

수치해석 Project 2

Due: 04.16. 15:00 (화)

1. Let us consider the cubic-spline approximation $S(x)$ (with the clamped boundary condition) of the function $f(x) = e^x$ on the interval $[0, 1]$, where $[0, 1] = \sqcup_{i=1}^n J_i$, $J_i = [(i-1)h, ih]$ with $h = \frac{1}{n}$.

$$Error_h = \max_{j=1, \dots, n} \left| f(jh - \frac{1}{2}h) - S(jh - \frac{1}{2}h) \right|.$$

a) show the table of convergence by taking $h = \frac{1}{2}, \frac{1}{4}, \dots, \frac{1}{32}$, which includes h , error, the order of convergence.

b) Draw the graphs of $S(x)$ (dotted) and $f(x)$ (solid) in the same coordinate when $h = \frac{1}{4}$.

* **Order of convergence:** $Error_h = ch^\alpha$, Therefore, $\alpha = \ln(Error_{2h}/Error_h)/\ln 2$

2. Let $f(x) = x \sin(x)$. Approximate the derivative $f'(1)$ by using

- a) the three point formula
- b) the five point formula

For each case show the table of convergence by taking $h = \frac{1}{2}, \frac{1}{4}, \dots, \frac{1}{32}$, which includes h , error, the order of convergence.

3. Let $f(x) = xe^x$. Approximate the integral $\int_0^1 f(x) dx = 1$ by using

- a) the composite trapezoidal rule
- b) the composite Simpson rule
- c) the composite 2-point Gaussian quadrature

with $h = \frac{1}{2}, \frac{1}{4}, \dots, \frac{1}{32}$. For each case show the table must contain h , error, the order of convergence.

d) Perform the Romberg integration for the case $h = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$