2015.05.08

Basic of Camera Calibration : Pt.1

강한솔

ISL Image System Laboratory

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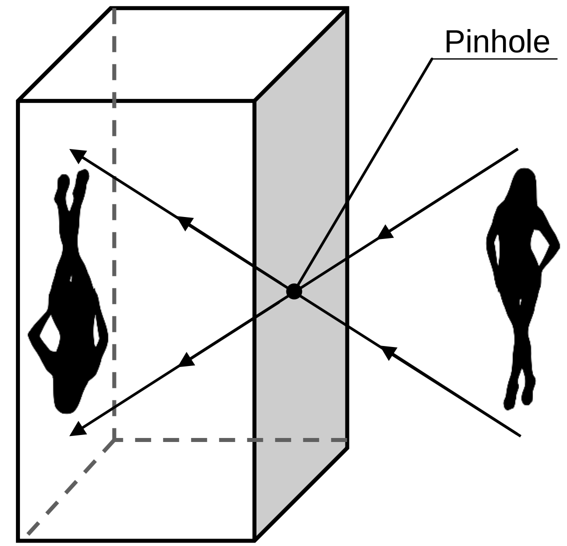
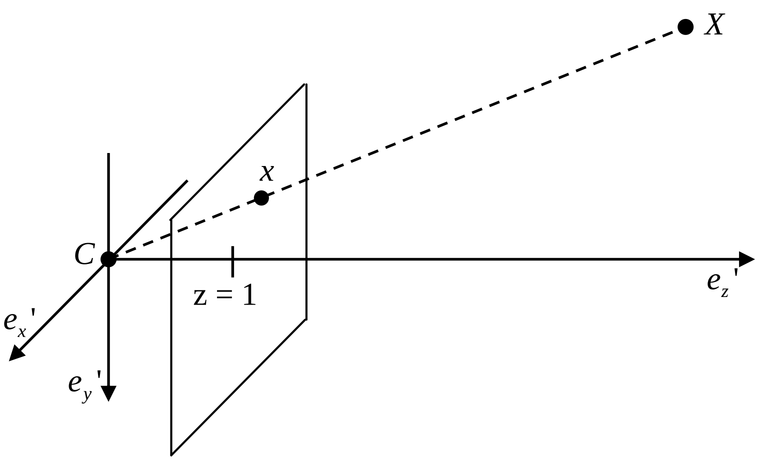
01 The Pinhole Camera Model

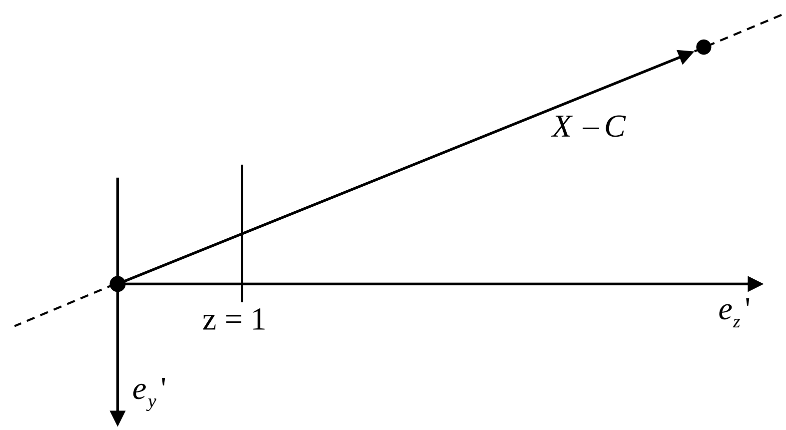
- What’s the pinhole camera?



01 The Pinhole Camera Model

- What's the pinhole camera?



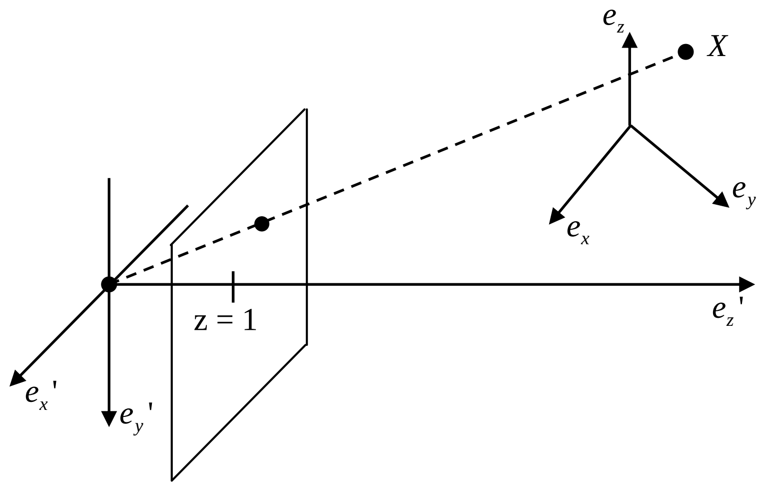


$$C + s(X - C) = sX, \quad s \in \mathbf{R}$$

$$X = \begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} \qquad \begin{matrix} sX_3' = 1 \\ s = \dfrac{1}{X_3'} \end{matrix} \qquad x = \begin{pmatrix} X_1'/X_3' \\ X_2'/X_3' \\ 1 \end{pmatrix}$$

01 The Pinhole Camera Model

- Moving Cameras

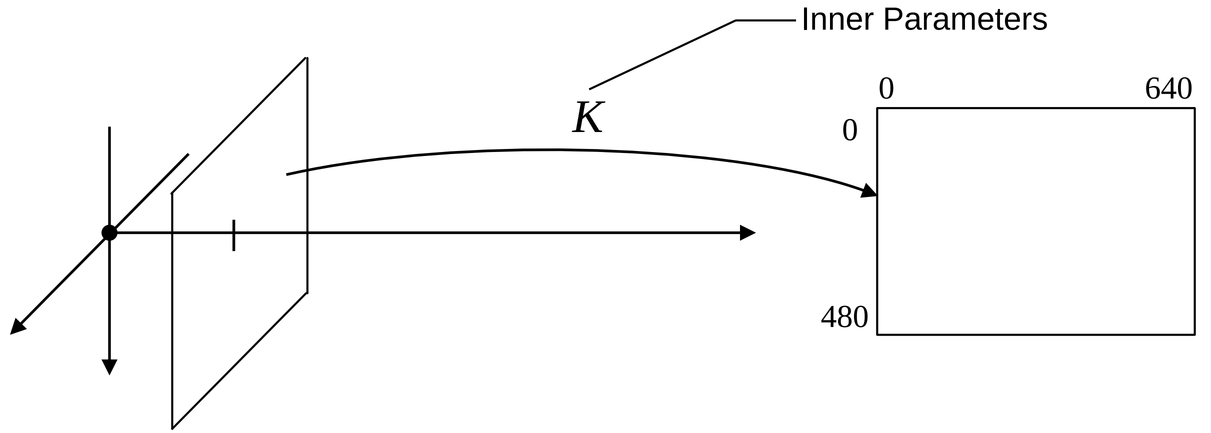


$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = R \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} + t \qquad X_3' \begin{pmatrix} X_1'/X_3' \\ X_2'/X_3' \\ 1 \end{pmatrix} = \begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = [R \quad t] \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ 1 \end{pmatrix} \qquad v = [R \quad t] \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$X_3' = [R_3 \quad t_3] \begin{bmatrix} X \\ 1 \end{bmatrix} \qquad \begin{cases} positive: in \;\; foront \;\; of \;\; the \;\; camera \\ negative: behind \;\; the \;\; camera \end{cases}$$

01 The Pinhole Camera Model

The Inner Parameters



$$x = \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} X_1'/X_3' \\ X_2'/X_3' \\ 1 \end{pmatrix} \qquad X_3'\begin{pmatrix} X_1'/X_3' \\ X_2'/X_3' \\ 1 \end{pmatrix} = \begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = [R \quad t]\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ 1 \end{pmatrix} \qquad \underbrace{\lambda \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}}_{\mathbf{x}} = \underbrace{K[R \quad t]}_{P} \underbrace{\begin{pmatrix} X_1' \\ X_2' \\ X_3' \\ 1 \end{pmatrix}}_{\mathbf{X}}$$

Camera Equation

$$\lambda \mathbf{x} = P\mathbf{X}$$

Camera matrix

02 Homogeneous Coordinates, Line and Conics

Homogeneous Coordinates

$$\lambda \mathbf{x} = P\mathbf{X}$$

ex)

$$v = P\mathbf{X}$$

$$v = (6,9,3) \xrightarrow{\text{projection}} (2,3)$$

$$v = (4,6,2) \xrightarrow{\text{projection}} (2,3)$$

➡ $(6,9,3)$ and $(4,6,2)$ are **homogeneous coordinates** of $(2,3)$

Given two vectors $\mathbf{x},\mathbf{y} \in \mathbf{R}^3$

$$\mathbf{x} = \lambda \mathbf{y} \ \ (\lambda \neq 0)$$

$$\mathbf{x} \sim \mathbf{y}$$

02 Homogeneous Coordinates, Line and Conics

Line and Points in $\mathbf{P}^2$

$$ax + by + c = 0$$

$$l = (a,b,c) \quad \mathbf{x} \sim (x,y,1) \ (\mathbf{x}: belong\ to\ l)$$

$$ax + by + c \longrightarrow (a,b,c) \bullet (x,y,1)$$

If we use $(\lambda x, \lambda y, \lambda)$

$$a\lambda x + b\lambda y + c\lambda = \lambda(ax + by + c) = 0$$

Represent l with $\left(\frac{a}{c}, \frac{b}{c}, 1\right)$ instead of (a,b,c)

➡ **Points are dual to lines in $\mathbf{P}^2$**

ex)

Two lines intersect each other in one point

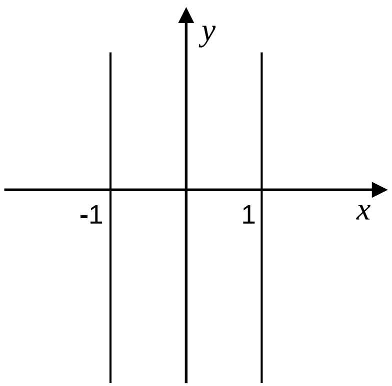
↕ **Is dual**

For any two points there is one line going through them both

02 Homogeneous Coordinates, Line and Conics

Vanishing Points

$$l_1 = (-1,0,1) \quad l_2 = (1,0,1)$$



$$\begin{cases} l_1^T \mathbf{x} = 0 \\ l_2^T \mathbf{x} = 0 \end{cases} \Leftrightarrow \begin{cases} -x + z = 0 \\ x + z = 0 \end{cases}$$

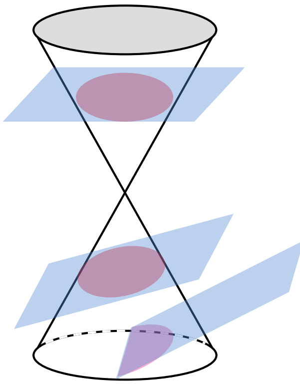
$$\begin{cases} x + z = 0 \\ 2z = 0 \\ y = t \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = t \\ z = 0 \end{cases}$$

ex) Intersection Point $(0,1,0)$

$$(0,1,\varepsilon) \xrightarrow{\div \varepsilon} \left(0, \frac{1}{\varepsilon}, 1\right)$$

02 Homogeneous Coordinates, Line and Conics

- Conics(2nd order curve)



$$\mathbf{x}^{\mathrm{T}} C \mathbf{x} = \mathbf{0}$$
$$(C : Symmetric\ matrix)$$

ex) circle

$$\begin{pmatrix} x & y & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = x^2 + y^2 - 1 = 0$$

- Dual Conic The set of all lines that tangent to the conic

$l = Cx$: tangent line to the conic at the point x

$$\begin{aligned} & x^T C x = 0 \\ &= x^T C C^{-1} C x \\ &= (Cx)^T C^{-1} C x \\ &= l^T C^{-1} l \end{aligned}$$

$$l^T C^{-1} l = 0$$

02 Homogeneous Coordinates, Line and Conics

Projective Transformation

Invertible mapping $\mathbf{P}^n \mapsto \mathbf{P}^n$ $x \sim Hy$

Where,

$x \in \mathbf{R}^{n+1}, \ y \in \mathbf{R}^{n+1}$ are homogeneous coordinates representing elements of $\mathbf{P}^n$ and $\mathbf{H}_{(n+1)\times(n+1)}$ is invertible

Projective transformations are called **homographies.**

In camera equation, K is Homography

$$\mathbf{x} \sim K\begin{bmatrix} R & t \end{bmatrix} \mathbf{X}$$

The required number of point

H 9 elements, but since these scale dose not matter ➡ **8 DOF**

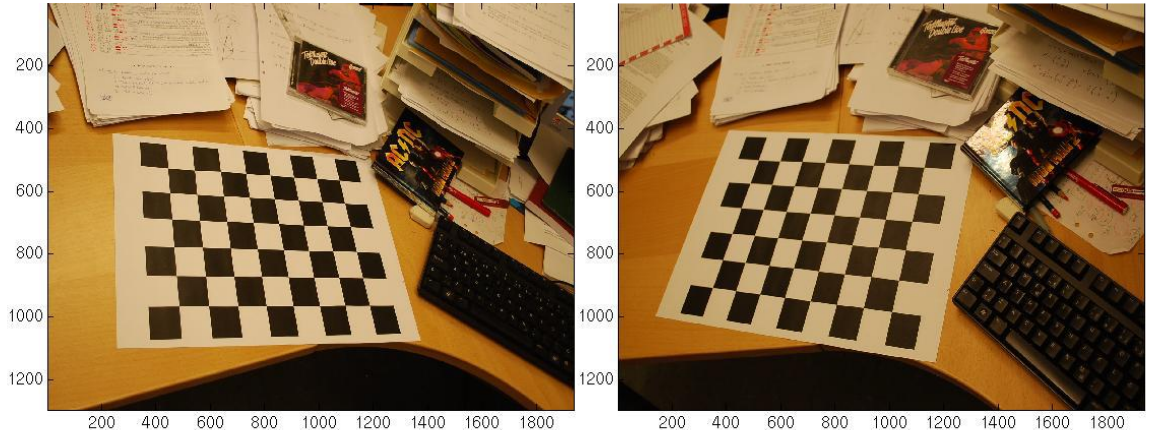
$\lambda_i \mathbf{x}_i = H\mathbf{y}_i \ (i = 1, \ldots, n)$ λ_i ➡ **3n equations**

➡ $3n \geq 8 + n \Leftrightarrow 2n \geq 8 \Leftrightarrow n \geq 4$

02 Homogeneous Coordinates, Line and Conics

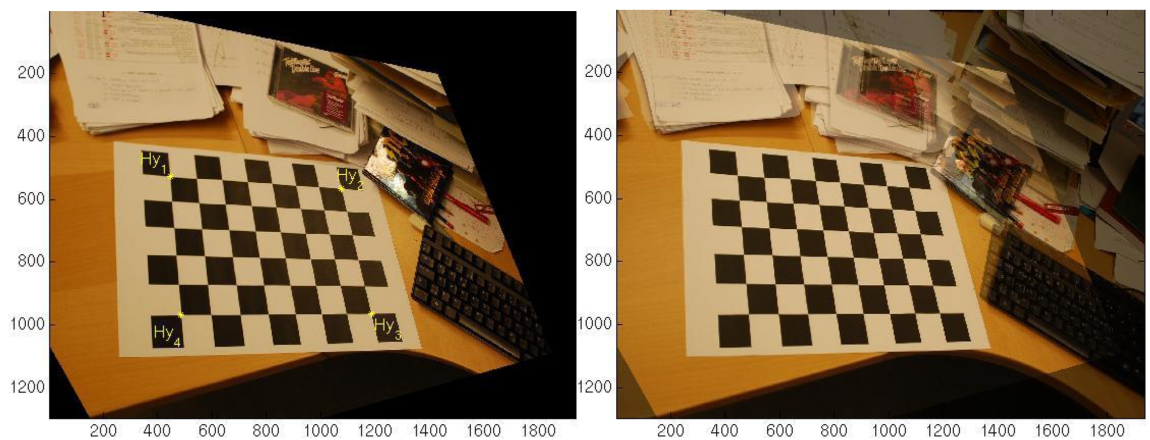
Projective Transformation

ex)



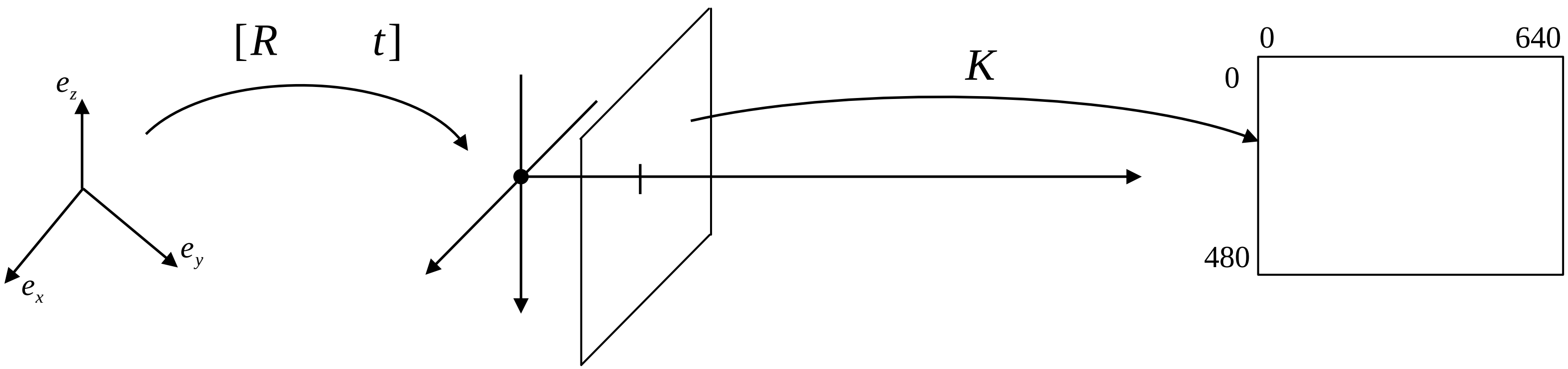
$$\mathbf{x_i} \sim P_1\mathbf{U_i},\ \mathbf{y_i} \sim P_2\mathbf{U_i}$$

$$\mathbf{x_i} \sim H\mathbf{y_i} \quad \lambda_i\mathbf{x}_i = H\mathbf{y}_i$$



03 The Inner Parameters

- K-matrix



$$K = \begin{pmatrix} \gamma f & s f & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

f : Focal length

x_0, y_0 : Principal point

γ : Aspect ratio

s : Skew

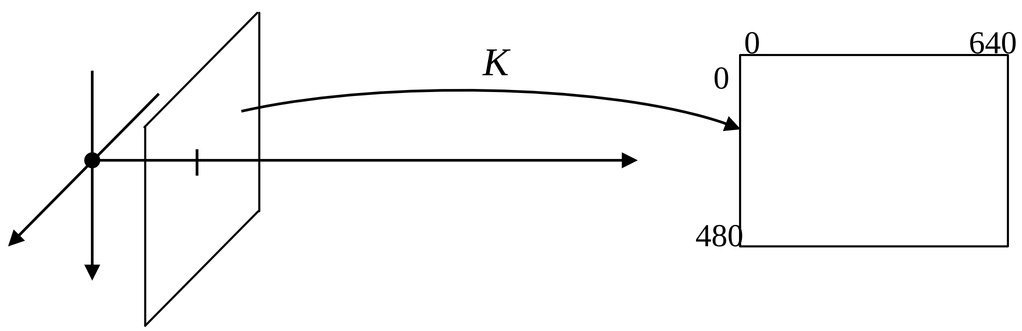
03 The Inner Parameters

- Focal length f   Re scales the images

$$\begin{pmatrix} fx \\ fy \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

- Principal point (x_0, y_0) Re centers the image

$$\begin{pmatrix} fx + x_0 \\ fy + y_0 \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$



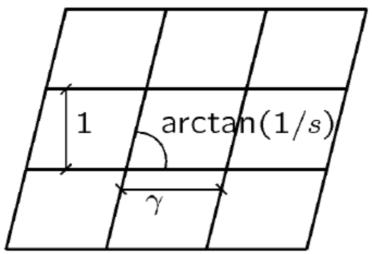
03 The Inner Parameters

- Aspect ratio Scaling in the x-direction

$$\begin{pmatrix} \gamma\, fx + x_0 \\ fy + y_0 \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma\, f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

- Skew Correct for tilted pixels.

$$\begin{pmatrix} \gamma\, f & s\, f & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$



03 The Inner Parameters

- Calibrated Cameras

$$P = K\begin{bmatrix} R & t \end{bmatrix}$$

$\downarrow$ If we know

Camera is calibrated

$$\lambda\tilde{\mathbf{x}} = K^{-1}K\begin{bmatrix} R & t \end{bmatrix}\mathbf{X} = \begin{bmatrix} R & t \end{bmatrix}\mathbf{X}$$

Where, $\tilde{\mathbf{x}} = K^{-1}\mathbf{x}$

normalized image points

normalized(calibrated) Camera

Q & A

Thank You