Searching

Terence Parr
University of San Francisco

UNIVERSITY OF SAN FRANCISCO

Common searching/membership strategies

- $O(mn)$ • **linear**: scan data structure looking for element(s)
- $O(m\log(n))$ • **binary search**: if array and sorted, split recursively in half
- $O(m\log(n))$ • **binary search tree**: subtree to left has elements less than current node and subtree to right has elements greater than
- $O(m)$ • **hash table**: function maps key to bucket, linear search in bucket; recall search index project from MSDS692; for word search, not arbitrary string search in document(s)
- $O(m)$ • **state machines** (graphs)

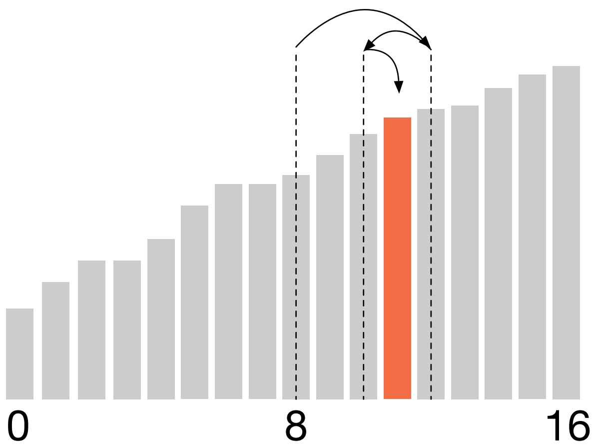
m = length of search string

UNIVERSITY OF SAN FRANCISCO

Binary search (review sort of)

- If we know data is sorted, we can search much faster than linearly
- Means we don't have to examine every element even worst-case

```
def binsearch(a,x):
    left = 0; right = len(a)-1
    while left<=right:
        mid = (left + right)//2
        if a[mid]==x: return mid
        if x < a[mid]: right = mid-1
        else: left = mid+1
    return -1
```



See http://interactivepython.org/runestone/static/pythonds/SortSearch/TheBinarySearch.html

Compare to (tail-)recursive version

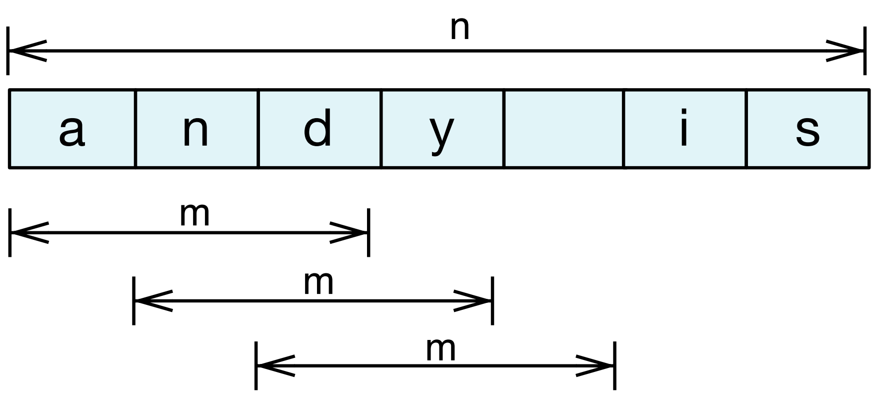
```
def binsearch(a,x,left,right):
    if left > right: return -1
    mid = (left + right)//2
    if a[mid]==x: return mid
    if x < a[mid]:
        return binsearch(a,x,left,mid-1)
    else:
        return binsearch(a,x,mid+1,right)
```

```
left = 0; right = len(a)-1
while left<=right:
    mid = (left + right)//2
    if a[mid]==x: return mid
    if x < a[mid]: right = mid-1
    else: left = mid+1
```

Bracket region with element

String matching

- **Problem**: Given a document of length n characters and a string of length m, find an occurrence or all occurrences
- Brute force algorithm is O(nm), but theoretical best case algorithm exists for O(n + m)
- **Exercise**: Describe brute force algorithm; why is it ”slow”?



Hash searches

- First, note that two equal strings have same hash code so we can compare int codes quickly even for huge strings
- Rabin-Karp* algorithm uses hash function to speed up but still O(nm) worst-case; works for any substring not just words
- **Idea**: h = hash search string s; compute hash for doc[i:i+m] and compare to h; if same, compare s to doc[i:i+m], return if found; move i from 0 to n-m
- Key is to avoid comparing strings unless the hash codes match

*https://en.wikipedia.org/wiki/Rabin%E2%80%93Karp_algorithm

Rabin-Karp (almost)

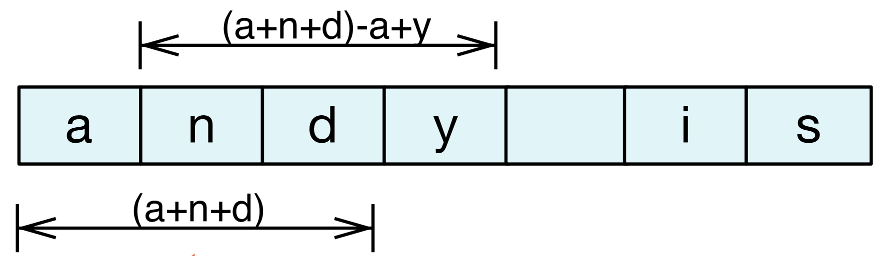
```
def search(doc, s) -> int:
    n = len(doc); m = len(s)
    hs = hash(s)
    for i in range(0,n-m+1):
        hdoc = hash(doc[i:i+m]) # slow O(m)
        if hdoc==hs: # fast
            if s==doc[i:i+m]: # slow
                return i
    return -1
```

```
def hash(s:str)->int:
    return sum(ord(c) for c in s)
```

Additive hashcode is important here

See searching notebook

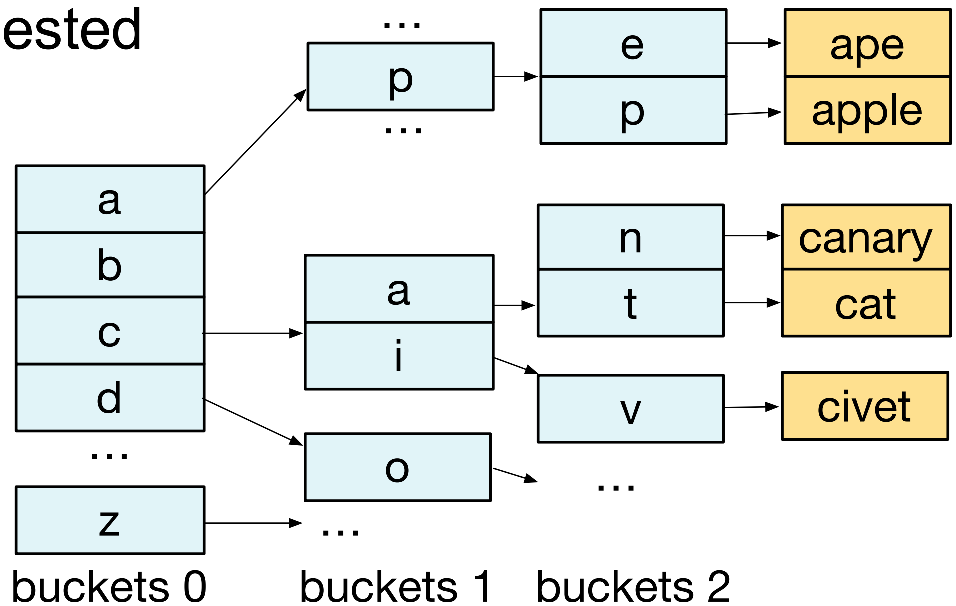
Issues



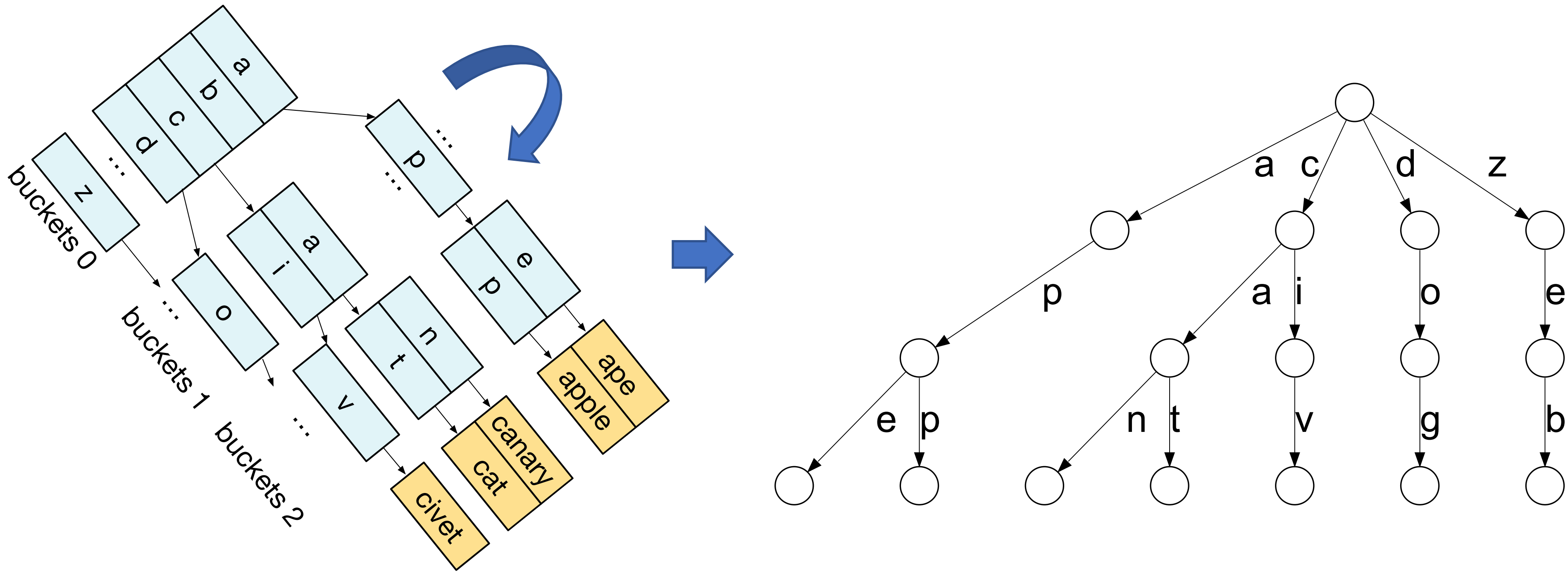
- Naïve hash(doc[i:i+m]) is O(m) for each i=1..n, so use rolling hash:
  - next hash is old hash minus doc[i] plus doc[i+m]
  - drop old one off, add in new char (see improved search() in notebook):
    `hdoc = hdoc - ord(doc[i]) + next  # roll it!`
- What about finding all occurrences?
- What if search string s is very long? Could still be expensive.
- Can we do better than O(nm) or even O(n+m) algorithms?
- Yes. I claim we can search for strings in doc in O(m) if we prepare a proper side data structure beforehand and you let me search for words instead of arbitrary strings
- How is this possible?!

Revisit recursive bucket sort

- Break up doc into words, make nested bucket structure as before
- To find a word, use s[i] to navigate and find final "leaf" with list of words with same prefix, linearly search leaf
- The index says how to navigate
- How long does it take to find s for n=len(doc), m=len(s)?
  T(n,m) = m



"*Tries*" or *Prefix Trees*

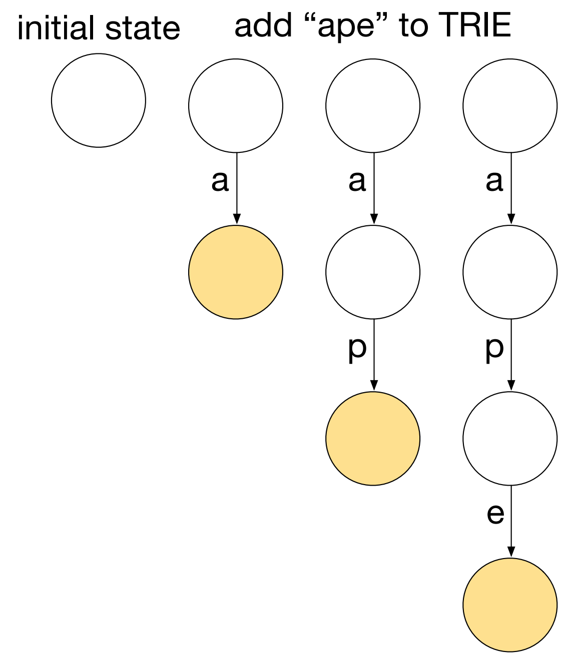


Convert buckets to nodes and rotate: we get a tree!

Adding string s to TRIE

```
class TrieNode:
    def __init__(self):
        self.edges = {}
```

- TRIE can hold a big set of words and we can search for a word superfast
- Note: Now that we're not sorting, order of edges is not important; can use dict()
- Starting at the root, add edge labeled with s[0] pointing to new node
- Traverse edge to child root.child[s[0]] and add subtree for s[1:] to that child
- Recurse until out of chars in string s

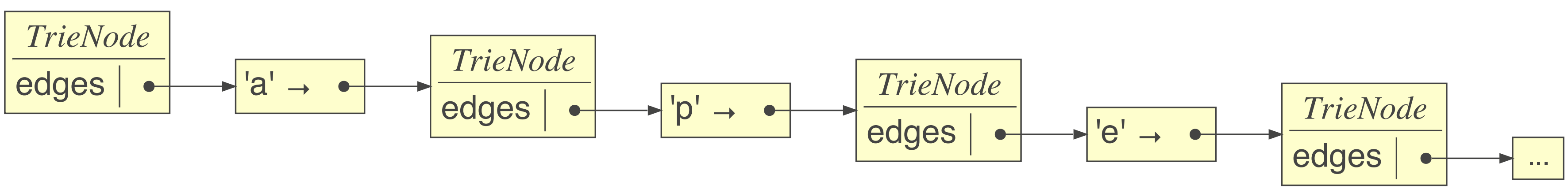


Construction

```
class TrieNode:
    def __init__(self):
        self.edges = {}
```

- add(root, “ape”)

```
def add(p:TrieNode, s:str, i=0) -> None:
    if i>=len(s): return
    if s[i] not in p.edges:
        p.edges[s[i]] = TrieNode()
    add(p.edges[s[i]], s, i+1)
```



Note that nodes have no values, edges contain the letters

Searching a Trie

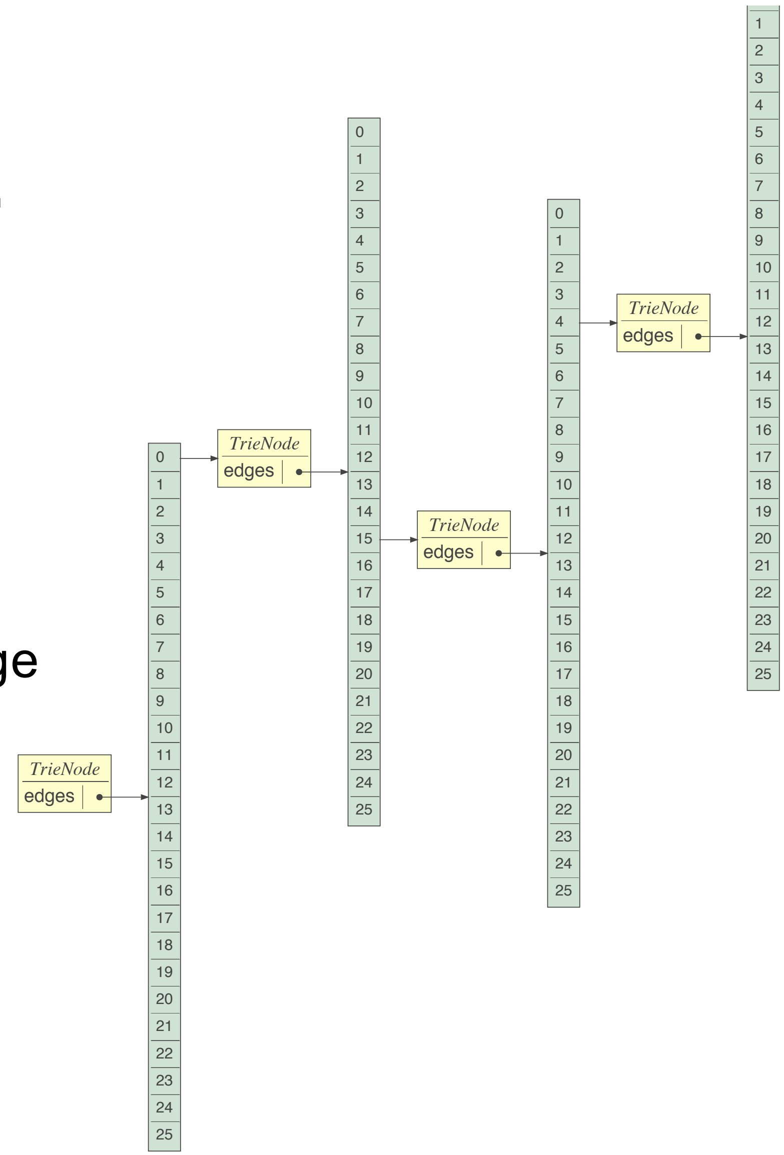
(was in a "big internet company" interview)

- Return true if s is prefix of word in Trie or full word in Trie
- Note that the search depends on <u>len(s)</u> NOT num words n in the vocabulary!!!

```
def search(root:TrieNode, s:str, i=0) -> bool:
    p = root
    while p is not None:
        if i>=len(s): return True
        if s[i] not in p.edges: return False
        p = p.edges[s[i]]
        i += 1
    return True
```

Dictionaries are O(1) but…

- …slower than array access via perfect hash function f(c) = ord(c) - ord('a')
- But we use 26 slots even for one edge
- How can we reduce memory costs?
  - Many nodes will have just one outgoing edge so we can optimize for that case with single pointer instead of an array
  - Switch to 26-element edge array if we need more than one edge



Exercise: Brute force dictionary search

- Load words from **/usr/share/dict/words** file (one per line) into list
- Search for each word in list of words; what is complexity?
- This takes almost 5 minutes on my fast computer. ugh

Exercise: Build Trie from dictionary of words

- From searching notebook, get Trie implementation
- Add each word to a trie, which takes about 6s on my machine
- Search the trie for each of 235,886 words; takes 0.75s for me!!
- Rejoice in your new super powers
- Cool interview question/task:
  How can you do fast spell checking on big documents?

See solution https://github.com/parrt/msds689/blob/master/notes/code/trie.py

Exercise: find all words starting with prefix

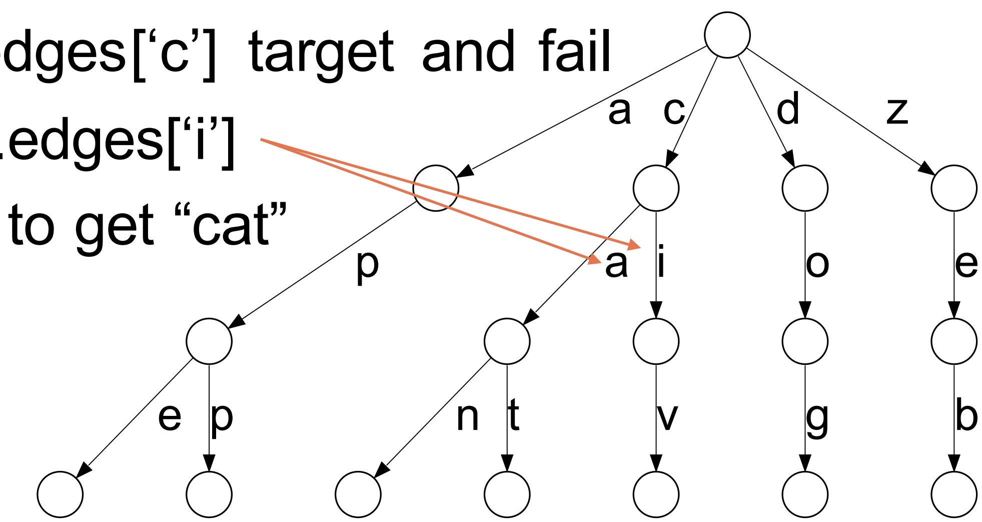
- Create a trie again from the word list
- Write a function that prints all words in trie that begin with a specific prefix like “app”; it should get “apple”, “application”, …
- Idea: trace prefix into trie, reaching specific non-leaf node p; find all reachable leaves; track string as recursion parameter for each path; print the string when you reach a leaf

Exercise: Build a suffix tree

- Simple: create trie from reversed strings or modify add() method to walk backwards through string

Exercise: Given misspelled words off by 1 letter only, find all possible words

- Trace word into trie until no edge exists for s[i]; this is node p
- Get list of words reachable from each node targeted by p starting with s[i+1]
- E.g., “cxt” would get to p=root.edges[‘c’] target and fail
- Find “t” from p.edges[‘a’] and p.edges[‘i’]
- We only find “t” matches via ‘a’ to get “cat”



Summary

- Lots of ways to search beyond linear and binary search
- String searching has some really efficient solutions such as Rabin-Karp; idea is to compare hash codes before doing string comparisons and do a rolling hash for the document substrings
- If we are willing to build a graph data structure, the TRIE is pretty hard to beat complexity and performance; looking up a word in the TRIE is O(m) for m character string!
- TRIE is just a nested pigeonhole sort turned into a graph
- Useful as prefix and suffix trees; can find even misspelled words