Rumor Spreading in Social Networks

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INTRODUCTION

How does a rumor spread in social networks? What's the role of personality? How does the spreading process vary in networks with different structures? These questions motivate our study of rumor spread in social networks. Specifically we are interested in exploring the following questions:

1: Will more talkative people spread rumor faster as compared to less talkative people?
2: How does rumor spread in a social network with "hubs" and "non-hubs"?
3: How could we slow down rumor spreading in social networks?

We come up with four models and compare the theoretical/numerical results with simulations on four types of networks: the Configuration model, Erdos-Renyi model, Watts-Strogatz model, and Barabasi-Albert Model. Our project may provide insight on rumor spread, and more generally, information spread in a society.

BASELINE MODEL

Model

We first implement an equivalent of SI model in homogeneous network. We assume that there are (S) spreaders and (I) ignorant people. We have the following parameters:

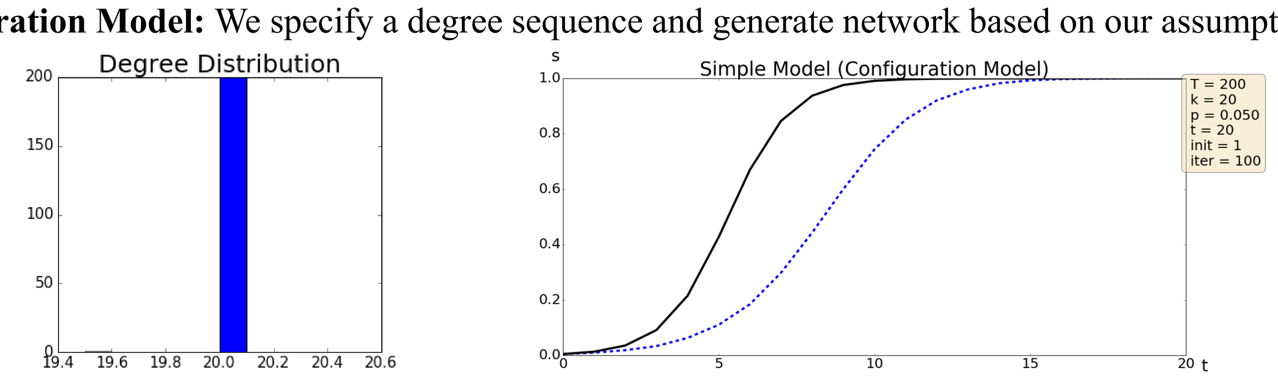
T : total population
s : proportion of the total population that has heard of the rumor
p : probability that a spreader spreads the rumor to an ignorant person
$<k>$: average degree of the network

We set up a differential equation:

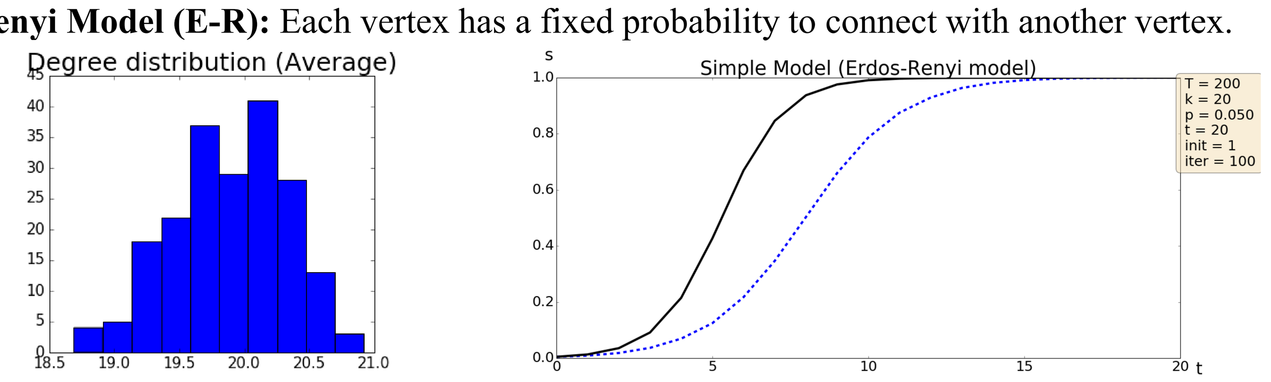
$$\frac{ds}{dt} = p\langle k\rangle s(1-s) \quad (1)$$

Results on Different Networks

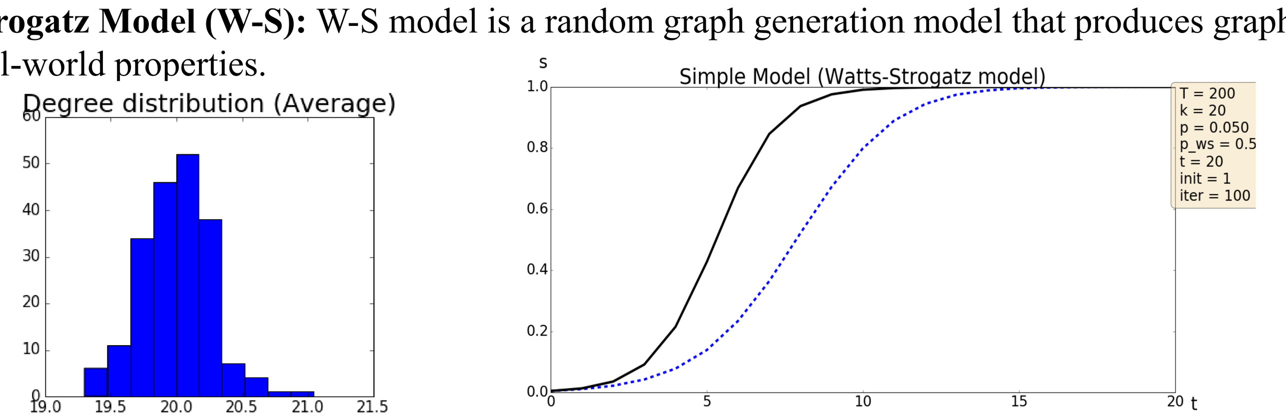
Configuration Model: We specify a degree sequence and generate network based on our assumption.



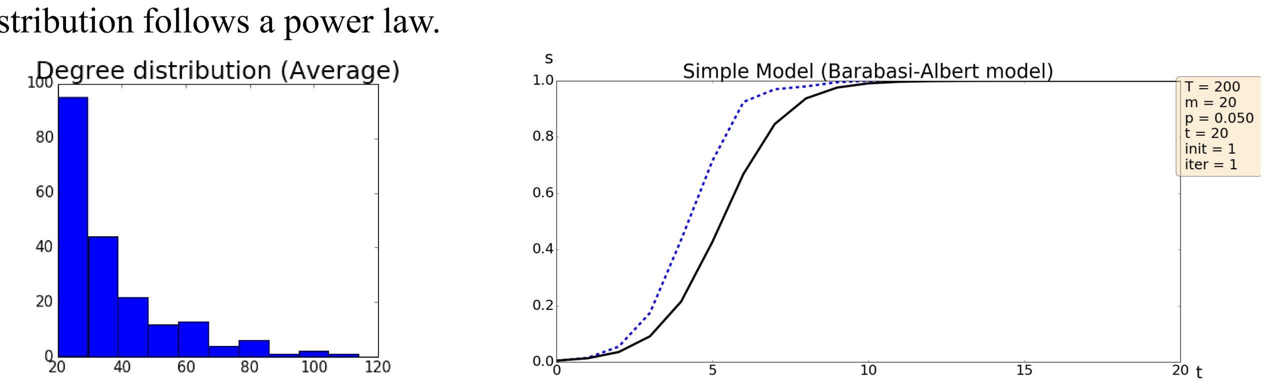
Erdos-Renyi Model (E-R): Each vertex has a fixed probability to connect with another vertex.



Watts-Strogatz Model (W-S): W-S model is a random graph generation model that produces graphs with small-world properties.



Barabasi-Albert Model (B-A): B-A model generates random scale-free networks, which means that the degree distribution follows a power law.



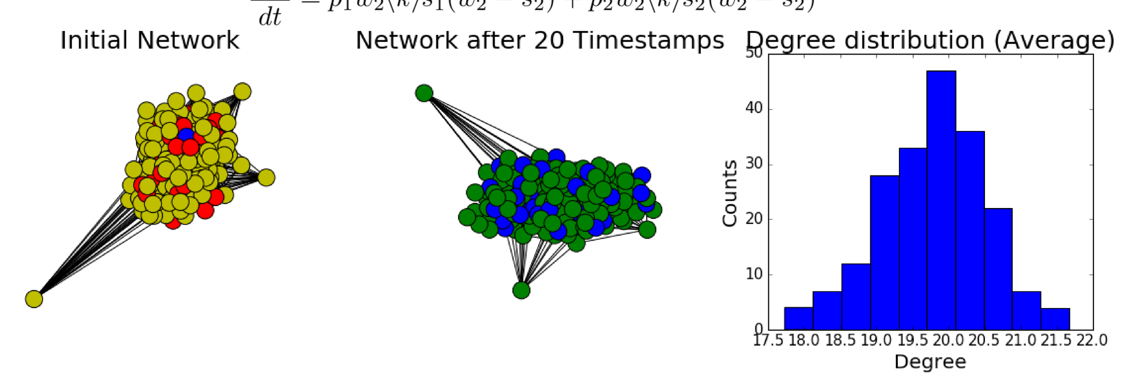
MODEL WITH TALKATIVE PEOPLE

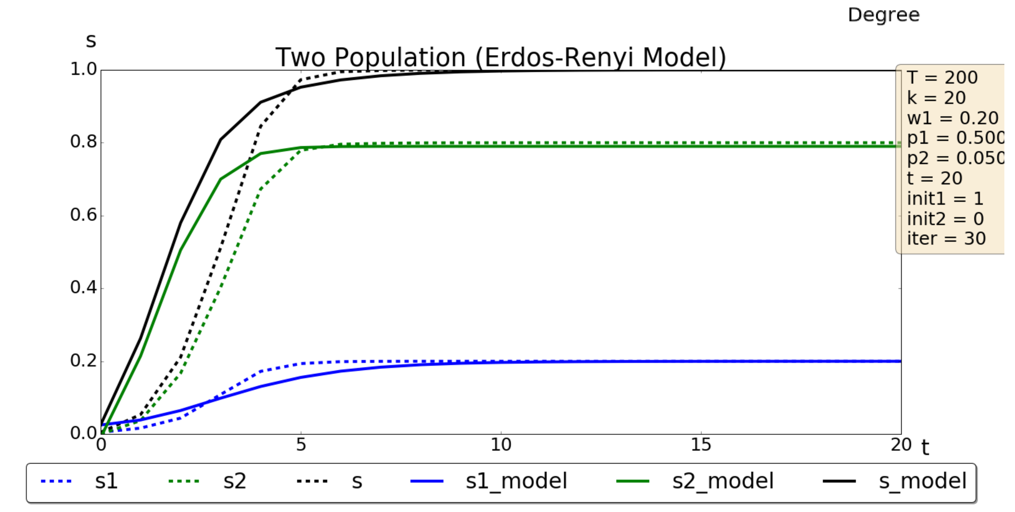
Next we divide the population into two groups based on talkativeness and we have the same set of parameters on each group. In this scenario, we have the following parameters:

T, T_1, T_2 : total population, population of the talkative group, population of the not talkative group
w_1, w_2 : weight of the total population that belongs to each group, respectively
s_1, s_2 : proportion of people in each group that has heard of the rumor, respectively
p_1, p_2 : probability that a talkative person and a non-talkative person spreads the rumor to an ignorant person, respectively
$<k>$: average degree of the network

In this scenario, an ignorant person may hear the rumor from spreaders from either group:

$$\begin{aligned} \frac{ds_1}{dt} &= p_1 w_1 \langle k\rangle s_1(w_1 - s_1) + p_2 w_1 \langle k\rangle s_2(w_1 - s_1) \\ \frac{ds_2}{dt} &= p_1 w_2 \langle k\rangle s_1(w_2 - s_2) + p_2 w_2 \langle k\rangle s_2(w_2 - s_2) \end{aligned} \quad (2)$$





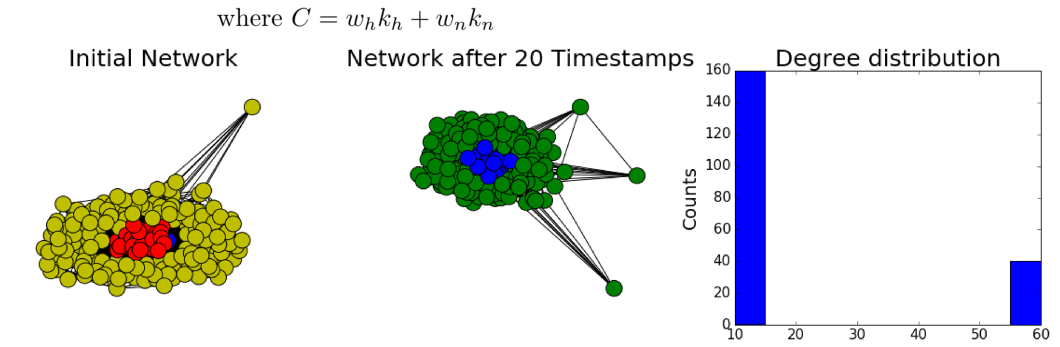
MODEL WITH HUBS

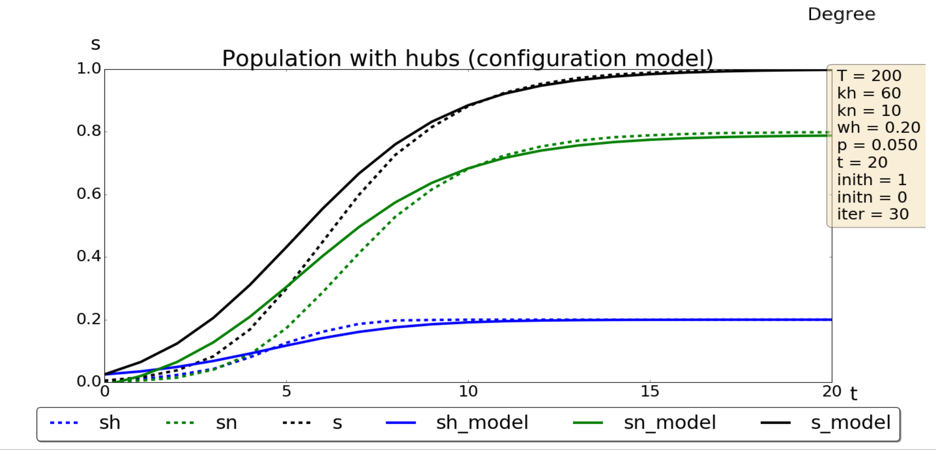
We then divide the population into hub group and non-hub group. People in the hub group have more connections than the people in the non-hub group. With this specification, we have different average number of degrees for each group. In this scenario, we have the following parameters:

T, T_h, T_n : total population, population of the hub group, population of the non-hub group
w_h, w_n : weights of the total population that belongs to hub group and non-hub group, respectively
s_h, s_n : proportions of people in each group out of total population that has heard of the rumor
p : probability that a spread spreads the information to an ignorant person
$<k_h>$, $<k_n>$: average degrees of the hub people and non-hub people, respectively

We set up a system of differential equations:

$$\begin{aligned} \frac{ds_h}{dt} &= p\frac{w_h\langle k_h\rangle}{C}\langle k_h\rangle s_h(w_h - s_h) + p\frac{w_h\langle k_h\rangle}{C}\langle k_n\rangle s_n(w_h - s_h) \\ \frac{ds_n}{dt} &= p\frac{w_n\langle k_n\rangle}{C}\langle k_n\rangle s_n(w_n - s_n) + p\frac{w_n\langle k_n\rangle}{C}\langle k_h\rangle s_h(w_n - s_n) \end{aligned} \quad (3)$$
$$\text{where } C = w_h k_h + w_n k_n$$





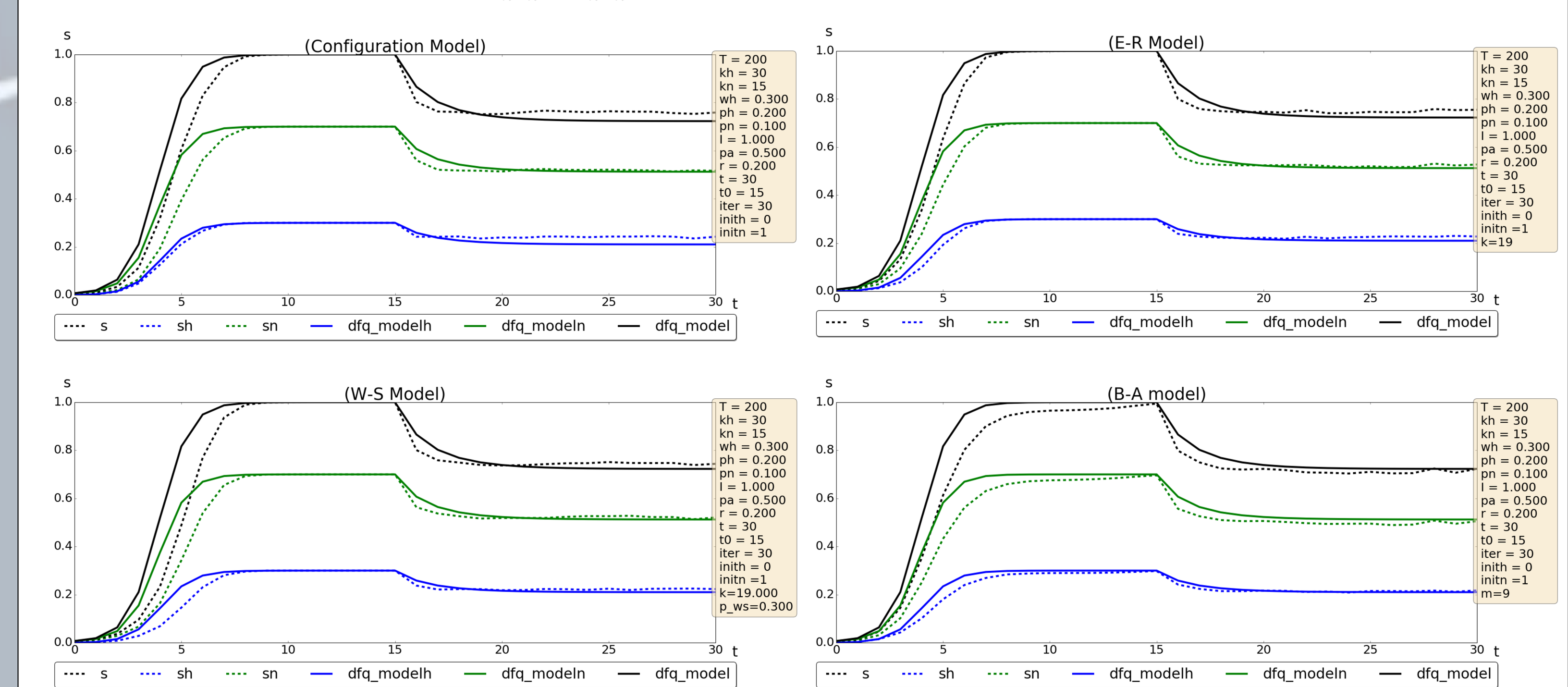
FINAL MODEL WITH ANTI-RUMOR

Finally we combine the previous models and introduce a few more parameters take into consideration the characteristics of a rumor as well as the possibility of policymakers trying to stop the rumor:

I : characteristic parameter of the rumor that takes on value between 0 and 1; the probability of spreading the rumor is multiplied by this parameter.
t_0 : time when anti-rumor is spread; anti-rumor instantly affects everyone
p_a : describes how convincing the anti-rumor is; probability of spreading the rumor to an ignorant person will be reduced to $p(1- p_a)$ after the anti-rumor is announced by policymakers
r : proportion (per t) of people who has heard the rumor but decides not to believe it anymore

We set up a system of differential equations:

$$\begin{aligned} \frac{ds_h}{dt} &= Ip_h(1-p_a)\frac{w_h\langle k_h\rangle}{C}\langle k_h\rangle s_h(w_h - s_h) + Ip_n(1-p_a)\frac{w_h\langle k_h\rangle}{C}\langle k_n\rangle s_n(w_h - s_h) - rs_h \\ \frac{ds_n}{dt} &= Ip_n(1-p_a)\frac{w_n\langle k_n\rangle}{C}\langle k_n\rangle s_n(w_n - s_n) + Ip_h(1-p_a)\frac{w_n\langle k_n\rangle}{C}\langle k_h\rangle s_h(w_n - s_n) - rs_n \end{aligned} \quad (5)$$
$$\text{where } C = w_h k_h + w_n k_n$$



CONCLUSIONS & FUTURE DIRECTIONS

Conclusions

Our results indicate that personal characteristics, specifically talkativeness, will influence rumor spreading in social networks. Assuming that more talkative people have a higher likelihood of spreading the rumor to their connections, we find out that it takes less time for a rumor to be completely spread out in a group with talkative people than in a group with only the non-talkative people.

We also find out that in a social network, rumor spreads faster if the spreading process starts from a hub person compared to the case when the spreading starts from a non-hub person.

Moreover, it is possible for someone (e.g. policymakers) to slow down the spread of a rumor, or to reduce the number of people believing the rumor, by spreading an anti-rumor. Our results show that after the anti-rumor is spread at t_0 , both hubs and non-hubs have fewer people believing in the rumor. As a result, it is no longer possible for rumor to be completely spread out in the entire population, as it does in the first three models.

Future Directions

Potential directions for further research include rumor spread in adaptive networks. In particular, if someone does not believe in the rumor, the rumor spreader might break the link from this person and connect with someone else. In addition, we could also conduct bifurcation analysis for the process of rumor spreading.

ACKNOWLEDGEMENTS

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