수치해석 HW1

Due: Mar. 22, 3:00 pm

1. Use Theorem 2.3 to show that $g(x) = \pi + \frac{1}{2}\sin(x/2)$ has a unique fixed point on $[0, 2\pi]$. Use fixed-point iteration to find an approximation to the fixed point that is accurate to within 10^{-2}. Use Corollary 2.5 to estimate the number of iterations required to achieve 10^{-2} accuracy, and compare this theoretical estimate to the number actually needed

2. The following four methods are proposed to compute $21^{1/3}$. Rank them in order, based on their apparent speed of convergence, assuming $p_0 = 1$.

a. $p_n = \dfrac{20p_{n-1} + 21/p_{n-1}^2}{21}$

b. $p_n = p_{n-1} - \dfrac{p_{n-1}^3 - 21}{3p_{n-1}^2}$

c. $p_n = \left(\dfrac{21}{p_{n-1}}\right)^{1/2}$

3. If $\{p_n\}_{n=0}^{\infty}$ are convergent Secant method approximations to p, the solution to $f(x) = 0$,

(i) then there exists a constant $C > 0$ exists with $p_{n+1} - p \approx C\,|p_n - p|\,|p_{n-1} - p|$

(ii) $\{p_n\}$ converges to p of order $\alpha = (1 + \sqrt{5})/2$

4. We consider numerical roof finding methods for the equation $x - \cos x = 0$

Perform your numerical tests, $p_0 = 0,\ p_1 = 1,\ |p_n - p_{n-1}| < tol.\ \ tol = 10^{-10}$,

i) Solve it by the bisection method ($a = 0,\ b = 1$)

ii) Solve it by a fixed point method ($p_0 = 0$)

iii) Solve it by the Newton's method ($p_0 = 0$)

iv) Solve it by the secant method ($p_0 = 0,\ p_1 = 1$)

v) Solve it by the method of false position. ($p_0 = 0,\ p_1 = 1$)

① Show the numerical results in tables as in the text book.

② Discuss your numerical observation based on the theory.

- Do not submit your programs