l_2-l_∞ Filtering for Discrete-Time Switched Fuzzy Systems with Missing Measurements

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***Abstract*—This paper focuses on the l_2-l_∞ filtering problem for a class of discrete-time switched fuzzy systems with missing measurements. The fuzzy plant considered in this paper incorporates characteristics of Takagi-Sugeno (T-S) fuzzy systems and switched systems simultaneously. The phenomenon of missing measurements is described by a stochastic variable that satisfies the Bernoulli binary distribution, which characterizes the effect of data loss in information transmitted from the plant to the filter. Based on a basis-dependent Lyapunov function, sufficient conditions are derived in terms of linear matrix inequalities, which ensure the stochastic stability as well as a given l_2-l_∞ performance for the filtering error system. Finally, two illustrative examples are presented to demonstrate the effectiveness of the proposed method.**

***Index Terms*—l_2-l_∞ filtering, switched systems, T-S fuzzy systems, missing measurements**

I. Introduction

Switched systems are a class of hybrid dynamical systems which consist of a set of subsystems and rules that orchestrate the switching among them. In recent years, considerable attention has been paid to the study of switched systems because of their significance both in theory and applications (see e.g., [1]–[5] and the references therein). The main advantage of switched systems is that many practical problems can be described as switched models, such as networked control systems [6]–[8], computer controlled systems and aircraft control systems. A lot of results focused on switched systems have been reported, for instance, switched systems with time delay are studied in [9]–[11], sampled-data control of switched systems is considered in [12], [13], and the authors investigate switched systems with average dwell time in [14]–[16].

It is noted that most of the results obtained in the existing literature are mainly concentrated on linear systems, however, systems are usually nonlinear in practical engineering, which bring great difficulties to analysis and synthesis of control problems. Fortunately, a fuzzy-model based approach has been introduced as an effective mathematical tool to model nonlinear systems. Utilizing the powerful approximation performance of the Takagi-Sugeno (T-S) fuzzy model, nonlinear systems can usually be represented equivalently as a set of linear subsystems with corresponding membership functions. Plenty of interesting results on T-S fuzzy systems have been obtained, to mention a few, output feedback controller design of T-S fuzzy systems is investigated in [17], [18], the study of uncertain nonlinear systems using T-S fuzzy models is presented in [19]–[21]. Fuzzy systems with infinite-distributed delays are considered in [22], [23] and stabilization of fuzzy systems under variable sampling is addressed in [24], [25].

On another research front line, filtering problem has received a lot of attention from researchers in recent decades. Besides the classical and well-known Kalman filtering [26], H_∞ and l_2-l_∞ filtering techniques have been counting among the most popular filtering techniques since their publication. Various results have been reported on H_∞ filtering in references [27]–[31] and l_2-l_∞ filtering in references [32]–[35]. It should be underscored that these filtering techniques are all rested on the basis that the information transmission between the plant and the filter are perfect, however, this is not the case in many practical problems, particularly for networked control systems where the signal y_{fk} received by the filter may not be the same as the output y_k of the plant. The phenomenon of missing measurements are caused by many reasons, e.g., a certain failure in the measurement, limited communication capacity of the network media, or intermittent sensor failures. Recently, a great deal of attention has been paid to this topic. Reference [36] investigates filtering problem for stochastic systems with missing measurements and [37] extends this result to discrete systems suffered with sensor delays which vary randomly. In [38], filtering for systems with intermittent measurements is presented while [39] considers quantization and packet dropouts simultaneously. So far in the previously mentioned literature, the l_2-l_∞ filtering problem for discrete-time switched fuzzy systems with missing measurements still remains open and has not been solved yet, this motivates us for the present study.

In this paper, we investigate the l_2-l_∞ filtering problem for a class of discrete-time switched fuzzy systems with missing measurements. The information transmission between the filter and the plant under consideration is not perfect and the phenomenon of the missing measurements is characterized by a stochastic variable that satisfies the Bernoulli binary distribution. By utilizing a basis-dependent Lyapunov function, sufficient conditions are established to guarantee the stochastic stability as well as a given l_2-l_∞ performance for the filtering error system. In additional, to simplify and facilitate the

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filter design procedures, we introduce some slack matrices to avoid the cross coupling between system matrices and Lyapunov matrices, which significantly reduce difficulties for the subsequent design. The resultant filter parameters are easily obtained by solving some LMIs using the MATLAB LMI toobox. Finally, two illustrative examples are provided to verify the effectiveness of the proposed method.

The remainder of the paper is constructed as follows. Section II briefly recalls some background material on the fuzzy plant and formulates the problem to be addressed subsequently. Section III contains our main results, which are sufficient conditions for stochastic stability with a prescribed l_2-l_∞ performance γ of the filtering error system and filter design synthesis . Two illustrative examples are given in Section IV and the paper is concluded in Section V.

Notation: The notations used throughout this paper are standard. The notation $X > 0$ $(X \geq Y)$ implies that X is positive definite (positive semidefinite). $||\cdot||_2$ denotes the Euclidean norm of a vector and its induced norm of a matrix. $l_2[0,+\infty)$ is the space of square-integrable vector functions over $[0,+\infty)$. Also, $E\{x\}$ and $E\{x|y\}$ represent the expectation of x and expectation of x conditional on y, respectively. $*$ within a matrix represents the symmetric terms. Finally, unless otherwise stated, all the matrices are assumed to have suitable dimensions for algebraic operations.

II. Preliminaries and problem formulation

The filtering problem discussed in this paper is shown in Fig.1. From which we can see intuitively, the physical plant under consideration is modeled by a switched T-S fuzzy system, and the communication link between the plant and the filter may not be reliable, that is, the system measurements may be unavailable (i.e., missing data) sometimes. In what follows, we model the whole problem mathematically.

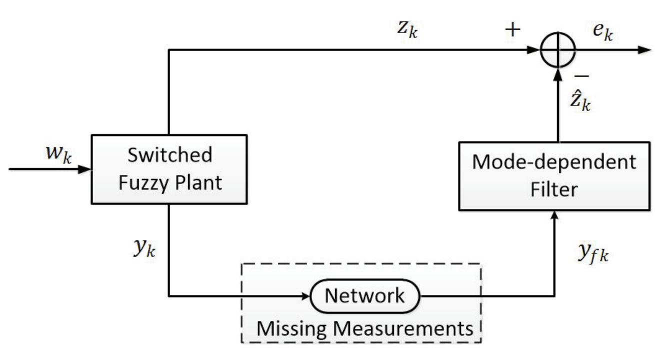

Fig. 1. Filtering system with missing measurements

A. Switched fuzzy model of nonlinear plant

The plant considered in this paper is represented by the following discrete-time switched T-S fuzzy model:

IF θ_{1k} is M_{i1}, θ_{2k} is M_{i2}, $\cdots$, and θ_{pk} is M_{ip}, THEN

$$\begin{cases} x_{k+1} = A_{r_k i}x_k + B_{r_k i}w_k \\ \quad y_k = C_{r_k i}x_k + D_{r_k i}w_k \\ \quad z_k = L_{r_k i}x_k \\ \qquad i \in \{1,2,\cdots,r\} \end{cases} \tag{1}$$

where M_{ij} is a fuzzy set, and $\theta_{jk} \in \{\theta_{1k},\theta_{2k},\cdots,\theta_{pk}\}$ is the premise variable and r denotes the number of IF-THEN rules; $x_k \in R^n$ is the state vector; $w_k \in R^m$ is the disturbance input vector that belongs to $l_2[0,\infty)$; $y_k \in R^q$ is the measurement output; $z_k \in R^v$ is the signal to be estimated; r_k is a switching variable. For $r_k = l(l \in \{1,2,3,\cdots,L\})$, A_{li}, B_{li}, C_{li}, D_{li} and L_{li} are known matrices with compatible dimensions.

In this paper, the stochastic variable r_k used to characterize the switching phenomena satisfying:

$$\Pr\{r_k = l\} = \pi_l. \tag{2}$$

Here r_k is state independent and π_l denotes "sojourn probability" [40], [41] of the corresponding subsystem, which satisfies

$$\sum_{l=1}^{L} \pi_l = 1. \tag{3}$$

On the other hand, some stochastic variables π_{kl} are defined as

$$\pi_{kl} = \begin{cases} 1, & r_k = l \\ 0, & r_k \neq l, \end{cases}$$

and it can be easily verified that

$$E\{\pi_{kl}\} = \pi_l.$$

It should be underscored that for switched systems studied in this paper, there is just one subsystem is triggered in a single moment, that is

$$\Pr\{r_k = l_1, r_k = l_2\} = 0, \text{ if } l_1 \neq l_2$$

and

$$E\{\pi_{kl_1}\pi_{kl_2}\} = \begin{cases} \pi_{l_1}, & l_1 = l_2 \\ 0, & l_1 \neq l_2 \end{cases}$$

where $l_1, l_2 \in \{1,2,3,\cdots,L\}$.

Next, in order to facilitate filter design procedures, by using a center-average defuzzifier, product-fuzzy inference, and singleton fuzzifier, we can obtain the following normalized fuzzy basis functions which are represented as

$$h_i(\theta_k) = \frac{\prod_{j=1}^{p} M_{ij}(\theta_{jk})}{\sum_{i=1}^{r}\prod_{j=1}^{p} M_{ij}(\theta_{jk})} \tag{4}$$

where $M_{ij}(\theta_{jk})$ denotes the grade of membership of θ_{jk} in M_{ij}. In what follows, we simplify the argument of $h_i(\theta_k)$ as h_i for simplicity. Hence, the following holds for all k:

$$\begin{cases} h_i \geq 0, \ i = \{1,2,...r\} \\ \displaystyle\sum_{i=1}^{r} h_i = 1. \end{cases} \tag{5}$$

Now, a more compact presentation of the switched T-S fuzzy system (1) can be given by:

$$\begin{cases} x_{k+1} = A_{lh}x_k + B_{lh}w_k \\ \quad y_k = C_{lh}x_k + D_{lh}w_k \\ \quad z_k = L_{lh}x_k \end{cases} \tag{6}$$

with

$$A_{lh}=\sum_{i=1}^{r}h_iA_{li},\quad B_{lh}=\sum_{i=1}^{r}h_iB_{li}$$
$$C_{lh}=\sum_{i=1}^{r}h_iC_{li},\quad D_{lh}=\sum_{i=1}^{r}h_iD_{li} \tag{7}$$
$$L_{lh}=\sum_{i=1}^{r}h_iL_{li}$$

and $h \triangleq (h_1, h_2, \cdots, h_r) \in \rho$, where ρ are basis functions satisfying (5).

B. Communication Link

Inspired by [36] [38], we assume in this paper that the measurements transferred from the plant to the filter may be unavailable sometimes, more specifically, the transmission of data is not always complete and the phenomenon of data loss is modeled as

$$y_{fk}=\alpha(k)y_k \tag{8}$$

where the stochastic variable $\{\alpha(k)\}$ denotes a Bernoulli-distributed white sequence that satisfies

$$\Pr\{\alpha(k)=1\}=E\{\alpha(k)\}=\bar{\alpha}$$
$$\Pr\{\alpha(k)=0\}=1-\bar{\alpha}.$$

It is obvious when $\bar{\alpha}=1$, that is, when $y_{fk}=y_k$, the information transmission between the filter and the plant is complete, otherwise, when $0\le\bar{\alpha}<1$, the phenomenon of data loss may occur.

C. Filtering error systems

In this paper, our objective is concentrated on the constructing of the following full-order filter:

$$\begin{cases}\hat{x}_{k+1}=A_{flh}\hat{x}_k+B_{flh}y_{fk}\\ \hat{z}_k=L_{flh}\hat{x}_k\end{cases} \tag{9}$$

where

$$A_{flh}=\sum_{i=1}^{r}h_iA_{fli},\ B_{flh}=\sum_{i=1}^{r}h_iB_{fli},\ E_{flh}=\sum_{i=1}^{r}h_iL_{fli}.$$

By defining $e_k=z_k-\hat{z}_k$, $\xi_k=\begin{bmatrix}x_k^{\mathrm{T}} & \hat{x}_k^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$, from (6), (8) and (9), the following filtering error system is obtained:

$$\begin{cases}\xi_{k+1}=(A_{1lh}+\tilde{\alpha}A_{2lh})\xi_k+(B_{1lh}+\tilde{\alpha}B_{2lh})w_k\\ e_k=\bar{L}_{lh}\xi_k\end{cases} \tag{10}$$

where

$$A_{1lh}=\begin{bmatrix}A_{lh} & 0\\ \bar{\alpha}B_{flh}C_{lh} & A_{flh}\end{bmatrix},\ A_{2lh}=\begin{bmatrix}0 & 0\\ B_{flh}C_{lh} & 0\end{bmatrix}$$
$$B_{1lh}=\begin{bmatrix}B_{lh}\\ \bar{\alpha}B_{flh}D_{lh}\end{bmatrix},\ B_{2lh}=\begin{bmatrix}0\\ B_{flh}D_{lh}\end{bmatrix}$$
$$\bar{L}_{lh}=\begin{bmatrix}L_{lh} & -L_{flh}\end{bmatrix}$$

and $\tilde{\alpha}(k)=\alpha(k)-\bar{\alpha}$. It can be easily verified that

$$E\{\tilde{\alpha}(k)\}=0,\quad E\{\tilde{\alpha}(k)\tilde{\alpha}(k)\}=\mu^2 \tag{11}$$

, where $\mu=\sqrt{\bar{\alpha}(1-\bar{\alpha})}$.

It can be found that the filtering error system is a system with stochastic parameters since the stochastic variable $\tilde{\alpha}(k)$ is involved in some of the parametric matrices in system (10). Hence, it is necessary to introduce the notion of stochastic stability in the mean square sense that will be instrumental in the sequel.

Definition 1: If for any initial condition when $w_k\equiv 0$, the following inequality is satisfied

$$E\left\{\sum_{k=0}^{\infty}\|\xi_k\|_2^2 \mid \xi_0\right\}<\infty, \tag{12}$$

then the filtering error system (9) is stochastically stable in the mean square.

Now, we can formulate the main problem to be addressed in this paper as follows:

For the switched fuzzy system (6) and a given scalar γ, design a filter of form (9) to ensure that the filtering error system (10) satisfies the following conditions :

1) (Stochastic stability) The filtering error system (10) is stochastically stable.

2) (l_2-l_∞ performance) Under the case of zero-initial conditions, the estimation error e_k satisfies

$$\sup_k\sqrt{E[||e_k||_2^2]}<\gamma\sqrt{\sum_{k=0}^{\infty}||\omega_k||_2^2} \tag{13}$$

for all nonzero $w_k\in l_2[0,\infty)$. Then, we can conclude that the filtering error system (10) is l_2-l_∞ stochastically stable with a guaranteed l_2-l_∞ performance γ.

III. Main results

In this section, the l_2-l_∞ filtering analysis and filter design synthesis for the switched fuzzy system (6) is concerned. In particular, sufficient conditions are derived to ensure the stochastic stability with a given l_2-l_∞ performance for the error system (10) and the resultant filter parameters can be easily achieved by solving a set of LMIs.

Theorem 1: Consider the fuzzy system (6) and assume the filter parameters of the system (9) are known. Then for a given $\gamma>0$, the error system (10) is said to be stochastically stable with the l_2-l_∞ performance γ, if we can find matrices $P_h>0$, $P_{h^+}>0$, $F_{lh}>0$ for any $l\in\{1,2,\cdots,L\}$ and h, $h^+\triangleq(h_1(\theta_{k+1}),h_2(\theta_{k+1}),\cdots,h_r(\theta_{k+1}))\in\rho$, satisfying:

$$\sum_{l=1}^{L}\pi_lF_{lh}<P_h \tag{14}$$

$$\begin{bmatrix}-P_{h^+} & 0 & P_{h^+}A_{1lh} & P_{h^+}B_{1lh}\\ * & -P_{h^+} & \mu P_{h^+}A_{2lh} & \mu P_{h^+}B_{2lh}\\ * & * & -F_{lh} & 0\\ * & * & * & -I\end{bmatrix}<0 \tag{15}$$

$$\begin{bmatrix}-P_h & -\bar{L}_{lh}^T\\ * & -\gamma^2I\end{bmatrix}\le 0. \tag{16}$$

Proof: For simplicity, the proof is divided into two steps, firstly, we prove the stochastic stability of the error system

(10). Toward this end, we choose the following Lyapunov function candidate

$$V_k = \xi_k^T P_h \xi_k. \tag{17}$$

When $w_k \equiv 0$, the filter error system (10) becomes

$$\begin{cases} \xi_{k+1} = (A_{1lh} + \tilde{\alpha} A_{2lh})\xi_k \\ e_k = \bar{L}_{lh}\xi_k \end{cases} \tag{18}$$

Define

$$\Delta V_k = E\{V_{k+1}|\xi_k\} - V_k, \quad V_{k+1} = \xi_{k+1}^T P_{h^+} \xi_{k+1}$$

and we obtain

$$\begin{aligned} \Delta V_k &= E\{V_{k+1}|\xi_k\} - V_k \\ &= E\{\xi_k^T (A_{1lh}^T + \tilde{\alpha}(k) A_{2lh}^T) P_{h^+} (A_{1lh} + \tilde{\alpha}(k) A_{2lh})\xi_k\} \\ &\quad - \xi_k^T P_h \xi_k. \end{aligned}$$

Considering $E\{\pi_{kl_1}\pi_{kl_2}\} = \begin{cases} \pi_{l_1}, & l_1 = l_2 \\ 0, & l_1 \neq l_2 \end{cases}$ and recalling (11), we have

$$\begin{aligned} \Delta V_k &= \xi_k^T [\sum_{l=1}^{L} \pi_l (A_{1lh}^T P_{h^+} A_{1lh} + \mu^2 A_{2lh}^T P_{h^+} A_{2lh})]\xi_k \\ &\quad - \xi_k^T P_h \xi_k. \end{aligned}$$

Applying the Schur complement to (15) we obtain

$$\begin{aligned} &\begin{bmatrix} A_{1lh}^T & \mu A_{2lh}^T \\ B_{1lh}^T & \mu B_{2lh}^T \end{bmatrix} \begin{bmatrix} P_{h^+} & 0 \\ * & P_{h^+} \end{bmatrix} \begin{bmatrix} A_{1lh} & B_{1lh} \\ \mu A_{2lh} & \mu B_{2lh} \end{bmatrix} \\ &\qquad - \begin{bmatrix} F_{lh} & 0 \\ * & I \end{bmatrix} < 0 \end{aligned} \tag{19}$$

which implies

$$A_{1lh}^T P_{h^+} A_{1lh} + \mu^2 A_{2lh}^T P_{h^+} A_{2lh} < F_{lh}$$

and thus

$$\sum_{l=1}^{L} \pi_l (A_{1lh}^T P_{h^+} A_{1lh} + \mu^2 A_{2lh}^T P_{h^+} A_{2lh}) < \sum_{l=1}^{L} \pi_l F_{lh}$$

which further implies

$$E\{\Delta V_k\} < \xi_k^T (\sum_{l=1}^{L} \pi_l F_{lh} - P_h)\xi_k < 0$$

by noting (14). Hence stochastic stability of the error system (10) is guaranteed.

Next, in order to achieve the l_2-l_∞ performance of the error system (10), the following index is introduced:

$$\begin{aligned} J &= \sum_{n=0}^{k-1} E\{\Delta V_n - w_n^T w_n\} \\ &= \sum_{n=0}^{k-1} E\left\{ \begin{bmatrix} \xi_n \\ w_n \end{bmatrix}^T \left(\sum_{l=1}^{L} \pi_l M_l - \begin{bmatrix} P_h & 0 \\ 0 & I \end{bmatrix} \right) \begin{bmatrix} \xi_n \\ w_n \end{bmatrix} \right\}. \end{aligned} \tag{20}$$

where

$$M_l \triangleq \begin{bmatrix} A_{1lh}^T & \mu A_{2lh}^T \\ B_{1lh}^T & \mu B_{2lh}^T \end{bmatrix} \begin{bmatrix} P_{h^+} & 0 \\ * & P_{h^+} \end{bmatrix} \begin{bmatrix} A_{1lh} & B_{1lh} \\ \mu A_{2lh} & \mu B_{2lh} \end{bmatrix}.$$

Then combining (14) with (19) we have

$$\sum_{l=1}^{L} \pi_l M_l - \begin{bmatrix} P_h & 0 \\ 0 & I \end{bmatrix} < 0$$

which implies $J < 0$ for any nonzero $w_k \in l_2[0, \infty)$ under zero-initial conditions, therefore we get

$$E\{\xi_k^T P_h \xi_k\} = E\{V_k\} < \sum_{n=0}^{k-1} w_n^T w_n. \tag{21}$$

On the other hand, according to Schur complement, it is easy to obtain from (16) that

$$\bar{L}_{lh}^T \bar{L}_{lh} < \gamma^2 P_h, \tag{22}$$

from which we know for all $k \geq 0$

$$\begin{aligned} E\{e_k^T e_k\} &= E\{\xi_k^T \bar{L}_{lh}^T \bar{L}_{lh} \xi_k\} \\ &< E\{\xi_k^T \gamma^2 P_h \xi_k\} \\ &< \gamma^2 \sum_{n=0}^{k-1} w_n^T w_n \\ &< \gamma^2 \sum_{n=0}^{\infty} w_n^T w_n, \end{aligned}$$

which means (13) holds under zero initial condition for any nonzero $w_k \in l_2[0, \infty)$, this completes the proof. ■

From the analysis above, we have the following observation. If there is no data loss in the communication channel, that is, signals are transmitted completely between the plant and the filter, then using the same notations as before, the following corollary whose proof is similar with that of Theorem 1 is derived immediately.

Corollary 1: Given the fuzzy system (6) and assume the filter parameters in (9) are known. The error system (10) will be stochastically stable with a given l_2-l_∞ performance γ when $\bar{\alpha} = 1$, if we can find some matrices $P_h > 0$, $P_{h^+} > 0$, $F_{lh} > 0$ for any $l \in \{1, 2, \cdots, L\}$, $h, h^+ \in \rho$ satisfying (14), (16) and

$$\begin{bmatrix} -P_{h^+} & P_{h^+} A_{1lh} & P_{h^+} B_{1lh} \\ * & -F_{lh} & 0 \\ * & * & -I \end{bmatrix} < 0$$

It can be found that there are some product terms between systems matrices and Lyapunov matrices P_{h^+} in the LMI (15), which would bring some difficulties for the filter synthesis in the sequel. To deal with this shortcoming, the following theorem is thus proposed by introducing an extra slack matrix G_l to avoid the cross coupling between systems matrices and Lyapunov matrices P_{h^+}.

Theorem 2: Consider the error system (10), the following two statements are equivalent:

(S1) There exist $P_{h^+} > 0, F_{lh}$ such that (15) hold.

(S1) There exist matrices $P_{h^+} > 0, F_{lh}, G_l$ such that the following inequality holds

$$\begin{bmatrix} \mathcal{S} & 0 & G_l^T A_{1lh} & G_l^T B_{1lh} \\ * & \mathcal{S} & \mu G_l^T A_{2lh} & \mu G_l^T B_{2lh} \\ * & * & -F_{lh} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{23}$$

where $\mathcal{S} := P_{h^+} - G_l - G_l^T$.

Proof: (S1) $\Rightarrow$ (S2): choosing G_l as $G_l = P_{h^+}$, we can conclude (23) from (15) immediately.

(S2) $\Rightarrow$ (S1): Note the inequality

$$[P_{h^+} - G_l]^T P_{h^+}^{-1} [P_{h^+} - G_l] \geq 0.$$

which is equivalent to

$$P_{h^+} - G_l - W_l^T \geq -G_l^T P_{h^+}^{-1} G_l,$$

this combines with (23) implies

$$\begin{bmatrix} -G_l^T P_{h^+}^{-1} G_l & 0 & G_l^T A_{1lh} & G_l^T B_{1lh} \\ * & -G_l^T P_{h^+}^{-1} G_l & \mu G_l^T A_{2lh} & \mu G_l^T B_{2lh} \\ * & * & -F_{lh} & 0 \\ * & * & * & -I \end{bmatrix} < 0.$$

Then by pre-multiplying $\text{diag}\{P_{h^+} G_l^{-T}, P_{h^+} G_l^{-T}, I, I\}$ and post-multiplying $\text{diag}\{G_l^{-1} P_{h^+}, G_l^{-1} P_{h^+}, I, I\}$ to the left and right sides of the above inequality respectively, we obtain the inequality (15). This completes the proof. ∎

Now, based on the results of Theorem 1 and Theorem 2, we are in a position to design the filter in the form of (9) and the detailed result is given as follows:

Theorem 3: Given the fuzzy system (1), there exists a suitable filter in the form of (9) such that error system (10) is stochastically stable with a prescribed l_2-l_∞ performance γ, if we can find matrices

$$\begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix} > 0, \quad \begin{bmatrix} F_{1li} & F_{2li} \\ * & F_{3li} \end{bmatrix} > 0 \tag{24}$$

$G_{1l}, G_{2l}, G_{3l}, \hat{A}_{fli}$, $\hat{B}_{fli}$ and $\hat{L}_{fli}$ for $i, j, s \in \{1, 2, \cdots, r\}$ and any $l \in \{1, 2, \cdots, L\}$, satisfying

$$\sum_{l=1}^{L} \pi_l \begin{bmatrix} F_{1li} & F_{2li} \\ * & F_{3li} \end{bmatrix} < \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix} \tag{25}$$

$$\Xi_{lijs} = \begin{bmatrix} \theta_{11} & \theta_{12} & 0 & 0 & \theta_{13} & \hat{A}_{fli} & \theta_{14} \\ * & \theta_{21} & 0 & 0 & \theta_{22} & \hat{A}_{fli} & \theta_{23} \\ * & * & \theta_{11} & \theta_{12} & \mu\hat{B}_{fli}C_{lj} & 0 & \mu\hat{B}_{fli}D_{lj} \\ * & * & * & \theta_{21} & \mu\hat{B}_{fli}C_{lj} & 0 & \mu\hat{B}_{fli}D_{lj} \\ * & * & * & * & -F_{1li} & -F_{2li} & 0 \\ * & * & * & * & * & -F_{3li} & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} -P_{1i} & -P_{2i} & -L_{li}^T \\ * & -P_{3i} & \hat{L}_{fli}^T \\ * & * & -\gamma^2 I \end{bmatrix} \leq 0 \tag{27}$$

where

$$\theta_{11} = P_{1s} - G_{1l} - G_{1l}^T, \ \theta_{12} = P_{2s} - G_{2l} - G_{3l}^T$$
$$\theta_{13} = G_{1l}A_{li} + \bar{a}\hat{B}_{fli}C_{lj}, \ \theta_{14} = G_{1l}B_{li} + \bar{a}\hat{B}_{fli}D_{lj}$$
$$\theta_{21} = P_{3s} - G_{2l} - G_{2l}^T, \ \theta_{22} = G_{3l}A_{li} + \bar{a}\hat{B}_{fli}C_{lj}$$
$$\theta_{23} = G_{3l}B_{li} + \bar{a}\hat{B}_{fli}D_{lj}.$$

Moreover, the parameters of the filter in (9) can be given by

$$A_{fli} = G_{2l}^{-1}\hat{A}_{fli}, \ B_{fli} = G_{2l}^{-1}\hat{B}_{fli}, \ L_{fli} = \hat{L}_{fli}. \tag{28}$$

Proof: Suppose that we can find matrices P_h, P_{h^+}, $G_{1l}, G_{2l}, G_{3l}, F_{lh}$, $\hat{A}_{flh}$, $\hat{B}_{flh}$, $\hat{L}_{flh}$ satisfying (26) and (27). Utilizing these matrices and $h, h^+ \in \rho$, it is possible for us to define the following matrices:

$$P_h = \sum_{i=1}^{r} h_i \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix}, P_{h^+} = \sum_{s=1}^{r} h_s^+ \begin{bmatrix} P_{1s} & P_{2s} \\ * & P_{3s} \end{bmatrix}$$
$$F_{lh} = \sum_{i=1}^{r} h_i \begin{bmatrix} F_{1li} & F_{2li} \\ * & F_{3li} \end{bmatrix}, \ \hat{A}_{flh} = \sum_{i=1}^{r} h_i \hat{A}_{fli}$$
$$\hat{B}_{flh} = \sum_{i=1}^{r} h_i \hat{B}_{fli}, \ \hat{L}_{flh} = \sum_{i=1}^{r} h_i \hat{L}_{fli},$$

then recalling (7), we have

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r} h_i h_j h_s^+ \Xi_{lijs}$$
$$= \begin{bmatrix} \Theta_{11} & \Theta_{12} & 0 & 0 & \Theta_{13} & \hat{A}_{flh} & \Theta_{14} \\ * & \Theta_{21} & 0 & 0 & \Theta_{22} & \hat{A}_{flh} & \Theta_{23} \\ * & * & \Theta_{11} & \Theta_{12} & \mu\hat{B}_{flh}C_{lh} & 0 & \mu\hat{B}_{flh}D_{lh} \\ * & * & * & \Theta_{21} & \mu\hat{B}_{flh}C_{lh} & 0 & \mu\hat{B}_{flh}D_{lh} \\ * & * & * & * & -F_{1lh} & -F_{2lh} & 0 \\ * & * & * & * & * & -F_{3lh} & 0 \\ * & * & * & * & * & * & -I \end{bmatrix}$$
$$= \begin{bmatrix} \mathcal{S} & 0 & G_l^T A_{1lh} & G_l^T B_{1lh} \\ * & \mathcal{S} & \mu G_l^T A_{2lh} & \mu G_l^T B_{2lh} \\ * & * & -F_{lh} & 0 \\ * & * & * & -I \end{bmatrix} < 0$$

where

$$\Theta_{11} = P_{1h^+} - G_{1l} - G_{1l}^T, \ \Theta_{12} = P_{2h^+} - G_{2l} - G_{3l}^T$$
$$\Theta_{13} = G_{1l}A_{lh} + \bar{a}\hat{B}_{flh}C_{lh}, \ \Theta_{14} = G_{1l}B_{lh} + \bar{a}\hat{B}_{flh}D_{lh}$$
$$\Theta_{21} = P_{3h^+} - G_{2l} - G_{2l}^T, \ \Theta_{22} = G_{3l}A_{lh} + \bar{a}\hat{B}_{flh}C_{lh}$$
$$\Theta_{23} = G_{3l}B_{lh} + \bar{a}\hat{B}_{flh}D_{lh}.$$

and (23) is clearly verified.

Now, it can be seen that (26) is equivalent to (15) according to Theorem 2. Moreover, it is obvious that (25) is equivalent to (14), (27) is equivalent to (16), respectively. This completes the proof. ∎

Corollary 2: Given the fuzzy system (6), it is possible to design a suitable filter of the form (9) such that the error system (10) is stochastically stable with a given l_2-l_∞ performance γ when $\bar{\alpha} = 1$, if we can find matrices $P_h, F_l h, W_{1l}, W_{2l}, W_{3l}, \hat{A}_{fli}$, $\hat{B}_{fli}$ and $\hat{L}_{fli}$ for $i, j, s \in \{1, 2, \cdots, r\}$ and any $l \in \{1, 2, \cdots, L\}$, satisfying (24), (27) and

$$\begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \hat{A}_{fli} & \theta_{14} \\ * & \theta_{21} & \theta_{22} & \hat{A}_{fli} & \theta_{23} \\ * & * & -F_{1li} & -F_{2li} & 0 \\ * & * & * & -F_{3li} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{29}$$

where

$$\theta'_{13} = G_{1l}A_{li} + \hat{B}_{fli}C_{lj}, \ \theta'_{22} = G_{3l}A_{li} + \hat{B}_{fli}C_{lj}.$$

Remark 1: From Theorem 3, it should be noticed that the l_2-l_∞ filtering synthesis discussed in this paper can be boiled down to solving LMIs (25), (26) and (27). Moreover, if these LMIs are feasible, the desired l_2-l_∞ filtering performance index γ^* can be achieved by solving the following:

$$\min \quad \sigma \quad \text{subject to } (25), (26) \text{ and } (27) \text{ with } \sigma = \gamma^2.$$

IV. Verification Examples

In this section, we provide both a practical example and a numerical example to show the effectiveness of the proposed filter design approaches.

A. Example 1

Consider a double-inverted pendulum connected by a spring [42], [43]. The dynamics of the pendulum system are described as

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = \dfrac{1}{100J_i}u_i - \dfrac{kr^2}{4J_i}x_{i1} + [\dfrac{m_i g r}{J_i} - \dfrac{kr^2}{4J_i}x_{i2}]sin(x_{i1}) \\ \qquad + \dfrac{1}{J_i}x_{i2} + \displaystyle\sum_{k=1,k\neq i}^{2} \dfrac{3kr^2}{4J_k}x_{i1} \end{cases}$$

where x_{i1} represents the angle of the pendulum from the vertical, x_{i2} represents the corresponding angular velocity and $i = 1, 2$. As in [43], $J_1 = 2$ kg and $J_2 = 2.5$ kg are moments of inertia; $m_1 = 2$ kg and $m_2 = 2.5$ kg are masses of two pendulums; $k = 8$ N$\cdot$ m/rad is the constant of connecting torsional spring; $r = 1$ denotes the length of the pendulum and $g = 9.8$ m/s^2 is the gravity constant.

Given $u_1 = -18x_{11} - 16x_{12}$ and $u_2 = -20x_{21} - 14x_{22}$, it can be checked immediately that the system is stable. Linearizing the physical system around $x_{i1} = (0, 0)$, $x_{i1} = (\pm 80^\circ, 0)$, $x_{i1} = (\pm 88^\circ, 0)$, then discretizing it with sampling period $T = 0.01$ s, finally, the following T-S fuzzy system is obtained by ignoring interconnected matrices:

Plant rule i: IF $|x_1(k)|$ is M^i, THEN

$$\begin{cases} x_{k+1} = A_{r_k i}x_k + B_{r_k i}w_k \\ y_k = C_{r_k i}x_k + D_{r_k i}w_k \\ z_k = L_{r_k i}x_k, r_k = \{1, 2\}, i = \{1, 2, 3\} \end{cases}$$

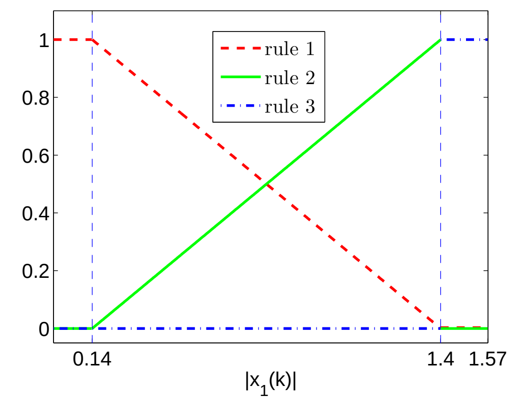

Fig. 2. Membership functions

where

$$A_{11} = \begin{bmatrix} 1 & 0.0120 \\ -1.3200 & -0.1540 \end{bmatrix}, A_{12} = \begin{bmatrix} 1 & 0.0120 \\ -1.1818 & -0.1658 \end{bmatrix}$$
$$A_{13} = \begin{bmatrix} 1 & 0.0120 \\ -1.3760 & -0.0352 \end{bmatrix}, B_{11} = B_{12} = B_{13} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$$
$$C_{11} = C_{12} = C_{13} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_{11} = D_{12} = D_{13} = 1$$
$$L_{11} = L_{12} = L_{13} = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

for the first subsystem and

$$A_{21} = \begin{bmatrix} 1 & 0.0120 \\ -1.3760 & -0.0352 \end{bmatrix}, A_{22} = \begin{bmatrix} 1 & 0.0120 \\ -1.2378 & -0.0447 \end{bmatrix}$$
$$A_{23} = \begin{bmatrix} 1 & 0.0120 \\ -1.2485 & -0.0448 \end{bmatrix}, B_{21} = B_{22} = B_{23} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}$$
$$C_{21} = C_{22} = C_{23} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_{21} = D_{22} = D_{23} = 1$$
$$L_{21} = L_{22} = L_{23} = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

for the second subsystem and membership functions are shown in Fig.2.

Choosing switching probabilities as $\pi_1 = 0.3, \pi_2 = 0.7$ (which is shown in Fig.3) and the expectation of the Bernoulli sequence is assumed to be $\bar{\alpha} = 0.8$. Then the following filter parameters are obtained by solving LMIs in Theorem 3:

Filter 1:

$$A_{f11} = \begin{bmatrix} 1.0009 & 0.0120 \\ -1.0807 & -0.1509 \end{bmatrix}, A_{f12} = \begin{bmatrix} 1.0003 & 0.0120 \\ -0.9397 & -0.1628 \end{bmatrix}$$
$$A_{f13} = \begin{bmatrix} 1.0011 & 0.0120 \\ -1.1484 & -0.0318 \end{bmatrix}, B_{f11} = \begin{bmatrix} 0.0002 \\ -0.4865 \end{bmatrix}$$
$$B_{f12} = \begin{bmatrix} -0.0001 \\ -0.4862 \end{bmatrix}, \quad B_{f13} = \begin{bmatrix} 0.0002 \\ -0.4829 \end{bmatrix}$$
$$L_{f11} = \begin{bmatrix} 0.0238 & -0.9991 \end{bmatrix}, \quad L_{f12} = \begin{bmatrix} 0.0260 & -0.9989 \end{bmatrix}$$
$$L_{f13} = \begin{bmatrix} 0.0201 & -0.9991 \end{bmatrix}.$$

Filter 2:

$$A_{f21} = \begin{bmatrix} 1.0010 & 0.0120 \\ -1.3991 & -0.0337 \end{bmatrix}, A_{f22} = \begin{bmatrix} 1.0007 & 0.0120 \\ -1.2583 & -0.0431 \end{bmatrix}$$

$$A_{f23} = \begin{bmatrix} 1.0010 & 0.0120 \\ -1.2684 & -0.0432 \end{bmatrix}, B_{f21} = \begin{bmatrix} 0.0007 \\ -0.2530 \end{bmatrix}$$

$$B_{f22} = \begin{bmatrix} 0.0005 \\ -0.2515 \end{bmatrix}, \quad B_{f23} = \begin{bmatrix} 0.0005 \\ -0.2499 \end{bmatrix}$$

$$L_{f21} = \begin{bmatrix} 0.0238 & -0.9991 \end{bmatrix}, \quad L_{f22} = \begin{bmatrix} 0.0260 & -0.9989 \end{bmatrix}$$

$$L_{f23} = \begin{bmatrix} 0.0201 & -0.9991 \end{bmatrix}$$

with the corresponding l_2-l_∞ performance $\gamma^* = 0.2254$.

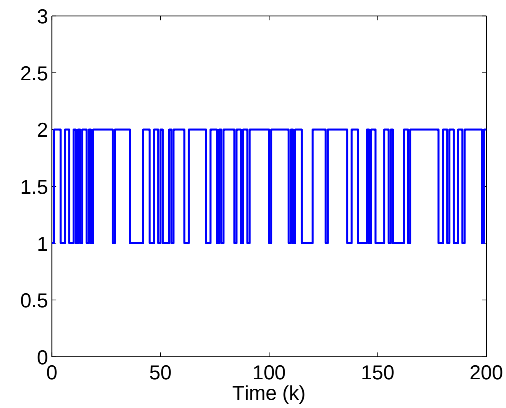

Fig. 3. The switching signal

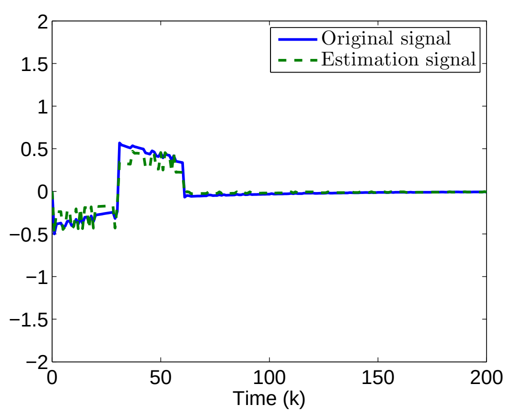

Fig. 4. Responses of signals

In order to validate the l_2-l_∞ performance of the designed filter, the external disturbance w_k is given as

$$w(k) = \begin{cases} -1, & 1 \leq k \leq 30 \\ 1, & 31 < k \leq 60 \\ 0, & \text{elsewhere} \end{cases}$$

Simulation results of the estimation signal and the filtering error with zero-initial conditions are depicted in Fig.4 and Fig.5, respectively, which show the good performance of the designed filter.

Next, to show the influence of the stochastic variable $\alpha(k)$ on the l_2-l_∞ optimal performance γ^* and the advantage of the basis-dependent Lyapunov function (BDLF) based method over the common Lyapunov function (CLF) based method, the follow-up numerical example is presented.

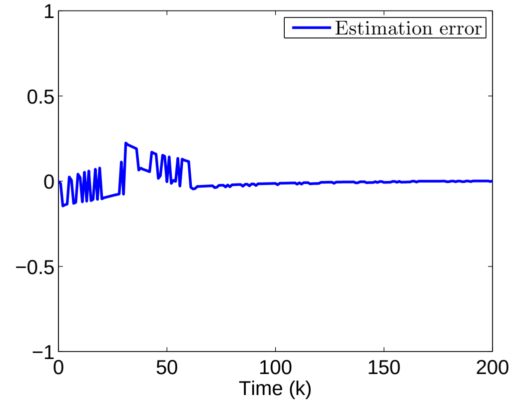

Fig. 5. The filtering error

B. Example 2

Consider a numerical example in the form of (6) with the following models:

Mode 1:

$$A_{11} = \begin{bmatrix} 0.60 & 0.41 \\ -0.11 & 0.43 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0.80 & 0.08 \\ -0.19 & 0.61 \end{bmatrix}$$

$$A_{13} = \begin{bmatrix} 0.88 & 0.09 \\ -0.17 & 0.69 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 0.11 \\ 0.19 \end{bmatrix}$$

$$B_{12} = \begin{bmatrix} 0.21 \\ 0.29 \end{bmatrix}, \quad B_{13} = \begin{bmatrix} 0.27 \\ 0.29 \end{bmatrix}$$

$$C_{11} = C_{12} = C_{13} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_{11} = D_{12} = D_{13} = 1$$

$$L_{11} = L_{12} = L_{13} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Mode 2:

$$A_{21} = \begin{bmatrix} 0.60 & 0.51 \\ -0.18 & 0.42 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0.91 & 0.09 \\ -0.13 & 0.70 \end{bmatrix}$$

$$A_{23} = \begin{bmatrix} 0.95 & 0.08 \\ -0.13 & 0.69 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 0.30 \\ 0.20 \end{bmatrix}$$

$$B_{22} = \begin{bmatrix} 0.10 \\ 0.20 \end{bmatrix}, \quad B_{23} = \begin{bmatrix} 0.21 \\ 0.24 \end{bmatrix}$$

$$C_{21} = C_{22} = C_{23} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_{21} = D_{22} = D_{23} = 1$$

$$L_{21} = L_{22} = L_{23} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The fuzzy weighting functions are given as

$$h_1 = \begin{cases} \dfrac{x_{1k}+4}{4}, & -4 < x_{1k} \leq 0 \\ \dfrac{-x_{1k}+4}{4}, & 0 < x_{1k} < 4 \\ 0, & \text{elsewhere} \end{cases}$$

$$h_2 = \frac{1}{2} h_1, \quad h_3 = 1 - h_1 - h_2.$$

By choosing switching probabilities as $\pi_1 = 0.2, \pi_2 = 0.8$ (which is shown in Fig.6) and the expectation of the Bernoulli sequence is assumed to be $\bar{\alpha} = 0.9$. Then the following filter parameters are achieved by solving LMIs in Theorem 3:

Filter 1:

$$A_{f11} = \begin{bmatrix} 0.6482 & 0.4417 \\ -0.1687 & 0.4746 \end{bmatrix}, A_{f12} = \begin{bmatrix} 0.6472 & 0.3773 \\ -0.2499 & 0.6504 \end{bmatrix}$$
$$A_{f13} = \begin{bmatrix} 0.6720 & 0.2947 \\ -0.2723 & 0.5539 \end{bmatrix}, B_{f11} = \begin{bmatrix} -0.0670 \\ -0.0308 \end{bmatrix}$$
$$B_{f12} = \begin{bmatrix} -0.1704 \\ -0.1168 \end{bmatrix}, \quad B_{f13} = \begin{bmatrix} -0.1878 \\ -0.1785 \end{bmatrix}$$
$$L_{f11} = \begin{bmatrix} -0.9714 & 0.0213 \end{bmatrix}, \quad L_{f12} = \begin{bmatrix} -0.9716 & 0.0218 \end{bmatrix}$$
$$L_{f13} = \begin{bmatrix} -0.9714 & 0.0212 \end{bmatrix}.$$

Filter 2:

$$A_{f21} = \begin{bmatrix} 0.3900 & 0.5310 \\ -0.2125 & 0.4262 \end{bmatrix}, A_{f22} = \begin{bmatrix} 0.6567 & 0.2189 \\ -0.2157 & 0.7310 \end{bmatrix}$$
$$A_{f23} = \begin{bmatrix} 0.6075, & 0.2867 \\ -0.3073 & 0.7304 \end{bmatrix}, B_{f21} = \begin{bmatrix} -0.2793 \\ -0.0709 \end{bmatrix}$$
$$B_{f22} = \begin{bmatrix} -0.2255 \\ -0.1538 \end{bmatrix}, \quad B_{f23} = \begin{bmatrix} -0.1574 \\ -0.2071 \end{bmatrix}$$
$$L_{f21} = \begin{bmatrix} -0.9714 & 0.0213 \end{bmatrix}, \quad L_{f22} = \begin{bmatrix} -0.9716 & 0.0218 \end{bmatrix}$$
$$L_{f23} = \begin{bmatrix} -0.9714 & 0.0212 \end{bmatrix}$$

with the corresponding l_2-l_∞ performance $\gamma^* = 0.1575$.

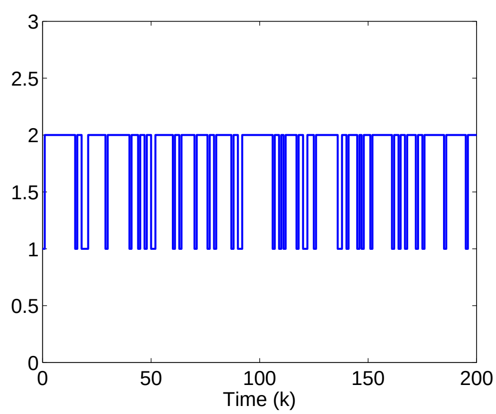

Fig. 6. The switching signal

The external disturbance w_k is assumed to be

$$w_k = 3\text{rand}(1)/(1 + 0.1k)$$

and response curves of the estimation signal and the filtering error with zero-initial conditions are depicted in Fig.7 and Fig.8, respectively.

It is noted that the optimal l_2-l_∞ performance based on BDLF can be calculated directly from Remark 1 and the calculation of the corresponding optimal performance based on CLF follows similarly by setting P_h and P_{h+} as constants. Moreover, by regulating the value of $\bar{\alpha}$, we can obtain the corresponding optimal l_2-l_∞ performance γ^* based on BDLF

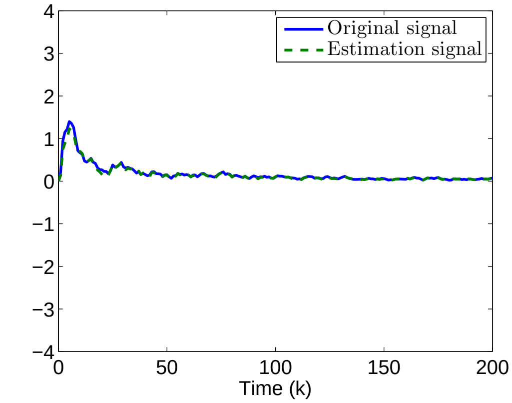

Fig. 7. Responses of signals

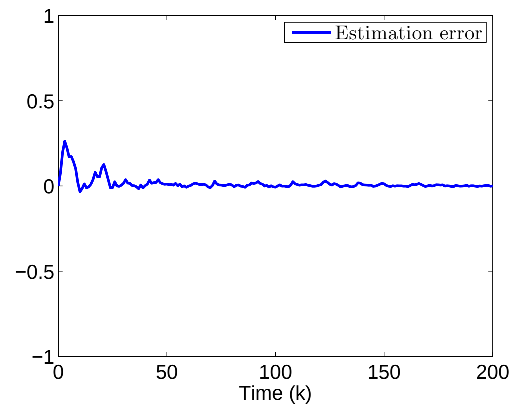

Fig. 8. The filtering error

and CLF, respectively. For ease of reference, the results are shown in Table I.

Finally, it can be noticed from Table I that under the same value of $\bar{\alpha}$, BDLF based method outperforms CLF based method indeed, in additional, we can see clearly that the performance γ^* increases as $\bar{\alpha}$ decreases, which is expected since the decrease of $\bar{\alpha}$ means data loss rate increases.

V. Conclusion

In this paper, the problem of l_2-l_∞ filter design problem for discrete-time nonlinear switched T-S fuzzy systems with missing measurements has been investigated. Switching phenomena and missing measurements are considered together. An LMI approach has been introduced to ensure the stochastic stability with a given l_2-l_∞ filtering performance index of the filtering error system. Meanwhile, the filter parameters can be achieved easily by solving a set of LMIs. Finally, two illustrative examples are presented to verify the effectiveness of the developed theoretical results. It should be noted that in practical NCSs, there exist many important topics such

TABLE I. Comparison of optimal performance with different $\bar{\alpha}$

$\bar{\alpha}$	0.9	0.8	0.7	0.6
γ^* of BDLF based method	0.1575	0.2098	0.2496	0.2848
γ^* of CLF based method	0.2124	0.2612	0.3025	0.3375

as uncertainties and time delay, which can also influence the performance of systems and even lead to systems instability. Throughout this paper we have considered the phenomenon of missing measurements only, hence the investigation of filtering for fuzzy systems with uncertainties and time delay deserve further study in our future work.

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