Delaunay Triangulation

안재원

목차

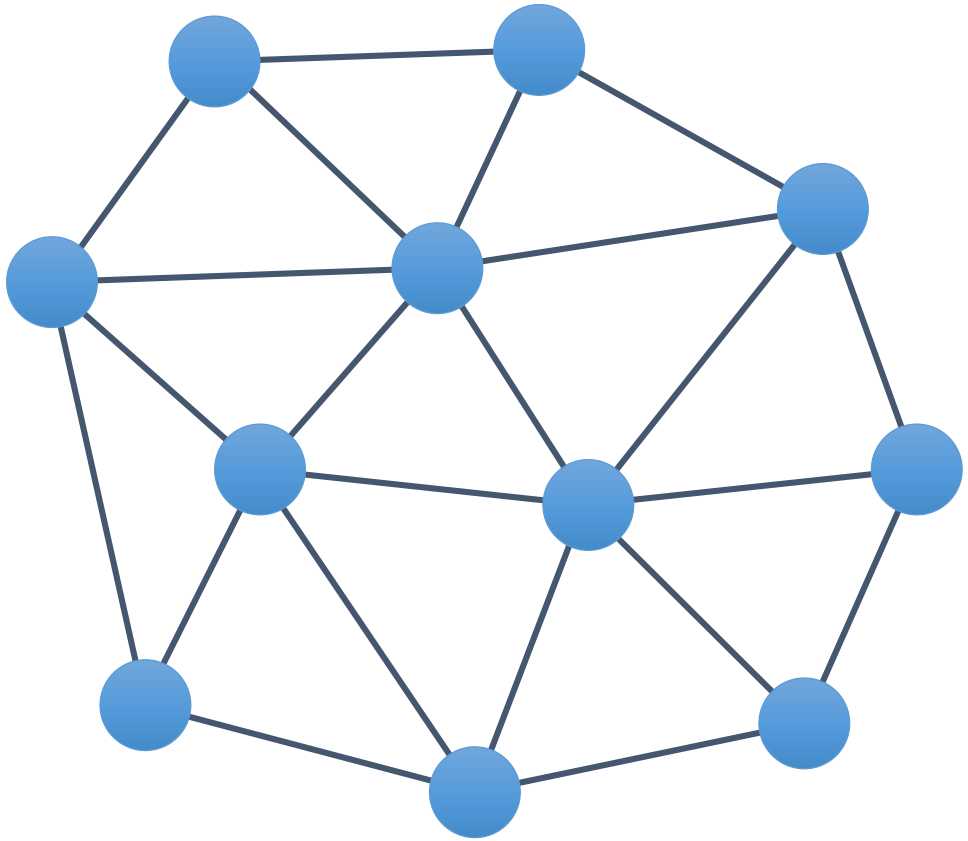
- Delaunay Triangulation
- How to Implement
- Result

- Boris Delaunay
- 1890.05 ~1980.07
- Russian

- Georgy Voronoi
- 1868.04 ~1908.11
- Russian

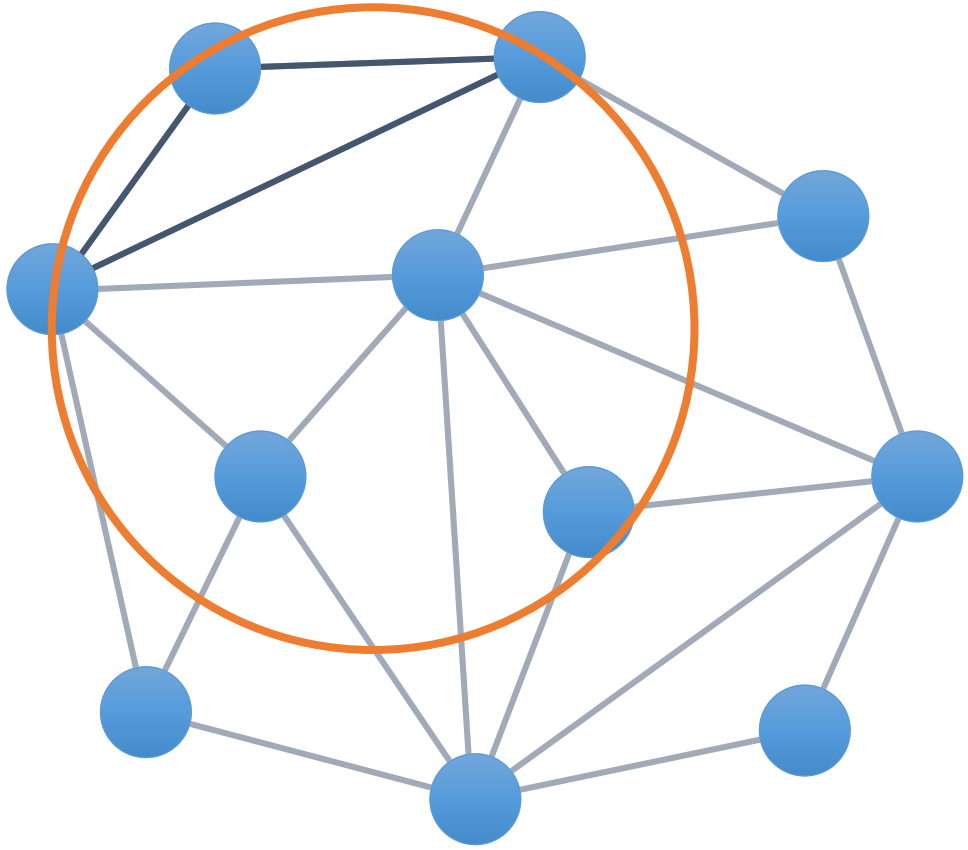
01 Delaunay Triangulation

- Intro

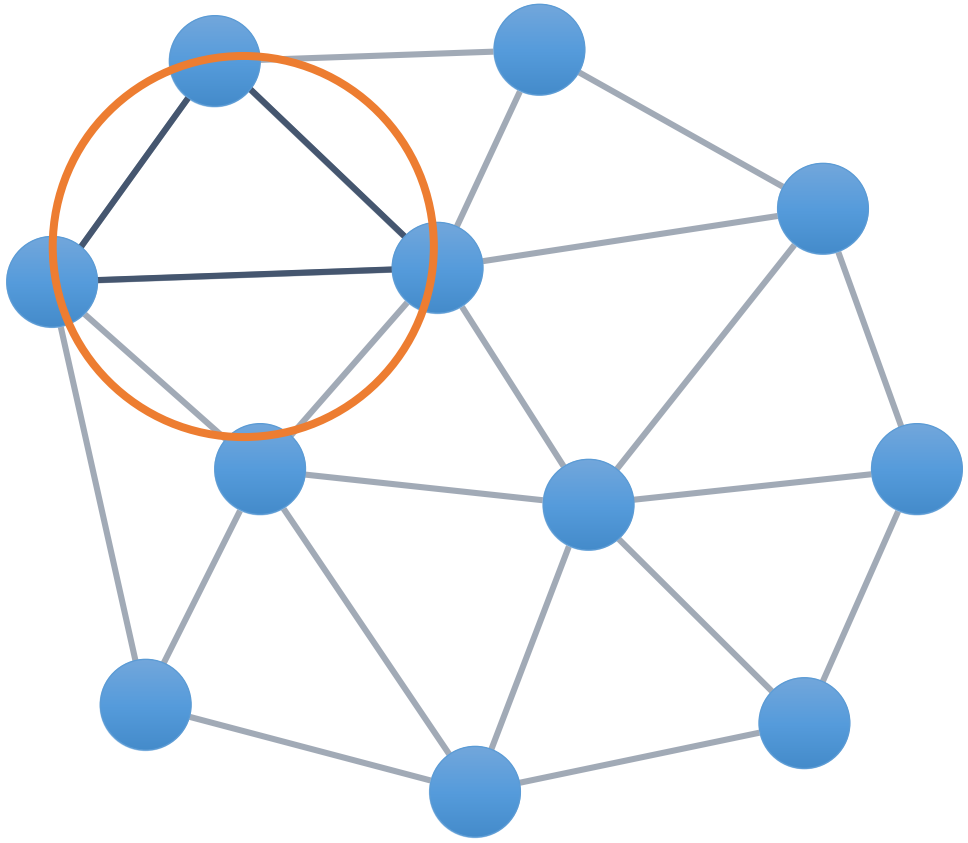


01 Delaunay Triangulation

- empty circumcircle property



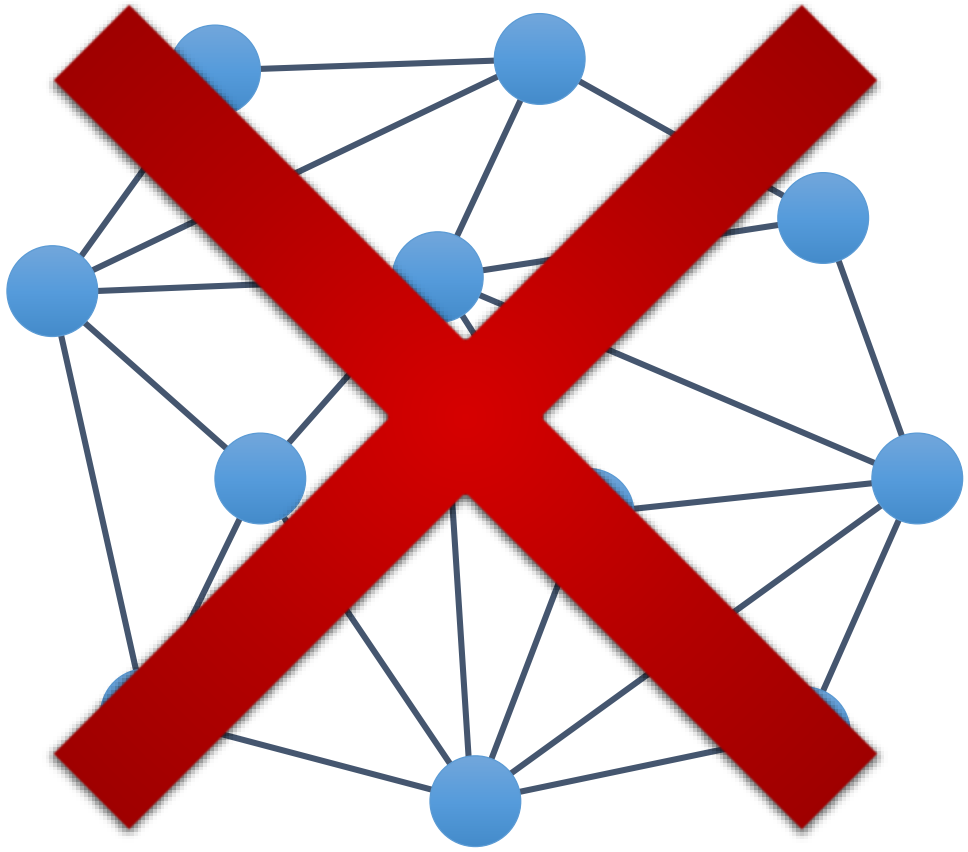
Non-Delaunay triangle



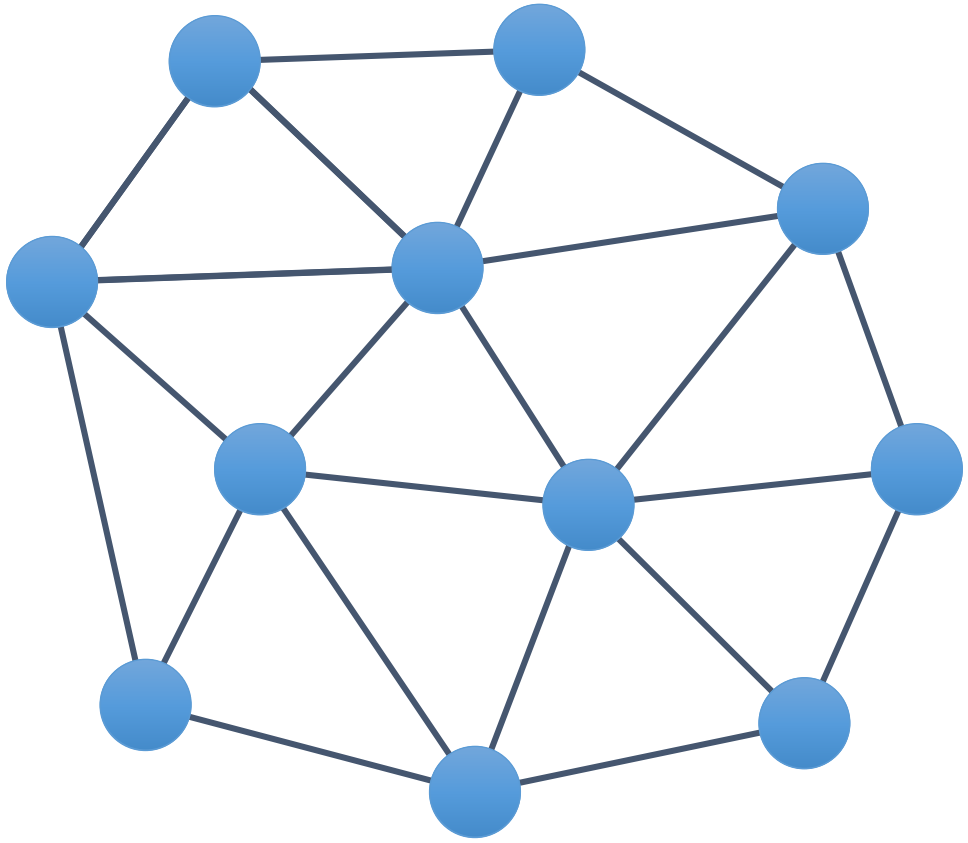
Delaunay triangle

01 Delaunay Triangulation

- empty circumcircle property



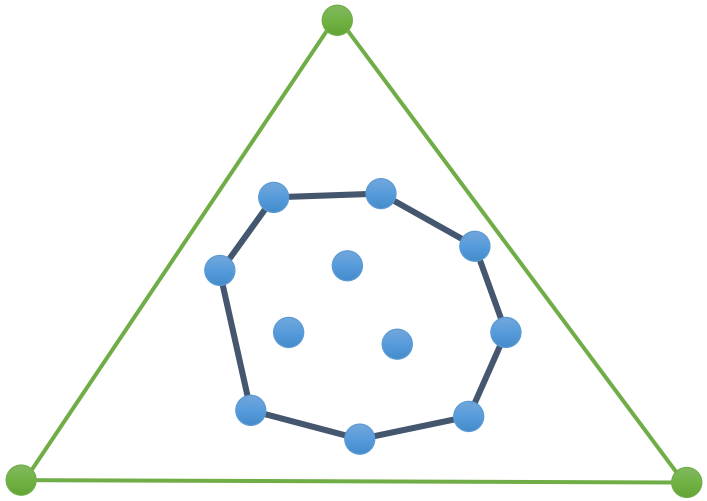
Non-Delaunay triangulation

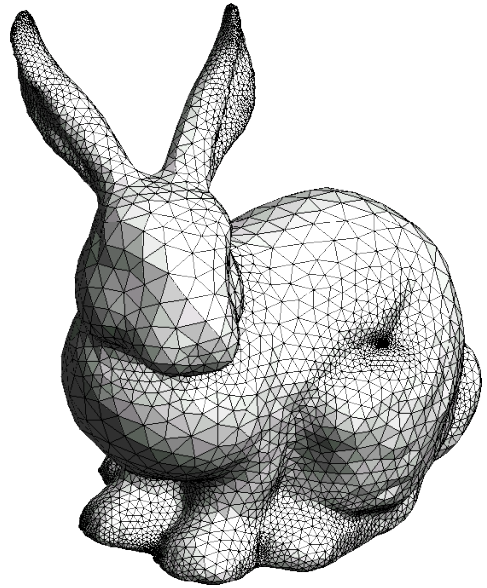


Delaunay triangulation

01 Delaunay Triangulation

- Applications

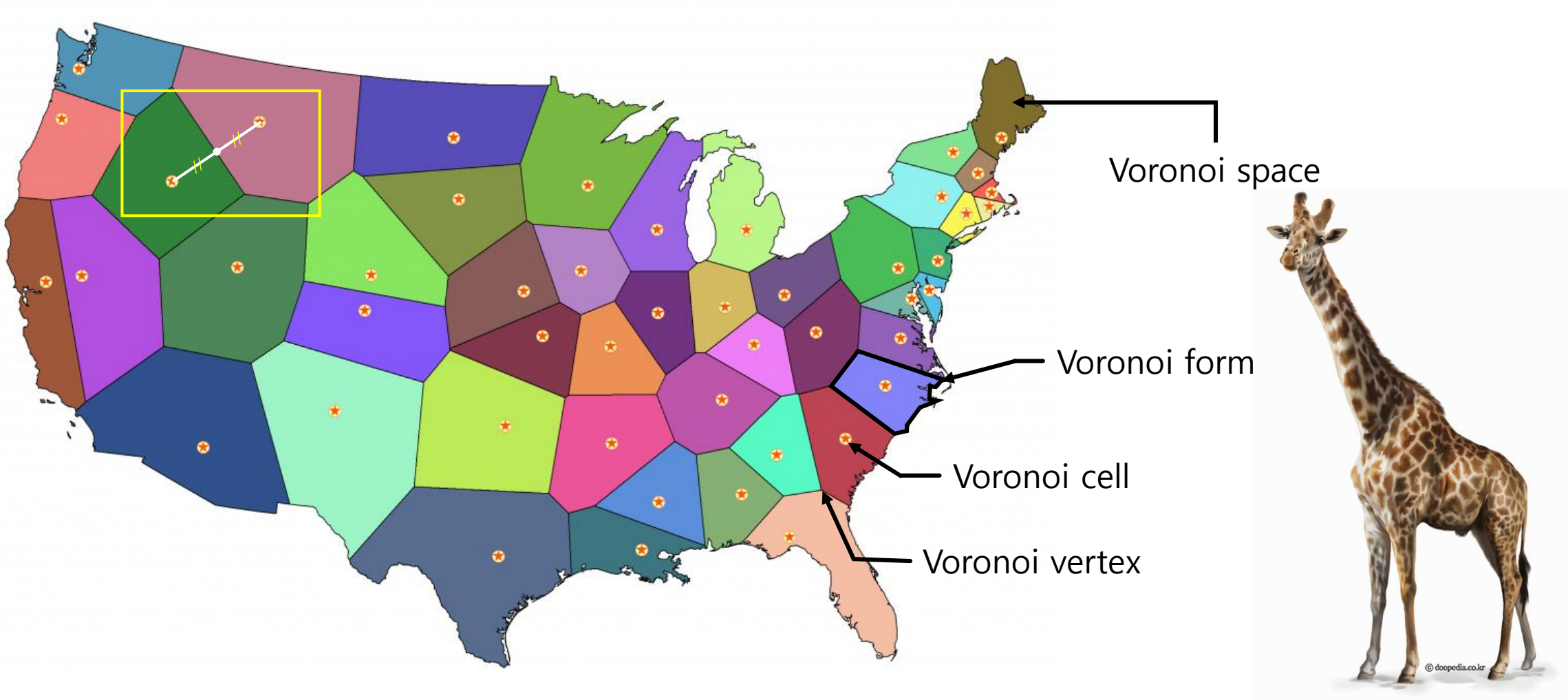




01

Delaunay Triangulation

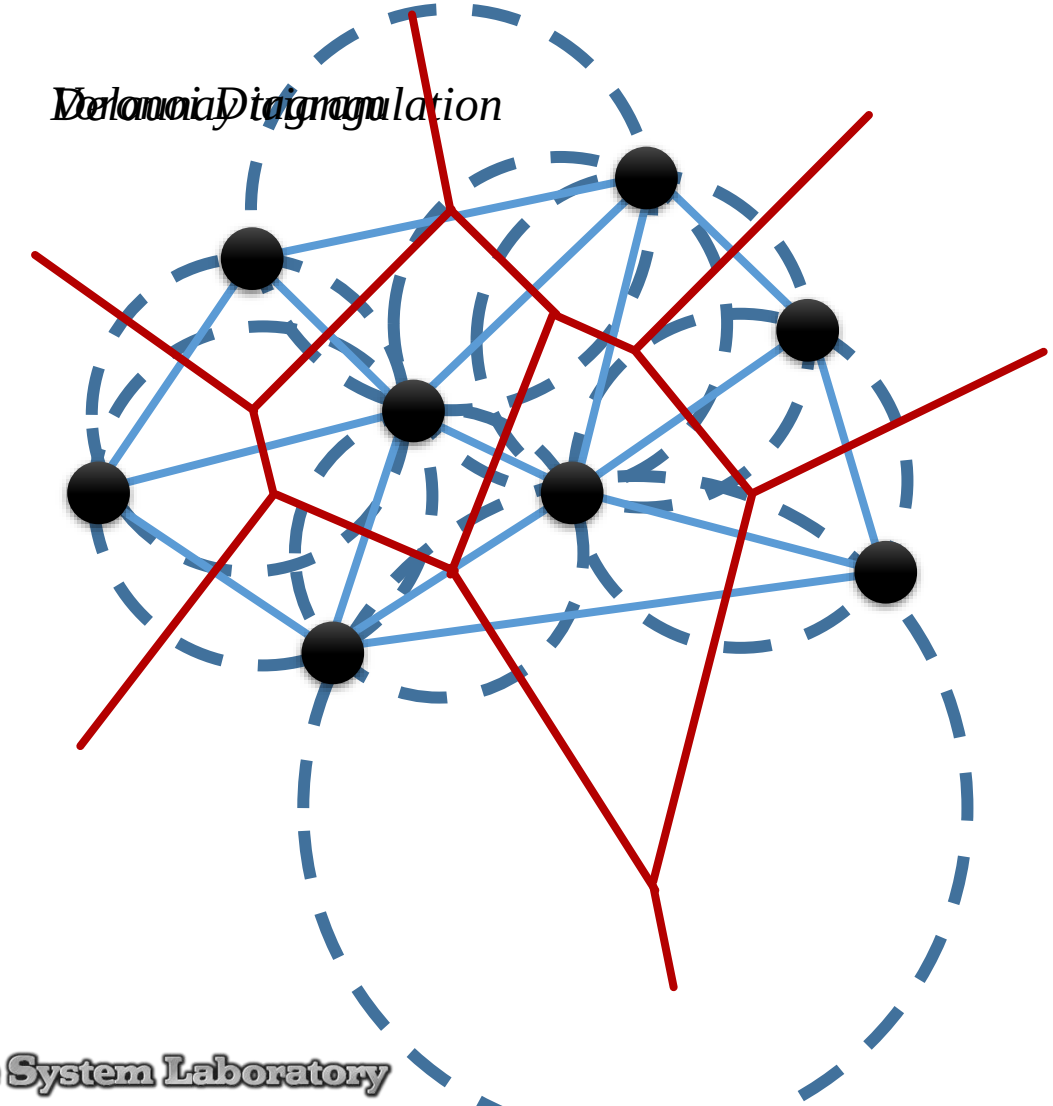
- Voronoi Diagram

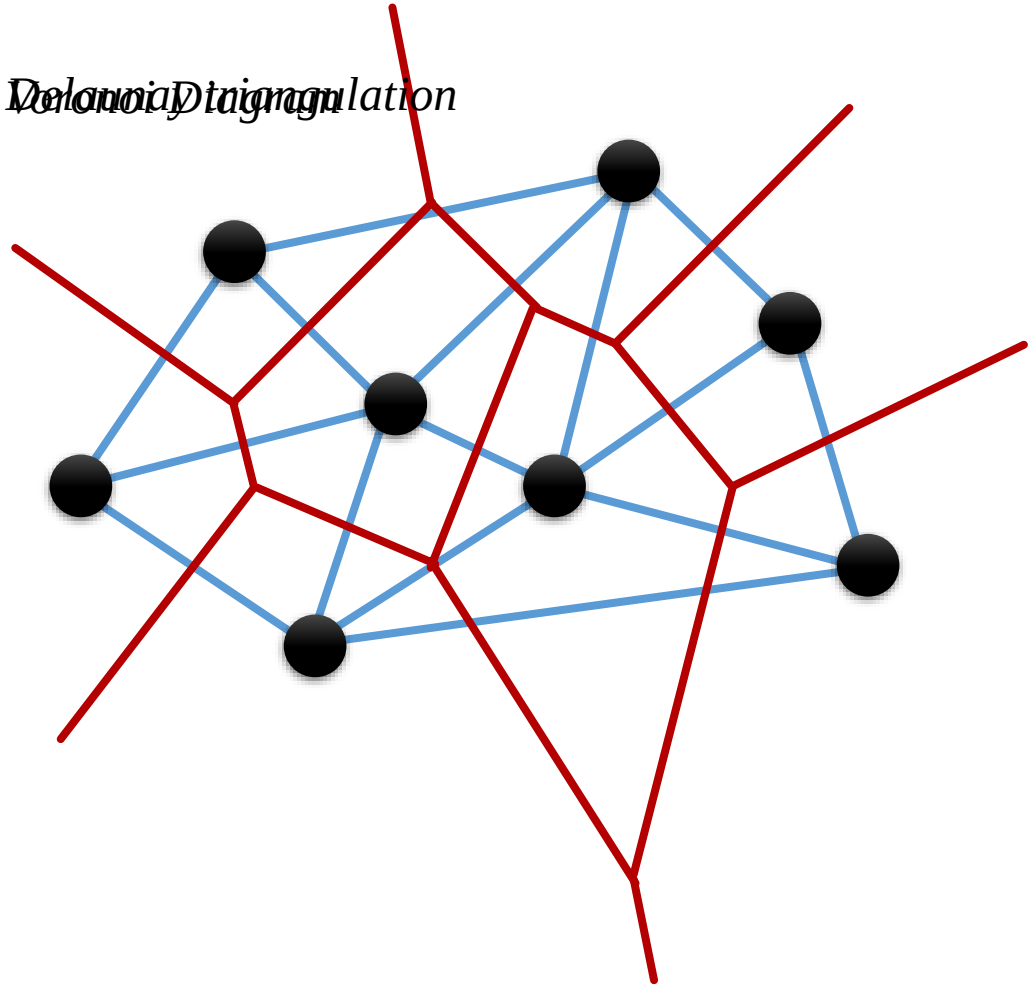


01

Delaunay Triangulation

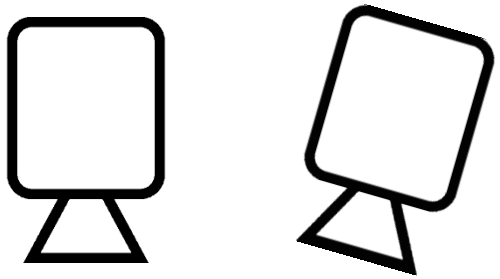
- Delaunay Triangulation & Voronoi Diagram

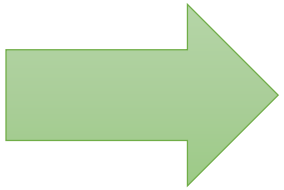




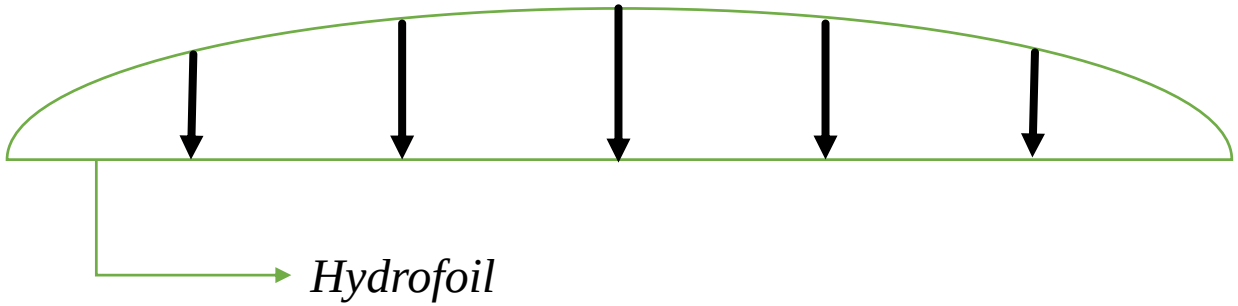
02 How to Implement

- Step 1





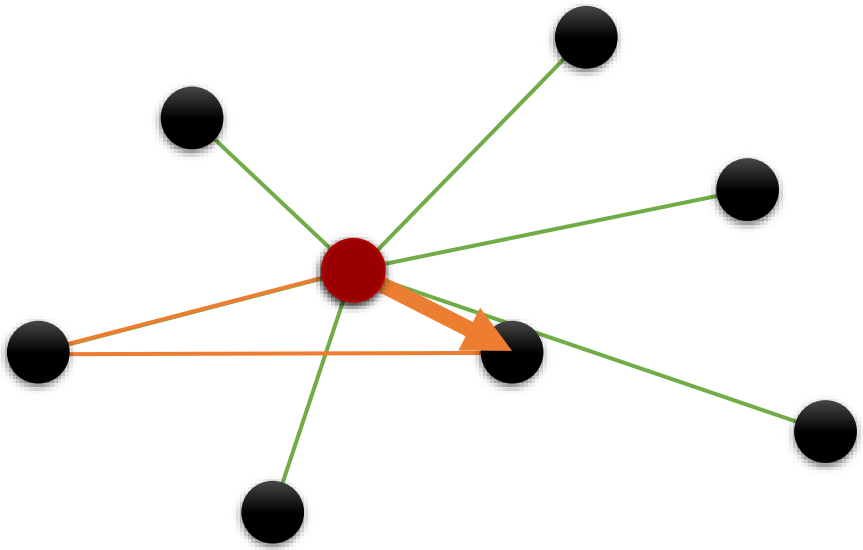
2D-Delaunay Triangulation



02

How to Implement

- Step 2



- Find the nearest point

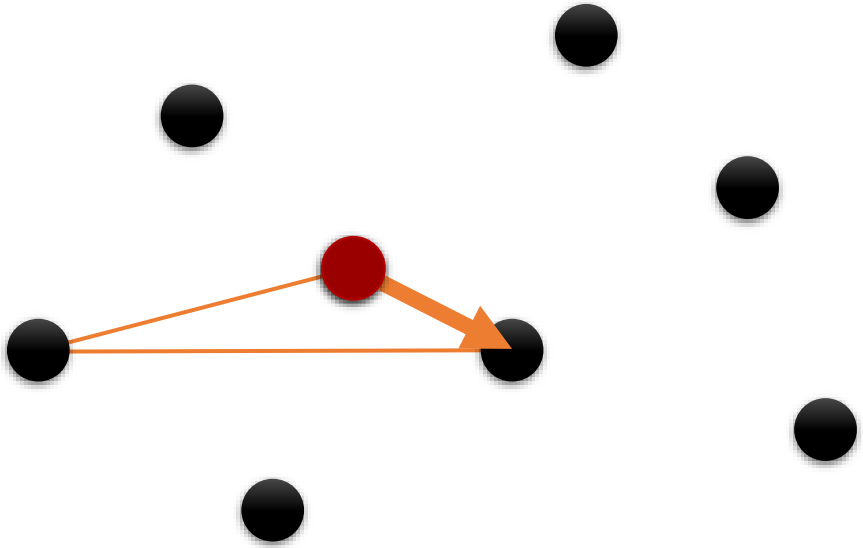
$$\sqrt{(x_1 - x_2)^2 + (z_1 - z_2)^2}$$

- Get vector of two points

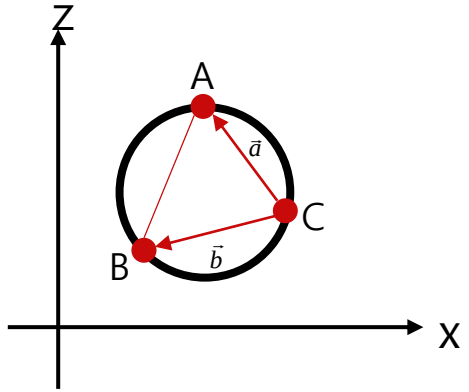
$$(x_2 - x_1, z_2 - z_1)$$

02 How to Implement

- Step 3



- Calculate circle property

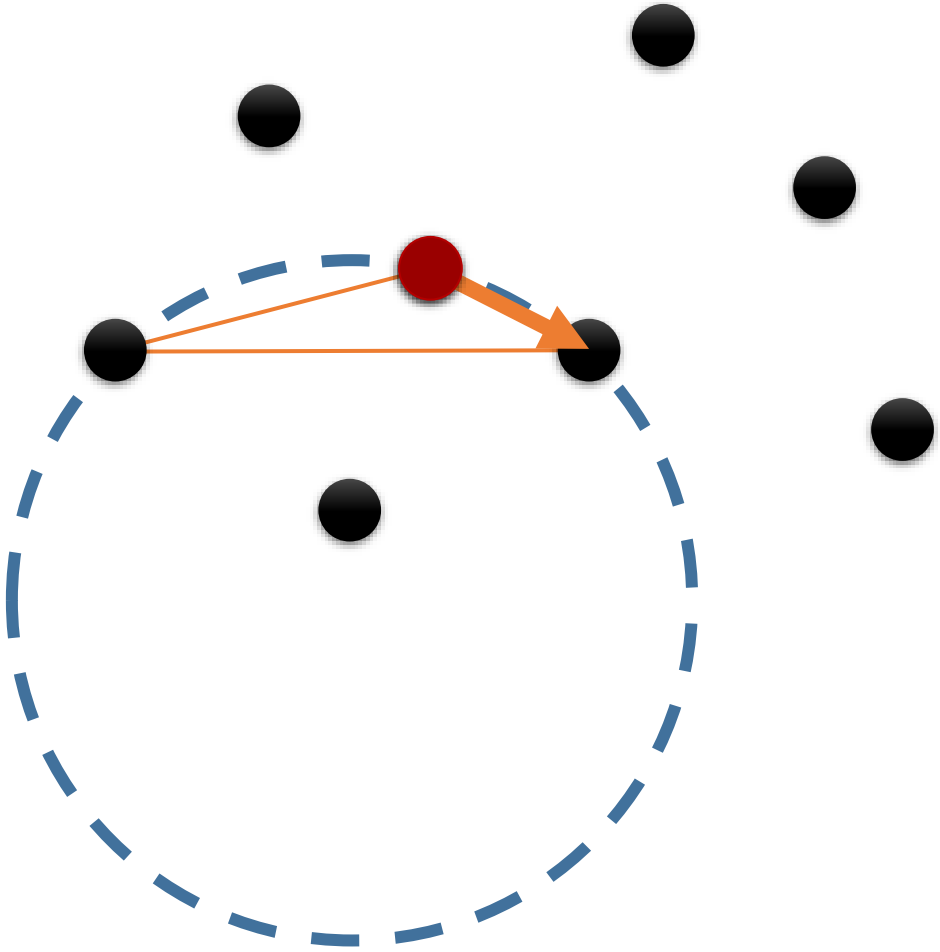


$$(x-x_c)^2+(z-z_c)^2-R^2=0$$

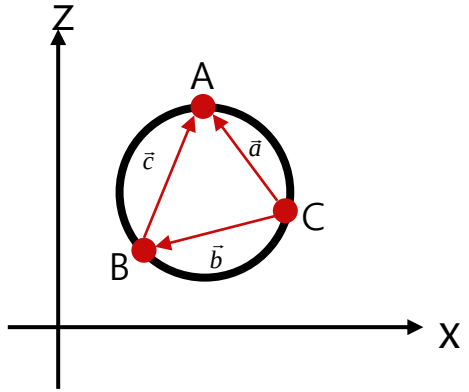
$$\begin{cases} x_c = C_x + \dfrac{b_y(a_x^{\,2}+a_y^{\,2})-a_y(b_x^{\,2}+b_y^{\,2})}{2(a_xb_y-a_yb_x)} \\ z_c = C_z + \dfrac{-b_x(a_x^{\,2}+a_y^{\,2})+a_x(b_x^{\,2}+b_y^{\,2})}{2(a_xb_y-a_yb_x)} \end{cases}$$

02 How to Implement

- Step 3



- Calculate circle property

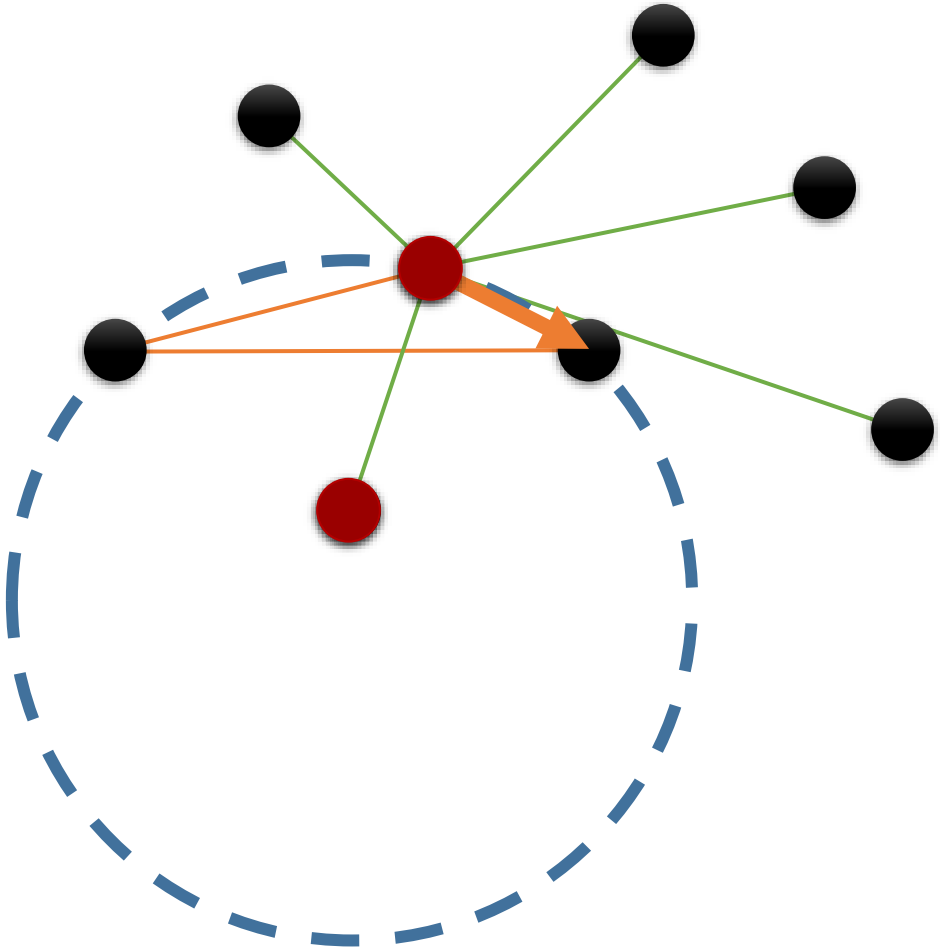


$$(x-x_c)^2+(z-z_c)^2-R^2=0$$

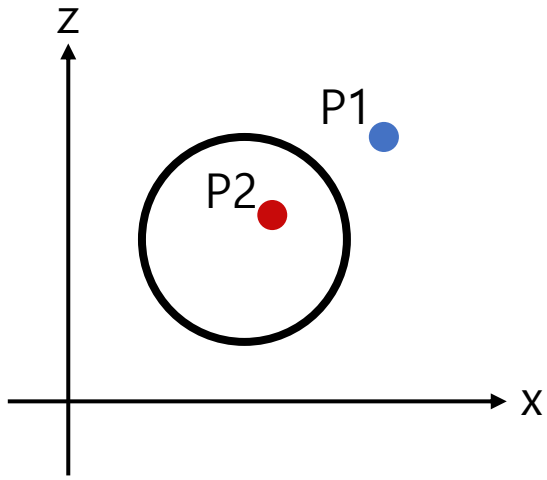
$$R=\frac{|\vec{a}||\vec{b}||\vec{c}|}{2|\vec{a}\times\vec{b}|}$$

02 How to Implement

- Step 4



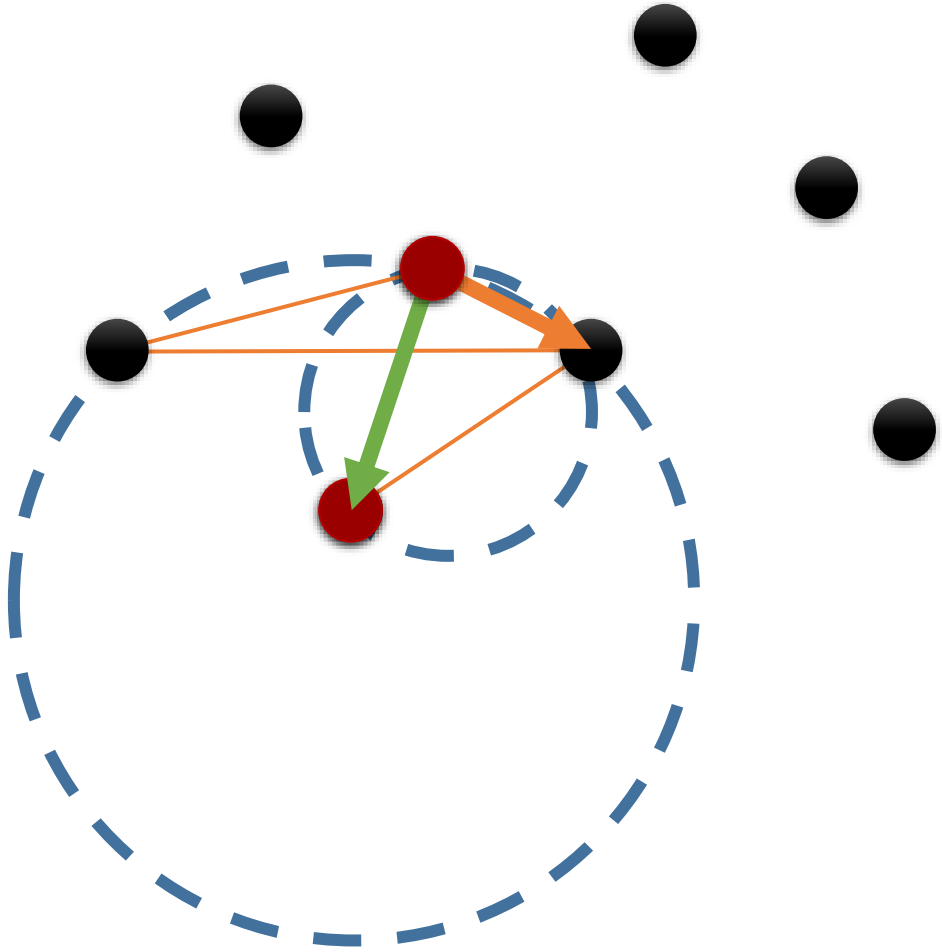
- Check empty circumcircle property



$$(x-x_c)^2+(z-z_c)^2-R^2 \begin{cases} <0 & \text{P2} \\ >0 & \text{P1} \end{cases}$$

02 How to Implement

- Step 5



- Check clockwise direction

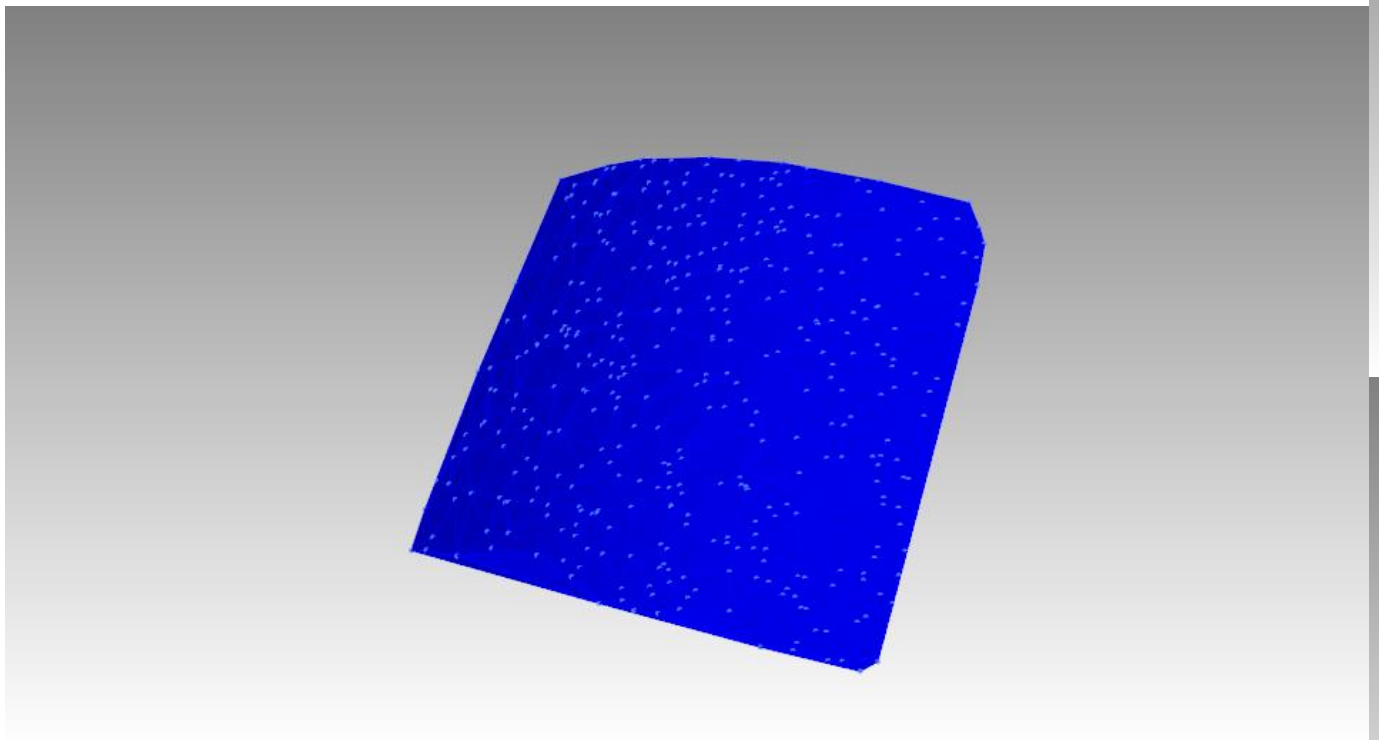
$$v_1 = (x_1, z_1)$$
$$v_2 = (x_2, z_2)$$

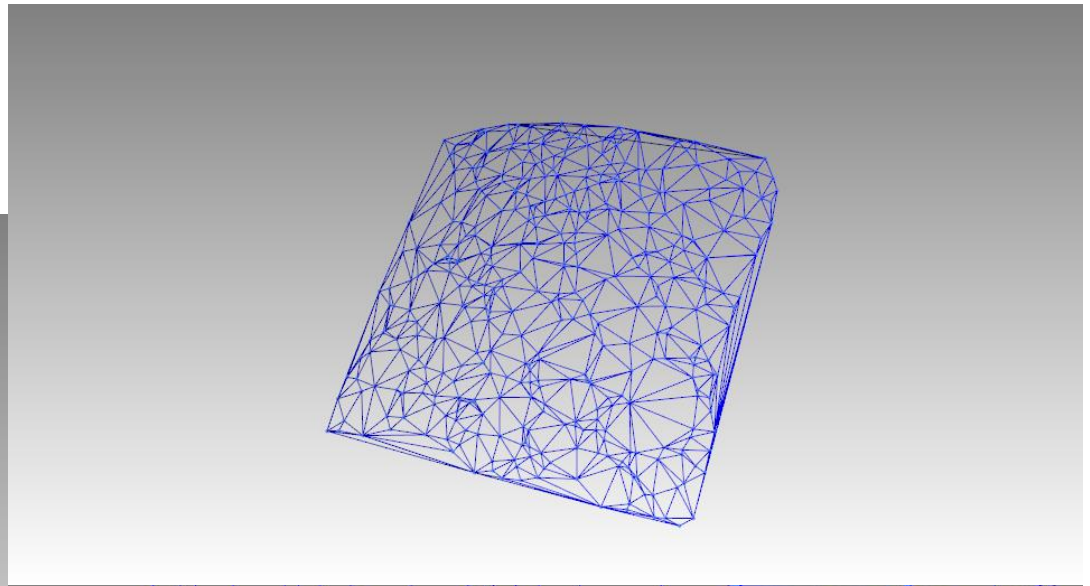
$$\theta = \sin^{-1}\left(\frac{x_1 z_2 - z_1 x_2}{\sqrt{x_1^{\,2} + z_1^{\,2}}\sqrt{x_2^{\,2} + z_2^{\,2}}}\right)$$

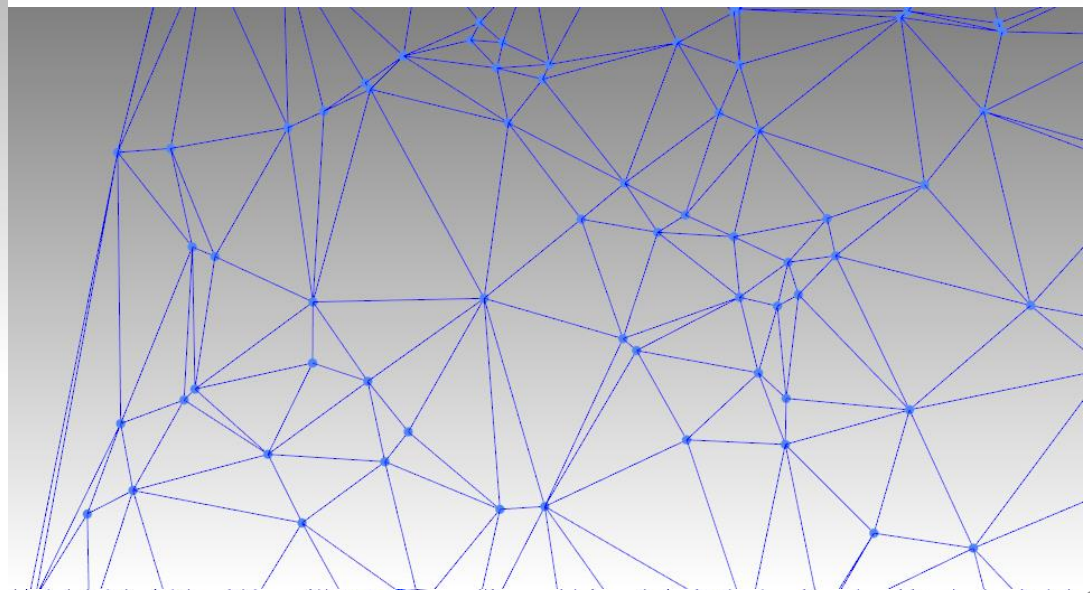
$$\theta \begin{cases} < 0 & \text{CW} \\ = 0 & \text{Co-linear} \\ > 0 & \text{CCW} \end{cases}$$

03 Result

- Hydrofoil

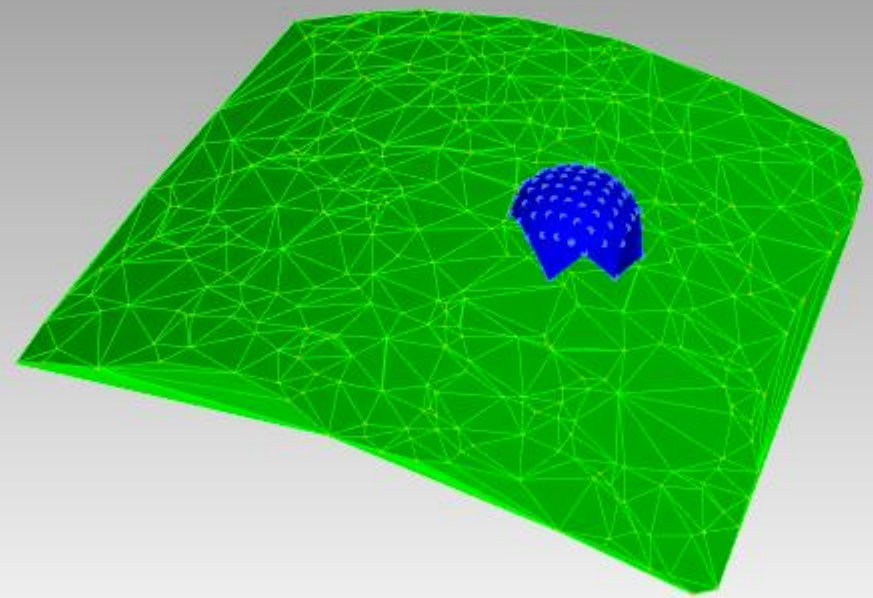






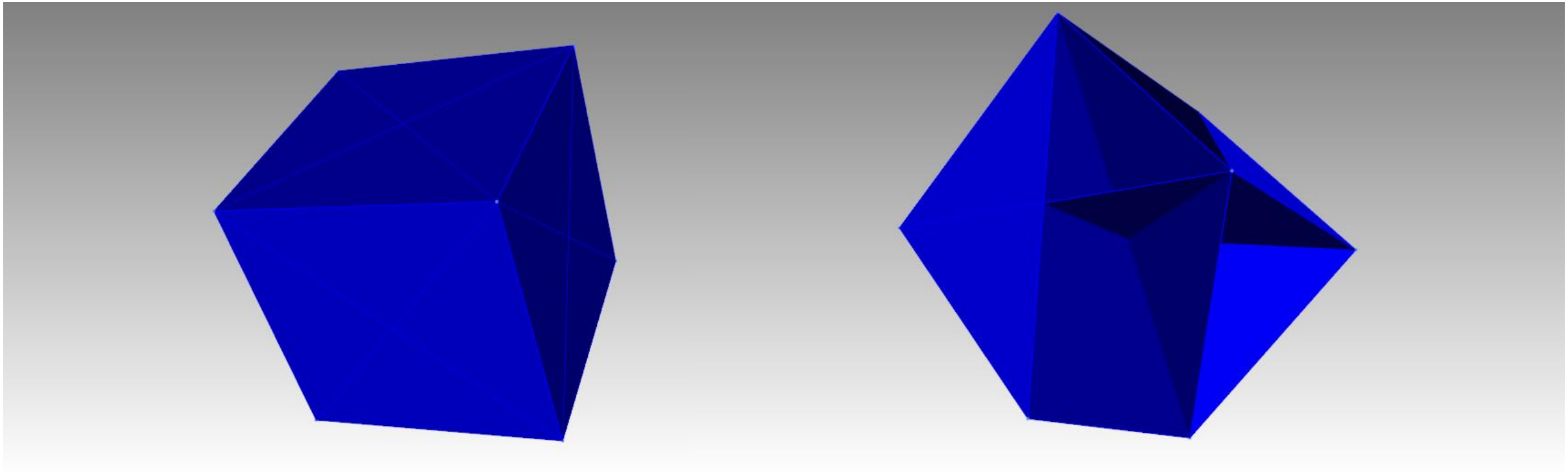
03 Result

- Cavitation



03 Result

- Other



-> 이런 형태의 PCD는 다른 과정이 추가로 필요하다.

감사합니다

Q & A

00 Normal