mathcal{V}_\chi\right) \geq 0$$
$$\left(X_{i\chi} - \mathcal{J}_\chi^T\right) X_{i\chi}^{-1} \left(X_{i\chi} - \mathcal{J}_\chi\right) \geq 0$$
$$\left(Z_{i\chi} - \mathcal{J}_\chi^T\right) Z_{i\chi}^{-1} \left(Z_{i\chi} - \mathcal{J}_\chi\right) \geq 0$$

it is concluded that the conditions (13)–(16) and (29) are ensured by the LMIs (41)–(45) and the conditions (39)–(40) are guaranteed by the LMIs (46) and (47), respectively. ■

Remark 5: In this paper, the SMC problem is investigated for a class of Markovian jump Lur'e systems under the event-triggered strategy (6) and the hidden Markov mode detection model (7). The proposed event-triggered asynchronous SMC law exhibits the following distinct features: 1) the Lur'e nonlinearities are contained for enhancing the control performance; 2) an event-triggered protocol is cooperated for the energy-saving purpose; and 3) the designed SMC law just utilizes the detected mode signal but not the system mode signal.

E. Algorithm

About the computational complexity of the conditions (41)–(47) in Theorem 3, it is required to solve $10N + 11NJ$ LMIs to get $6N + 4J + 7NJ + 1$ decision variables, or $(N + 2NJ + 1)[(n_x(n_x+1))/2] + N([(m(m+1))/2] + J[(q(q+1))/2] + J[(n_v(n_v+1))/2]) + J(mn_x + mq + n_x^2 + q^2) + (3NJ + N)q + 3N$ scalar variables.

For the prescribed event-triggered parameter α in (6), we seek to find a minimum H_∞ performance level γ. As per Theorem 3, the design procedures of the SMC law with the minimum H_∞ performance can be summarized as follows.

Step 1: Given the parameter $\alpha \in [0, 1]$ in (6). Select the parameters β_i properly such that the matrix X_i is nonsingular for any $i \in \mathbb{S}$. Compute the scalars δ_i and ϱ in (10).

Step 2: Get the matrices $\tilde{T}$, $\mathcal{K}_\chi$, $\mathcal{L}_\chi$, $\mathcal{J}_\chi$, and $\mathcal{V}_\chi$ by solving the following optimization problem:

$$\min_{\tilde{P}_i,\tilde{T},\tilde{X}_{i\chi},\tilde{Z}_{i\chi},\tilde{Q}_i,\tilde{Y}_{i\chi},\tilde{W}_{i\chi},\tilde{\Psi}_{i\chi},\tilde{\Xi}_{i\chi},\tilde{\Gamma}_{i\chi},\tilde{\Delta}_i,\tilde{\varsigma}_i,\tilde{\omega}_i,\vartheta_i} \gamma^* \quad \text{subject to: LMIs (41)} - (47) \text{ with } \gamma^* \triangleq \gamma^2. \tag{48}$$

Step 3: Produce the event-triggered protocol (6) with $T = \tilde{T}^{-1}$, and the asynchronous SMC law (9) with matrices $K_\chi = \mathcal{K}_\chi \mathcal{J}_\chi^{-1}$ and $L_\chi = \mathcal{L}_\chi \mathcal{V}_\chi^{-1}$.

Step 4: Apply the obtained asynchronous SMC scheme (9) to the Markovian jump Lur'e system (1) under the event-triggered protocol (6).

IV. Illustrative Example

In this section, a modified practical experiment from [25] and [32] is employed to illustrate the proposed event-triggered asynchronous SMC scheme, where the angular velocity of a dc motor device is controlled subject to abrupt failures. The Markov jump variable $r(k) = i$ characterize the random failures occurred on the power, which are generated by a computer. The power modes jump between three modes, that is, 0% of rotary [normal mode, $r(k) = 1$], +20% of rotary for improving the power [low mode, $r(k) = 2$], and −40% of rotary for decreasing the power [medium mode, $r(k) = 3$]. Denote the system state as $x(k) = [\,x_1(k) \quad x_2(k) \quad x_3(k)\,]^T$, where $x_1(k)$, $x_2(k)$, and $x_3(k)$ stand for the angular velocity of the rotor, the electrical current consumed by the motor, and the integrative term written as a discrete sum, respectively.

Now, the networked dc motor device with Markov-driven power failures can be modeled as the discrete-time Markovian jump Lur'e system (1) with

$$A_i = \begin{bmatrix} a_{11}^{(i)} & a_{12}^{(i)} & 0 \\ a_{21}^{(i)} & a_{22}^{(i)} & 0 \\ a_{31}^{(i)} & 0 & a_{33}^{(i)} \end{bmatrix}, \quad B_i = \begin{bmatrix} b_1^{(i)} \\ b_2^{(i)} \\ 0 \end{bmatrix}$$

$$G_i = 0.1I,\ C_i = \mathrm{diag}\{1, 1.2, 1.1\},\ i \in \{1, 2, 3\}$$

$$M_1 = \begin{bmatrix} 0.9 & 0.5 & 0.12 \end{bmatrix}^T, \quad M_2 = \begin{bmatrix} 0.1 & 0.1 & 0.4 \end{bmatrix}^T$$

$$M_3 = \begin{bmatrix} -0.9 & -0.1 & 0.1 \end{bmatrix}^T, \ E_1 = \begin{bmatrix} -0.1 & 0.7 & 0.11 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0.2 & 0.2 & 0.13 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0.14 & 0.17 & 0.01 \end{bmatrix}$$

$$\Phi_1(k) = \frac{1}{1+k^2},\ \Phi_2(k) = \cos(0.03k),\ \Phi_3(k) = \sin(20k)$$

and the other parameters chosen as in Table I. The transition probability matrix for the Markov jump failures is set as [25]

$$\Pi = \begin{bmatrix} 0.95 & 0.05 & 0 \\ 0.36 & 0.6 & 0.04 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}.$$

The following nonlinearity functions $\varphi_i(H_i x(k))$ as in [7] and [40] are considered for the Markovian jump Lur'e system (1):

$$D_1 = \begin{bmatrix} 0.6 & 0.8 & 1 \end{bmatrix}^T,\ D_2 = \begin{bmatrix} 0.4 & 0.7 & 0.1 \end{bmatrix}^T$$

$$D_3 = \begin{bmatrix} 0.4 & 0.5 & -0.3 \end{bmatrix}^T,\ H_1 = \begin{bmatrix} 0.5 & 0.3 & 0.1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0.4 & -0.5 & 0.3 \end{bmatrix},\ H_3 = \begin{bmatrix} 1.5 & -0.9 & 0.4 \end{bmatrix}$$

$$\Omega_1 = 0.8,\ \Omega_2 = 1.5,\ \Omega_3 = 0.7$$

$$\varphi_1(H_1x(k)) = 0.5\Omega_1 H_1 x(k)(1 + \cos(H_1 x(k)))$$

$$\varphi_2(H_2x(k)) = 0.5\Omega_2 H_2 x(k)\left(1 - e^{-0.1(H_2x(k))^2}\right)$$

$$\varphi_3(H_3x(k)) = 0.5\Omega_3 H_3 x(k)(1 - \sin(H_3 x(k))).$$

In this example, we suppose that the system state $x(k)$ is transmitted to the controller under the event-triggered protocol (6) with the design parameter $\alpha = 0.4$, and the information of the power failure mode is emitted to the controller via a mode detector with the following MDPM:

$$\Theta = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.7 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}.$$

Our purpose now is to synthesize an event-triggered asynchronous SMC law (9) to stochastically stabilize the networked dc motor device subject to the actuator disturbances $f_i(x(k), k) = 0.3\sqrt{x_1^2 + x_2^2 + x_3^2}$. Letting $\beta_1 = \beta_2 = \beta_3 = (1/3)$ in (8), we have the sliding matrix $Z = [\,0.8010 \quad 1.1010 \quad 0\,]$. It is easily verified that the nonsingularity of the matrix $X_i = ZB_i$ can be ensured for every mode i. By solving optimization problem (48), we obtain the minimum H_∞ disturbance attenuation performance $\gamma = 4.2650$ with the following event-triggered matrix and asynchronous SMC law for $k \in [l_n, l_{n+1})$:

$$T = \begin{bmatrix} 156.2101 & 0.5267 & -3.6297 \\ 0.5267 & 153.5401 & 0.4952 \\ -3.6297 & 0.4952 & 194.4801 \end{bmatrix} \tag{49}$$

$$u(k) = \begin{cases} \begin{bmatrix} 0.2856 & -0.4651 & 0.0729 \end{bmatrix} x(l_n) \\ -0.0079\varphi_1(H_1 x(l_n)) \\ -0.7179\|x(l_n)\| \cdot \mathrm{sgn}(s(l_n)), \quad \chi = 1 \\ \begin{bmatrix} 1.8481 & -1.0199 & 0.0676 \end{bmatrix} x(k) \\ -0.0098\varphi_2(H_2 x(l_n)) \\ -0.7179\|x(l_n)\| \cdot \mathrm{sgn}(s(l_n)), \quad \chi = 2 \\ \begin{bmatrix} 0.1896 & -0.4268 & 0.0339 \end{bmatrix} x(k) \\ -0.0069\varphi_3(H_3 x(l_n)) \\ -0.7179\|x(l_n)\| \cdot \mathrm{sgn}(s(l_n)), \quad \chi = 3. \end{cases} \tag{50}$$

The simulation results are shown in Figs. 2–7. Under $v(k) = 0$, the initial condition $x(0) = [\,0.3 \quad -0.1 \quad 0.5\,]^T$ and a possible sequence of system and controller modes as depicted in Fig. 2, it is easily verified that the dc motor device in open-loop case is unstable subject to the Markov-driven power failures. However, under the same scenarios, the MSS of the networked dc motor device can be attained by the event-triggered asynchronous SMC law (50) as shown in Fig. 3. The

TABLE I
PARAMETERS IN THE NETWORKED DC MOTOR DEVICE

Parameters	$i=1$	$i=2$	$i=3$
$a_{11}^{(i)}$	-0.4799	-1.6026	0.6346
$a_{12}^{(i)}$	0.51546	0.91632	0.9178
$a_{21}^{(i)}$	-0.38162	-0.5918	-0.5056
$a_{22}^{(i)}$	1.44723	0.30317	0.24811
$a_{31}^{(i)}$	0.1399	0.0740	0.3865
$a_{33}^{(i)}$	-0.9255	-0.4338	0.0982
$b_1^{(i)}$	0.58705	1.02851	0.7874
$b_2^{(i)}$	1.55010	0.22282	1.5302

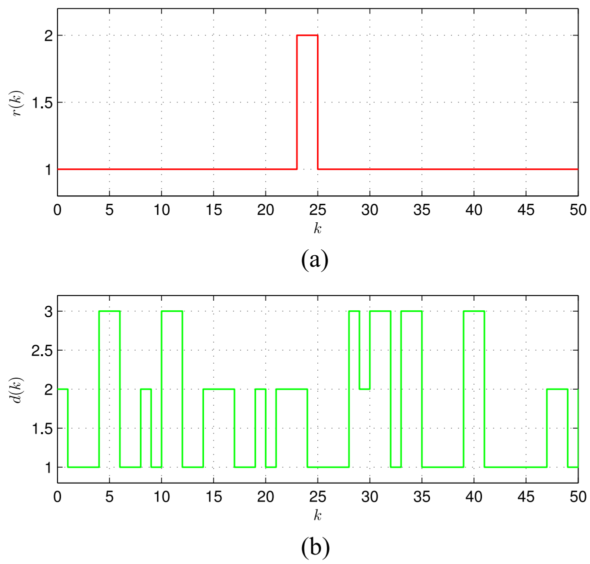

Fig. 2. Possible sequence of (a) system and (b) controller modes.

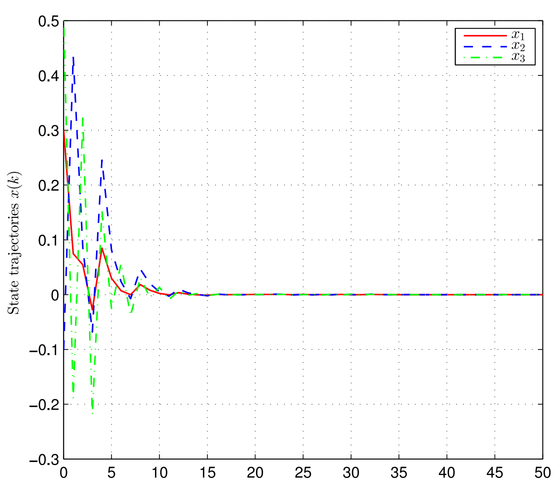

Fig. 3. State trajectories $x(k)$ in closed-loop case.

evolutions of the release instants and release interval, the sliding variable $s(k)$ and the event-triggered asynchronous SMC input $u(k)$ are depicted in Figs. 4–6, respectively.

In order to illustrate the H_∞ disturbance attenuation performance of the obtained event-triggered asynchronous SMC law (50), we define an auxiliary

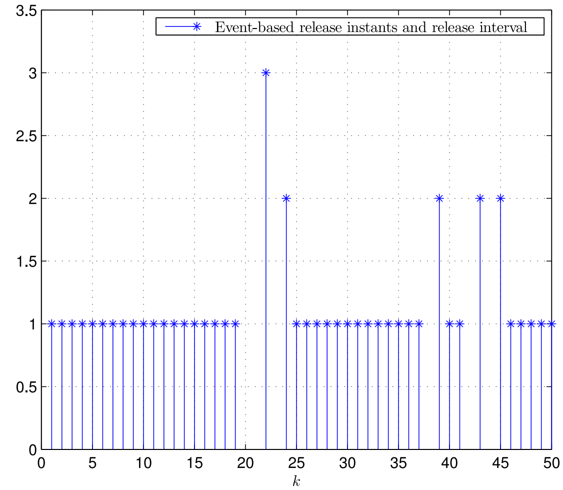

Fig. 4. Release instants and release intervals for the event-triggered condition (6).

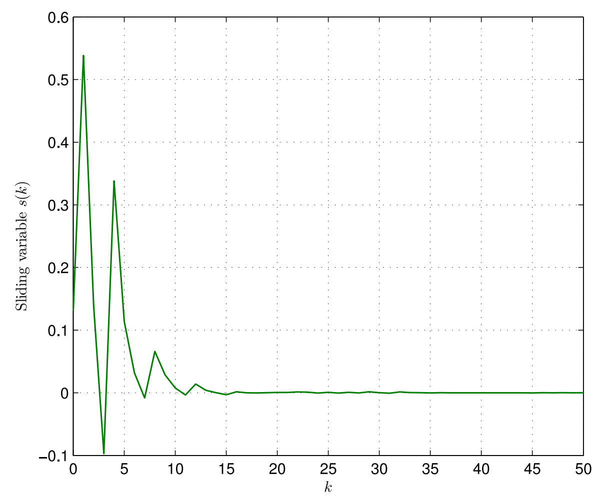

Fig. 5. Sliding variable $s(k)$.

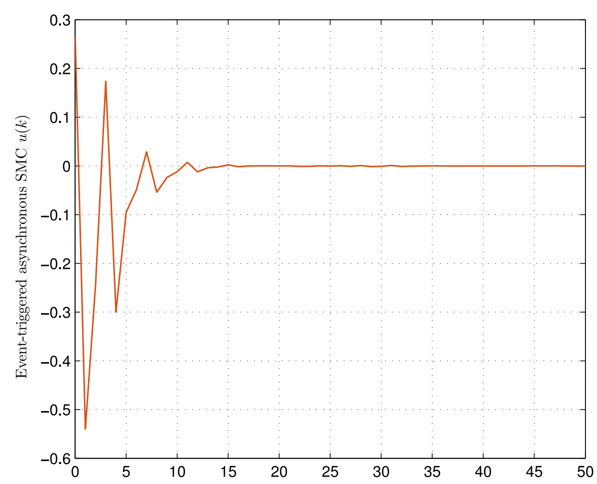

Fig. 6. Event-triggered asynchronous SMC input $u(k)$.

function as $\gamma_D(k) = \sqrt{[(\sum_{j=0}^{k}\|z(j)\|^2)/(\sum_{j=0}^{k}\|v(j)\|^2)]}$. Under the zero initial condition and the external disturbance $v(k) = [[(\sin(100k))/((k+1)^2)] - (0.9/(k+1)^2)$

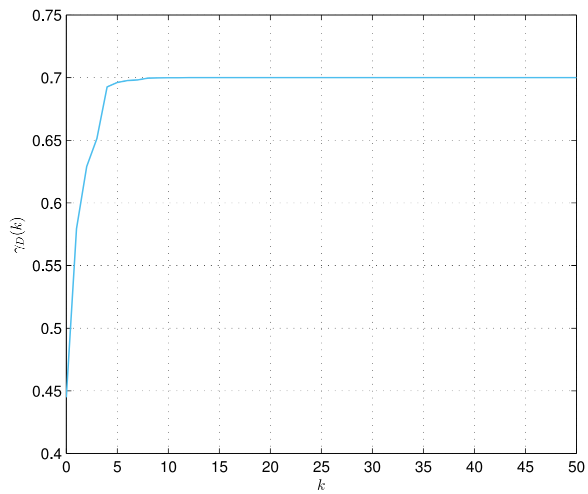

Fig. 7. Time response of the function $\gamma_D(k)$.

$[(\cos(0.1k^2))/(k+1)^3]]^T$, Fig. 7 shows the time response of $\gamma_D(k)$ for the closed-loop dc motor device subject to the system and controller modes as shown in Fig. 2. All simulation results have confirmed the effectiveness of the proposed event-triggered asynchronous H_∞ SMC scheme for the networked Markovian jump Lur'e system (1).

V. Conclusion

In this paper, we have addressed the event-triggered asynchronous SMC problem for networked Markovian jump Lur'e systems. By *only* using the detected modes, an asynchronous SMC scheme has been proposed with the event-triggered protocol. The stochastic Lur'e-type Lyapunov functional and the HMM approach have been exploited to derive the sufficient conditions to ensure the MMS of the sliding mode dynamics with a prescribed H_∞ disturbance attenuation performance. It is interesting for us to extend the proposed results to the semi-MJSs [9], [26] and nonhomogeneous MJSs [12] in near future.

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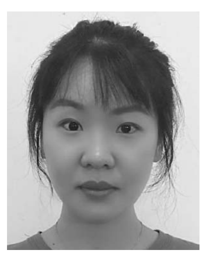

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