

Asynchronous Control for Markov Jump Lure's Systems With Control Saturation

Abstract—The abstract here.

I. INTRODUCTION

II. PRELIMINARIES

Consider a class of discrete-time MJLS on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{cases} x_{k+1} = A_{r(k)}x_k + F_{r(k)}\varphi(y_k) + B_{r(k)}u_k \\ \quad + E_{r(k)}^x w_k, \quad y_k = C_{r(k)}x_k, \\ z_k = C_{r(k)}^z x_k + G_{r(k)}^z \varphi(y_k) + D_{r(k)}^z u_k \\ \quad + E_{r(k)}^z w_k \end{cases} \quad (1)$$

where $x_k \in \mathbb{R}^{n_x}, u_k \in \mathbb{R}^{n_u}, y_k \in \mathbb{R}^{n_y}, z_k \in \mathbb{R}^{n_z}$, and $w_k \in \mathbb{R}^{n_w}$ represent the state, the control input, the output related to the nonlinearity, the controlled output and the exogenous disturbance vector respectively. $A_{r(k)}, F_{r(k)}, B_{r(k)}, E_{r(k)}^x, C_{r(k)}^z, G_{r(k)}^z, D_{r(k)}^z$ and $E_{r(k)}^z$ represent the time-varying system matrices, all of which are pre-known and real. $\{r(k), k \geq 0\}$ is a Markov chain taking values in a positive integer set $\mathcal{N} = \{1, 2, \dots, N\}$ with mode TPs:

$$\Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij} \quad (2)$$

Clearly, for all $i, j \in \mathcal{N}$, $\pi_{ij} \in [0, 1]$, and $\sum_{j=0}^N \pi_{ij} = 1$ for each mode i . The system matrices in (1) at instant k can be expressed as $A_i, F_i, B_i, E_i^x, C_i^z, G_i^z, D_i^z$ and E_i^z , which are real known constant matrices with appropriate dimensions, and the related transition probability matrix is described as $\Pi = \{\pi_{ij}\}$.

Assumption 1: The function $\varphi(\cdot) : \mathbb{R}^{n_y} \rightarrow \mathbb{R}^{n_y}$ satisfies:

- (1) $\varphi(0) = 0$ and
- (2) there exists a diagonal positive definite matrix $\Omega \in \mathbb{R}^{p \times p}$ such that:

$$\varphi_{(\ell)}(y)[\varphi(y) - \Omega y]_{(\ell)} \leq 0. \quad (3)$$

where for all $y \in \mathbb{R}^p$, $\ell \in \{1, \dots, p\}$. And in this case we say that the nonlinearity $\varphi(\cdot)$ satisfies it's own cone bounded sector conditions and to be decentralized. According to (3) we have:

$$SC(y, \Lambda) := \varphi'(y)\Lambda[\varphi(y) - \Omega y] \leq 0, \quad (4)$$

where Λ is any diagonal positive semidefinite matrix, and Ω is pre-given. From the Assumption 1, we get:

$$[\Omega y]_{(\ell)}[\varphi(y) - \Omega y]_{(\ell)} \leq 0, \quad (5)$$

which implies:

$$0 \leq \varphi'(y)\Lambda\varphi(y) \leq \varphi'(y)\Lambda\Omega y \leq y' \Omega \Lambda \Omega y \quad (6)$$

Definition 1: System (1) is said to be locally stochastically stable if for $w(k) = 0$ and any initial condition $x_0 \in \mathcal{D}_0$, $r_0 \in \mathcal{N}$, the following formulation holds:

$$\|x\|_2^2 = \sum_{k=0}^{\infty} \mathbb{E}[\|x_k\|^2] < \infty. \quad (7)$$

Where in this case the set $\mathcal{D}_0 \subset \mathbb{R}^p$ is said to be the domain of stochastic stability of the origin.

Define \mathcal{F}_k as the σ -field generated by the random variables x_k and $r(k)$. We define next the class of exogenous disturbances with bounded energy and the finite ℓ_2 -induced norm.

Definition 2: For $\zeta > 0$, set \mathcal{W}_ζ is defined as follows:

$$\begin{aligned} \mathcal{W}_\zeta &:= \{w = \{w_k\}_{k \in \mathbb{N}}; w_k \in \mathbb{R}^{n_w}, k \in \mathbb{N}, w_k \text{ is} \\ &\mathcal{F}_k\text{-measurable, and } \|w\|_2^2 = \sum_{k=0}^{\infty} \mathbb{E}(\|w_k\|)^2 < \zeta. \end{aligned} \quad (8)$$

The finite ℓ_2 -induced gain associated with the closed-loop system (1) with $x_0 = 0$ between the disturbance $w = \{w_k\}_{k \in \mathbb{N}}$ and controlled output $z = \{z_k\}_{k \in \mathbb{N}}$ is equal or less than γ if for every $w \in \mathcal{W}_\zeta$ then we have

$$\|z\|_2^2 = \sum_{k=0}^{\infty} \mathbb{E}[\|z_k\|^2] \leq \gamma^2 \|w\|_2^2 = \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}[\|w_k\|^2]. \quad (9)$$

In this paper, we consider the following controller:

$$u_k = K_{\sigma(k)}x_k + \Gamma_{\sigma(k)}\varphi(y_k) \quad (10)$$

where $K_{\sigma(k)} \in \mathbb{R}^{n_u \times n_x}$ is a time-varying controller gain matrix, and $\Gamma_{\sigma(k)} \in \mathbb{R}^{n_u \times n_y}$ is a time-varying nonlinear output feedback gain matrix. The parameter $\{\sigma(k), k \geq 0\}$ takes values in another pre-given positive integer set, which is marked as $\mathcal{M} = \{1, 2, \dots, M\}$ subject to the pre-known conditional probability matrix $\Phi = \{\mu_{im}\}$, the probabilities of which are defined by

$$\Pr\{\sigma(k) = \phi | r(k) = i\} = \mu_{i\phi}. \quad (11)$$

Where for all $i \in \mathcal{N}, \phi \in \mathcal{M}, \mu_{i\phi} \in [0, 1]$, and $\sum_{\phi=1}^M \mu_{i\phi} = 1$ for each $i \in \mathcal{N}$.

Combing the asynchronous controller (10) and system (1) we have the following closed system:

$$\begin{cases} x_{k+1} = \bar{A}_{i\theta}x_k + \bar{F}_{i\theta}\varphi_i(C_i x_k) + E_i^x w_k \\ z_k = \bar{C}_{i\theta}x_k + \bar{G}_{i\theta}\varphi_i(C_i x_k) + E_i^z w_k \end{cases} \quad (12)$$

Where, for $i \in \mathcal{N}, \phi \in \mathcal{M}$

$$\begin{aligned}\bar{A}_{i\phi} &= A_i + B_i K_\phi, & \bar{F}_{i\phi} &= F_i + B_i \Gamma_{i\phi} \\ \bar{C}_{i\phi}^z &= C_i^z + D_i^z K_\phi, & \bar{G}_{i\phi}^z &= G_i^z + D_i^z \Gamma_\phi\end{aligned}$$

III. MAIN RESULT

$$\begin{cases} x_{k+1} = \bar{A}_{i\theta} x_k + \bar{F}_{i\theta} \varphi_i(C_i x_k) + E_i^x w_k \\ z_k = \bar{C}_{i\theta} x_k + \bar{G}_{i\theta} \varphi_i(C_i x_k) + E_i^z w_k \end{cases}$$

System (12) with $x_0 = 0$ and $w_k = 0$ is stochastically stable, if for all $i \in \mathcal{N}$ and $\phi \in \mathcal{M}$, there exist positive definite matrices $\bar{P}_i \in \mathbb{R}^{n_x \times n_x}$, $R_{i\phi} \in \mathbb{R}^{(n_x+n_y) \times (n_x+n_y)}$, matrices $K_\phi \in \mathbb{R}^{n_u \times n_x}$, $\Gamma_\phi \in \mathbb{R}^{n_u \times n_y}$ and positive semidefinite matrix $T_i \in \mathbb{R}^{n_y}$ to ensure (13) and (14) hold.

$$\begin{bmatrix} -R_{i\theta} & \mathcal{H}_{i\phi} \\ * & \mathcal{P}_i \end{bmatrix} < 0 \quad (13)$$

$$\begin{bmatrix} \mathcal{S}_{i\phi} & \mathcal{N}_{i\phi} \\ * & \mathcal{L}_{i\phi} \end{bmatrix} < 0 \quad (14)$$

$$\begin{bmatrix} -R_{i\theta} & \sqrt{\pi_{i1}} \hat{A}_{i\theta}' & \cdots & \sqrt{\pi_{iN}} \hat{A}_{i\theta}' \\ \sqrt{\pi_{i1}} \hat{A}_{i\theta} & -\bar{P}_1 & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \sqrt{\pi_{iN}} \hat{A}_{i\theta} & 0 & 0 & -\bar{P}_N \end{bmatrix} < 0$$

$$\begin{bmatrix} \mathcal{S}_{i\phi} & \sqrt{u_{i1}} \mathcal{N}_{i1} & \cdots & \sqrt{u_{iM}} \mathcal{N}_{iM} \\ \sqrt{u_{i1}} \mathcal{N}_{i1} & L_{i1} & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \sqrt{u_{iM}} \mathcal{N}_{iM} & 0 & 0 & L_{iM} \end{bmatrix} < 0$$

where

$$\begin{aligned}\mathcal{H}_{i\phi} &= \begin{bmatrix} \sqrt{\pi_{i1}} \hat{A}_{i\phi} \\ \sqrt{\pi_{i2}} \hat{A}_{i\phi} \\ \vdots \\ \sqrt{\pi_{iN}} \hat{A}_{i\phi} \end{bmatrix}', \quad \mathcal{S}_{i\phi} = \begin{bmatrix} -\bar{P}_i & 0 \\ * & H_{i\phi} \end{bmatrix} \\ H_{i\theta} &= \begin{bmatrix} -I_{n_x} & C_i' \Omega_i T_i \\ * & -2T_i \end{bmatrix}, \quad \hat{A}_{i\theta} = [\bar{A}_{i\theta} \quad \bar{F}_{i\theta}] \\ \mathcal{P}_i &= \text{diag}\{-\bar{P}_1, -\bar{P}_2, \dots, -\bar{P}_N\} \\ \mathcal{N}_{i\phi} &= \begin{bmatrix} \sqrt{u_{i1}} \begin{bmatrix} \bar{P}_i & 0 \\ * & R_{i1} \end{bmatrix} & \cdots & \sqrt{u_{iM}} \begin{bmatrix} \bar{P}_i & 0 \\ * & R_{iM} \end{bmatrix} \end{bmatrix} \\ \mathcal{L}_{i\phi} &= \text{diag}\left\{ \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i1} \end{bmatrix}, \dots, \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{iM} \end{bmatrix} \right\}\end{aligned}$$

Proof: Construct a Lyapunov function in the form of (15).

$$V(k, x_k, r(k)) = x_k' P_{r(k)} x_k \quad (15)$$

Where $P_{r(k)} = \bar{P}_{r(k)}^{-1}$. Let $\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{V(k+1, x_{k+1}, r(k+1)) - V(k, x_k, i)\}$ and it is easy to find that

$$\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{x_{k+1}' X_i x_{k+1}\} - V(k, x_k, i) \quad (16)$$

Where $X_i = \sum_{j=1}^N \pi_{ij} P_j$. Based on system (12) we can further obtain

$$\begin{aligned}\mathbb{E}\{x_{k+1}' X_i x_{k+1}\} &= \sum_{\phi=1}^M u_{i\phi} \hat{x}_k' \hat{A}_{i\phi}' X_i \hat{A}_{i\phi} \hat{x}_k \\ &= \hat{x}_k' \left(\sum_{\phi=1}^M u_{i\phi} \hat{A}_{i\phi}' X_i \hat{A}_{i\phi} \right) \hat{x}_k\end{aligned} \quad (17)$$

Where $\hat{x}_k = (x_k', \varphi_i'(C_i x_k))'$. Which implies

$$\mathbb{E}\{\Delta V(k)\} = \hat{x}_k' \left(\sum_{\phi=1}^M u_{i\phi} \hat{A}_{i\phi}' X_i \hat{A}_{i\phi} \right) \hat{x}_k - V(k, x_k, i) \quad (18)$$

Denoting $h_{i\phi} = \text{diag}\{P_i, I_{n_x+n_y}, \{\text{diag}(I_{n_x}, R_{i\phi}^{-1})_{\phi=1}^M\}\}$ and using $h_{i\phi}$ to pre- and post-multiplying the inequality given in (14), respectively, then we can get

$$\begin{bmatrix} \hat{\mathcal{S}}_{i\phi} & \sqrt{u_{i1}} I & \cdots & \sqrt{u_{iM}} I \\ \sqrt{u_{i1}} I & L_{i1} & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \sqrt{u_{iM}} I & 0 & 0 & L_{iM} \end{bmatrix} < 0$$

Where

$$\hat{\mathcal{S}}_{i\phi} = \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix}, \quad L_{i\phi} = \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i\phi}^{-1} \end{bmatrix}$$

Applying Schur complement, we can get

$$\sum_{\phi=1}^M u_{i\phi} \begin{bmatrix} I & 0 \\ * & R_{i\phi} \end{bmatrix} + \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix} < 0 \quad (19)$$

(19) multiply $(x_k', \hat{x}_k')'$ on left and its transpose on the right. We get

$$\hat{x}_k' \sum_{\phi=1}^M u_{i\phi} R_{i\phi} \hat{x}_k - x_k' P_i x_k - 2SC(i, x_k, T_i) < 0 \quad (20)$$

Applying Schur complement to inequality (13). We can obtain

$$\hat{A}_{i\phi} X_i \hat{A}_{i\phi} < R_{i\phi} \quad (21)$$

Combining inequalities (20) and (21), it's easy to find that

$$\begin{aligned}\hat{x}_k' \sum_{\phi=1}^M u_{i\phi} (\hat{A}_{i\phi} X_i \hat{A}_{i\phi}) \hat{x}_k - x_k' P_i x_k \\ - 2SC(i, x_k, T_i) < 0\end{aligned} \quad (22)$$

Combining inequalities (18) and (22), it is easy to obtain that

$$\mathbb{E}\{\Delta V(k)\} - 2SC(i, x_k, T_i) < 0 \quad (23)$$

Furthermore, noticing the condition given in (4), we further have $\mathbb{E}\{\Delta V(k)\} < 0$, which completes the proof.

Theorem 2: For each mode $i \in \mathcal{N}$ and $\phi \in \mathcal{M}$, if there exist positive-definite matrices $\bar{P}_i \in \mathbb{R}^{n_x \times n_x}$ and $R_{i\phi} \in \mathbb{R}^{(n_x+n_y+n_w) \times (n_x+n_y+n_w)}$, matrices $K_\phi \in \mathbb{N}^{n_u \times n_x}$, $\Gamma_\phi \in \mathbb{R}^{n_u \times n_y}$, positive semi-definite matrix $T_i \in \mathbb{R}^{n_y \times n_y}$ and a positive scalar γ , such that the following LMIs are verified

$$\begin{bmatrix} -R_{i\theta} & \mathcal{H}_{i\phi} \\ * & \mathcal{P}_i \end{bmatrix} < 0 \quad (24)$$

$$\begin{bmatrix} \mathcal{S}_{i\phi} & \mathcal{N}_{i\phi} \\ * & \mathcal{L}_{i\phi} \end{bmatrix} < 0 \quad (25)$$

$$\begin{bmatrix} -R_{i\theta} & \sqrt{\pi_{i1}}\hat{A}_{i\theta} & \cdots & \sqrt{\pi_{iN}}\hat{A}_{i\theta} & \hat{A}_{i\phi}^z \\ \sqrt{\pi_{i1}}\hat{A}_{i\theta} & -\bar{P}_1 & 0 & 0 & 0 \\ \vdots & 0 & \ddots & 0 & 0 \\ \sqrt{\pi_{iN}}\hat{A}_{i\theta} & 0 & 0 & -\bar{P}_N & 0 \\ \hat{A}_{i\phi}^z & 0 & 0 & 0 & -I_{n_w} \end{bmatrix} < 0$$

Where

$$\begin{aligned} \mathcal{H}_{i\phi} &= \begin{bmatrix} \sqrt{\pi_{i1}}\hat{A}_{i\phi} \\ \sqrt{\pi_{i2}}\hat{A}_{i\phi} \\ \vdots \\ \sqrt{\pi_{iN}}\hat{A}_{i\phi} \\ \hat{A}_{i\phi}^z \end{bmatrix}, \quad \mathcal{S}_{i\phi} = \begin{bmatrix} -\bar{P}_i & 0 \\ * & H_{i\phi} \end{bmatrix} \\ \mathcal{P}_i &= \text{diag}\{-\bar{P}_1, -\bar{P}_2, \dots, -\bar{P}_N, -I_{n_w}\} \\ \hat{A}_{i\theta} &= [\bar{A}_{i\theta} \quad \bar{F}_{i\theta} \quad E_i^x], \quad \hat{A}_{i\theta}^z = [\bar{C}_{i\theta}^z \quad \bar{G}_{i\theta}^z \quad E_i^z] \\ \mathcal{N}_{i\phi} &= \begin{bmatrix} \sqrt{u_{i1}} \begin{bmatrix} \bar{P}_i & 0 \\ * & R_{i1} \end{bmatrix} & \cdots & \sqrt{u_{iM}} \begin{bmatrix} \bar{P}_i & 0 \\ * & R_{iM} \end{bmatrix} \end{bmatrix} \\ \mathcal{L}_{i\phi} &= \text{diag}\left\{ \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i1} \end{bmatrix}, \dots, \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{iM} \end{bmatrix} \right\} \\ H_{i\theta} &= \begin{bmatrix} -I_{n_x} & C_i' \Omega_i T_i & 0 \\ * & -2T_i & 0 \\ * & * & -\gamma^2 I_{n_w} \end{bmatrix} \end{aligned}$$

then system (12) with $x_0 = 0$ and $w \in \mathcal{W}_\gamma$ is stochastically stable and the l_2 -gain of the closed system is strictly less or equal to γ .

Proof: We first select (16) as the Lyapunov function and $P_{r(k)} = \bar{P}_{r(k)}^{-1}$. Similar to the proof of theorem 1, we can obtain that

$$\mathbb{E}\{\Delta V(k)\} = \hat{x}_k' \left(\sum_{\phi=1}^M u_{i\phi} \hat{A}_{i\phi}' X_i \hat{A}_{i\phi} \right) \hat{x}_k - V(k, x_k, i) \quad (26)$$

Where $\hat{x}_k = (x_k', \varphi_i'(C_i x_k), w_k')'$, $X_i = \sum_{j=1}^N \pi_{ij} P_j$. Denoting $h_{i\phi} = \text{diag}\{P_i, I_{n_x+n_y+n_w}, \{\text{diag}(I_{n_x}, R_{i\phi}^{-1})_{\phi=1}^M\}\}$

and using $h_{i\phi}$ to pre- and post-multiplying the inequality given in (25), respectively, then we can obtain

$$\begin{bmatrix} \hat{\mathcal{S}}_{i\phi} & \sqrt{u_{i1}}I & \cdots & \sqrt{u_{iM}}I \\ \sqrt{u_{i1}}I & L_{i1} & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \sqrt{u_{iM}}I & 0 & 0 & L_{iM} \end{bmatrix} < 0$$

Where

$$\hat{\mathcal{S}}_{i\phi} = \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix}, L_{i\phi} = \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i\phi}^{-1} \end{bmatrix}$$

Applying Schur complement, we can get

$$\sum_{\phi=1}^M u_{i\phi} \begin{bmatrix} I & 0 \\ * & R_{i\phi} \end{bmatrix} + \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix} < 0 \quad (27)$$

(27) multiply $(x_k', \hat{x}_k')'$ on left and its transpose on the right. We get

$$\hat{x}_k' \left(\sum_{\phi=1}^M u_{i\phi} R_{i\phi} \right) \hat{x}_k - x_k' P_i x_k - 2SC(i, x_k, T_i) - \gamma^2 w_k' w_k < 0 \quad (28)$$

Applying Schur complement to inequality (24). We can obtain

$$\hat{A}_{i\phi} X_i \hat{A}_{i\phi} + (\hat{A}_{i\phi}^z)' \hat{A}_{i\phi}^z < R_{i\phi} \quad (29)$$

Combining inequalities (28) and (29), it's easy to find that

$$\begin{aligned} \hat{x}_k' \left(\sum_{\phi=1}^M u_{i\phi} (\hat{A}_{i\phi} X_i \hat{A}_{i\phi} + (\hat{A}_{i\phi}^z)' \hat{A}_{i\phi}^z) \right) \hat{x}_k - x_k' P_i x_k \\ - 2SC(i, x_k, T_i) - \gamma^2 w_k' w_k < 0 \end{aligned} \quad (30)$$

Which implies

$$\mathbb{E}\{\Delta V(k)\} + \mathbb{E}\{z_k' z_k\} - \gamma^2 w_k' w_k - 2SC(i, x_k, T_i) < 0 \quad (31)$$

Furthermore, noticing the condition given in (4), we further have $\mathbb{E}\{\Delta V(k)\} + \mathbb{E}\{z_k' z_k\} - \gamma^2 w_k' w_k < 0$, which means

$$\mathbb{E}\{\Delta V(k) + z_k' z_k - \gamma^2 w_k' w_k | x_k, r(k) = i\} < 0 \quad (32)$$

Summing up from $k = 0$ to $k = T$, and recalling that $x_0 = 0$, that $\mathbb{E}\{k+1, x_{k+1}, r(k+1)\} \geq 0$, we get that $\mathbb{E}\{\|z_k\|^2\}$. Noting the zero initial condition, we can conclude that $\frac{\|z\|_2}{\|w\|_2} < \gamma$ is satisfied, which completes the proof.

Remark: It's easy to find that, if (24) and (25) holds, the asynchronous controller can be designed based on Theorem 2. The optimal l_2 -gain performance γ^* can be obtained via dealing with the optimal problem as follows:

$$\min \quad \sigma \quad \text{subject to (24) and (25) with } \sigma = \gamma^2 \quad (33)$$

IV. NUMERICAL EXAMPLE

In this section, we provide an example to verify the validity of proposed methods. Consider the Markov jump Lur'e system (1) with following data: $\mathcal{N} = \{1, 2\}$, $\mathcal{M} = \{1, 2\}$

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.4 & 0.4 \\ 0.2 & 1 \end{bmatrix}, & A_2 &= \begin{bmatrix} 1.1 & 0.6 \\ 0.3 & 0.4 \end{bmatrix} \\ B_1 &= \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0.7 \\ 0.5 \end{bmatrix}, & F_1 &= \begin{bmatrix} 1 \\ 1.2 \end{bmatrix}, & F_2 &= \begin{bmatrix} 1.2 \\ 1 \end{bmatrix} \\ C_1 &= \begin{bmatrix} 0.9 \\ 0.5 \end{bmatrix}, & C_2 &= \begin{bmatrix} -1 \\ 0.7 \end{bmatrix}, & C_1^z &= \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, & C_1^z &= \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \\ G_1^z &= 0.3, & G_2^z &= 0.4, & D_1^z &= 0, & D_2^z &= 0.3; \\ E_1^z &= 1.3, & E_2^z &= -0.8, & E_1^x &= E_2^x = \begin{bmatrix} 1 & 0.5 \end{bmatrix}' \\ \Omega_1 &= 1.3, & \varphi_1(y) &= 0.5\Omega_1 y(1 + \cos(25y)) \\ \Omega_2 &= 1.5, & \varphi_2(y) &= 0.5\Omega_2 y(1 - \sin(20y)) \\ \Pi &= \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} & \Phi &= \begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \end{bmatrix} \end{aligned}$$

By solving the l_2 -gain optimization problem, it is easy to find that $\gamma^* = 2.41$ and following asynchronous controller gain matrices can be obtained.

$$\begin{aligned} K_1 &= \begin{bmatrix} -1.0266 & -1.2008 \end{bmatrix}, & \Gamma_1 &= -2.4363 \\ K_2 &= \begin{bmatrix} -1.1812 & -0.9191 \end{bmatrix}, & \Gamma_2 &= -2.2642 \end{aligned}$$

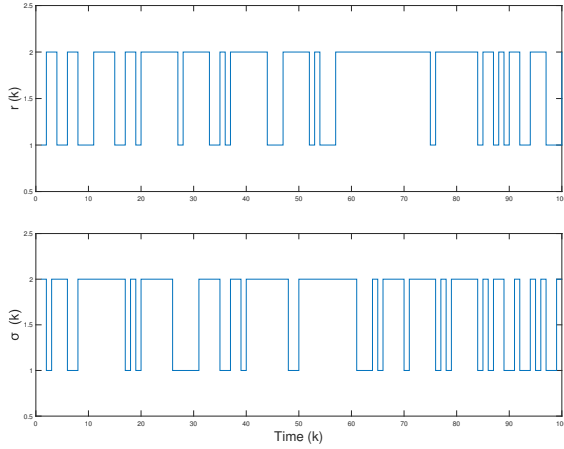


Fig. 1. Modes of system and controller

Fig.1 gives a possible time sequences of the mode of system and asynchronous controller. We select the initial state $x_0 = \begin{bmatrix} 2 & -2.5 \end{bmatrix}'$, and the external disturbance is assumed to be $w_k = \sin(k) * 0.9^k$. Then we can get state response of system, which is depicted in Fig.2.

V. CONCLUSIONS

The stochastic stability analysis and l_2 -gain optimization problems for a class of discrete-time nonlinear Markov systems with asynchronous controller have been discussed in this paper.

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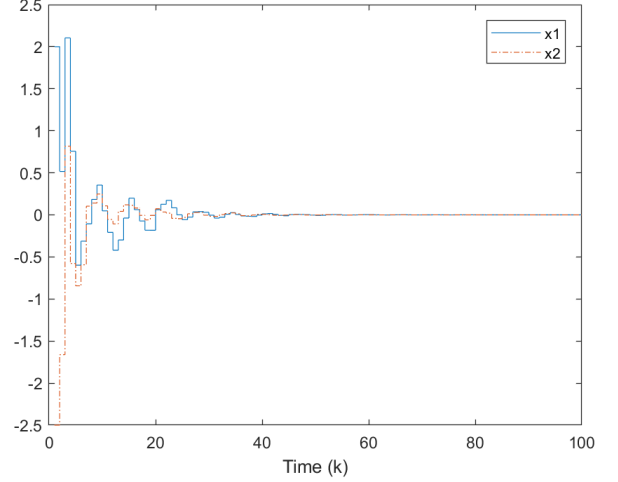


Fig. 2. System state

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