Asychronous Control for Markov Jump Lure's Systems

***Abstract*—This paper is concerned with the control problems for a class of discrete-time Lur'e systems with asynchronous controller. A hidden Markov model (HHM) is introduced to describe the asynchronous phenomenon between the systems mode and controller modes. The linear matrix inequality (LMI) approach is utilized to analyze the stability of the closed-loop system and l_2-gain performance. Then a sufficient condition is proposed in the form of LMI to guarantee the stochastic stability of the closed-loop system, and to minimize the obtained l_2-gain from the disturbance to output, further more the asynchronous controller which contains a linear state feedback and a cone-bounded nonlinear output feedback can be designed by solving the given conditions. A simulation example is given to demonstrate the effectiveness of the proposed method.**

I. Introduction

As an important class of stochastic switching systems, Markov jump systems (MJSs) have received considerable attention because of its powerful ability in modeling systems with abrupt change in parameters or system structures, for instance, environmental disturbances, actuators failures, and variations in subsystems interconnections. Over the past few decades, a large number of works for stability analysis and the controller/filter design have been published[1234]. For example,

In most existing works, it was implicitly assumed that the information of system modes can be fully accessible for the controller/filter all the time, so that controller/filter modes can run synchronously with system modes. Unfortunately, in practical applications, it is rather difficult to satisfy this ideal assumption because of some unexpected factors, for instance, time delays, data dropouts and quantization in network control systems. In order to overcome this strict limitation, two research approach on control/filtering of MJSs were proposed, namely, mode-independent and asynchronous methods. In mode-independent methods[2][3], the controller modes is independent with plant modes, which means have no use of system information and may result in some conservatism. Therefore, asynchronous methods on control/filtering problems for MJSs gain more attention in recent years. For example, a piecewise homogeneous Markov chain was utilized to design an asynchronous l_2-l_∞ filter for discrete-time MJSs with randomly occurred sensor nonlinearities in [5]. After that, in [4], a new strategy based on HMM framework was adopted in passivity-based asynchronous control problems, which covers the synchronous situation. Very recently, many works on asynchronous control/filtering problem of MJSs have been published based on this powerful model. For instance, asynchronous filter design of discrete-time Markov Jump Systems was discussed in [7][9], the same issue was discussed in continuous-time domain in [8], and asynchronous l_2-l_∞ control problem for discrete-time Markovian jump linear systems with partly accessible controller mode information was addressed in [10]. On the other hand, nonlinearity is another common phenomenon universally exists in the practical control systems. Among many different descriptions of nonlinear system, a special one called Lur'e system[11], which is consisted of a linear part and a cone-bounded nonlinearity, has received a lot of attentions in recent years[some references]. Many excellent works have been published on stability analysis and controller design problems for Lur'e system. For example, the issue of absolute stabilization for Lur'e system with a sector bounded nonlinearity under control saturations was discussed in [12] and [13], respectively, in discrete-time and continues-time domain. The flexibility was increased due to a mode-dependent controller which consisted of a linear state-feedback and a cone-bound nonlinear output feedback was introduced, and was also universally adopted in many later works. In [14], stability and l_2-gain problem was discussed for a class of discrete-time nonlinear MJSs with actuator saturation and incomplete knowledge of transition probabilities by construct a stochastic quadratic candidate Lyapunov function. Almost at the same time, a new class of Lyapunov functions including the cone-bounded nonlinearity, Lur'e type Lyapunov function, was proposed to the stability analysis for discrete-time Lur'e systems in [15]. Then, the stochastic Lur's type Lyapunov function was applied to solve the stochastic stability problem for a class of discrete-time Markov jump Lur'e systems in [16] and [17]. However, as we can know, the issue of stability and stabilization for discrete-time Lur'e systems with asynchronous controller have not been investigated yet, which motivates us for the current work.

In this paper, the stochastic stabilization and l_2-gain minimization problems was discussed for a class of discrete-time Markov Jump Lur'e system with asynchronous controller. The contribution of this paper are: (1) Design a asynchronous controller which consists of a liner state feedback and a cone-bound nonlinear output feedback(similar with [15]) based on a Hidden Markov model. (2) A sufficient theorem in the from LMI was given to make sure the system is stochastic stabilize and minimize the l_2-gain performance. The rest of this paper is organized as follows: In Section II, we introduce the system model and give some preliminaries related to this paper. Section III analysis the stochastic stability for a class of discrete-time Markov Jump Lur'e system. Section IV deals with the l_2-gain minimization problem and design the asynchronous controller. A numerical example was provided to illustrate the effectiveness of the given theorems in Section V. Finally, we summarize the works of this paper in Section VI.

II. Preliminaries

Consider a class of discrete-time MJLS on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{cases} x_{k+1} = A_{r(k)}x_k + F_{r(k)}\varphi(y_k) + B_{r(k)}u_k \\ \qquad + E^x_{r(k)}w_k, \quad y_k = C_{r(k)}x_k, \\ z_k = C^z_{r(k)}x_k + G^z_{r(k)}\varphi(y_k) + D^z_{r(k)}u_k \\ \qquad + E^z_{r(k)}w_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}, u_k \in \mathbb{R}^{n_u}, y_k \in \mathbb{R}^{n_y}, z_k \in \mathbb{R}^{n_z}$, and $w_k \in \mathbb{R}^{n_w}$ represent the state, the control input, the output related to the nonlinearity, the controlled output and the exogenous disturbance vector respectively. $A_{r(k)}, F_{r(k)}, B_{r(k)}, E^x_{r(k)}, C^z_{r(k)}, G^z_{r(k)}, D^z_{r(k)}$ and $E^z_{r(k)}$ represent the time-varying system matrices, all of which are pre-known and real. $\{r(k), k \geq 0\}$ is a Markov chain taking values in a positive integer set $\mathcal{N} = \{1, 2, \dots, N\}$ with mode TPs:

$$\Pr\{r(k+1) = j|r(k) = i\} = \pi_{ij} \tag{2}$$

Clearly, for all $i, j \in \mathcal{N}$, $\pi_{ij} \in [0, 1]$, and $\sum_{j=1}^{N} \pi_{ij} = 1$ for each mode i. The system matrices in (1) at instant k can be expressed as $A_i, F_i, B_i, E^x_i, C^z_i, G^z_i, D^z_i$ and E^z_i, which are real known constant matrices with appropriate dimensions, and the related transition probability matrix is described as $\Pi = \{\pi_{ij}\}$.

Assumption 1: The function $\varphi(\cdot) : \mathbb{R}^{n_y} \to \mathbb{R}^{n_y}$ satisfies:
(1) $\varphi(0) = 0$ and
(2) there exists a diagonal positive definite matrix $\Omega \in \mathbb{R}^{n_y \times n_y}$ such that:

$$\varphi_{(\ell)}(y)[\varphi(y) - \Omega y]_{(\ell)} \leq 0. \tag{3}$$

where for all $y \in \mathbb{R}^{n_y}$, $\ell \in \{1, \dots, n_y\}$. And in this case we say that the nonlinearity $\varphi(\cdot)$ satisfies it's own cone bounded sector conditions and to be decentralized. According to (3) we have:

$$SC(y, \Lambda) := \varphi^{'}(y)\Lambda[\varphi(y) - \Omega y] \leq 0, \tag{4}$$

where Λ is any diagonal positive semidefinite matrix, and Ω is pre-given. From the Assumption 1, we get:

$$[\Omega y]_{(\ell)}[\varphi(y) - \Omega y]_{(\ell)} \leq 0, \tag{5}$$

which implies:

$$0 \leq \varphi^{'}(y)\Lambda\varphi(y) \leq \varphi^{'}(y)\Lambda\Omega y \leq y^{'}\Omega\Lambda\Omega y \tag{6}$$

Definition 1: System (1) is said to be locally stochastically stable if for $w_(k) = 0$ and any initial condition $x_0 \in \mathcal{D}_0$, $r_0 \in \mathcal{N}$, the following formulation holds:

$$\|x\|_2^2 = \sum_{k=0}^{\infty} \mathbb{E}[\|x_k\|^2] < \infty. \tag{7}$$

Where in this case the set $\mathcal{D}_0 \subset \mathbb{R}^p$ is said to be the domain of stochastic stability of the origin.

Define $\mathcal{F}_k$ as the σ-field generated by the random variables x_k and $r(k)$. We define next the class of exogenous disturbances with bounded energy and the finite ℓ_2-induced norm.
Definition 2: For $\zeta > 0$, set $\mathcal{W}_\zeta$ is defined as follows:

$$\begin{aligned} &\mathcal{W}_\zeta := \{w = \{w_k\}_{k\in\mathbb{N}}; w_k \in \mathbb{R}^m, k \in \mathbb{N}, w_k \text{ is} \\ &\qquad \mathcal{F}_k\text{-measureable, and } \|w\|_2^2 = \sum_{k=0}^{\infty} \mathbb{E}(\|w_k\|)^2 < \zeta. \end{aligned} \tag{8}$$

The finite ℓ_2-induced gain associated with the closed-loop system (1) with $x_0 = 0$ between the disturbance $w = \{w_k\}_{k\in\mathbb{N}}$ and controlled output $z = \{z_k\}_{k\in\mathbb{N}}$ is equal or less than γ if for every $w \in \mathcal{W}_\zeta$ then we have

$$\|z\|_2^2 = \sum_{k=0}^{\infty} \mathbb{E}\left[\|z_k\|^2\right] \leq \gamma^2\|w\|_2^2 = \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\left[\|w_k\|^2\right]. \tag{9}$$

In this paper, we consider the following controller:

$$u_k = K_{\sigma(k)}x_k + \Gamma_{\sigma(k)}\varphi(y_k) \tag{10}$$

where $K_{\sigma(k)} \in \mathbb{N}^{n_u \times n_x}$ is a time-varying controller gain matrix, and $\Gamma_{\sigma(k)} \in \mathbb{N}^{n_u \times n_y}$ is a time-varying nonlinear output feedback gain matrix. The parameter $\{\sigma(k), k \geq 0\}$ takes values in another pre-given positive integer set, which is marked as $\mathcal{M} = \{1, 2, \dots, M\}$ subject to the pre-known conditional probability matrix $\Phi = \{\mu_{im}\}$, the probabilities of which are defined by

$$\Pr\{\sigma(k) = \phi|r(k) = i\} = \mu_{i\phi}. \tag{11}$$

Where for all $i \in \mathcal{N}, \phi \in \mathcal{M}, \mu_{i\phi} \in [0, 1]$, and $\sum_{\phi=1}^{M} \mu_{i\phi} = 1$ for each $i \in \mathcal{N}$.

Remark 1. *Similar with [4], a HHM was introduced to describe the asynchronous phenomenon appears between the controller and the plant. Its easy to find that, the controller designed here consists a linear state feedback and a non-linear output feedback which satisfy cone-bounded condition introduced in assumption 1. Different form the synchronous controller introduced in [14] and [16], the controller adopted in this paper is a asynchronous controller, which means the controller mode and the plant mode may different at the same time. Obviously, when the probability matrix Φ is unit matrix, the controller mode run synchronously with the plant, asynchronous controller becomes a synchronous controller, which means the asynchronous controller considered in this paper takes the synchronization as a special case.*

Combing the asynchronous controller (10) and system (1) we have the following closed-loop system:

$$\begin{cases} x_{k+1} = \bar{A}_{i\theta}x_k + \bar{F}_{i\theta}\varphi_i(C_ix_k) + E^x_iw_k \\ z_k = \bar{C}^z_{i\theta}x_k + \bar{G}^z_{i\theta}\varphi_i(C_ix_k) + E^z_iw_k \end{cases} \tag{12}$$

Where, for $i \in \mathcal{N}, \phi \in \mathcal{M}$

$$\begin{aligned} \bar{A}_{i\phi} &= A_i + B_iK_\phi, & \bar{F}_{i\phi} &= F_i + B_i\Gamma_{i\phi} \\ \bar{C}^z_{i\phi} &= C^z_i + D^z_iK_\phi, & \bar{G}^z_{i\phi} &= G^z_i + D^z_i\Gamma_\phi \end{aligned}$$

III. Main Result

A. stochastic stability analysis

In this subsection, we focus on the stochastic stability analysis for the closed-loop system (12). A sufficient condition presented in the form of LMI was derived to guarantee the considered Lur'e system stochastically stable.

Theorem1. System (12) with $x_0 = 0$ and $w_k = 0$ is stochastically stable, if for all $i \in \mathcal{N}$ and $\phi \in \mathcal{M}$, there exist positive definite matrices $\bar{P}_i \in \mathbb{R}^{n_x \times n_x}$, $R_{i\phi} \in \mathbb{R}^{(n_x+n_y)\times(n_x+n_y)}$, matrices $K_\phi \in \mathbb{R}^{n_u \times n_x}$, $\Gamma_\phi \in \mathbb{R}^{n_u \times n_y}$ and positive semidefinite matrix $T_i \in \mathbb{R}^{n_y}$ to ensure (13) and (14) hold.

$$\begin{bmatrix} -R_{i\theta} & \mathscr{H}_{i\phi} \\ * & \mathscr{P}_i \end{bmatrix} < 0 \tag{13}$$

$$\begin{bmatrix} \mathscr{S}_{i\phi} & \mathscr{N}_{i\phi} \\ * & \mathscr{L}_{i\phi} \end{bmatrix} < 0 \tag{14}$$

where

$$\mathscr{H}_{i\phi} = \begin{bmatrix} \sqrt{\pi_{i1}}\hat{A}_{i\phi} \\ \sqrt{\pi_{i2}}\hat{A}_{i\phi} \\ \vdots \\ \sqrt{\pi_{i_N}}\hat{A}_{i\phi} \end{bmatrix}', \mathscr{S}_{i\phi} = \begin{bmatrix} -\bar{P}_i & 0 \\ * & H_{i\phi} \end{bmatrix}$$

$$H_{i\theta} = \begin{bmatrix} -I_{n_x} & C_i' \Omega_i T_i \\ * & -2T_i \end{bmatrix}, \hat{A}_{i\theta} = \begin{bmatrix} \bar{A}_{i\theta} & \bar{F}_{i\theta} \end{bmatrix}$$

$$\mathscr{P}_{i\phi} = \mathrm{diag}\{-\bar{P}_1, -\bar{P}_2, \dots, -\bar{P}_N\}$$

$$\mathscr{N}_{i\phi} = \begin{bmatrix} \sqrt{u_{i1}} \begin{bmatrix} \bar{P}_i & 0 \\ * & R_{i1} \end{bmatrix} & \cdots & \sqrt{u_{iM}} \begin{bmatrix} \bar{P}_i & 0 \\ * & R_{iM} \end{bmatrix} \end{bmatrix}$$

$$\mathscr{L}_{i\phi} = \mathrm{diag}\left\{ \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i1} \end{bmatrix}, \dots, \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{iM} \end{bmatrix} \right\}$$

Proof: We first construct a Lyapunov function in the form of (15).

$$V(k, x_k, r(k)) = x_k' P_{r(k)} x_k \tag{15}$$

And $P_{r(k)} = \bar{P}_{r(k)}^{-1}$. Let $\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{V(k+1, x_{k+1}, r(k+1) = j | x_k, r(k) = i\} - V(k, x_k, i)$ and it is easy to find that

$$\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{x_{k+1}' X_i x_{k+1}\} - V(k, x_k, i) \tag{16}$$

Where $X_i = \sum_{j=1}^{N} \pi_{ij} P_j$. Based on system (12) we can further obtain

$$\begin{aligned} \mathbb{E}\{x_{k+1}' X_i x_{k+1}\} &= \sum_{\phi=1}^{M} u_{i\phi} \hat{x}_k' \hat{A}_{i\phi}' X_i \hat{A}_{i\phi} \hat{x}_k \\ &= \hat{x}_k' \Big(\sum_{\phi=1}^{M} u_{i\phi} \hat{A}_{i\phi}' X_i \hat{A}_{i\phi} \Big) \hat{x}_k \end{aligned} \tag{17}$$

Where $\hat{x}_k = (x_k', \varphi_i'(C_i x_k))'$. Which implies

$$\mathbb{E}\{\Delta V(k)\} = \hat{x}_k' \Big(\sum_{\phi=1}^{M} u_{i\phi} \hat{A}_{i\phi}' X_i \hat{A}_{i\phi} \Big) \hat{x}_k - x_k' P_i x_k \tag{18}$$

Denoting $h_{i\phi} = \mathrm{diag}\{P_i, I_{n_x+n_y}, \{\mathrm{diag}(I_{n_x}, R_{i\phi}^{-1})_{\phi=1}^{M}\}\}$ and using $h_{i\phi}$ to pre- and post-multiplying the inequality given in (14), respectively, then we can get

$$\begin{bmatrix} \hat{\mathscr{S}}_{i\phi} & \sqrt{u_{i1}} I & \cdots & \sqrt{u_{iM}} I \\ \sqrt{u_{i1}} I & L_{i1} & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \sqrt{u_{iM}} I & 0 & 0 & L_{iM} \end{bmatrix} < 0$$

Where, for all $\phi \in \mathcal{M}$

$$\hat{\mathscr{S}}_{i\phi} = \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix}, L_{i\phi} = \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i\phi}^{-1} \end{bmatrix}$$

Applying Schur complement to the inequality above, we can further obtain

$$\sum_{\phi=1}^{M} u_{i\phi} \begin{bmatrix} I & 0 \\ * & R_{i\phi} \end{bmatrix} + \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix} < 0 \tag{19}$$

(19) multiply $(x_k', \hat{x}_k')'$ on left and it's transpose on the right. We get

$$\begin{aligned} &\hat{x}_k' \sum_{\phi=1}^{M} u_{i\phi} R_{i\phi} \hat{x} + x_k' (\sum_{\phi=1}^{M} u_{i\phi} I_{n_x}) x_k - x_k' I_{n_x} x_k \\ &- x_k' P_i x_k - 2SC(i, x_k, T_i) < 0 \end{aligned} \tag{20}$$

Noting that $\sum_{\phi=0}^{M} u_{i\phi} = 1$, then $\sum_{\phi=0}^{M} u_{i\phi} I_{n_x} = I_{n_x}$, that implies the (20) is equivalent to:

$$\hat{x}_k' \sum_{\phi=1}^{M} u_{i\phi} R_{i\phi} \hat{x}_k - x_k' P_i x_k - 2SC(i, x_k, T_i) < 0 \tag{21}$$

Applying Schur complement to inequality (13). We can obtain

$$\hat{A}_{i\phi} X_i \hat{A}_{i\phi} < R_{i\phi} \tag{22}$$

Combining inequalities (21) and (22), it's easy to find that

$$\begin{aligned} &\hat{x}_k' \sum_{\phi=1}^{M} u_{i\phi} (\hat{A}_{i\phi} X_i \hat{A}_{i\phi}) \hat{x}_k - x_k' P_i x_k \\ &\qquad - 2SC(i, x_k, T_i) < 0 \end{aligned} \tag{23}$$

Consider the condition given in (18), it is easy to obtain that

$$\mathbb{E}\{\Delta V(k)\} - 2SC(i, x_k, T_i) < 0 \tag{24}$$

Furthermore, noticing the condition given in (4), we further have $\mathbb{E}\{\Delta V(k)\} < 0$, which means the closed-loop system (12) is stochastically stable.

Remark 2. *19-20łŁł Noting that,*

Remark 3. *R*

B. l_2-gain minimization and controller design

The l_2-gain minimization problem for closed-loop system (12) will be discussed in this subsection. The asynchronous controller (10) can be designed by solving the l_2-gain optimization problem, and the following theorem provides a sufficient condition to make sure the closed-loop system is stochastically stable and l_2-gain is less or equal to γ.

Theorem 2: For each mode $i \in \mathcal{N}$ and $\phi \in \mathcal{M}$, if there exist positive-definite matrices $\bar{P}_i \in \mathbb{R}^{n_x \times n_x}$ and $R_{i\phi} \in \mathbb{R}^{(n_x+n_y+n_w)\times(n_x+n_y+n_w)}$, matrices $K_\phi \in \mathbb{N}^{n_u \times n_x}, \Gamma_\phi \in \mathbb{R}^{n_u \times n_y}$, positive semi-definite matrix $T_i \in \mathbb{R}^{n_y \times n_y}$ and a positive scalar γ, such that the following LMIs are verified

$$\begin{bmatrix} -R_{i\theta} & \mathscr{H}_{i\phi} \\ * & \mathscr{P}_i \end{bmatrix} < 0 \tag{25}$$

$$\begin{bmatrix} \mathscr{S}_{i\phi} & \mathscr{N}_{i\phi} \\ * & \mathscr{L}_{i\phi} \end{bmatrix} < 0 \tag{26}$$

Where

$$\mathscr{H}_{i\phi} = \begin{bmatrix} \sqrt{\pi_{i1}}\hat{A}_{i\phi} \\ \sqrt{\pi_{i2}}\hat{A}_{i\phi} \\ \vdots \\ \sqrt{\pi_{iN}}\hat{A}_{i\phi} \\ \hat{A}^z_{i\phi} \end{bmatrix}', \quad \mathscr{S}_{i\phi} = \begin{bmatrix} -\bar{P}_i & 0 \\ * & H_{i\phi} \end{bmatrix}$$

$$\mathscr{P}_{i\phi} = \mathrm{diag}\{-\bar{P}_1, -\bar{P}_2, \ldots, -\bar{P}_N, -I_{n_w}\}$$

$$\hat{A}_{i\theta} = \begin{bmatrix} \bar{A}_{i\theta} & \bar{F}_{i\theta} & E^x_i \end{bmatrix}, \quad \hat{A}^z_{i\theta} = \begin{bmatrix} \bar{C}^z_{i\theta} & \bar{G}^z_{i\theta} & E^z_i \end{bmatrix}$$

$$\mathscr{N}_{i\phi} = \begin{bmatrix} \sqrt{u_{i1}} \begin{bmatrix} \bar{P}_i & 0 \\ * & R_{i1} \end{bmatrix} & \cdots & \sqrt{u_{iM}} \begin{bmatrix} \bar{P}_i & 0 \\ * & R_{iM} \end{bmatrix} \end{bmatrix}$$

$$\mathscr{L}_{i\phi} = \mathrm{diag}\left\{ \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i1} \end{bmatrix}, \ldots, \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{iM} \end{bmatrix} \right\}$$

$$H_{i\theta} = \begin{bmatrix} -I_{n_x} & C_i' \Omega_i T_i & 0 \\ * & -2T_i & 0 \\ * & * & -\gamma^2 I_{n_w} \end{bmatrix}$$

then system (12) with $x_0 = 0$ and $w \in \mathcal{W}_\gamma$ is stochastically stable and the l_2-gain of the closed-loop system is strictly less or equal to γ.

Proof: We first select (16) as the Lyapunov function and $P_{r(k)} = \bar{P}^{-1}_{r(k)}$. Similar to the proof of theorem 1, it's easy to find that

$$\mathbb{E}\{\Delta V(k)\} = \hat{x}_k' \Big(\sum_{\phi=1}^{M} u_{i\phi} \hat{A}'_{i\phi} X_i \hat{A}_{i\phi} \Big) \hat{x}_k - V(k, x_k, i) \tag{27}$$

Where $\hat{x}_k = (x_k', \varphi_i'(C_i x_k), w_k')'$, $X_i = \sum_{j=1}^N \pi_{ij} P_j$

Denoting $h_{i\phi} = \mathrm{diag}\{P_i, I_{n_x+n_y+n_w}, \{\mathrm{diag}(I_{n_x}, R^{-1}_{i\phi})_{\phi=1}^M\}\}$ and using $h_{i\phi}$ to pre- and post-multiplying the inequality given in (26), respectively, then we can obtain

$$\begin{bmatrix} \hat{\mathscr{S}}_{i\phi} & \sqrt{u_{i1}}I & \cdots & \sqrt{u_{iM}}I \\ \sqrt{u_{i1}}I & L_{i1} & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \sqrt{u_{iM}}I & 0 & 0 & L_{iM} \end{bmatrix} < 0$$

Where

$$\hat{\mathscr{S}}_{i\phi} = \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix}, L_{i\phi} = \begin{bmatrix} -I_{n_x} & 0 \\ * & -R^{-1}_{i\phi} \end{bmatrix}$$

Noting that $L_{i\phi} < 0$ for any $\phi \in \mathcal{M}$, apply Schur complement lemma to the inequality above, we can get

$$\sum_{\phi=1}^{M} u_{i\phi} \begin{bmatrix} I & 0 \\ * & R_{i\phi} \end{bmatrix} + \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix} < 0 \tag{28}$$

(28) multiply $(x_k', \hat{x}_k')'$ on left and it's transpose on the right. We get

$$\begin{aligned} &\hat{x}_k'(\sum_{\phi=1}^{M} u_{i\phi} R_{i\phi})\hat{x}_k + \hat{x}_k'(\sum_{\phi=1}^{M} u_{i\phi} I_{n_x})\hat{x}_k - \hat{x}_k' I_{n_x} \hat{x}_k \\ &- x_k' P_i x_k - 2SC(i, x_k, T_i) - \gamma^2 w_k' w_k < 0 \end{aligned} \tag{29}$$

Noting that $\sum_{\phi=0}^M u_{i\phi} = 1$, then $\sum_{\phi=0}^M u_{i\phi} I_{n_x} = I_{n_x}$, that implies the (29) is equivalent to:

$$\hat{x}_k'(\sum_{\phi=1}^{M} u_{i\phi} R_{i\phi})\hat{x}_k - x_k' P_i x_k - 2SC(i, x_k, T_i) - \gamma^2 w_k' w_k < 0 \tag{30}$$

Applying Schur complement to inequality (25). We can obtain

$$\hat{A}_{i\phi} X_i \hat{A}_{i\phi} + (\hat{A}^z_{i\phi})' \hat{A}^z_{i\phi} < R_{i\phi} \tag{31}$$

Combining inequalities (30) and (31), it's easy to find that

$$\begin{aligned} &\hat{x}_k' \Big(\sum_{\phi=1}^{M} u_{i\phi} (\hat{A}_{i\phi} X_i \hat{A}_{i\phi} + (\hat{A}^z_{i\phi})' \hat{A}^z_{i\phi}) \Big) \hat{x}_k - x_k' P_i x_k \\ &- 2SC(i, x_k, T_i) - \gamma^2 w_k' w_k < 0 \end{aligned} \tag{32}$$

Which implies

$$\mathbb{E}\{\Delta V(k)\} + \mathbb{E}\{z_k' z_k\} - \gamma^2 w_k' w_k - 2SC(i, x_k, T_i) < 0 \tag{33}$$

Furthermore, noticing the condition given in (4), we further have $\mathbb{E}\{\Delta V(k)\} + \mathbb{E}\{z_k' z_k\} - \gamma^2 w_k' w_k < 0$, which means

$$\mathbb{E}\{\Delta V(k) + z_k' z_k - \gamma^2 w_k' w_k | x_k, r(k) = i\} < 0 \tag{34}$$

Summing up from $k = 0$ to $k = T$, and recalling that $x_0 = 0$, that $\mathbb{E}\{k+1, x_{k+1}, r(k+1)\} \geq 0$, we have that $\sum_{k=0}^T \mathbb{E}\{\|z_k\|^2\} - \gamma^2 \sum_{k=0}^T \mathbb{E}\{\|w_k\|^2\} \leq 0$. Taking the limit as $k \to \infty$, we can conclude that $\|z\|_2 \leq \gamma \|w\|_2$, which means, for all $w \in \mathcal{W}_\zeta$, the l_2-gain of the closed-loop system (12) is less or equal to γ, and this completes the proof of theorem 2.

Remark 4. *It's easy to find that, if (25) and (26) holds, the asynchronous controller can be designed based on Theorem 2. The optimal l_2-gain performance γ^* can be obtained via dealing with the optimal problem as follows:*

$$\min \quad \sigma \quad \text{subject to (25) and (26) with } \sigma = \gamma^2 \tag{35}$$

IV. Numerical Example

In this section, we provide an example to verify the validity of the proposed methods. Consider the Markov jump Lur'e system (1) with following parameters: $\mathcal{N} = \{1, 2\}$, $\mathcal{M} = \{1, 2\}$

$$
\begin{aligned}
&A_1 = \begin{bmatrix} 0.4 & 0.4 \\ 0.2 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.1 & 0.6 \\ 0.3 & 0.4 \end{bmatrix} \\
&B_1 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.7 \\ 0.5 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 1 \\ 1.2 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 1.2 \\ 1 \end{bmatrix} \\
&C_1 = \begin{bmatrix} 0.9 \\ 0.5 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -1 \\ 0.7 \end{bmatrix}, \quad C_1^z = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \quad C_1^z = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \\
&G_1^z = 0.3, \quad G_2^z = 0.4, \quad D_1^z = 0, \quad D_2^z = 0.3; \\
&E_1^z = 1.3, \quad E_2^z = -0.8, \quad E_1^x = E_2^x = \begin{bmatrix} 1 & 0.5 \end{bmatrix}' \\
&\mathit{\Omega}_1 = 1.3, \quad \varphi_1(y) = 0.5\mathit{\Omega}_1 y(1 + cos(25y)) \\
&\mathit{\Omega}_2 = 1.5, \quad \varphi_2(y) = 0.5\mathit{\Omega}_2 y(1 - sin(20y)) \\
&\mathit{\Pi} = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \quad \mathit{\Phi} = \begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \end{bmatrix}
\end{aligned}
$$

By solving the l_2-gain minimization problem in (35), it is easy to find that $\gamma^* = 2.41$ and following asynchronous controller gain matrices can be obtained.

$$
\begin{aligned}
K_1 &= \begin{bmatrix} -1.0266 & -1.2008 \end{bmatrix}, \quad \mathit{\Gamma}_1 = -2.4363 \\
K_2 &= \begin{bmatrix} -1.1812 & -0.9191 \end{bmatrix}, \quad \mathit{\Gamma}_2 = -2.2642
\end{aligned}
$$

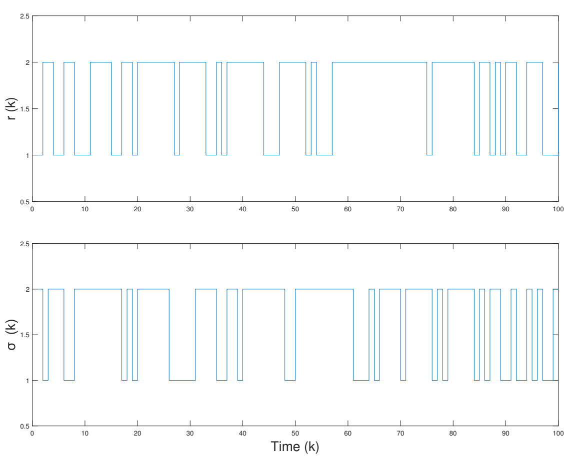

Fig. 1. Modes of system and controller

Fig.1 gives a possible time sequences of the mode of system and asynchronous controller. We select the initial state $x_0 = \begin{bmatrix} 2 & -2.5 \end{bmatrix}'$, and the external disturbance is assumed to be $w_k = sin(k) * 0.9^k$. Then we can get state response of system, which is depicted in Fig.2.

V. Conclusions

The stochastic stabilization and l_2-gain minimization problems for a class of discrete-time nonlinear Markov systems with asynchronous controller have been discussed in this paper.

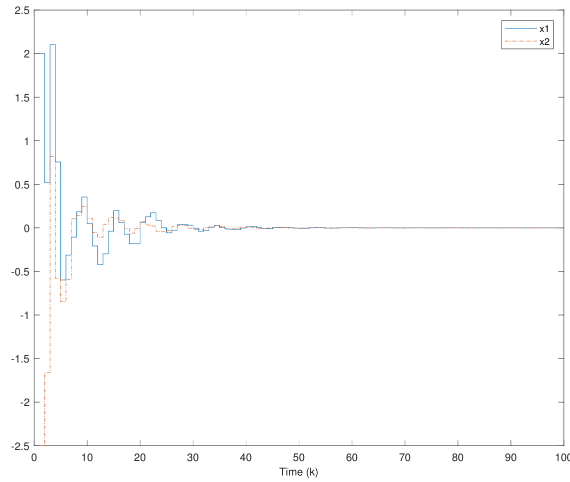

Fig. 2. System state

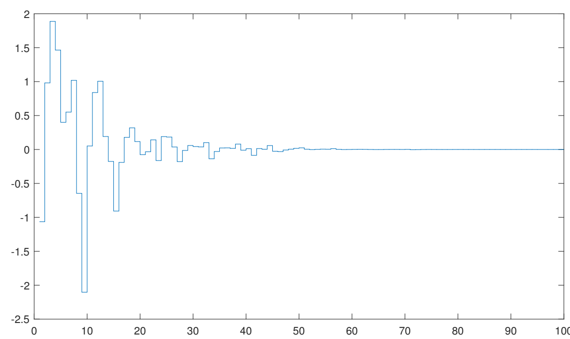

Fig. 3. System input

Acknowledgment

The authors would like to thank...

References

[1] Zhu Y, Zhang L, Zheng W X. Distributed $\mathcal{H}_\infty$ Filtering for a Class of Discrete-Time Markov Jump Lure Systems With Redundant Channels[J]. IEEE Transactions on Industrial Electronics, 2016, 63(3):1876-1885.

[2] Todorov M G, Fragoso M D. New methods for mode-independent robust control of Markov jump linear systems[J]. Systems & Control Letters, 2016, 90:38-44.

[3] Wu, H. N., & Cai, K. Y. (2006). Mode-independent robust stabilization for uncertain markovian jump nonlinear systems via fuzzy control. IEEE Transactions on Systems Man & Cybernetics Part B Cybernetics A Publication of the IEEE Systems Man & Cybernetics Society, 36(3), 509-19.

[4] Wu, Zheng Guang, et al. "Passivity-Based Asynchronous Control for Markov Jump Systems." IEEE Transactions on Automatic Control 62.4(2017):2020-2025.

[5] Z.-G. Wu, P. Shi, H. Su, and J. Chu, Asynchronous l_2-l_∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities, Automatica, vol. 50, no. 1, pp. 180186, 2014.

[6] Z.-G. Wu, S. Dong, H. Su, and C. Li, Asynchronous dissipative control for fuzzy Markov jump systems, IEEE Trans. Cybern., to be published, doi: 10.1109/TCYB.2017.2739754.

[7] Shen Y, Wu Z G, Shi P, et al. Asynchronous Filtering for Markov Jump Neural Networks With Quantized Outputs[J]. IEEE Transactions on Systems Man & Cybernetics Systems, 2018, PP(99):1-11.

[8] Dong, Shanling, et al. "Hidden-Markov-Model-Based Asynchronous Filter Design of Nonlinear Markov Jump Systems in Continuous-Time Domain." IEEE Transact ions on Cybernetics PP.99:1-11.

[9] Zhang M, Shi P, Liu Z, et al. Fuzzy Model-based Asynchronous H ∞ Filter Design of Discrete-Time Markov Jump Systems[J]. Journal of the Franklin Institute, 2017, 354(18).

[10] Song, Jun, et al. "Finite-time l_2-l_∞ control of Markovian jump linear systems with partly accessible hidden information via asynchronous output feedback." Control Conference IEEE, 2018:2447-2452.

[11] Khalil, H. K. (2002). Nonlinear systems (3rd ed.). Prentice Hall

[12] Castelan E B, Moreno U F, Pieri E R D. Absolute stabilization of discrete-time systems with a sector bounded nonlinearity under control saturations[C]// IEEE International Symposium on Circuits and Systems, 2006. ISCAS 2006. Proceedings. IEEE, 2006:4 pp.

[13] Castelan E B, Tarbouriech S, Queinnec I. Control design for a class of nonlinear continuous-time systems [J]. Automatica, 2008, 44(8):2034-2039.

[14] Song G, Zhang Y, Xu S. Stability and l 2 -gain analysis for a class of discrete-time non-linear Markovian jump systems with actuator saturation and incomplete knowledge of transition probabilities[J]. Iet Control Theory & Applications, 2012, 6(17):2716-2723.

[15] Gonzaga, Carlos A. C., M. Jungers, and J. Daafouz. "Stability analysis of discrete-time Lure systems ." Automatica 48.9(2012):2277-2283.

[16] Gonzaga C A C, Jungers M, Daafouz J. Stability analysis of discrete-time Lure systems [J]. Automatica, 2012, 48(9):2277-2283.

[17] Gonzaga C A C, Costa O L V. Stochastic stabilization and induced 2 -gain for discrete-time Markov jump Lur'e systems with control saturation[M]. Pergamon Press, Inc. 2014.