Asychronous Control for Markov Jump Lure's Systems

Abstract—**This paper is concerned with the control problems for a class of discrete-time Lur'e systems with asynchronous controller. A hidden Markov model (HHM) is introduced to describe the asynchronous phenomenon between the systems mode and controller modes. The linear matrix inequality (LMI) approach is utilized to analyze the stability of the closed-loop system and l_2-gain performance. Then a sufficient condition is proposed in the form of LMI to guarantee the stochastic stability of the closed-loop system, and to minimize the obtained l_2-gain from the disturbance to output, further more the asynchronous controller can be designed by solving the given conditions. A simulation example is given to demonstrate the effectiveness of the proposed method.**

I. Introduction

As an important class of stochastic switching systems, Markov jump systems (MJSs) have received considerable attention because of its powerful ability in modeling systems with abrupt change in parameters or system structures, for instance, environmental disturbances, actuators failures, and variations in subsystems interconnections. Over the past few decades, a large number of works for stability analysis and the controller/filter design have been published[1234]. For example,

In most existing works, it was implicitly assumed that the information of system modes can be fully accessible for the controller/filter all the time, so that controller/filter modes can run synchronously with system modes. Unfortunately, in practical applications, it is rather difficult to satisfy this ideal assumption because of some unexpected factors, for instance, time delays, data dropouts and quantization in network control systems. In order to overcome this strict limitation, two research approach on control/filtering of MJSs were proposed, namely, mode-independent and asynchronous methods. In mode-independent methods[2][3], the controller modes is independent with plant modes, which means have no use of system information and may result in some conservatism. Therefore, asynchronous methods on control/filtering problems for MJSs gain more attention in recent years. For example, a piecewise homogeneous Markov chain was utilized to design an asynchronous l_2-l_∞ filter for discrete-time MJSs with randomly occurred sensor nonlinearities in [5]. After that, in [4], a new strategy based on HMM framework was adopted in passivity-based asynchronous control problems, which covers the synchronous situation. Very recently, many works on asynchronous control/filtering problem of MJSs have been published based on this powerful model. For instance, asynchronous filter design of discrete-time Markov Jump Systems was discussed in [7][9], the same issue was discussed in continuous-time domain in [8], and asynchronous l_2-$l\infty$ control problem for discrete-time Markovian jump linear systems with partly accessible controller mode information was addressed in [10]

However, as we can know, the issue of stability and stabilization for discrete-time Lur'e systems with asynchronous controller have not been discussed yet, which remains to be explored.

On the other hand, the so called Lure system [7], which is consisted of a linear part and a sector-bounded nonlinearity part, has gained considerable attentions.

II. Preliminaries

Consider a class of discrete-time MJLS on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{cases} x_{k+1} = A_{r(k)}x_k + F_{r(k)}\varphi(y_k) + B_{r(k)}u_k \\ \qquad + E^x_{r(k)}w_k, \quad y_k = C_{r(k)}x_k, \\ z_k = C^z_{r(k)}x_k + G^z_{r(k)}\varphi(y_k) + D^z_{r(k)}u_k \\ \qquad + E^z_{r(k)}w_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}, u_k \in \mathbb{R}^{n_u}, y_k \in \mathbb{R}^{n_y}, z_k \in \mathbb{R}^{n_z}$, and $w_k \in \mathbb{R}^{n_w}$ represent the state, the control input, the output related to the nonlinearity, the controlled output and the exogenous disturbance vector respectively. $A_{r(k)}, F_{r(k)}, B_{r(k)}, E^x_{r(k)}, C^z_{r(k)}, G^z_{r(k)}, D^z_{r(k)}$ and $E^z_{r(k)}$ represent the time-varying system matrices, all of which are pre-known and real. $\{r(k), k \geq 0\}$ is a Markov chain taking values in a positive integer set $\mathcal{N} = \{1, 2, \ldots, N\}$ with mode TPs:

$$\Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij} \tag{2}$$

Clearly, for all $i, j \in \mathcal{N}$, $\pi_{ij} \in [0, 1]$, and $\sum_{j=0}^{N} \pi_{ij} = 1$ for each mode i. The system matrices in (1) at instant k can be expressed as $A_i, F_i, B_i, E^x_i, C^z_i, G^z_i, D^z_i$ and E^z_i, which are real known constant matrices with appropriate dimensions, and the related transition probability matrix is described as $\Pi = \{\pi_{ij}\}$.

Assumption 1: The function $\varphi(\cdot) : \mathbb{R}^{n_y} \to \mathbb{R}^{n_y}$ satisfies:
(1) $\varphi(0) = 0$ and
(2) there exists a diagonal positive definite matrix $\Omega \in \mathbb{R}^{p\times p}$ such that:

$$\varphi_{(\ell)}(y)[\varphi(y) - \Omega y]_{(\ell)} \leq 0. \tag{3}$$

where for all $y \in \mathbb{R}^p$, $\ell \in \{1, \ldots, p\}$. And in this case we say that the nonlinearity $\varphi(\cdot)$ satisfies it's own cone bounded sector conditions and to be decentralized. According to (3) we have:

$$SC(y,\Lambda) := \varphi^{'}(y)\Lambda[\varphi(y) - \Omega y] \leq 0, \tag{4}$$

where Λ is any diagonal positive semidefinite matrix, and Ω is pre-given. From the Assumption 1, we get:

$$[\Omega y]_{(\ell)}[\varphi(y) - \Omega y]_{(\ell)} \leq 0, \tag{5}$$

which implies:

$$0 \leq \varphi^{'}(y)\Lambda\varphi(y) \leq \varphi^{'}(y)\Lambda\Omega y \leq y^{'}\Omega\Lambda\Omega y \tag{6}$$

Definition 1: System (1) is said to be locally stochastically stable if for $w_(k) = 0$ and any initial condition $x_0 \in \mathcal{D}_0$, $r_0 \in \mathcal{N}$, the following formulation holds:

$$\|x\|_2^2 = \sum_{k=0}^{\infty} \mathbb{E}[\|x_k\|^2] < \infty. \tag{7}$$

Where in this case the set $\mathcal{D}_0 \subset \mathbb{R}^p$ is said to be the domain of stochastic stability of the origin.

Define $\mathcal{F}_k$ as the σ-field generated by the random variables x_k and $r(k)$. We define next the class of exogenous disturbances with bounded energy and the finite ℓ_2-induced norm.

Definition 2: For $\zeta > 0$, set $\mathcal{W}_\zeta$ is defined as follows:

$$\begin{aligned} &\mathcal{W}_\zeta := \{w = \{w_k\}_{k\in\mathbb{N}}; w_k \in \mathbb{R}^m, k \in \mathbb{N}, w_k \text{ is} \\ &\quad \mathcal{F}_k\text{-measureable, and } \|w\|_2^2 = \sum_{k=0}^{\infty}\mathbb{E}(\|w_k\|)^2 < \zeta. \end{aligned} \tag{8}$$

The finite ℓ_2-induced gain associated with the closed-loop system (1) with $x_0 = 0$ between the disturbance $w = \{w_k\}_{k\in\mathbb{N}}$ and controlled output $z = \{z_k\}_{k\in\mathbb{N}}$ is equal or less than γ if for every $w \in \mathcal{W}_\zeta$ then we have

$$\|z\|_2^2 = \sum_{k=0}^{\infty}\mathbb{E}\left[\|z_k\|^2\right] \leq \gamma^2\|w\|_2^2 = \gamma^2\sum_{k=0}^{\infty}\mathbb{E}\left[\|w_k\|^2\right]. \tag{9}$$

In this paper, we consider the following controller:

$$u_k = K_{\sigma(k)}x_k + \Gamma_{\sigma(k)}\varphi(y_k) \tag{10}$$

where $K_{\sigma(k)} \in \mathbb{N}^{n_u\times n_x}$ is a time-varying controller gain matrix, and $\Gamma_{\sigma(k)} \in \mathbb{N}^{n_u\times n_y}$ is a time-varying nonlinear output feedback gain matrix. The parameter $\{\sigma(k), k \geq 0\}$ takes values in another pre-given positive integer set, which is marked as $\mathcal{M} = \{1, 2, \ldots, M\}$ subject to the pre-known conditional probability matrix $\Phi = \{\mu_{im}\}$, the probabilities of which are defined by

$$\Pr\{\sigma(k) = \phi | r(k) = i\} = \mu_{i\phi}. \tag{11}$$

Where for all $i \in \mathcal{N}, \phi \in \mathcal{M}, \mu_{i\phi} \in [0,1]$, and $\sum_{\phi=1}^{M}\mu_{i\phi} = 1$ for each $i \in \mathcal{N}$.

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Combing the asynchronous controller (10) and system (1) we have the following closed system:

$$\begin{cases} x_{k+1} = \bar{A}_{i\theta}x_k + \bar{F}_{i\theta}\varphi_i(C_ix_k) + E_i^x w_k \\ z_k = \bar{C}_{i\theta}x_k + \bar{G}_{i\theta}\varphi_i(C_ix_k) + E_i^z w_k \end{cases} \tag{12}$$

Where,for $i \in \mathcal{N}, \phi \in \mathcal{M}$

$$\begin{aligned} \bar{A}_{i\phi} &= A_i + B_iK_\phi, & \bar{F}_{i\phi} &= F_i + B_i\Gamma_{i\phi} \\ \bar{C}_{i\phi}^z &= C_i^z + D_i^zK_\phi, & \bar{G}_{i\phi}^z &= G_i^z + D_i^z\Gamma_\phi \end{aligned}$$

III. Main Result

$$\begin{cases} x_{k+1} = \bar{A}_{i\theta}x_k + \bar{F}_{i\theta}\varphi_i(C_ix_k) + E_i^x w_k \\ z_k = \bar{C}_{i\theta}x_k + \bar{G}_{i\theta}\varphi_i(C_ix_k) + E_i^z w_k \end{cases}$$

System (12) with $x_0 = 0$ and $w_k = 0$ is stochastically stable, if for all $i \in \mathcal{N}$ and $\phi \in \mathcal{M}$, there exist positive definite matrices $\bar{P}_i \in \mathbb{R}^{n_x\times n_x}$, $R_{i\phi} \in \mathbb{R}^{(n_x+n_y)\times(n_x+n_y)}$, matrices $K_\phi \in \mathbb{R}^{n_u\times n_x}$, $\Gamma_\phi \in \mathbb{R}^{n_u\times n_y}$ and positive semidefinite matrix $T_i \in \mathbb{R}^{n_y}$ to ensure (13) and (14) hold.

$$\begin{bmatrix} -R_{i\theta} & \mathscr{H}_{i\phi} \\ * & \mathscr{P}_i \end{bmatrix} < 0 \tag{13}$$

$$\begin{bmatrix} \mathscr{S}_{i\phi} & \mathscr{N}_{i\phi} \\ * & \mathscr{L}_{i\phi} \end{bmatrix} < 0 \tag{14}$$

where

$$\mathscr{H}_{i\phi} = \begin{bmatrix} \sqrt{\pi_{i1}}\hat{A}_{i\phi} \\ \sqrt{\pi_{i2}}\hat{A}_{i\phi} \\ \vdots \\ \sqrt{\pi_{i_N}}\hat{A}_{i\phi} \end{bmatrix}^{'}, \mathscr{S}_{i\phi} = \begin{bmatrix} -\bar{P}_i & 0 \\ * & H_{i\phi} \end{bmatrix}$$

$$H_{i\theta} = \begin{bmatrix} -I_{n_x} & C_i^{'}\Omega_iT_i \\ * & -2T_i \end{bmatrix}, \hat{A}_{i\theta} = \begin{bmatrix} \bar{A}_{i\theta} & \bar{F}_{i\theta} \end{bmatrix}$$

$$\mathscr{P}_{i\phi} = \operatorname{diag}\{-\bar{P}_1, -\bar{P}_2, \ldots, -\bar{P}_N\}$$

$$\mathscr{N}_{i\phi} = \begin{bmatrix} \sqrt{u_{i1}}\begin{bmatrix} \bar{P}_i & 0 \\ * & R_{i1} \end{bmatrix} & \cdots & \sqrt{u_{iM}}\begin{bmatrix} \bar{P}_i & 0 \\ * & R_{iM} \end{bmatrix} \end{bmatrix}$$

$$\mathscr{L}_{i\phi} = \operatorname{diag}\left\{ \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i1} \end{bmatrix}, \ldots, \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{iM} \end{bmatrix} \right\}$$

Proof: Construct a Lyapunov function in the form of (15).

$$V(k, x_k, r(k)) = x_k^{'}P_{r(k)}x_k \tag{15}$$

Where $P_{r(k)} = \bar{P}_{r(k)}^{-1}$. Let $\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{V(k+1, x_{k+1}, r(k+1) = j | x_k, r(k) = i\} - V(k, x_k, i)$ and it is easy to find that

$$\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{x_{k+1}^{'}X_ix_{k+1}\} - V(k, x_k, i) \tag{16}$$

Where $X_i = \sum_{j=1}^{N}\pi_{ij}P_j$. Based on system (12) we can further obtain

$$\begin{aligned} \mathbb{E}\{x_{k+1}^{'}X_ix_{k+1}\} &= \sum_{\phi=1}^{M}u_{i\phi}\hat{x}_k^{'}\hat{A}_{i\phi}^{'}X_i\hat{A}_{i\phi}\hat{x}_k \\ &= \hat{x}_k^{'}\Big(\sum_{\phi=1}^{M}u_{i\phi}\hat{A}_{i\phi}^{'}X_i\hat{A}_{i\phi}\Big)\hat{x}_k \end{aligned} \tag{17}$$

Where $\hat{x}_k = (x_k^{'}, \varphi_i^{'}(C_i x_k))^{'}$. Which implies

$$\mathbb{E}\{\varDelta V(k)\} = \hat{x}_k^{'}\Big(\sum_{\phi=1}^{M} u_{i\phi}\hat{A}_{i\phi}^{'}X_i\hat{A}_{i\phi}\Big)\hat{x}_k - V(k, x_k, i) \quad (18)$$

Denoting $h_{i\phi} = \mathrm{diag}\{P_i, I_{n_x+n_y}, \{\mathrm{diag}(I_{n_x}, R_{i\phi}^{-1})_{\phi=1}^{M}\}\}$ and using $h_{i\phi}$ to pre- and post-multiplying the inequality given in (14), respectively, then we can get

$$\begin{bmatrix} \hat{\mathscr{S}}_{i\phi} & \sqrt{u_{i1}}I & \cdots & \sqrt{u_{iM}}I \\ \sqrt{u_{i1}}I & L_{i1} & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \sqrt{u_{iM}}I & 0 & 0 & L_{iM} \end{bmatrix} < 0$$

Where

$$\hat{\mathscr{S}}_{i\phi} = \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix}, L_{i\phi} = \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i\phi}^{-1} \end{bmatrix}$$

Applying Schur complement, we can get

$$\sum_{\phi=1}^{M} u_{i\phi}\begin{bmatrix} I & 0 \\ * & R_{i\phi} \end{bmatrix} + \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix} < 0 \quad (19)$$

(19) multiply $(x_k^{'}, \hat{x}_k^{'})^{'}$ on left and it's transpose on the right. We get

$$\hat{x}_k^{'}\sum_{\phi=1}^{M} u_{i\phi}R_{i\phi}\hat{x}_k - x_k^{'}P_i x_k - 2SC(i, x_k, T_i) < 0 \quad (20)$$

Applying Schur complement to inequality (13). We can obtain

$$\hat{A}_{i\phi}X_i\hat{A}_{i\phi} < R_{i\phi} \quad (21)$$

Combining inequalities (20) and (21), it's easy to find that

$$\begin{aligned} &\hat{x}_k^{'}\sum_{\phi=1}^{M} u_{i\phi}(\hat{A}_{i\phi}X_i\hat{A}_{i\phi})\hat{x}_k - x_k^{'}P_i x_k \\ &\qquad - 2SC(i, x_k, T_i) < 0 \end{aligned} \quad (22)$$

Combining inequalities (18) and (22), it is easy to obtain that

$$\mathbb{E}\{\varDelta V(k)\} - 2SC(i, x_k, T_i) < 0 \quad (23)$$

Furthermore, noticing the condition given in (4), we further have $\mathbb{E}\{\varDelta V(k)\} < 0$, which completes the proof.

Theorem 2: For each mode $i \in \mathcal{N}$ and $\phi \in \mathcal{M}$, if there exist positive-definite matrices $\bar{P}_i \in \mathbb{R}^{n_x \times n_x}$ and $R_{i\phi} \in \mathbb{R}^{(n_x+n_y+n_w)\times(n_x+n_y+n_w)}$, matrices $K_\phi \in \mathbb{N}^{n_u \times n_x}, \Gamma_\phi \in \mathbb{R}^{n_u \times n_y}$, positive semi-definite matrix $T_i \in \mathbb{R}^{n_y \times n_y}$ and a positive scalar γ, such that the following LMIs are verified

$$\begin{bmatrix} -R_{i\theta} & \mathscr{H}_{i\phi} \\ * & \mathscr{P}_i \end{bmatrix} < 0 \quad (24)$$

$$\begin{bmatrix} \mathscr{S}_{i\phi} & \mathscr{N}_{i\phi} \\ * & \mathscr{L}_{i\phi} \end{bmatrix} < 0 \quad (25)$$

Where

$$\mathscr{H}_{i\phi} = \begin{bmatrix} \sqrt{\pi_{i1}}\hat{A}_{i\phi} \\ \sqrt{\pi_{i2}}\hat{A}_{i\phi} \\ \vdots \\ \sqrt{\pi_{iN}}\hat{A}_{i\phi} \\ \hat{A}_{i\phi}^{z} \end{bmatrix}^{'}, \quad \mathscr{S}_{i\phi} = \begin{bmatrix} -\bar{P}_i & 0 \\ * & H_{i\phi} \end{bmatrix}$$

$$\mathscr{P}_{i\phi} = \mathrm{diag}\{-\bar{P}_1, -\bar{P}_2, \ldots, -\bar{P}_N, -I_{n_w}\}$$

$$\hat{A}_{i\theta} = \begin{bmatrix} \bar{A}_{i\theta} & \bar{F}_{i\theta} & E_i^{x} \end{bmatrix}, \quad \hat{A}_{i\theta}^{z} = \begin{bmatrix} \bar{C}_{i\theta}^{z} & \bar{G}_{i\theta}^{z} & E_i^{z} \end{bmatrix}$$

$$\mathscr{N}_{i\phi} = \begin{bmatrix} \sqrt{u_{i1}}\begin{bmatrix} \bar{P}_i & 0 \\ * & R_{i1} \end{bmatrix} & \cdots & \sqrt{u_{iM}}\begin{bmatrix} \bar{P}_i & 0 \\ * & R_{iM} \end{bmatrix} \end{bmatrix}$$

$$\mathscr{L}_{i\phi} = \mathrm{diag}\left\{\begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i1} \end{bmatrix}, \ldots, \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{iM} \end{bmatrix}\right\}$$

$$H_{i\theta} = \begin{bmatrix} -I_{n_x} & C_i^{'}\varOmega_i T_i & 0 \\ * & -2T_i & 0 \\ * & * & -\gamma^2 I_{n_w} \end{bmatrix}$$

then system (12) with $x_0 = 0$ and $w \in \mathcal{W}_\gamma$ is stochastically stable and the l_2-gain of the closed-loop system is strictly less or equal to γ.

Proof: We first select (16) as the Lyapunov function and $P_{r(k)} = \bar{P}_{r(k)}^{-1}$. Similar to the proof of theorem 1, it's easy to find that

$$\mathbb{E}\{\varDelta V(k)\} = \hat{x}_k^{'}\Big(\sum_{\phi=1}^{M} u_{i\phi}\hat{A}_{i\phi}^{'}X_i\hat{A}_{i\phi}\Big)\hat{x}_k - V(k, x_k, i) \quad (26)$$

Where $\hat{x}_k = (x_k^{'}, \varphi_i^{'}(C_i x_k), w_k^{'})^{'}$, $X_i = \sum_{j=1}^{N}\pi_{ij}P_j$

Denoting $h_{i\phi} = \mathrm{diag}\{P_i, I_{n_x+n_y+n_w}, \{\mathrm{diag}(I_{n_x}, R_{i\phi}^{-1})_{\phi=1}^{M}\}\}$ and using $h_{i\phi}$ to pre- and post-multiplying the inequality given in (25), respectively, then we can obtain

$$\begin{bmatrix} \hat{\mathscr{S}}_{i\phi} & \sqrt{u_{i1}}I & \cdots & \sqrt{u_{iM}}I \\ \sqrt{u_{i1}}I & L_{i1} & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \sqrt{u_{iM}}I & 0 & 0 & L_{iM} \end{bmatrix} < 0$$

Where

$$\hat{\mathscr{S}}_{i\phi} = \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix}, L_{i\phi} = \begin{bmatrix} -I_{n_x} & 0 \\ * & -R_{i\phi}^{-1} \end{bmatrix}$$

Noting that $L_{i\phi} < 0$ for $\phi \in \mathcal{M}$, apply Schur complement to the inequality above, we can get

$$\sum_{\phi=1}^{M} u_{i\phi}\begin{bmatrix} I & 0 \\ * & R_{i\phi} \end{bmatrix} + \begin{bmatrix} -P_i & 0 \\ * & H_{i\phi} \end{bmatrix} < 0 \quad (27)$$

(27) multiply $(x_k^{'}, \hat{x}_k^{'})^{'}$ on left and it's transpose on the right. We get

$$\hat{x}_k^{'}(\sum_{\phi=1}^{M} u_{i\phi}R_{i\phi})\hat{x}_k - x_k^{'}P_i x_k - 2SC(i, x_k, T_i) - \gamma^2 w_k^{'}w_k < 0 \quad (28)$$

Applying Schur complement to inequality (24). We can obtain

$$\hat{A}_{i\phi}X_i\hat{A}_{i\phi} + (\hat{A}^z_{i\phi})'\hat{A}^z_{i\phi} < R_{i\phi} \tag{29}$$

Combining inequalities (28) and (29), it's easy to find that

$$\begin{aligned}\hat{x}_k'\Big(\sum_{\phi=1}^{M}u_{i\phi}(\hat{A}_{i\phi}X_i\hat{A}_{i\phi} + (\hat{A}^z_{i\phi})'\hat{A}^z_{i\phi})\Big)\hat{x}_k - x_k'P_ix_k \\ - 2SC(i,x_k,T_i) - \gamma^2 w_k'w_k < 0\end{aligned} \tag{30}$$

Which implies

$$\mathbb{E}\{\varDelta V(k)\} + \mathbb{E}\{z_k'z_k\} - \gamma^2 w_k'w_k - 2SC(i,x_k,T_i) < 0 \tag{31}$$

Furthermore, noticing the condition given in (4), we further have $\mathbb{E}\{\varDelta V(k)\} + \mathbb{E}\{z_k'z_k\} - \gamma^2 w_k'w_k < 0$, which means

$$\mathbb{E}\{\varDelta V(k) + z_k'z_k - \gamma^2 w_k'w_k | x_k, r(k) = i\} < 0 \tag{32}$$

Summing up from $k = 0$ to $k = T$, and recalling that $x_0 = 0$, that $\mathbb{E}\{k+1, x_{k+1}, r(k+1)\} \geq 0$, we get that $\mathbb{E}\{\|z_k\|^2\}$ Noting the zero initial condition , we can conclude that $\frac{\|z\|_2}{\|w\|_2} < \gamma$ is satisfied, which completes the proof.
Remark: It's easy to find that, if (24) and (25) holds, the asynchronous controller can be designed based on Theorem 2. The optimal l_2-gain performance γ^* can be obtained via dealing with the optimal problem as follows:

$$\min \quad \sigma \quad \text{subject to (24) and (25) with } \sigma = \gamma^2 \tag{33}$$

IV. Numerical Example

In this section, we provide an example to verify the validity of proposed methods. Consider the Markov jump Lur'e system (1) with following data: $\mathcal{N} = \{1,2\}$, $\mathcal{M} = \{1,2\}$

$$A_1 = \begin{bmatrix}0.4 & 0.4\\0.2 & 1\end{bmatrix}, \quad A_2 = \begin{bmatrix}1.1 & 0.6\\0.3 & 0.4\end{bmatrix}$$

$$B_1 = \begin{bmatrix}0.5\\0.5\end{bmatrix}, \quad B_2 = \begin{bmatrix}0.7\\0.5\end{bmatrix}, \quad F_1 = \begin{bmatrix}1\\1.2\end{bmatrix}, \quad F_2 = \begin{bmatrix}1.2\\1\end{bmatrix}$$

$$C_1 = \begin{bmatrix}0.9\\0.5\end{bmatrix}, \quad C_2 = \begin{bmatrix}-1\\0.7\end{bmatrix}, \quad C_1^z = \begin{bmatrix}0.2\\0\end{bmatrix}, \quad C_1^z = \begin{bmatrix}0.1\\0.1\end{bmatrix}$$

$$G_1^z = 0.3, \quad G_2^z = 0.4, \quad D_1^z = 0, \quad D_2^z = 0.3;$$

$$E_1^z = 1.3, \quad E_2^z = -0.8, \quad E_1^x = E_2^x = \begin{bmatrix}1 & 0.5\end{bmatrix}'$$

$$\varOmega_1 = 1.3, \quad \varphi_1(y) = 0.5\varOmega_1 y(1 + cos(25y))$$

$$\varOmega_2 = 1.5, \quad \varphi_2(y) = 0.5\varOmega_2 y(1 - sin(20y))$$

$$\varPi = \begin{bmatrix}0.6 & 0.4\\0.2 & 0.8\end{bmatrix} \quad \varPhi = \begin{bmatrix}0.4 & 0.6\\0.3 & 0.7\end{bmatrix}$$

By solving the l_2-gain minimization problem in (33), it is easy to find that $\gamma^* = 2.41$ and following asynchronous controller gain matrices can be obtained.

$$K_1 = \begin{bmatrix}-1.0266 & -1.2008\end{bmatrix}, \quad \varGamma_1 = -2.4363$$
$$K_2 = \begin{bmatrix}-1.1812 & -0.9191\end{bmatrix}, \quad \varGamma_2 = -2.2642$$

Fig.1 gives a possible time sequences of the mode of system and asynchronous controller. We select the initial state $x_0 = \begin{bmatrix}2 & -2.5\end{bmatrix}'$, and the external disturbance is assumed to be $w_k = sin(k) * 0.9^k$. Then we can get state response of system, which is depicted in Fig.2.

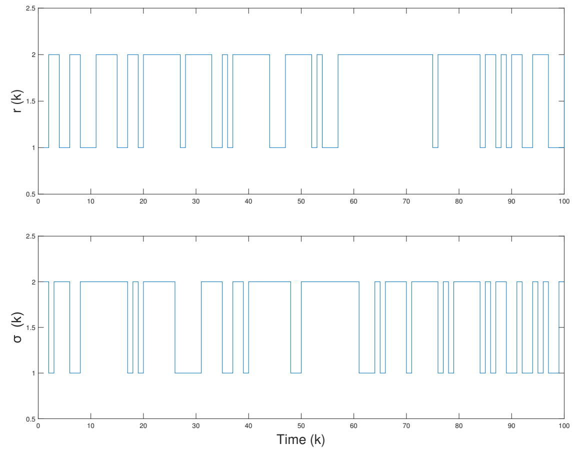

Fig. 1. Modes of system and controller

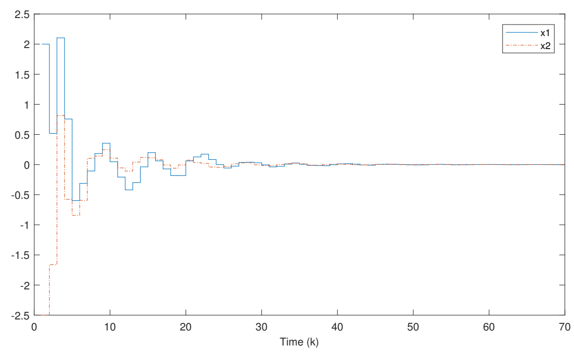

Fig. 2. System state

V. Conclusions

The stochastic stabilization and l_2-gain minimization problems for a class of discrete-time nonlinear Markov systems with asynchronous controller have been discussed in this paper.

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