Asychronous Control for Markov Jump Lure's Systems With Control Saturation

***Abstract*—The abstract here.**

I. Introduction

II. Preliminaries

Consider a class of discrete-time MJLS with control saturation on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{cases} x_{k+1} = A_{r(k)}x_k + F_{r(k)}\varphi(y_k) + B_{r(k)}\mathrm{sat}(u_k) \\ \qquad + E^x_{r(k)}w_k, \quad y_k = C_{r(k)}x_k, \\ z_k = C^z_{r(k)}x_k + G^z_{r(k)}\varphi(y_k) + D^z_{r(k)}\mathrm{sat}(u_k) \\ \qquad + E^z_{r(k)}w_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n, u_k \in \mathbb{R}^m, y_k \in \mathbb{R}^p, z_k \in \mathbb{R}^q$, and $w_k \in \mathbb{R}^r$ represent the state, the control input, the output related to the nonlinearity, the controlled output and the exogenous disturbance vector respectively. $A_{r(k)}, F_{r(k)}, B_{r(k)}, E^x_{r(k)}, C^z_{r(k)}, G^z_{r(k)}, D^z_{r(k)}$ and $E^z_{r(k)}$ represent the time-varying system matrices, all of which are pre-known and real. The saturation function $\mathrm{sat}(\cdot)$ is defined as follows:

$$\mathrm{sat}(u)_{(\ell)} = \mathrm{sign}(u_{(\ell)})\min(\rho_{(\ell)}, |u_{(\ell)}|), \tag{2}$$

where $\ell \in \{1, \cdots, m\}$ represent the ℓth element of the vector and the pre-given vector $\rho > 0$. $\{r(k), k \geq 0\}$ is a Markov chain taking values in a positive integer set $\mathcal{N} = \{1, 2, \ldots, N\}$ with mode TPs:

$$\Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij} \tag{3}$$

Clearly, for all $i, j \in \mathcal{N}$, $\pi_{ij} \in [0, 1]$, and $\sum_{j=0}^{N}\pi_{ij} = 1$ for each mode i. The system matrices in (1) at instant k can be expressed as $A_i, F_i, B_i, E^x_i, C^z_i, G^z_i, D^z_i$ and E^z_i, which are real known constant matrices with appropriate dimensions, and the related transition probability matrix is described as $\Pi = \{\pi_{ij}\}$.

Assumption 1: The function $\varphi(\cdot) : \mathbb{R}^p \to \mathbb{R}^p$ satisfies:
(1) $\varphi(0) = 0$ and
(2) there exists a diagonal positive definite matrix $\Omega \in \mathbb{R}^{p\times p}$ such that:

$$\varphi_{(\ell)}(y)[\varphi(y) - \Omega y]_{(\ell)} \leq 0. \tag{4}$$

where for all $y \in \mathbb{R}^p$, $\ell \in \{1, \ldots, p\}$. And in this case we say that the nonlinearity $\varphi(\cdot)$ satisfies it's own cone bounded sector conditions and to be decentralized. According to (4) we have:

$$SC(y, \Lambda) := \varphi^{'}(y)\Lambda[\varphi(y) - \Omega y] \leq 0, \tag{5}$$

where Λ is any diagonal positive semidefinite matrix, and Ω is pre-given. From the Assumption 1, we get:

$$[\Omega y]_{(\ell)}[\varphi(y) - \Omega y]_{(\ell)} \leq 0, \tag{6}$$

which implies:

$$0 \leq \varphi^{'}(y)\Lambda\varphi(y) \leq \varphi^{'}(y)\Lambda\varphi(y) \leq y^{'}\Omega\Lambda\Omega y \tag{7}$$

Definition 1: System (1) is said to be locally stochastically stable if for $w(k) = 0$ and any initial condition $x_0 \in \mathcal{D}_0$, $r(0) \in \mathcal{N}$, the following formulation holds:

$$\|x\|_2^2 = \sum_{k=0}^{\infty}\mathbb{E}[\|x_k\|^2] < \infty. \tag{8}$$

Where in this case the set $\mathcal{D}_0 \subset \mathbb{R}^p$ is said to be the domain of stochastic stability of the origin.

Define $\mathcal{F}_k$ as the σ-field generated by the random variables x_k and $r(k)$. We define next the class of exogenous disturbances with bounded energy and the finite ℓ_2-induced norm.
Definition 2: For $\gamma > 0$, set $\mathcal{W}_\gamma$ is defined as follows:

$$\begin{aligned} \mathcal{W}_\gamma := \{ & w = \{w_k\}_{k\in\mathbb{N}}; w_k \in \mathbb{R}^m, k \in \mathbb{N}, w_k \text{ is} \\ & \mathcal{F}_k\text{-measureable, and } \|w\|_2^2 = \sum_{k=0}^{\infty}\mathbb{E}(\|w_k\|)^2 < \frac{1}{\gamma}\}. \end{aligned} \tag{9}$$

The finite ℓ_2-induced gain associated with the closed-loop system (1) with $x_0 = 0$ between the disturbance $w = \{w_k\}_{k\in\mathbb{N}}$ and controlled output $z = \{z_k\}_{k\in\mathbb{N}}$ is equal or less than $\sqrt{\varrho}$ if for every $w \in \mathcal{W}_\gamma$ then we have

$$\frac{1}{\varrho}\|z\|_2^2 = \frac{1}{\varrho}\sum_{k=0}^{\infty}\mathbb{E}\left[\|z_k\|^2\right] \leq \|w\|_2^2 = \sum_{k=0}^{\infty}\mathbb{E}\left[\|w_k\|^2\right]. \tag{10}$$

In this paper, we consider the following controller:

$$u_k = K_{\sigma(k)}x_k + \Gamma_{\sigma(k)}\varphi(y_k) \tag{11}$$

where $K_{\sigma(k)} \in \mathbb{N}^{m\times n}$ is a time-varying controller gain matrix, and $\Gamma_{\sigma(k)} \in \mathbb{N}^{m\times p}$ is a time-varying nonlinear output feedback gain matrix. The parameter $\{\sigma(k), k \geq 0\}$ takes values in another pre-given positive integer set, which is marked as $\mathcal{M} = \{1, 2, \ldots, M\}$ subject to the pre-known conditional probability matrix $\Phi = \{\mu_{im}\}$, the probabilities of which are defined by

$$\Pr\{\sigma(k) = m | r(k) = i\} = \mu_{im}. \tag{12}$$

Where for all $i \in \mathcal{N}, m \in \mathcal{M}, \mu_{im} \in [0,1]$, and $\sum_{m=1}^{M} \mu_{im} = 1$ for each $i \in \mathcal{N}$.

In this paper, we define the dead-zone nonlinearity $\delta(u)$ as follows:$\delta(u) = u - \mathrm{sat}(u)$, for any $u \in \mathbb{R}^m$. Combing the asynchronous controller (11) and system (1) we have the following closed system:

$$\begin{cases} x_{k+1} = (A_i + B_i K_m)x_k + (F_i + B_i \mathit{\Gamma}_m)\varphi(y_k) \\ \qquad\qquad - B\delta(u_k) + E_i^x w_k \\ z_k = (C_i^z + D_i^z K_m)x_k + (G_i^z + D_i^z \mathit{\Gamma}_m)\varphi(y_k) \\ \qquad - D_i^z \delta(u_k) + E_i^z w_k \end{cases} \tag{13}$$

We consider a generalized local sector condition $\delta(u) = u - \mathrm{sat}(u)$, and use ρ to denote the bounds of saturation function $\mathrm{sat}(u)$. For given matrices $H_m \in \mathbb{R}^{n+p}$, we define the set $\mathcal{S}(H_m, \rho) := \{\xi \in \mathbb{R}^{n_x+n_y}; -\rho \leq H_m\xi \leq \rho, \forall m \in \mathcal{M}\}$.
Lemma 3. Consider matrices $\widehat{K_m} = [K_m \ \mathit{\Gamma}_m] \in \mathbb{N}^{m\times(n+p)}$ and $\widehat{J_m} = [J_{1,m} \ J_{2,m}] \in \mathbb{N}^{m\times(n+p)}$. If $\widehat{x} = [x^{'} \ \varphi^{'}(C_i x)]^{'} \in \mathcal{S}(\widehat{K_m} - \widehat{J_m}, \rho)$, for $u = K_m x + \mathit{\Gamma}_m \varphi(C_i x)$ we have that the nonlinearity $\delta(u)$ satisfies the following sector condition for any diagonal positive definite matrix $T_m \in \mathbb{R}^{m\times m}$:

$$SC_u(i, m, u, x, T_m) := \delta^{'}(u) T_m^{-1} [\delta(u) - \widehat{J_m}\widehat{x}] \leq 0. \tag{14}$$

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REFERENCES

[1] H. Kopka and P. W. Daly, *A Guide to LaTeX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.