Asynchronous Siding Mode Control of Two-dimensional Markov Jump Systems in Roesser Model

Abstract—**abstract**

***Index Terms*—Markov jump systems, 2D systems, Siding mode control, Hidden Markov model**

I. Introduction

This part is introduciton.

II. Preliminaries

In this paper, we consider the following two-dimensional Markov jump systems in Roesser model:

$$\begin{cases} \boldsymbol{x(i,\,j)} = A_{r(i,j)}x(i,j) + E_{r(i,j)}w(i,j) \\ \qquad + B_{r(i,j)}[(u(i,j) + f(x(i,j),r(i,j))] \\ y(i,j) = C_{r(i,j)}x(i,j) + D_{r(i,j)}w(i,j) \end{cases} \tag{1}$$

where

$$\boldsymbol{x(i,\,j)} = \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix}, \quad x(i,j) = \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix},$$

$x^h(i,h) \in \mathbb{R}^{n_h}$ and $x^v(i,h) \in \mathbb{R}^{n_v}$ represent horizontal and vertical states respectively, $u(i,j) \in \mathbb{R}^{n_u}$ and $y(i,j) \in \mathbb{R}^{n_y}$ represent the controlled input and output respectively, and $w(i,j) \in \mathbb{R}^{n_w}$ represents the exogenous disturbance which belongs to $\ell_2\{[0,\infty),[0,\infty)\}$. $A_{r(i,j)}$,$B_{r(i,j)}$,$C_{r(i,j)}$,$D_{r(i,j)}$ and $E_{r(i,j)}$ represent the time-varying system matrices, all of which are real known constant matrices with appropriate dimensions. Besides, we assume that the matrix $B_{r(i,j)}$ is full column rank for each $r(i,j) \in \mathcal{N}_1$, that is, rank($B_{r(i,j)}$)= n_u. The nonlinear function $f(x(i,j),r(i,j))$ satisfying the following property:

$$\|f(x(i,j),r(i,j)\| \le \delta_{r(i,j)}\|x(i,j)\| \tag{2}$$

where $\delta_{r(i,j)}$ is a known scalar, $\|\cdot\|$ denotes the Euclidean norm of a vector. The parameter $r(i,j)$ takes values in a finite set $\mathcal{N}_1 = \{1,2...,N_1\}$ with transition probability matrix $\Lambda = \{\lambda_{k\tau}\}$, and the related transition probability from mode k to mode τ is given by

$$\begin{aligned} &\Pr\{r(i+1,j) = \tau | r(i,j) = k\} \\ =&\Pr\{r(i,j+1) = \tau | r(i,j) = k\} = \lambda_{k\tau},\ \forall k,\tau \in \mathcal{N}_1 \end{aligned} \tag{3}$$

where $\lambda_{k\tau} \in [0,1]$, for all $k,\tau \in \mathcal{N}_1$, and $\sum_{\tau=1}^{N_1} \lambda_{k\tau} = 1$ for every mode k.

We define the boundary condition (X_0, Γ_0) of system (1), as follows:

$$\begin{cases} X_0 = \{x^h(0,j), x^v(i,0)|\ i,j = 0,1,2...\} \\ \Gamma_0 = \{r(0,j), r(i,0)|\ i,j = 0,1,2...\} \end{cases} \tag{4}$$

And the corresponding zero boundary condition is assumed as $x^h(0,j) = 0, x^v(i,0) = 0, i,j = 0,1,2....$ Besides, we further impose following assumption on X_0.

Assumption 1. *The boundary condition X_0 satisfies:*

$$\lim_{L\to\infty} \mathbb{E}\Big\{\sum_{\ell=1}^{L}(\|x^h(0,\ell)\|^2 + \|x^v(\ell,0)\|^2)\Big\} < \infty \tag{5}$$

where $\mathbb{E}\{\cdot\}$ stands for mathematical expectation.

In practical applications, the complete information of $r(i,j)$ can not always be available to the controller. Hence, in this paper, the hidden Markov model $(r(i,j),\sigma(i,j),\Lambda,\Psi)$ as in [refto] is introduced to characterize the asynchronous phenomenon between the controller and the system. The parameter $\sigma(i,j)$, refers to controller mode, takes values in another finite set $\mathcal{N}_2 = \{1,2...N_2\}$, and satisfies the conditional probability matrix $\Psi = \{\mu_{ks}\}$ with conditional mode transition probabilities

$$\Pr\{\sigma(i,j) = s | r(i,j) = k\} = \mu_{ks}, \tag{6}$$

where $\mu_{ks} \in [0,1]$ for all $k \in \mathcal{N}_1, s \in \mathcal{N}_2$, and $\sum_{s=1}^{N2} \mu_{ks} = 1$ for any mode k.

Next, the definitions of asymptotically mean square stable and H_∞ performance for 2D systems will be given in Definition 1 and Definition 2, respectively.

Definition 1. *The 2D Markov jump system* (1) *with $w(i,j) \equiv 0$ is said to be asymptotically mean square stable if the following holds:*

$$\lim_{i+j\to\infty} \mathbb{E}\{\|x(i,j)\|^2\} = 0 \tag{7}$$

for any boundary condition X_0 with Assumption 1.

Definition 2. *Given a scalar $\gamma > 0$, the 2D Markov jump system* (1) *is said to be asymptotically mean square stable with an H_∞ disturbance attenuation performance γ if the system satisfies* (7)*, and under zero boundary condition, the following holds:*

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\{\|y(i,j)\|^2\} < \gamma^2 \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\{\|w(i,j)\|^2\} \tag{8}$$

for all $w(i,j) \in \ell_2\{[0,\infty),[0,\infty)\}$.

Now, we will make some notational simplification for convenience. The parameter $r(i,j)$ is represented by k, $r(i+1,j)$

and $r(i,j+1)$ are represented by τ, $\sigma(i,j)$ is represented by s.

The objective of this work is to devise an asynchronous SMC law $u(i,j)$, such that the 2D Markov jump system (1) is asymptotically mean square stable with an H_∞ disturbance attenuation performance γ.

III. Main Result

A. Sliding surface and sliding mode controller

In this paper, an asynchronous sliding surface function is constructed as follows:

$$s(i,j)=\begin{bmatrix} s^h(i,j) \\ s^v(i,j) \end{bmatrix}=Gx(i,j) \tag{9}$$

where $G=\sum_{k=1}^{N_1}\beta_k G_k^T$, and scalars β_k should be chosen such that GB_k is nonsingular for any $k\in\mathcal{N}_1$. Based on the the assumption that B_k is full column rank for any $k\in\mathcal{N}_1$, we can find that the above condition can be guaranteed easily with the properly selected parameter β_k.

An asynchronous 2D-SMC law is designed as follows:

$$u(i,j)=K_s x(i,j)-\rho(i,j)\frac{s(i,j)}{\|s(i,j)\|}, \tag{10}$$

for any $s\in\mathcal{N}_2$, where the matrix $K_s\in\mathbb{R}^{n_u\times n_x}$ with $n_x=n_h+n_v$ will be determined later, and the parameter $\rho(i,j)$ is given as

$$\rho(i,j)=\varrho_1\|x(i,j)\|+\varrho_2\|w(i,j)\| \tag{11}$$

with $\varrho_1=\max_{k\in\mathcal{N}_1}\{\delta_k\}$, $\varrho_2=\max_{k\in\mathcal{N}_1}\{\|(GB_k)^{-1}GE_k\|\}$, and the parameter δ_k is given in (2).

Combining system (1) and the asynchronous 2D-SMC low (9), the closed-loop 2D markov jump system can be obtained easily as follows:

$$\boldsymbol{x(i,j)}=\bar{A}_{ks}x(i,j)+B_k\bar{\rho}_k(i,j)+E_k w(i,j) \tag{12}$$

where $\bar{A}_{ks}=A_k+B_kK_s$, and $\bar{\rho}_k(i,j)$ as follows

$$\bar{\rho}_k(i,j)=f_k(x(i,j))-(\varrho_1\|x(i,j)\|+\varrho_2\|w(i,j)\|)\cdot\frac{s(i,j)}{\|s(i,j)\|}$$

Then, based on the properties of norm, the following condition can be deduced easily

$$\|\bar{\rho}_k(i,j)\|\le(\varrho_1+\delta_k)\|x(i,j)\|+\varrho_2\|w(i,j)\|. \tag{13}$$

B. Analysis of Stability and H_∞ attenuation performance

In this subsection, we focus on the stability and H_∞ attenuation performance analysis for the closed-loop 2D system (12). A sufficient condition will be derived to guarantee the considered system is asymptotically mean square stable with an H_∞ attenuation performance γ.

Theorem 1. *Consider the Markov jump system (1) under the Assumption (1) and with the asynchronous 2D-SMC low (10). For a given scalar $\gamma>0$, if there exist matrices $K_s\in\mathbb{R}^{n_u\times n_x}$, $R_k=\mathrm{diag}\{R_k^h,R_k^v\}>0$, $Q_{ks}>0$, $T_{ks}>0$ and scalars $\epsilon_k>0$, for any $k\in\mathcal{N}_1, s\in\mathcal{N}_2$, such that the following inequalities hold:*

$$B_k^T\mathcal{R}_kB_k-\epsilon_kI\le 0 \tag{14}$$

$$\mathcal{A}+2\Big(\sum_{s=0}^{N_2}\mu_{ks}\mathrm{diag}\{Q_{ks},T_{ks}\}\Big)\le 0 \tag{15}$$

$$\hat{A}_{ks}^T\mathcal{R}_k\hat{A}_{ks}-\mathrm{diag}\{Q_{ks},T_{ks}\}\le 0 \tag{16}$$

where

$$\mathcal{A}=\begin{bmatrix} \mathit{\Pi}_1 & \mathit{\Pi}_3 \\ * & \mathit{\Pi}_2 \end{bmatrix}$$

with

$$\begin{cases} \mathit{\Pi}_1=-R_k+4(\delta_k+\varrho_1)^2\epsilon_kI+C_k^TC_k \\ \mathit{\Pi}_2=-\gamma^2I+D_k^TD_k+4\varrho_2^2\epsilon_kI \\ \mathit{\Pi}_3=C_k^TD_k \end{cases}$$

and $\mathcal{R}_k=\sum_{\tau=1}^{N1}\lambda_{k\tau}R_\tau$, $\hat{A}_{ks}=\begin{bmatrix}\bar{A}_{ks} & E_k\end{bmatrix}$, then, the closed-loop system (12) is asymptotically mean square stable with an H_∞ disturbance attenuation performance γ.

Proof. Select the Lyapunov candidate as follows
Theroem proof. □

Remark 1. *Remark.*

C. l_2-gain minimization

subsection introduction.

Theorem 2. *Theroem.*

Proof. Proof of theorem. □

Remark 2. *Remark.*

IV. Numerical Example

In this section, we provide an example to verify the validity of the proposed method.

V. Conclusions

References

[1] Zhang, Guangming, et al. "Finite-time H static output control of Markov jump systems with an auxiliary approach." Applied Mathematics & Computation 273.C(2016):553-561.