Adaptive Actuator Failure Compensation Control of Second-order Nonlinear Systems with Unknown Time Delay

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***Abstract*—A robust adaptive control law is proposed based on backstepping technique for unknown actuator failure and unknown time delay in this paper. Seperate from existing results, the bounding function of time delay term $h_i(x_1, x_2)$ in system (1) depends only on all system states, which triangular structure is destroyed here. By maintaining a linear projection between the state vector (x_1, x_2) and its transformed vector (z_1, z_2) in controller design, triangular structure requirement of backstepping technology is no longer needed. Simulation results show that the designed controller can ensure the stability of the closed-loop system.**

***Index Terms*—Adaptive control, Time delay, Actuator failure, Backsteppting.**

I. Introduction

TIME delay exists in great number of practical systems, especially in mechanics, physics, biology and engineering systems and so on. With the problem of instability, even catastrophic accident, time delay are inevitably to be considered in the design and anal of control systems. In recent twenty years, many schemes have been proposed to solve this problem. For example, see [1]-[7], including coupled neural networks [1], uncertain neural systems [2], Markovian jump neural networks [3], and csome ways trying to compensate the effect of time delays in discrete-time neural networks [4]-[5]. For uncertain nonlinear systems, especially parametric strict feedback systems, the existence of unknown time delay make the controller design and stability analysis become difficult. The triangular time delay term is considered in [6]-[7] and the control laws have been proposed by using backstepping technology.

Meanwhile, some common cases such as actuator failures [8]-[10], fading [11], uncertain kinematics and dynamics to robotic systems [12] and electronic circuits systems [13], dead-zone [14]-[17], input and sensor nonlinearities [18]-[20], backlash [21], state constraint [22] of practical systems have great effect on the performance of the controlled system. Actuator failures generally occurec in all practical systems with uncertainty at the same time, both with value and pattern. Therefore, the uncertainties caused by unknown failures are difficult to compensate. Especially, the existence of time delays renders such problem much more complex. In this paper, we address such a problem by considering controlling a second-order nonlinear systems with time delays. Different from [6]-[7], time delays considered in this paper do not need triangular structure condition. Hence standard backstepping technology [23] is not used in the controller design. In order to deal with the situation, the effects of such time delays are not considered in every step, but accumulated to the last step, and compensated by choosing appropriate design parameters in control laws. Simultaneously, the existence of unknown actuator failures make the controller design and stability analysis become more difficult. Consequently, the compositive effects caused by time delays and unknown failures must be compensated by the same controller. To confront the problems, Online estimator for unknown actuator failures for time delay systerms has been designed. Based on maintaining a linear relation between the state vector (x_1, x_2) and its transformed vector (z_1, z_2), the uncertainties of actuator failures and unknown time delays have been transfered easy to handle, and the stability of systems is protected. Simulations show the effectiveness.

The main contributions of this paper, compared with the existing results, are as follows: (1) The control problem is explored for time delay systems with unknown actuator failures and unknown parameters. (2) In addition, the online estimator of unknown actuator failures for time delay systerms has been designed to compensate the compositive effects caused by unknown time delays and unknown actuator failures. (3) Moreover, unlike existing results of time delay systems by using backstepping technology triangular structure requirement is no longer needed in this paper. Namely, the bounding function of time delay term $h_i(x_1, x_2)$ in system (1) is allowed to depend on all system states and then triangular structure is destroyed.

The rest of the paper is organized as follows. In section 2, we formulate the second order time delay system by differential equations and give the model of unknown actuator failures. Adaptive control scheme and stability analyses of the closed-loop system are given in section 3. In section 4, some simulation results are shown in detail. Finally, the paper is concluded in section 5.

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II. Models and problem statement

To illustrate our design ideas, following class of second-order nonlinear systems with uncertain parameters and unknown time delay is considered. And The system model is given by

$$\begin{aligned} &\dot{x}_1 = x_2 + h_1(x_1(t-\tau), x_2(t-\tau)) \\ &\dot{x}_2 = \theta^T f(x) + \sum_{i=1}^{m} b_i u_i + h_2(x_1(t-\tau), x_2(t-\tau)) \\ &y = x_1 \end{aligned} \tag{1}$$

where $x = (x_1, x_2)^T$ is system state and $u_i \in R$ $(i = 1, 2\cdots, m)$ is input, $y = x_1$ is output. $f(x) \in R^p$ is known function and $b_i \in R$ $(i = 1, 2\cdots, m)$, $\theta \in R^p$ are unknown parameters. $h_1(\cdot)$, $h_2(\cdot) \in R$ are known smooth functions and $\tau \in R$ is unknown time delay of state x.

We now consider the ith actuator may fail during its operation. As in [8] and [9], the failure of the ith actuator at time instant t_{if} can be modelled as follows

$$\begin{aligned} &u_i = \rho_i u_{ci} + u_{ki}, \quad (\forall t \geq t_{if}) \\ &\rho_i u_{ki} = 0 \end{aligned} \tag{2}$$

where u_{ci} is the input of the ith actuator and $\rho_i \in [0, 1]$, u_{ki} and t_{if} are unknown constant. The following three cases are discussed in this paper:

- $\rho_i = 1$,
  It indicates $u_i = u_{ci}$. The ith actuator works normally.
- $0 < \rho_i < 1$,
  It indicates $u_i = \rho_i u_{ci}$. The ith actuator is called partial loss of effectiveness.
- $\rho_i = 0$,
  It indicates $u_i = u_{ki}$. The ith actuator is called total loss of effectiveness.

The control objective is to design adaptive control scheme for the system (1) with unknown actuator failures given in (2) to guarantee all signals bounded. To design adaptive control scheme, the following Assumptions for system functions, unknown time delay and actuator failures are ordered.

Assumption 1: Nonlinear functions $h_i(x_1, x_2)$ $\;i = 1, 2$ satisfy following condition

$$|h_i(x_1, x_2)| \leq \delta_i ||(x_1, x_2)||_2 \quad (i = 1, 2) \tag{3}$$

where $\delta_i > 0$ is a known constant.

Remark 1: In the context of adaptive control of strict-feedback systems with state delays by using backstepping techniques, the time delay term in the ith state differential equation is required to only depend on states $x_1, \cdots, x_i$, for examples [6] and [7]. Such a requirement can keep the control system has triangular structure which is needed by backstepping. However according to Assumption 1, the bounding function of time delay term $h_i(x_1, x_2)$ in system (1) can depend all system states. Thus it means that such triangular structure requirement is no longer needed in this paper.

Remark 2: δ_i can be interpreted as the gain or strength of the time delay term. In this paper, we will discuss how to obtain the adaptive controller of system (1). The existence of time delay and unknown actuator failures given in (2) render the controller design much more complex and difficult, therefore the result given in this paper will be aimed at enough small δ_i. It makes our results limited.

Assumption 2: The number of total fail actuators is up to $m - 1$ and the control objectives can be achieved by other normal actuators. Also any actuator can change only from normal to partial failure or total failure once.

Remark 3: Assumption 2 is a basic assumption required in adaptive failure compensation. It can be concluded that total number of actuator failures is finite and there exists a finite time instant T_f after which no new failure will occur.

Assumption 3: Unknown parameter $b_i \neq 0$ and the sign of b_i is known.

With actuator failure model given in (2), system (1) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2 + h_1(x_1(t-\tau), x_2(t-\tau)) \\ \dot{x}_2 &= \theta^T f(x) + \sum_{i=1}^{m} b_i(\rho_i u_{ci} + u_{ki}) \\ &\quad + h_2(x_1(t-\tau), x_2(t-\tau)) \\ y &= x_1 \end{aligned} \tag{4}$$

III. Design and analysis of adaptive controllers

To obtain available control law and update laws based on the backstepping approach, the following projection are introduced.

$$\begin{aligned} z_1 &= y = x_1 \\ z_2 &= x_2 - \alpha_1 \end{aligned} \tag{5}$$

where α_1 is the virtual control in step 1. Below we will give the design details following the recursive backstepping procedure.

Step 1: From (1) and (5) the derivative of z_1 can be rewritten as

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 \\ &= x_2 + h_1(x_1(t-\tau), x_2(t-\tau)) \\ &= z_2 + \alpha_1 + h_1(x_1(t-\tau), x_2(t-\tau)) \end{aligned} \tag{6}$$

We define a positive definite Lyapunov function as follows

$$\bar{V}_1 = \frac{1}{2} z_1^2 + \frac{e_1}{2} \int_{t-\tau}^{t} U_1(x_1(s), x_2(s)) ds \tag{7}$$

where $e_1 > 0$ is a design parameter. Smooth function $U_1(x_1(s), x_2(s))$ is chosen as

$$U_1(x_1(s), x_2(s)) = h_1^2(x_1(s), x_2(s)) \tag{8}$$

Then we can get the derivative of $\bar{V}_1$ is

$$\begin{aligned} \dot{\bar{V}}_1 &= z_1 \dot{z}_1 + \frac{e_1}{2} U_1(x_1(s), x_2(s)) \Big|_{t-\tau}^{t} \\ &= z_1(z_2 + \alpha_1 + h_1(x_1(t-\tau), x_2(t-\tau))) \\ &\quad + \frac{e_1}{2}(h_1^2(x_1(t), x_2(t)) - h_1^2(x_1(t-\tau), x_2(t-\tau))) \end{aligned} \tag{9}$$

Note that

$$z_1 h_1(x_1(t-\tau), x_2(t-\tau)) \leq \frac{z_1^2}{2e_1} + \frac{e_1}{2} h_1^2(x_1(t-\tau), x_2(t-\tau)) \tag{10}$$

So we have

$$\dot{V}_1 \leq z_1(z_2+\alpha_1)+\frac{z_1^2}{2e_1}+\frac{e_1}{2}h_1^2(x_1(t),x_2(t)) \tag{11}$$

Virtual control a_1 can be chosen as

$$\alpha_1=-k_1z_1-\frac{1}{2e_1}z_1 \tag{12}$$

where k_1 is a positive design parameter. Then from (11) and (12) we have

$$\begin{aligned}\dot{V}_1 &\leq z_1(z_2-k_1z_1-\frac{1}{2e_1}z_1)+\frac{z_1^2}{2e_1}+\frac{e_1}{2}h_1^2(x_1(t),x_2(t))\\ &= z_1z_2-k_1z_1^2+\frac{e_1}{2}h_1^2(x_1(t),x_2(t)) \quad (13)\end{aligned}$$

Step 2: From (1) and (5), we have

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2-\dot{\alpha}_1 \quad (14)\\ &= \theta^Tf(x)+\sum_{i=1}^m b_iu_i+h_2(x_1(t-\tau),x_2(t-\tau))-\dot{\alpha}_1\end{aligned}$$

Note that

$$\begin{aligned}\dot{\alpha}_1 &= \frac{d\alpha_1}{dx_1}\dot{x}_1 \quad (15)\\ &= -(k_1+\frac{1}{2e_1})(x_2+h_1(x_1(t-\tau),x_2(t-\tau)))\end{aligned}$$

Then

$$\begin{aligned}\dot{z}_2 &= \theta^Tf(x)+\sum_{i=1}^m b_iu_i+h_2(x_1(t-\tau),x_2(t-\tau))\\ &\quad+(k_1+\frac{1}{2e_1})(x_2+h_1(x_1(t-\tau),x_2(t-\tau))) \quad (16)\end{aligned}$$

From (1) and (5)

$$\begin{aligned}z_2\dot{z}_2 &= z_2(\theta^Tf(x)+\sum_{i=1}^m b_iu_i)\\ &\quad+z_2h_2(x_1(t-\tau),x_2(t-\tau)) \quad (17)\\ &\quad+(k_1+\frac{1}{2e_1})z_2(x_2+h_1(x_1(t-\tau),x_2(t-\tau)))\end{aligned}$$

Where

$$\begin{aligned}&z_2h_2(x_1(t-\tau),x_2(t-\tau))\\ &\leq \frac{z_2^2}{2e_{22}}+\frac{e_{22}}{2}h_2^2(x_1(t-\tau),x_2(t-\tau))\\ &z_2h_1(x_1(t-\tau),x_2(t-\tau))\\ &\leq \frac{z_2^2}{2e_{21}}+\frac{e_{21}}{2}h_1^2(x_1(t-\tau),x_2(t-\tau)) \quad (18)\end{aligned}$$

where e_{21} and e_{22} are positive design parameters. Then we have

$$\begin{aligned}z_2\dot{z}_2 &\leq z_2(\theta^Tf(x)+\sum_{i=1}^m b_iu_i)+\frac{z_2^2}{2e_{22}}+\frac{z_2^2}{2e_{21}}\\ &\quad+(k_1+\frac{1}{2e_1})z_2x_2+\frac{e_{22}}{2}h_2^2(x_1(t-\tau),x_2(t-\tau))\\ &\quad+\frac{e_{21}}{2}(k_1+\frac{1}{2e_1})h_1^2(x_1(t-\tau),x_2(t-\tau)) \quad (19)\end{aligned}$$

Different from the standard backstepping approach, a virtual control α in this step is designed as follows

$$\begin{aligned}\alpha &= -z_1-k_2z_2-\hat{\theta}^Tf(x)-(k_1+\frac{1}{2e_1})(\frac{1}{2e_{21}}z_2+x_2)\\ &\quad-\frac{1}{2e_{22}}z_2 \quad (20)\end{aligned}$$

where k_2 is a positive design parameter and $\hat{\theta}$ is the estimation of θ. Similar to [8] and [9], the control law and update laws are designed as follows

Control Law:

$$u_{ci}=sign(b_i)\hat{\varrho}^T\gamma \tag{21}$$

where $\hat{\varrho}$ is the estimation of ϱ. ϱ is an unknown parameter and γ is a known function. Both are $m+1$ dimensional vectors and can be denoted as

$$\varrho=(\varrho_1,\varrho_{21},\ldots,\varrho_{2m})^T, \gamma=(\gamma_1,\gamma_{21},\ldots,\gamma_{2m})^T \tag{22}$$

Update Laws:

$$\begin{aligned}\dot{\hat{\theta}} &= \Gamma_\theta f(x)z_2\\ \dot{\hat{\varrho}} &= -\Gamma_\varrho \gamma z_2 \quad (23)\end{aligned}$$

where $\Gamma_\theta, \Gamma_\varrho$ are positive definite matrices.

IV. Stability analysis

We now analyze the stability of closed loop system with control law and update laws in (21) and (23). We use T_j to represent the time instant at which the actuator failure occurred. Therefore, all actuators working condition do not change between two adjacent time point T_i and T_{i+1}. Namely, no new normal actuator fails in time interval (T_i,T_{i+1}). Let the set U_{iT} denotes the actuators of total failure in interval (T_i,T_{i+1}). Clearly, with Assumption 2 there exists a time instant T_f such that no new failure will occur after it. Let $T_0=0$, $T_{f+1}=\infty$, so we can get $U_{0T}=\{1,2,\cdots,m\}$ and U_{fT} is not empty. Our mail result is given as follows

Theorem 1: Consider the time delay system (1) with unknown actuator failures described by (2) with control law (21) and the update laws (23). Under Assumption 1 to Assumption 3, there exist a positive constant δ^* such that for all $\delta_i<\delta^*$ All the signals in the closed-loop system are globally bounded. Also output y satisfies

$$lim_{t\to\infty}|y|=0 \tag{24}$$

Proof. Firstly, we consider the following Lyapunov function in time interval $[T_i,T_{i+1})$

$$\begin{aligned}V_i &= \bar{V}_1+\frac{1}{2}z_2^2+\frac{1}{2}\tilde{\theta}^T\Gamma_\theta^{-1}\tilde{\theta}+\sum_{i\in\bar{U}_{it}}\frac{\rho_i|b_i|}{2}\tilde{\varrho}^T\Gamma_\varrho^{-1}\tilde{\varrho}\\ &\quad+\frac{e_{21}}{2}(k_1+\frac{1}{2e_1})\int_{t-\tau}^t U_{21}(x_1(s),x_2(s))ds\\ &\quad+\frac{e_{22}}{2}\int_{t-\tau}^t U_{22}(x_1(s),x_2(s))ds \quad (25)\end{aligned}$$

where $\tilde{\varrho}=\hat{\varrho}-\varrho$ and $\tilde{\theta}=\hat{\theta}-\theta$ are estimation errors of ϱ and θ. Let $U_{21}(x_1(s),x_2(s))=h_1^2(x_1(s),x_2(s))$ and

$U_{22}(x_1(s), x_2(s)) = h_2^2(x_1(s), x_2(s))$. Then the derivative of V_i is

$$\begin{aligned}\dot{V}_i &= \dot{\bar{V}}_1 + z_2\dot{z}_2 + \tilde{\theta}^T\Gamma_\theta^{-1}\dot{\hat{\theta}} + \sum_{i\in\bar{U}_{it}}\rho_i|b_i|\tilde{\varrho}^T\Gamma_\varrho^{-1}\dot{\hat{\varrho}} \\ &+\frac{e_{21}}{2}(k_1+\frac{1}{2e_1})(h_1^2(t)-h_1^2(t-\tau)) \\ &+\frac{e_{22}}{2}(h_2^2(t)-h_2^2(t-\tau))\end{aligned} \tag{26}$$

where we use $h_i^2(t)$ and $h_i^2(t-\tau)$ represent $h_i^2(x_1(t), x_2(t))$ and $h_i^2(x_1(t-\tau), x_2(t-\tau))$, respectively. With (13) and (19) we have

$$\begin{aligned}\dot{V}_i &\leq z_1z_2 - k_1z_1^2 + \frac{e_1}{2}h_1^2(t) + z_2(\sum_{i=1}^m b_i(\rho_i u_{ci} + u_{ki}) \\ &+\theta^T f(x) + (k_1+\frac{1}{2e_1})x_2) + \frac{z_2^2}{2e_{22}} + (k_1+\frac{1}{2e_1}) \\ &\frac{z_2^2}{2e_{21}} + \tilde{\theta}^T\Gamma_\theta^{-1}\dot{\hat{\theta}} + \sum_{i\in\bar{U}_{it}}\rho_i|b_i|\tilde{\varrho}^T\Gamma_\varrho^{-1}\dot{\hat{\varrho}} \\ &+\frac{e_{21}}{2}(k_1+\frac{1}{2e_1})h_1^2(t) + \frac{e_{22}}{2}h_2^2(t)\end{aligned} \tag{27}$$

Take ϱ as

$$\begin{aligned}&\varrho_1 = \frac{1}{\sum_{i\in\bar{U}_{iT}}\rho_i|b_i|} \\ &\varrho_{2i} = 0, (i\in\bar{U}_{iT}) \\ &\varrho_{2i} = -\frac{b_i u_{ki}}{\sum_{i\in\bar{U}_{iT}}\rho_i|b_i|}(i\in U_{iT})\end{aligned} \tag{28}$$

and γ

$$\gamma_1 = \alpha; \quad \gamma_{2i} = 1 \tag{29}$$

Clearly, we have

$$\begin{aligned}&\sum_{i=1}^m \rho_i b_i sign(b_i)\varrho^T\gamma \\ &= \sum_{i=1}^m \rho_i|b_i|\Big(\frac{1}{\sum_{i\in\bar{U}_{iT}}\rho_i|b_i|}\alpha + \sum_{i\in U_{iT}}(-\frac{b_iu_{ki}}{\sum_{i\in\bar{U}_{iT}}\rho_i|b_i|})\Big) \\ &= \sum_{i\in\bar{U}_{iT}} \rho_i|b_i|\Big(\frac{1}{\sum_{i\in\bar{U}_{iT}}\rho_i|b_i|}\alpha + \sum_{i\in U_{iT}}(-\frac{b_iu_{ki}}{\sum_{i\in\bar{U}_{iT}}\rho_i|b_i|})\Big) \\ &= \alpha - \sum_{i\in U_T} b_iu_{ki}\end{aligned} \tag{30}$$

With control law in (21)

$$\begin{aligned}&\sum_{i=1}^m b_i(\rho_i u_{ci} + u_{ki}) = \sum_{i\in\bar{U}_{iT}} b_i\rho_i u_{ci} + \sum_{i\in U_{iT}} b_iu_{ki} \\ &= \sum_{i\in\bar{U}_{iT}} b_i\rho_i sign(b_i)\hat{\varrho}^T\gamma + \sum_{i\in U_{iT}} b_iu_{ki} \\ &= \sum_{i\in\bar{U}_{iT}} b_i\rho_i sign(b_i)(\tilde{\varrho}+\varrho)^T\gamma + \sum_{i\in U_{iT}} b_iu_{ki} \\ &= \sum_{i\in\bar{U}_{iT}} b_i\rho_i sign(b_i)(\tilde{\varrho}+\varrho)^T\gamma + \sum_{i\in U_{iT}} b_iu_{ki}\end{aligned} \tag{31}$$

From (30), we have

$$\sum_{i=1}^m b_i(\rho_i u_{ci} + u_{ki}) = \sum_{i\in\bar{U}_{iT}} |b_i|\rho_i\tilde{\varrho}^T\gamma + \alpha \tag{32}$$

Then the derivative of V_i can be rewritten as

$$\begin{aligned}\dot{V}_i &\leq z_1z_2 - k_1z_1^2 + \frac{e_1}{2}h_1^2(t) + z_2(\sum_{i\in\bar{U}_{iT}}|b_i|\rho_i\tilde{\varrho}^T\gamma + \alpha \\ &+\theta^T f(x) + (k_1+\frac{1}{2e_1})x_2) + \frac{z_2^2}{2e_{22}} + (k_1+\frac{1}{2e_1}) \\ &\frac{z_2^2}{2e_{21}} + \tilde{\theta}^T\Gamma_\theta^{-1}\dot{\hat{\theta}} + \sum_{i\in\bar{U}_{it}}\rho_i|b_i|\tilde{\varrho}^T\Gamma_\varrho^{-1}\dot{\hat{\varrho}} \\ &+\frac{e_{21}}{2}(k_1+\frac{1}{2e_1})h_1^2(t) + \frac{e_{22}}{2}h_2^2(t) \qquad (33) \\ &= -k_1z_1^2 - k_2z_2^2 + \sum_{i\in\bar{U}_{it}}\rho_i|b_i|\tilde{\varrho}^T\Gamma_\varrho^{-1}(\dot{\hat{\varrho}} + \Gamma_\varrho\gamma z_2) \\ &+\tilde{\theta}^T\Gamma_\theta^{-1}(\dot{\hat{\theta}} - \Gamma_\theta f(x)z_2) + \frac{e_{22}}{2}h_2^2(t) \\ &+(\frac{e_1}{2} + \frac{e_{21}}{2}(k_1+\frac{1}{2e_1}))h_1^2(t)\end{aligned}$$

With update laws given in (23), we can get

$$\begin{aligned}\dot{V}_i &\leq -k_1z_1^2 - k_2z_2^2 + \frac{e_{22}}{2}h_2^2(t) \\ &+(\frac{e_1}{2} + \frac{e_{21}}{2}(k_1+\frac{1}{2e_1}))h_1^2(t)\end{aligned} \tag{34}$$

From Assumption 1, we have

$$\begin{aligned}\dot{V}_i &\leq -\sum_{i=1,2}k_iz_i^2 + \frac{e_{22}}{2}\delta_2^2||x||_2^2 \\ &+(\frac{e_1}{2} + \frac{e_{21}}{2}(k_1+\frac{1}{2e_1}))\delta_1^2||x||_2^2\end{aligned} \tag{35}$$

From projection in (5) and virtual controls (12), we have

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -(k_1+\frac{1}{2e_1}) & 1 \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \tag{36}$$

The following inequality can be obtained

$$||x||_2 \leq (2 + (k_1+\frac{1}{2e_1})^2)^{\frac{1}{2}}||z||_2 \tag{37}$$

So inequality (35) can be rewritten as

$$\begin{aligned}\dot{V}_i \leq &-\sum_{i=1,2}k_iz_i^2 + (2+(k_1+\frac{1}{2e_1})^2) \\ &[(\frac{e_1}{2}+\frac{e_{21}}{2}(k_1+\frac{1}{2e_1}))\delta_1^2 + \frac{e_{22}}{2}\delta_2^2]||z||_2^2\end{aligned} \tag{38}$$

In order to stabilize the system, the following requirement on design parameters can be established

$$\begin{cases} k_1 \geq [(\frac{e_1}{2}+\frac{e_{21}}{2}(k_1+\frac{1}{2e_1}))\delta_1^2 + \frac{e_{22}}{2}\delta_2^2](2+(k_1+\frac{1}{2e_1})^2) \\ k_2 \geq [(\frac{e_1}{2}+\frac{e_{21}}{2}(k_1+\frac{1}{2e_1}))\delta_1^2 + \frac{e_{22}}{2}\delta_2^2](2+(k_1+\frac{1}{2e_1})^2) \end{cases} \tag{39}$$

Clearly, the second inequality is easy to be realized by choosing k_2 big enough. So the key to solve the problem relies on how to select k_2, such that the first inequality is satisfied. First about the existence of a positive constant δ^* such that for all $\delta_i < \delta^*$, design parameters k_1 can be choosen to make the first inequality hold. For a group of design parameters $k_1^*, e_1^*, e_{21}^*, e_{22}^*$, clearly there exists a δ^* making

$$\Phi^* = [(\frac{e_1^*}{2}+\frac{e_{21*}}{2}(k_1^*+\frac{1}{2e_1^*}))(\delta^*)^2 + \frac{e_{22}^*}{2}(\delta^*)^2](2+(k_1^*+\frac{1}{2e_1^*})^2) \tag{40}$$

small enough and then satisfying

$$k_1^* > \Phi^* \tag{41}$$

So for all $\delta_i < \delta^*$, we have

$$\begin{aligned}&[(\frac{e_1^*}{2} + \frac{e_{21^*}}{2}(k_1^* + \frac{1}{2e_1^*}))(\delta_1)^2 + \frac{e_{22}^*}{2}(\delta_2)^2] \\ &(2 + (k_1^* + \frac{1}{2e_1^*})^2) \leq \Phi^* < k_1^* \end{aligned}\tag{42}$$

Namely, for above δ^* when $\delta_i < \delta^*$ design parameters $k_1^*, e_1^*, e_{21}^*, e_{22}^*$ can make the first inequality of (39) hold. Letting

$$k^* \triangleq k_2^* = k_1^* \tag{43}$$

we can get

$$\dot{V}_i \leq -\ell \sum_{i=1}^{2} z_i^2 \tag{44}$$

where

$$\ell = k^* - \Phi^* > 0 \tag{45}$$

From (44), we know V_i is non increasing in time interval $[T_i, T_{i+1})$. So we can get $V_j(T_{i+1}^-) \leq V_i(T_i^+)$. When $i = 0$, it is $V_0(T_1^-) \leq V_0(0)$. It can be conclude that all signals z_i, $\tilde{\theta}$, $\tilde{\varrho}$ are bounded in the time interval $[0, T_1)$. Noting that the difference between $V_1(T_1^+)$ and $V_0(T_1^-)$ is only the coefficients in front of the term $\tilde{\varrho}^T \Gamma_\varrho \tilde{\varrho}$. Since all the possible jumping on ϱ are bounded, $V_1(T_1^+)$ is bounded, then $V_1(T_2^-)$ is bounded. Similar to the above analysis, we can get $V_i(T_{i+1}^-)$ is bounded from the bound of $V_i(T_i^+)$ and $V_{i-1}(T_i^-)$. Also we can get $V_f(T_f^+)$ is bounded by the bound of $V_{f-1}(T_f^-)$ and $V_{f-1}(T_{f-1}^+)$. Therefore, V_f is non increasing in time interval (T_f, ∞). Namely, it can be shown that $V_f(t) \leq V_f(T_f^+)$. Then we can get $z, \tilde{\theta}, \tilde{\varrho}$ are all bounded in $[0, \infty]$. Easily, we can get $lim_{t\to\infty}|y| = 0$ from (44). □□

Remark 4:

- In order to prove the derivative of V_i being non-positive in time interval $[T_i, T_{i+1})$, linear transformation between vector x and z is established in the controller design. Such analysis is quite different from standard backstepping procedures.
- The value of parameter δ^* is difficult to obtain for a general uncertain nonlinear system, especially for high order nonlinear systems. If so, the significance of our results is to show the existence of such a parameter. In other words, when magnitude δ_i of time delay term $h_i(\cdot)$ satisfying the condition that $\delta_i < \delta^*$, the controller shown in this paper can stabilize system (1).

V. Simulation Studies

In this section, the results of simulation are presented to verify the effectiveness of the proposed robust adaptive neural network tracking control law with hysteresis input. Following second-order system as considered:

$$\begin{aligned}\dot{x}_1 &= x_2 + h_1(x_1(t-\tau), x_2(t-\tau)) \\ \dot{x}_2 &= (2 + cos(x_1 x_2))\theta + u + h_2(x_1(t-\tau), x_2(t-\tau)) \\ y &= x_1 \end{aligned}\tag{46}$$

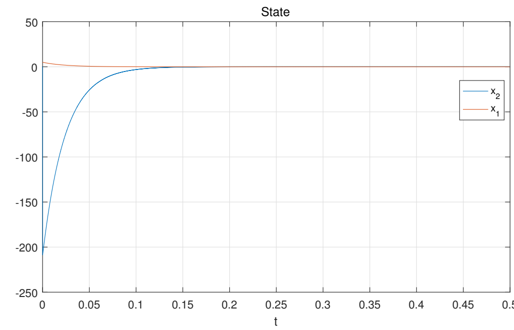

Fig. 1. States $x_1(t)$ and $x_2(t)$

where x_1, x_2 are system states and u is the input. θ is an unknown parameter and $h_1(x_1(t-\tau), x_2(t-\tau))$, $h_2(x_1(t-\tau), x_2(t-\tau))$ are known smooth functions and $\tau \in R$ is unknown time delay of state x. Consider h_1, h_2 as

$$h_1(t-\tau) = 0.05(sinx_1(t-\tau) + cosx_2(t-\tau)) \tag{47}$$

$$h_2(t-\tau) = 0.04(cosx_1(t-\tau) + sinx_2(t-\tau)) \tag{48}$$

Such that we have

$$h_1(t-\tau) < 0.1 \tag{49}$$

$$h_2(t-\tau) < 0.1 \tag{50}$$

Satisfy Assumption 1. In the simulation, following parameters are choosen: $theta = 0.8$, $\tau = 0.2$, $e_1 = 0.1$, $e_{21} = 0.04$, $e_{22} = 0.3$. $\delta_1 = \delta_2 = 0.1$. Then we choose $k_1 = k_2 = 40$ such that equation group (39) holds. Γ_θ and Γ_ϱ are update law gain, in order to make simulation faster, we could choose them as big as possible. Fig.1 shows the states x_1 and x_2 with time.

Remember $y = x_1(t)$, $x_1(t)$ and $x_2(t)$ tend to 0 with time. Therefore verify Theorem 1.

VI. Conclusion

A robust adaptive state feedback control scheme is proposed by using backstepping technique for second oreder time delay systems with unknown actuator failures. unlike existing results of time delay systems by using backstepping technology, the bounding function of time delay term $h_i(x_1, x_2)$ in system (1) is allowed to depend on all system states. The boundedness of all signals of the closed-loop system are ensured by the proposed control law and corresponding update laws. Simulation studies verify the effectiveness of the proposed control scheme.

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