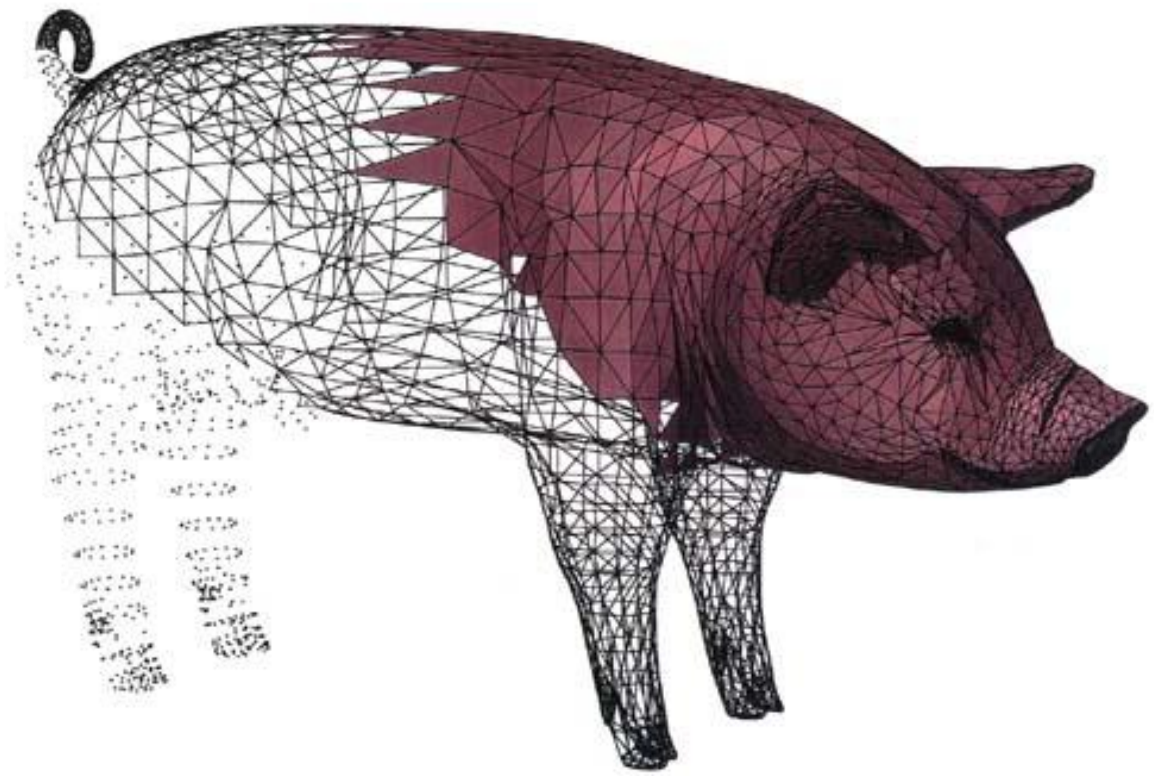
Poisson surface reconstruction

※ Kazhdan, Michael, Matthew Bolitho, and Hugues Hoppe. "Poisson surface reconstruction." *Proceedings of the fourth Eurographics symposium on Geometry processing*. Vol. 7. 2006.

안재원

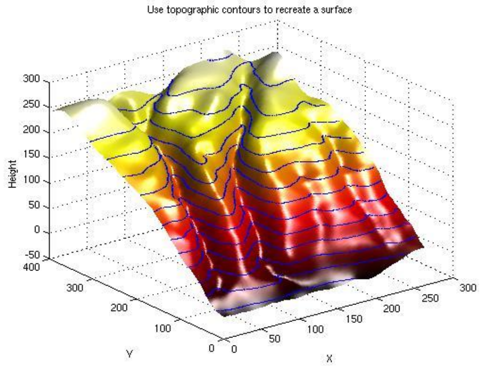
목차

01 Surface Reconstruction

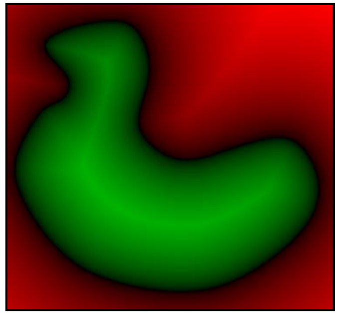


<General approaches>

- Computational Geometry
- Surface Fitting



- Implicit Function Fitting



02 Poisson Surface Reconstruction

- Intro

Poisson equation

- Second order PDE(Partial Differential Equation)
- n차원 다양체 M위에서, f가 M위에 주어진 함수라 할 때, 미지의 함수 φ는 다음을 만족한다.

$$\Delta\varphi = \nabla^2\varphi = f$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\varphi(x, y, z) = f(x, y, z)$$

※ In 3-Dimensional

Gradient

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \cdots, \frac{\partial f}{\partial x_n}\right)^T$$

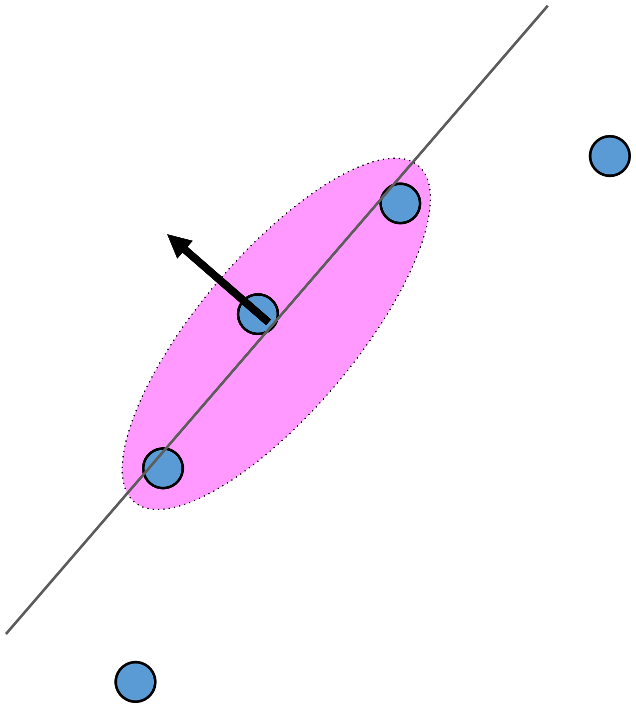
Divergence

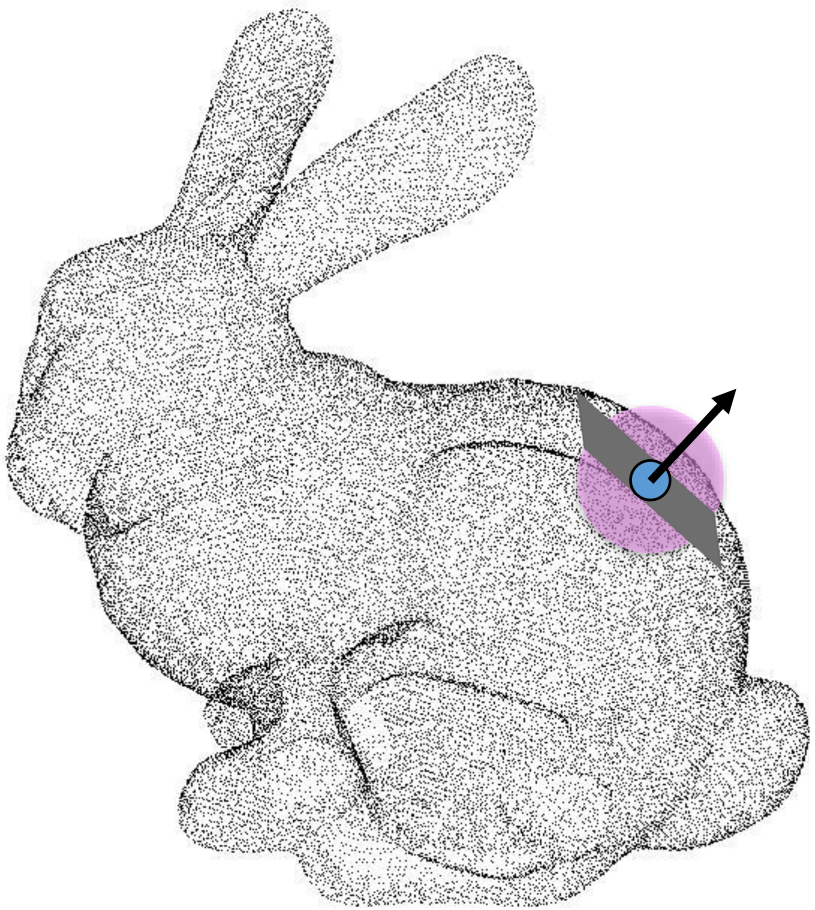
$$\vec{\nabla} \cdot \vec{f} = \frac{\partial \vec{f}}{\partial x_1} + \cdots + \frac{\partial \vec{f}}{\partial x_n}$$

02 Poisson Surface Reconstruction

- *Intro*

Normal



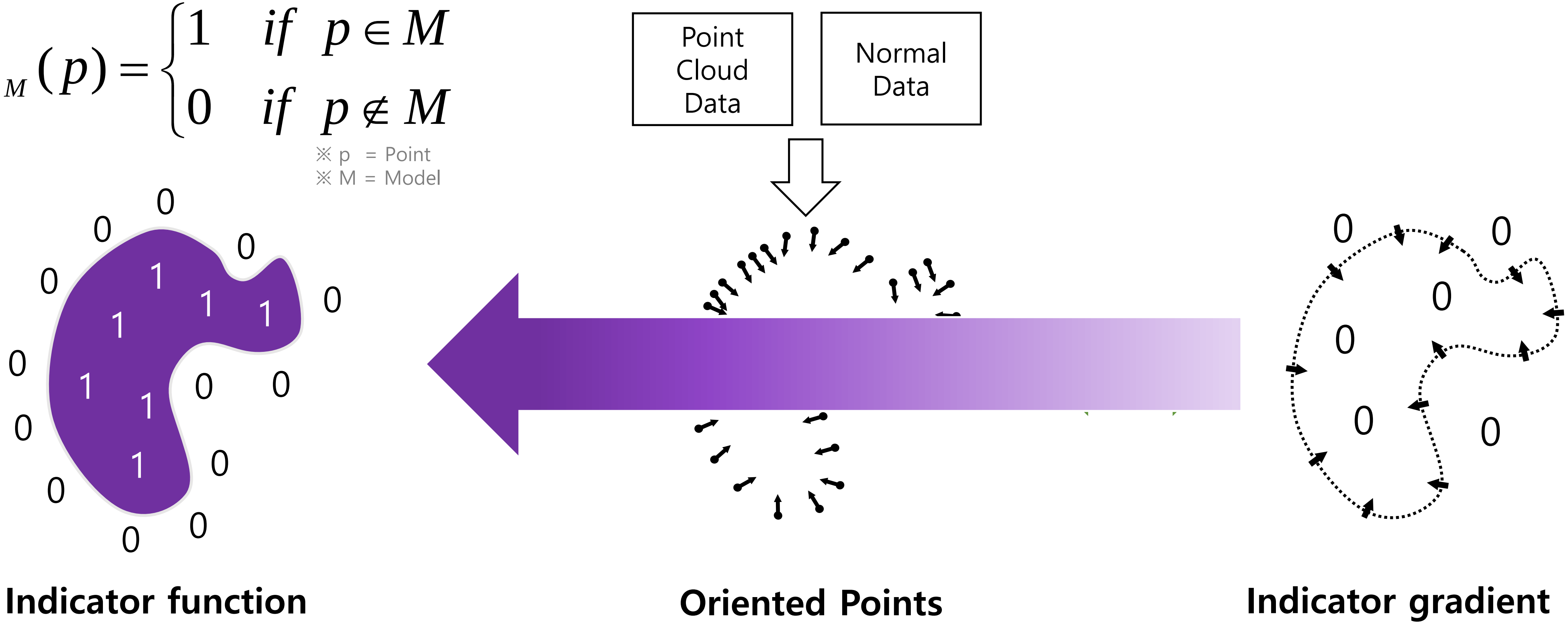


02 Poisson Surface Reconstruction

- *Get Indicator function*

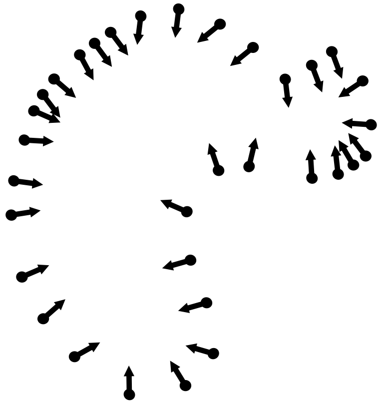
$$X_M(p) = \begin{cases} 1 & \text{if } \; p \in M \\ 0 & \text{if } \; p \notin M \end{cases}$$

※ p = Point
※ M = Model



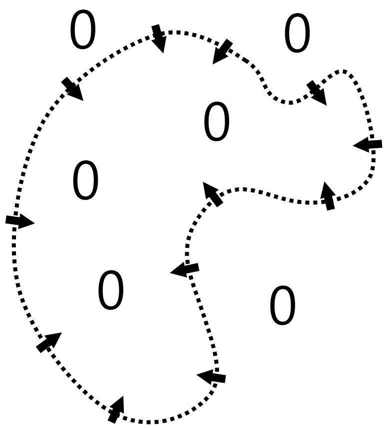
02 Poisson Surface Reconstruction

- Get Indicator Gradient



Oriented Points

$\vec{V}$



Indicator gradient

∇X_M

$$\min_X \left\| \nabla X - \vec{V} \right\|$$

Apply the divergence operator

$$\nabla \cdot \vec{V} = \nabla \cdot \nabla X \equiv \Delta X$$

Poisson equation

02 Poisson Surface Reconstruction

- Relationship between Gradient of the indicator function and Surface normal field

$$\nabla \cdot \vec{V} = \nabla \cdot \nabla X \equiv \Delta X$$

-Lemma

$$\nabla (X_M * \tilde{F})(q_0) = \int_{\partial M} \tilde{F}_p(q_0) \vec{N}_{\partial M}(p) dp$$

Model M

Boundary ∂M

Indicator function X_M

Surface normal $\vec{N}_{\partial M}$

Smoothing filter $\tilde{F}(q)$

Translation to point p $\tilde{F}_p(q) = \tilde{F}(q - p)$

02 Poisson Surface Reconstruction

- Relationship between Gradient of the indicator function and Surface normal field

$$\nabla \cdot \vec{V} = \nabla \cdot \nabla X \equiv \Delta X$$

-Lemma

$$\nabla(X_M * \tilde{F})(q_0) = \int_{\partial M} \tilde{F}_p(q_0)\vec{N}_{\partial M}(p)dp$$

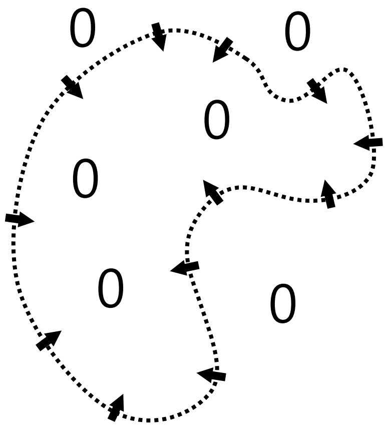
※ In x-components

$$\left.\frac{\partial}{\partial x}\right|_{q_0}(X_M * \tilde{F}) = \left.\frac{\partial}{\partial x}\right|_{q=q_0}\int_M \tilde{F}(q-p)dp$$

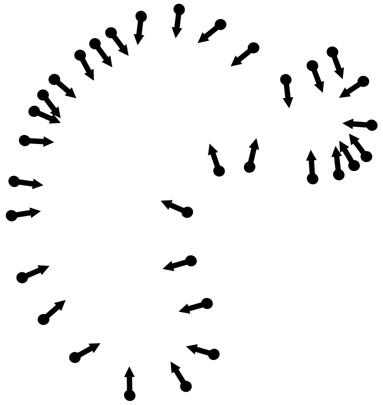
$$= \int_M \left(-\frac{\partial}{\partial x}\tilde{F}(q_0 - p)\right)dp$$

$$= -\int_M \nabla \cdot (\tilde{F}(q_0 - p),0,0)dp$$

$$= \int_{\partial M}\left\langle (\tilde{F}_p(q_0),0,0), \vec{N}_{\partial M}(p)\right\rangle dp$$



Indicator gradient



Oriented Points

02 Poisson Surface Reconstruction

- Relationship between Gradient of the indicator function and Surface normal field

$$\nabla(X_M * \tilde{F})(q_0) = \int_{\partial M} \tilde{F}_p(q_0)\vec{N}_{\partial M}(p)dp$$

$$\Rightarrow \nabla(X_M * \tilde{F})(q) = \sum_{s \in S} \int_{\mathcal{P}_s} \tilde{F}_p(q)\vec{N}_{\partial M}(p)dp \qquad \mathcal{P}_s \subset \partial M$$

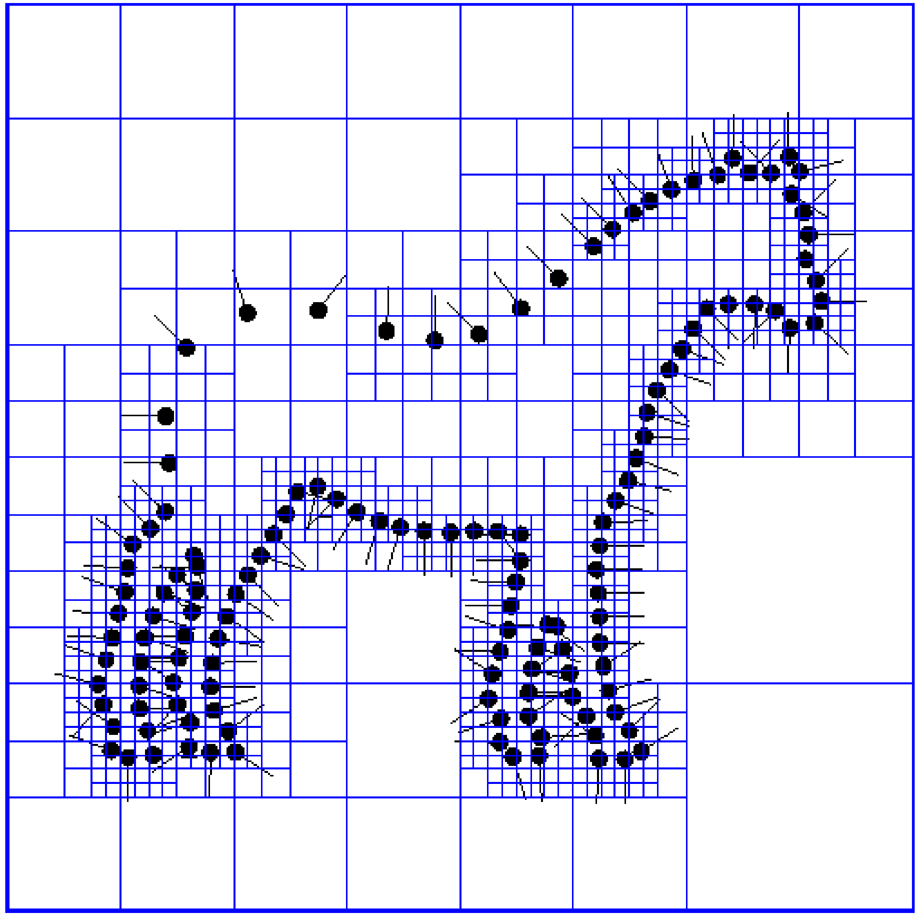
$$\approx \sum_{s \in S} |\mathcal{P}_s| \tilde{F}_{s.p}(q) s.\vec{N}(p)$$

$$\equiv \vec{V}(q)$$

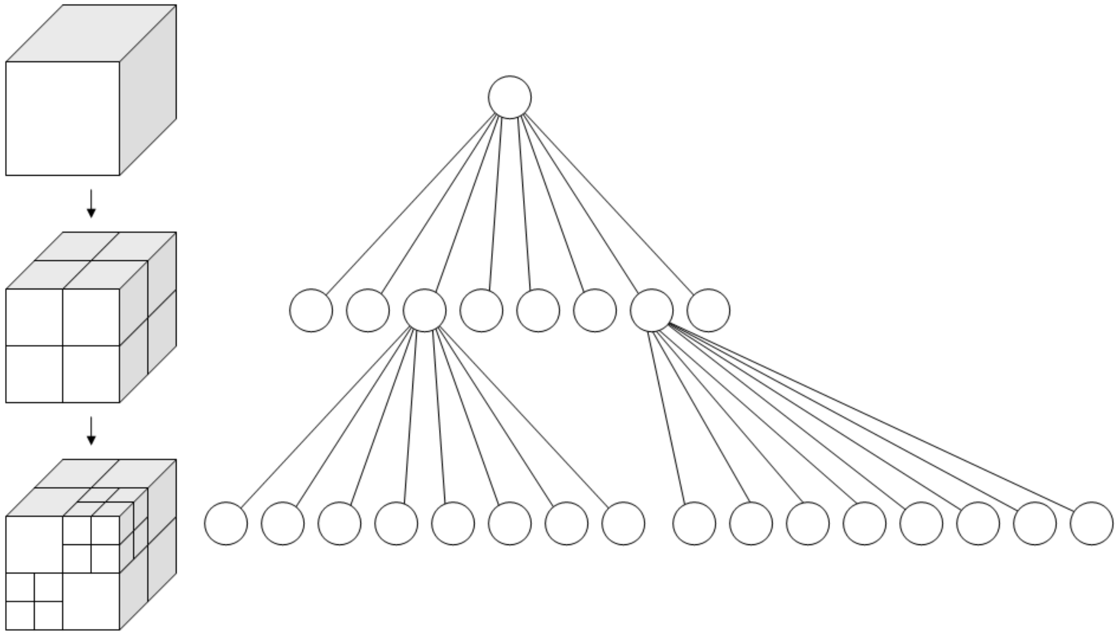
$$\Rightarrow \vec{V} = \nabla X$$

02 Poisson Surface Reconstruction

- Implementation

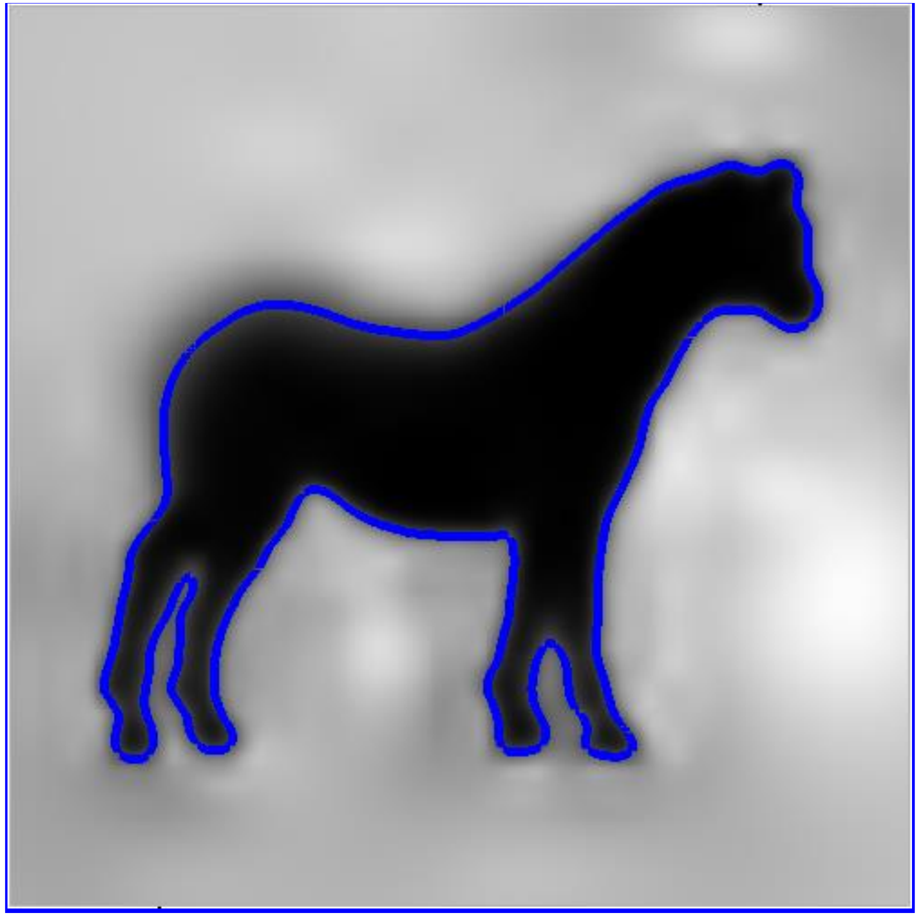


- **Octree**



02 Poisson Surface Reconstruction

- Implementation



- **Approximate** $\vec{\hat{V}}(q)$

$$\vec{\hat{V}}(q) \equiv \sum_{s\in S} \sum_{o\in Ngbr_D(s)} \alpha_{o,s} F_o(q) s.\vec{N}$$

- **Poisson equation problem**

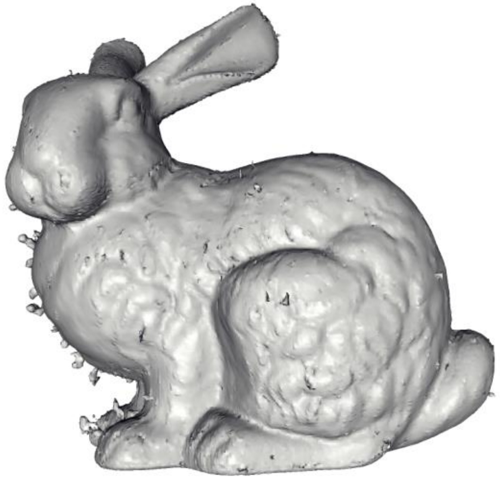
$$\nabla \cdot \vec{\hat{V}} = \Delta \hat{X}$$

03 Performance Test

- *Stanford Bunny*

Power Crust

FastRBF



MPU

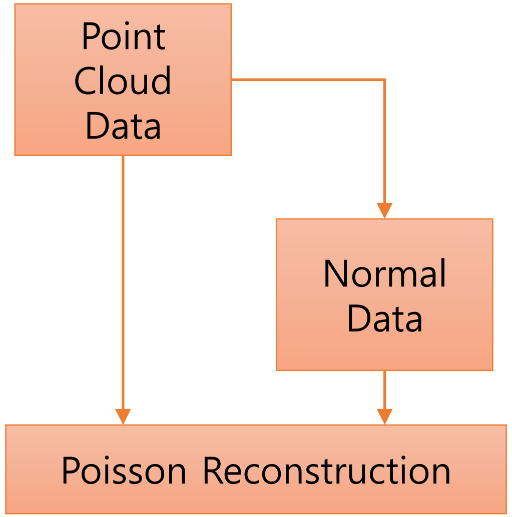
VRIP

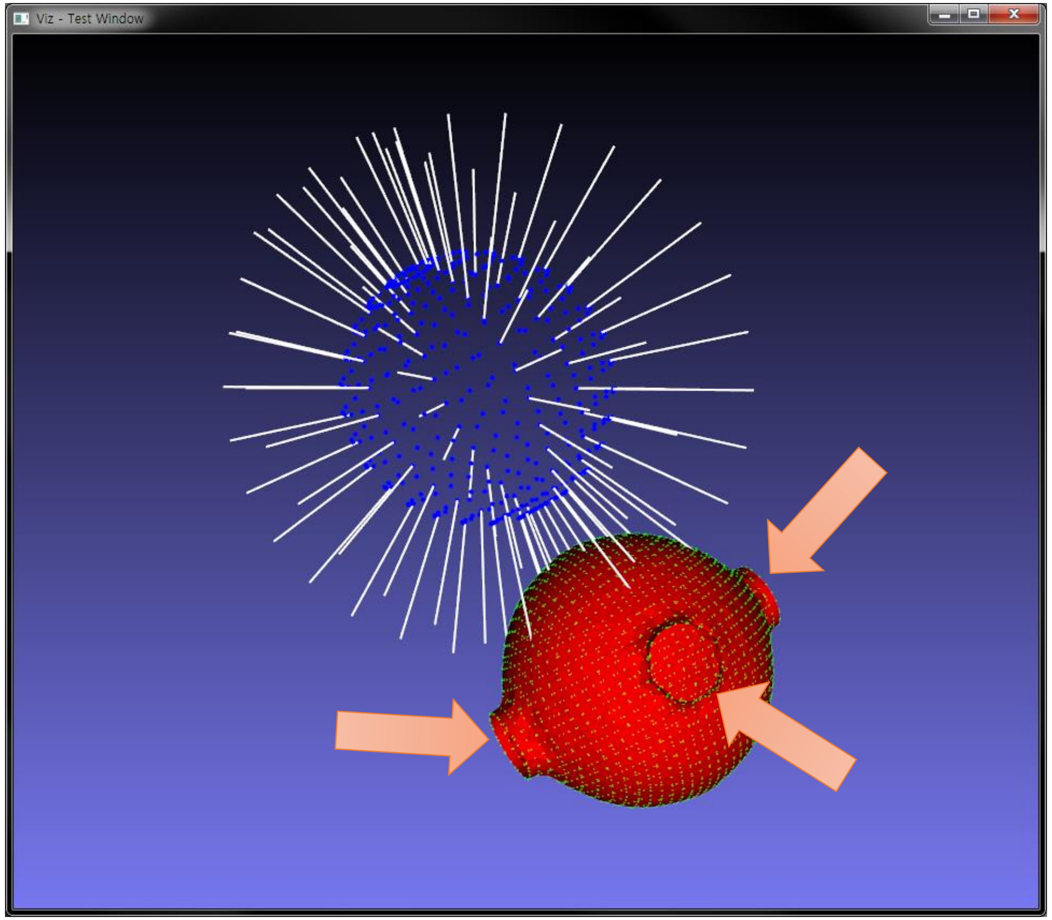
FFT Reconstruction

Our Method

03 Performance Test

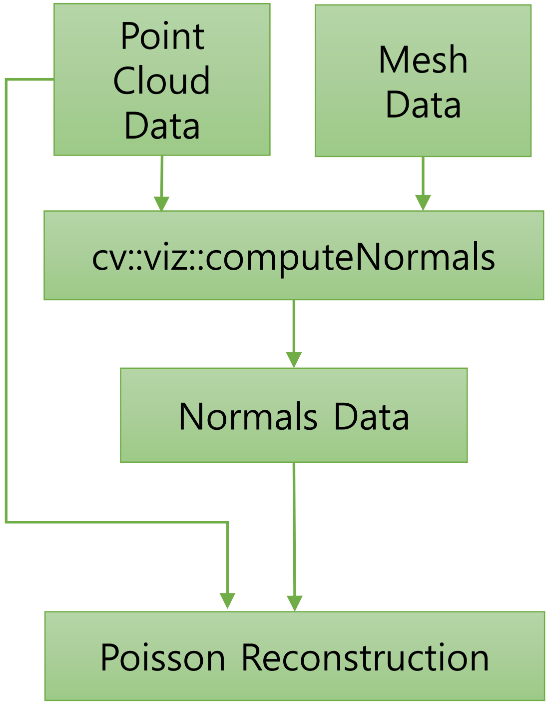
- Set Normal(By myself)

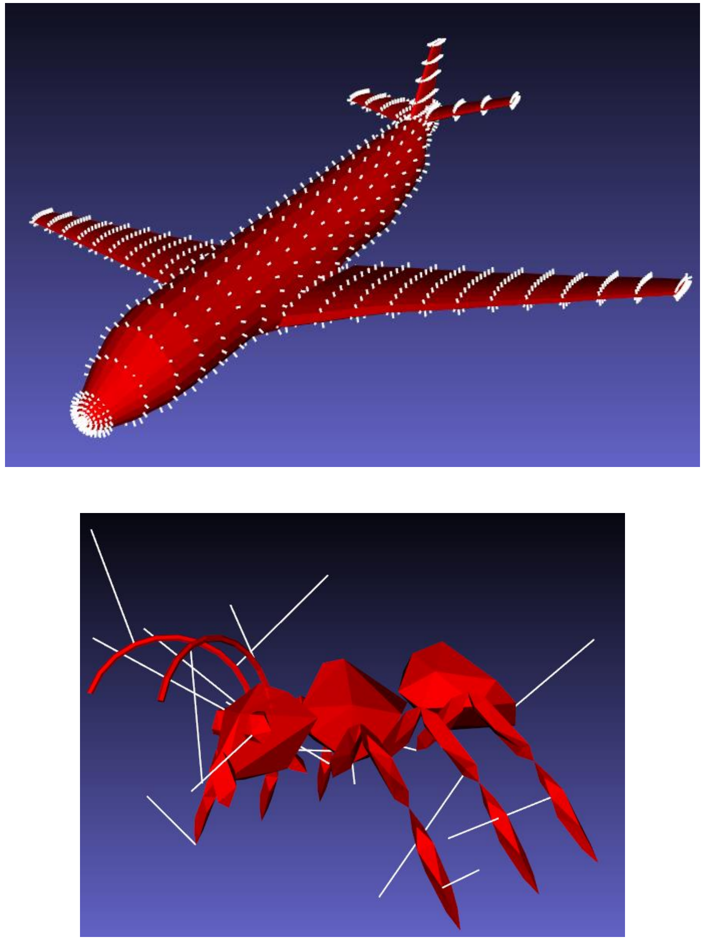




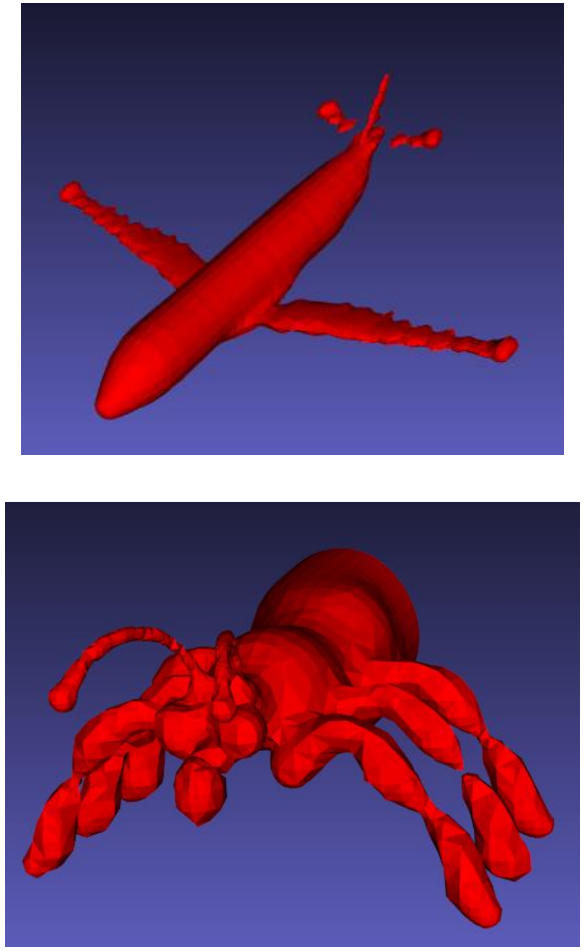
03 Performance Test

- Set Normal(Use OpenCV Library)





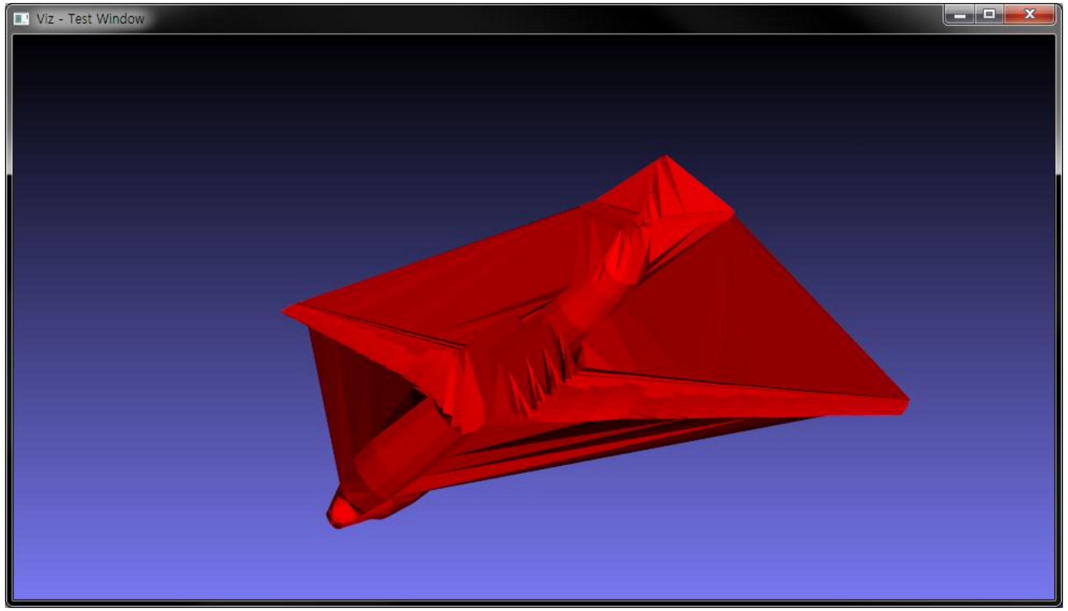
<Origin Data>

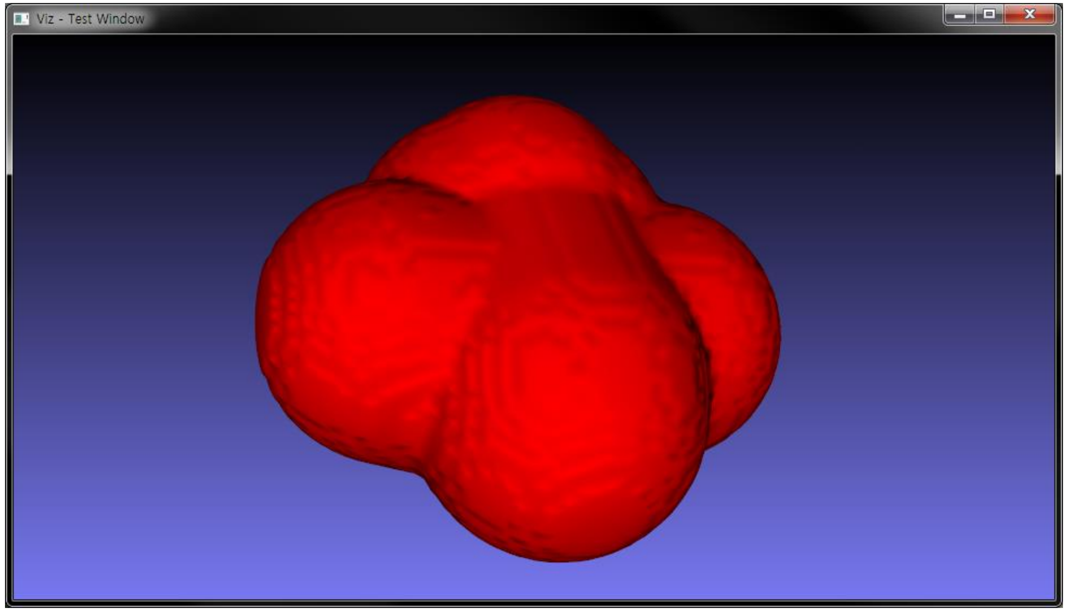


<Poisson Reconstruction Result>

04 Etc

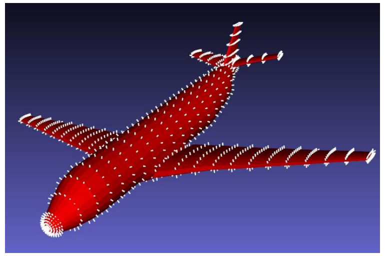
- VTK Library





<Triangulate a Terrain Map>

<Gaussian Splat>



감사합니다

Q & A