# Geometric Transformation Introduction

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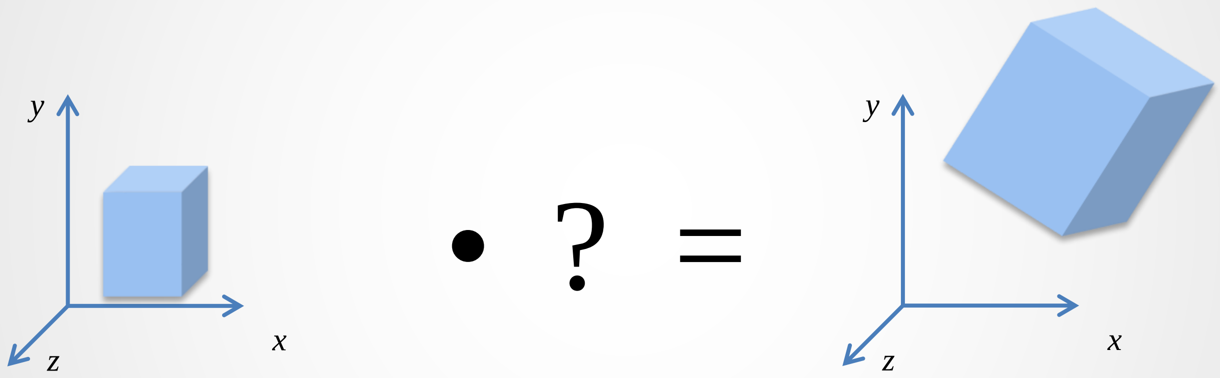
# Geometric Transformation



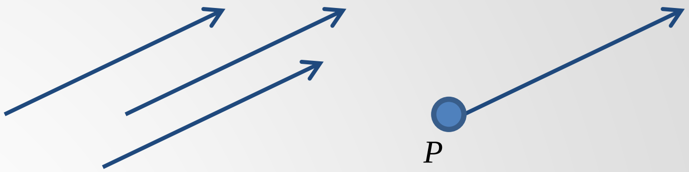

- 물체의 이동, 회전, 크기조절 등의 작업을 **기하변환(Geometric Transformation)** or **변환(Transformation)** 이라 한다.

# Geometric Transformation

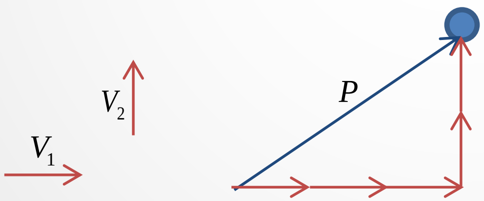
- **점의 표현**

벡터 : 크기와 방향을 동시에 지님

어파인(Affine) 공간 : 벡터에 점을 추가




기반 벡터 : 자신의 합성에 의해 다른 모든 벡터를 표시할 수 있는 벡터




$$p = 4V_1 + 2V_2$$

기반 벡터를 이용한 표현

# Geometric Transformation

- **점의 표현**

원점(Origin) : 기반 벡터의 시작 위치를 모은 한 점

좌표계(Coordinate System) : 원점과 기반 벡터로 구성되는 프레임

3차원 좌표계에서 점과 벡터의 표현

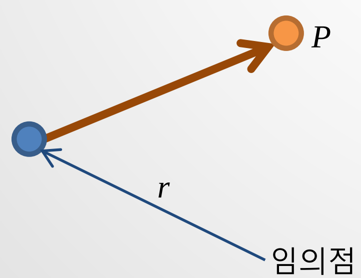
$$v = 4V_1 + 2V_2 + V_3$$

$$p = r + 4V_1 + 2V_2 + V_3$$

동차(Homogeneous Coordinates) 좌표계

$$v = 4V_1 + 2V_2 + V_3 + 0 \cdot r$$

$$p = 4V_1 + 2V_2 + V_3 + 1 \cdot r$$



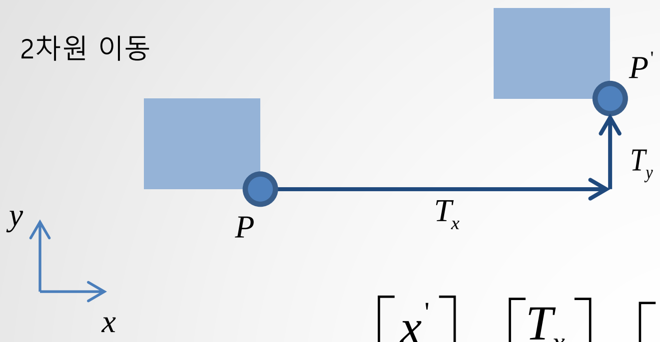

$$v = \begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \end{bmatrix} \quad p = \begin{bmatrix} 4 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

# Geometric Transformation

## - Translation

$$P' = T \cdot P$$

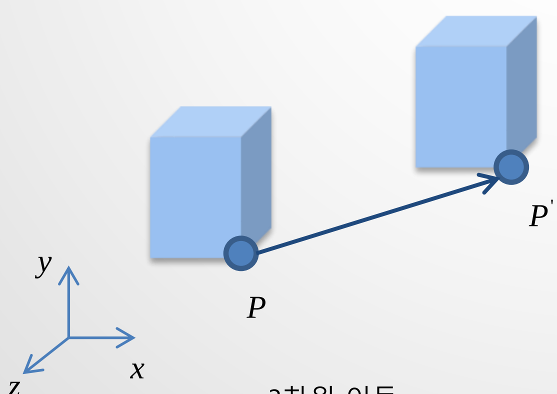
2차원 이동




$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



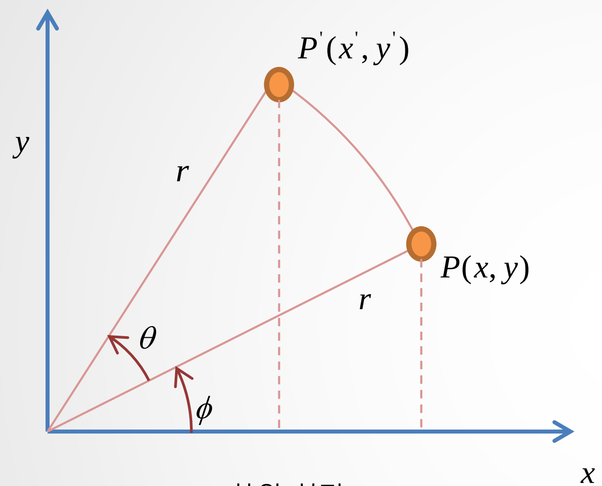

3차원 이동

# Geometric Transformation

- **Rotation**

$$P' = R \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$




2차원 회전

$$P' = R_z(\theta) \cdot F$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = r\cos(\phi+\theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta = x\cos\theta - y\sin\theta$$

$$y' = r\sin(\phi+\theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta = x\sin\theta + y\cos\theta$$

# Geometric Transformation

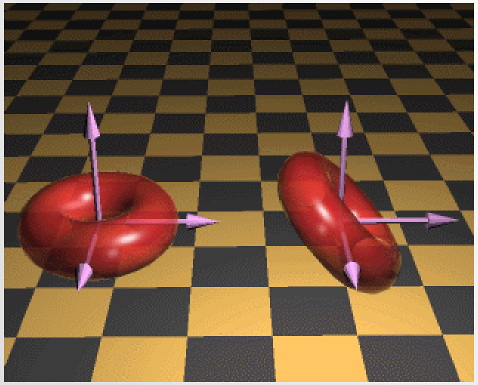
- **Rotation**

$$P^{'} = R_x(\theta) \cdot F$$

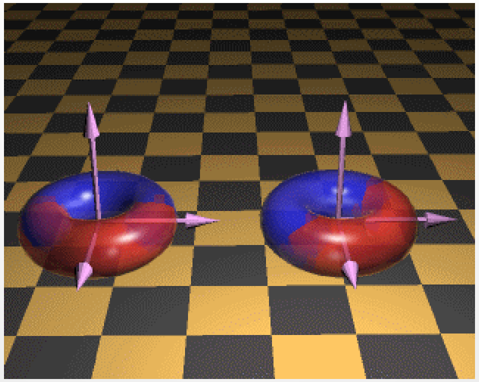
$$\begin{bmatrix} x^{'} \\ y^{'} \\ z^{'} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P^{'} = R_y(\theta) \cdot F$$

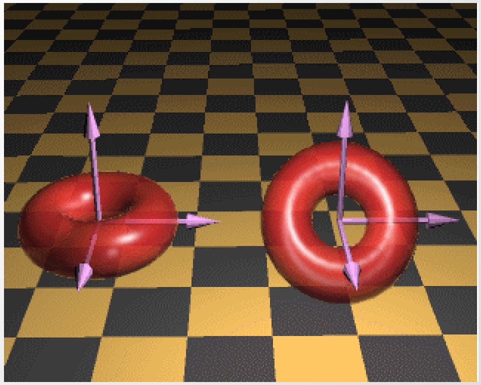
$$\begin{bmatrix} x^{'} \\ y^{'} \\ z^{'} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Z-axis



Y-axis

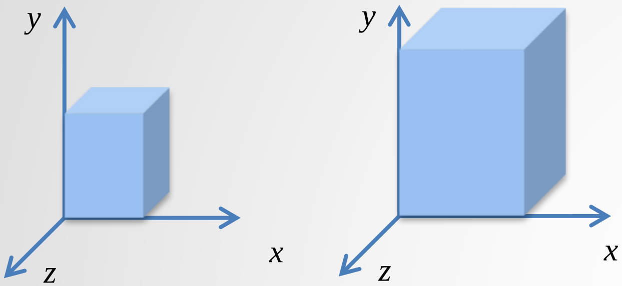


X-axis

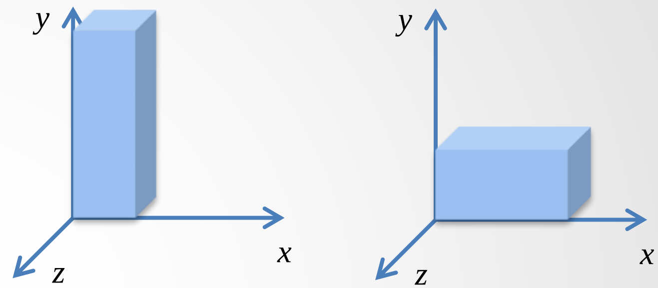
Rotation Transformation

# Geometric Transformation

- **Scaling**




Uniform scaling



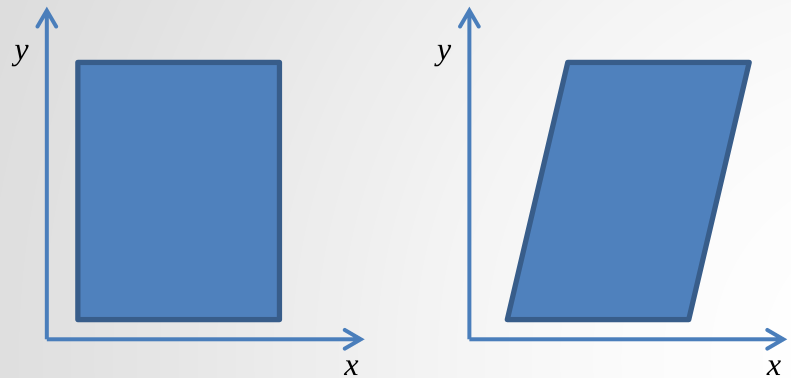

Differential Scaling

$$P' = S \cdot P$$

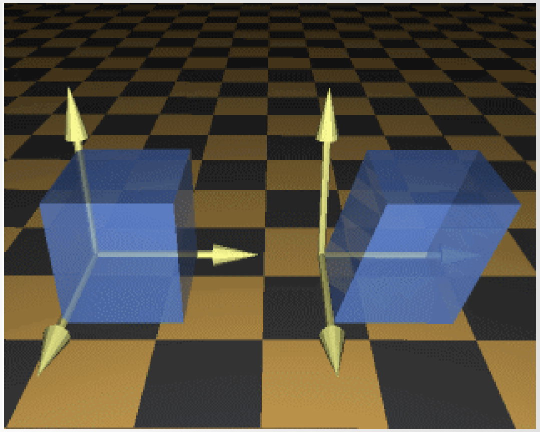
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Geometric Transformation

## - Shearing




Shearing( $Sh_x = 0$ )



$$P' = Sh \cdot P$$

* $Sh_x, Sh_y$ : Shearing factor

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_y & 1 & 0 \\ Sh_x & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

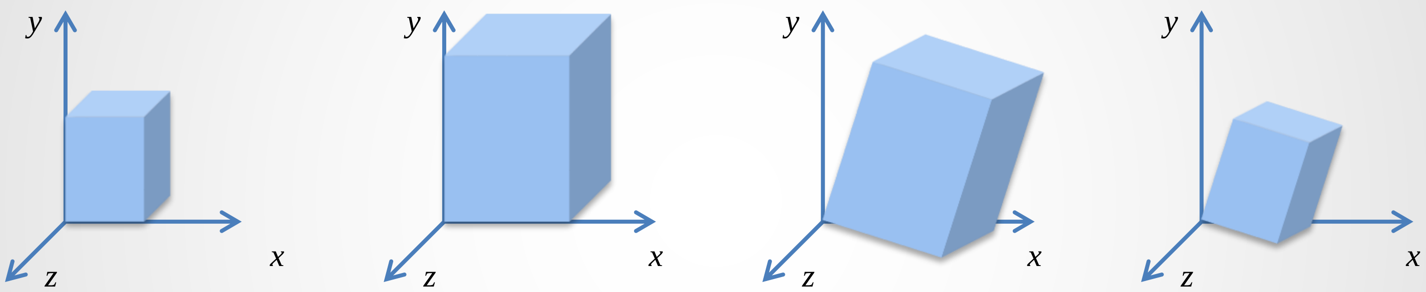
x-y 평면을 따라서 가해진 전단 변환 행렬식

# Geometric Transformation

- **복합 변환**

  일반적인 변환은 연속적

Ex 1) 크기조절(S1) → 회전(R1) → 크기조절(S2)




$$P' = S2 \cdot R1 \cdot S1 \cdot P$$

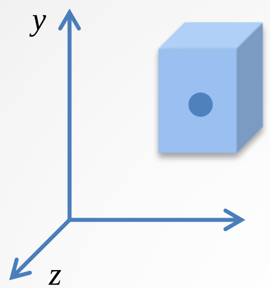
$$P' = C \cdot P$$

*복합 행렬을 이용하여 데이터 변환 처리 시간 단축

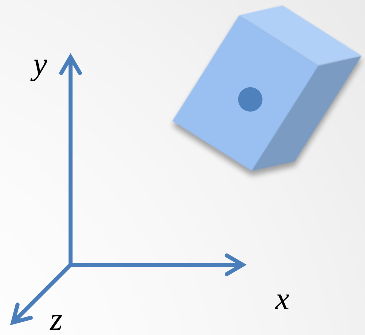
# Geometric Transformation

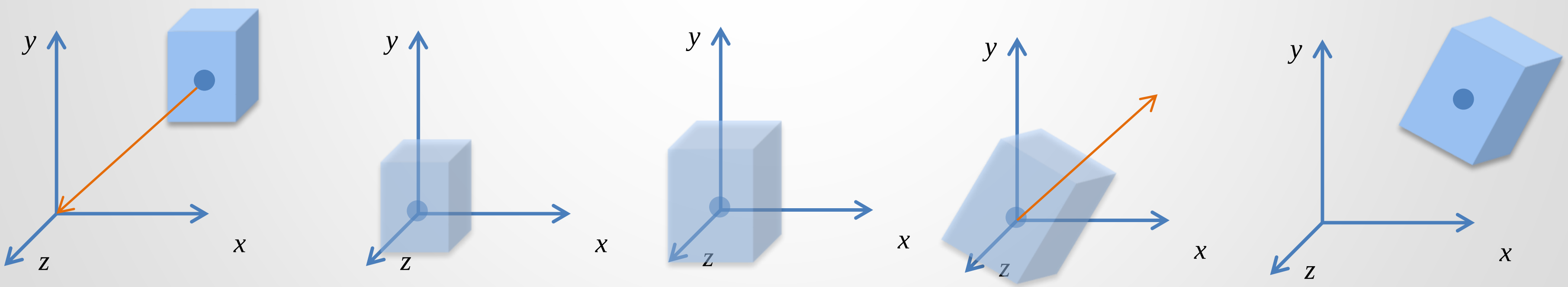
## - 복합 변환

Ex 2) Pivot 중심의 회전




$\cdot ? =$



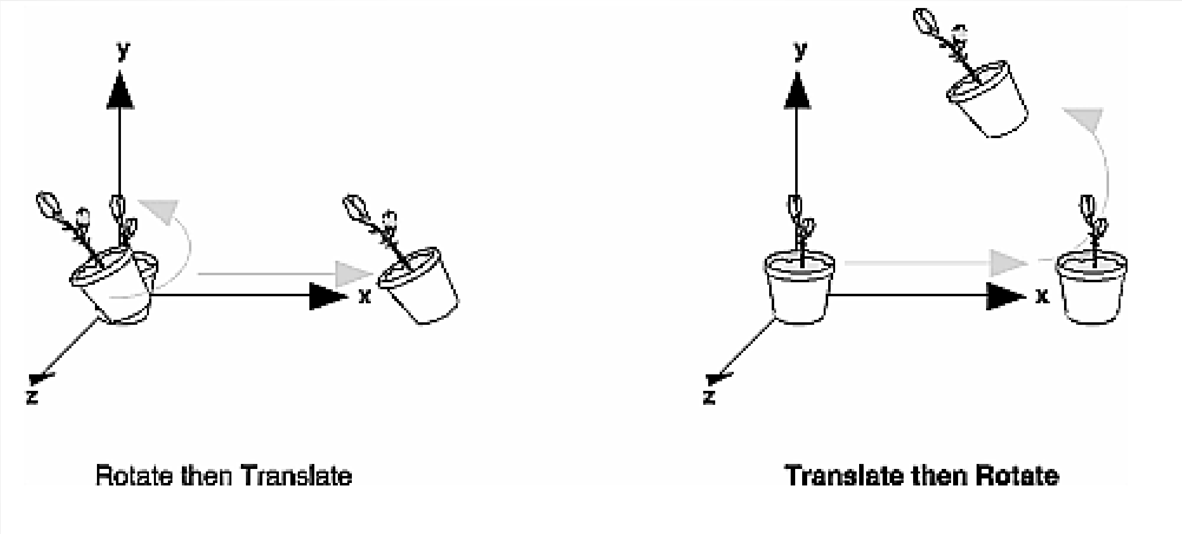




$$C = T(X_p, Y_p, Z_p) \cdot R_z(\theta) \cdot S \cdot T(-X_p, -Y_p, -Z_p)$$

# Geometric Transformation

- **복합 변환**

$$P^{'} = T \cdot R \cdot P \qquad P^{'} = R \cdot T \cdot P$$



# Geometric Transformation

## - 변환의 종류

강체 변환(Rigid Body Transformation) : Translation + Rotation

유사 변환(Similarity Transformation) : Translation + Rotation + Uniform scaling

변환 전후 물체면 사이의 각이 유지되고 물체 내 정점간 거리가 일정한 비율로 유지

어파인 변환(Affine Transformation) : Translation + Rotation + Scaling + Shearing

변환 전후 물체의 타입(직선, 다각형, 곡면) 유지

마지막 행은 항상 (0,0,0,1)

원근 변환(Perspective Transformation)

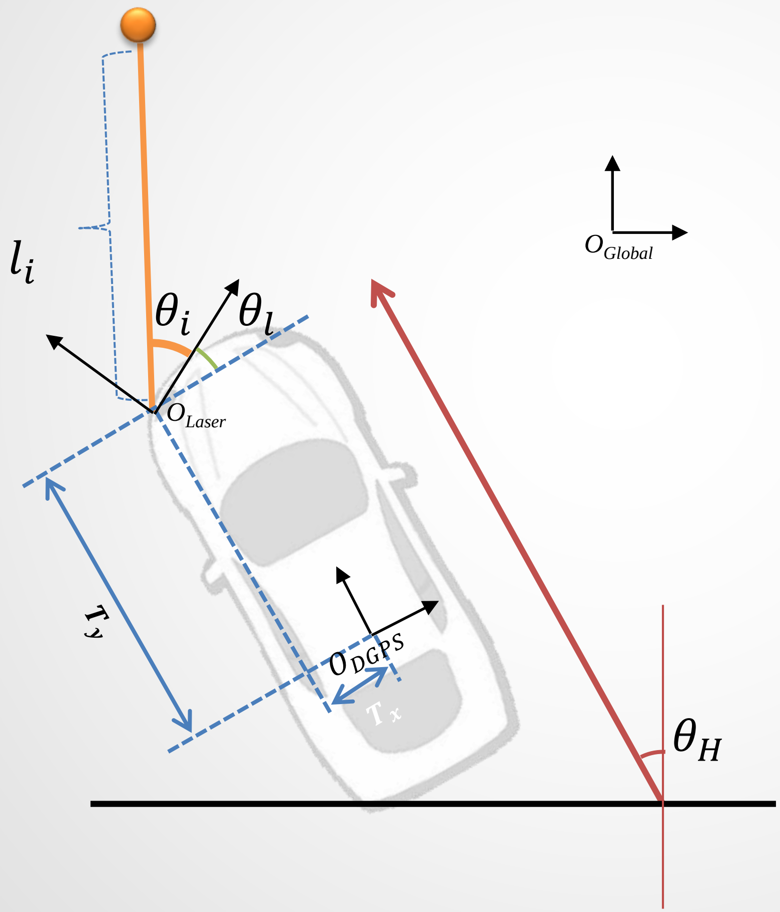
변환 전후 평행했던 선분이 만날 수 있음

$$
\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
$$

- Rotation
- Translation
- Scaling
- Shearing

# Geometric Transformation

- **Example**




$$P_{laser} = \begin{bmatrix} l_i \cos(\theta_i) \\ l_i \sin(\theta_i) \end{bmatrix}$$

$$P'_{DGPS} = T_l \cdot R_l \cdot P_{laser}$$

$$P'_{Global} = T_H \cdot R_H \cdot P_{DPGS}$$

$$P'_{Global} = T_H \cdot R_H \cdot T_l \cdot R_l \cdot P_{laser}$$