수치해석 Project 3

Due: 6.7. 15:00 (Friday)

(1) Let us consider a problem

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

The exact solution is $X = [3\ 5\ 6\ 6\ 5\ 3]^t$ for $AX = b$.

a) Solve the above system by the Jacobi method

b) by the Gauss-Seidel method

c) by the SOR method with the relaxation parameter $\omega = 1.4$

The stopping criterion is $\|X_n - X_{n-1}\|_\infty < 10^{-4}$ and $X_0 = [0\,0\,0\,0\,0\,0]^t$.

Show the results as in the following table form for each problem

n	$\|X_n - X\|_\infty$	$\|X_n - X\|_\infty / \|X_{n-1} - X\|_\infty$
1		
2		

(2) Solve the ordinary differential equation

$$y' = -y^2, \quad y(0) = 1 \quad \text{on } [0, 1].$$

The exact solution is $y(t) = \dfrac{1}{t+1}$

a) by the (forward) Euler method with $h = 0.05\ 0.1$

$$y_{n+1} = y_n + h\ f(t_n, y_n)$$

b) by the mid-point method with $h = 0.05\ 0.1$

$$z = y_n + \frac{h}{2}\ f(t_n, y_n)$$

$$y_{n+1} = y_n + h\ f(t_{n+1/2}, z)$$

For (a) and (b)

** Draw the graphs of numerical solutions in one frame.

** Draw the graphs of errors in one frame.