

# MLE / MAP

Jeonghun Yoon

# Terms

Bayes rule

Maximum Likelihood Estimate (MLE)

Maximum A Posteriori Estimate (MAP)

# Bayes rule

The diagram shows the Bayes' rule formula  $p(\theta|\mathbb{x}) = \frac{p(\mathbb{x}|\theta)p(\theta)}{\sum p(\mathbb{x}|\theta)p(\theta)}$  enclosed in a blue rectangular box. Three labels with arrows point to parts of the formula: 'likelihood (우도 값)' points to the numerator's first term  $p(\mathbb{x}|\theta)$ ; 'prior (사전 확률)' points to the numerator's second term  $p(\theta)$ ; and 'posteriori (사후 확률)' points to the entire left side of the equation  $p(\theta|\mathbb{x})$ .

$$p(\theta|\mathbb{x}) = \frac{p(\mathbb{x}|\theta)p(\theta)}{\sum p(\mathbb{x}|\theta)p(\theta)}$$

- 사후 확률 : 관찰 값들이 관찰 된 후에 모수(parameter)의 발생 확률을 구한다.
- 사전 확률 : 관찰 값들이 관찰 되기 전에 모수의 발생 확률을 구한다.
- 우도 값 : 모수의 값이 주어졌을 때 관찰 값들이 발생할 확률

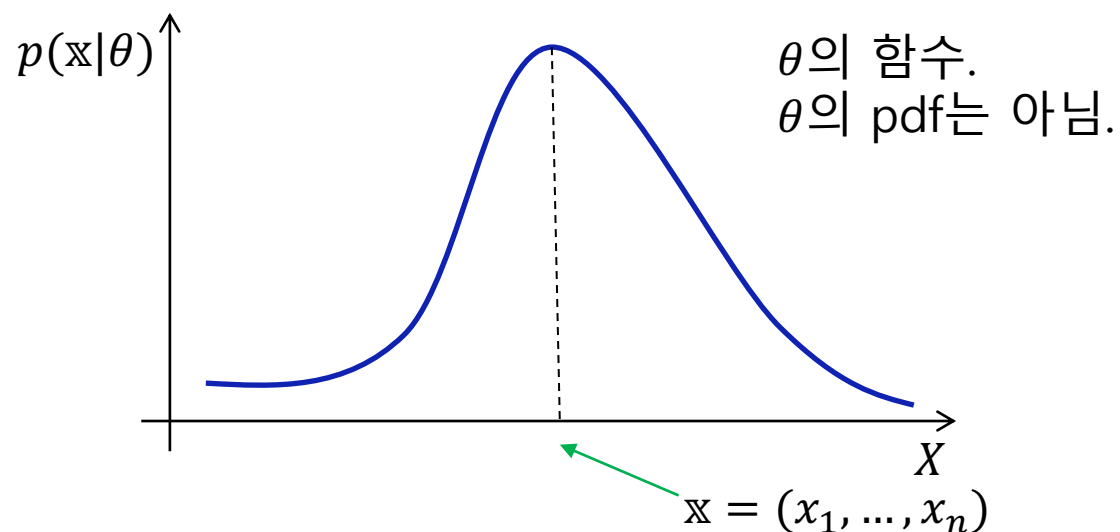
# Maximum Likelihood Estimate

$$\mathbb{x} = (x_1, \dots, x_n)$$

우도(likelihood)는 다음과 같이 정의 된다.

$$\mathcal{L}(\theta) = p(\mathbb{x}|\theta)$$

변수(parameter)  $\theta$ 가 주어졌을 때, data set  $\mathbb{x} = (x_1, \dots, x_n)$  (관찰 된, observed) 를 얻을 수 있는(obtaining) 확률

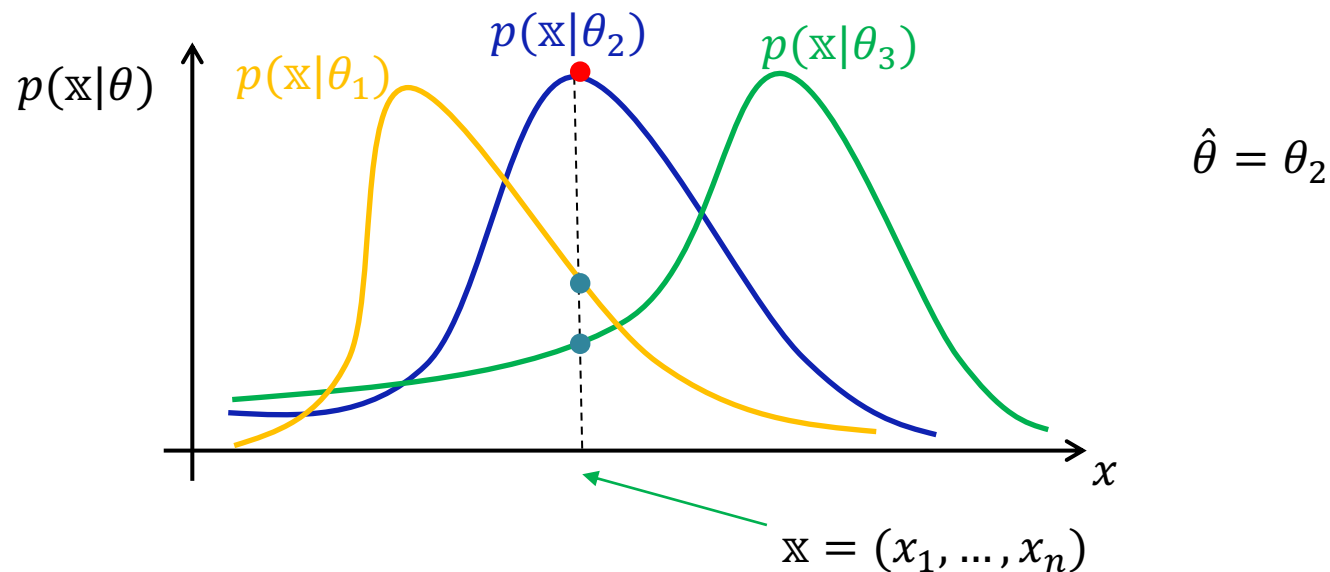


# Maximum Likelihood Estimate

Maximum Likelihood Estimate는 다음과 같이 정의 된다.

$$\hat{\theta} = \arg \max_{\theta} \mathcal{L}(\theta) = \arg \max_{\theta} p(\mathbb{x}|\theta)$$

관찰 된 data set  $\mathbb{x} = (x_1, \dots, x_n)$ 을 얻을 수 있는 확률이 가장 큰  $\theta$ 가 MLE이다.



# Maximum A Posteriori Estimate

우리가 likelihood function  $p(\mathbf{x}|\theta)$ 와 prior  $p(\theta)$ 를 알 때, Bayes rule에 의하여 posteriori function의 값을 구할 수 있다.

A diagram enclosed in a blue rectangular box. It shows the relationship between likelihood, prior, and posterior probabilities. The text 'likelihood (우도 값)' has an arrow pointing to the numerator of the fraction  $p(\mathbf{x}|\theta)p(\theta)$ . The text 'prior (사전 확률)' also has an arrow pointing to the numerator. The text 'posteriori (사후 확률)' has an arrow pointing to the entire fraction  $p(\theta|\mathbf{x}) = \frac{p(\mathbf{x}|\theta)p(\theta)}{\sum p(\mathbf{x}|\theta)p(\theta)}$ .

$$p(\theta|\mathbf{x}) = \frac{p(\mathbf{x}|\theta)p(\theta)}{\sum p(\mathbf{x}|\theta)p(\theta)}$$



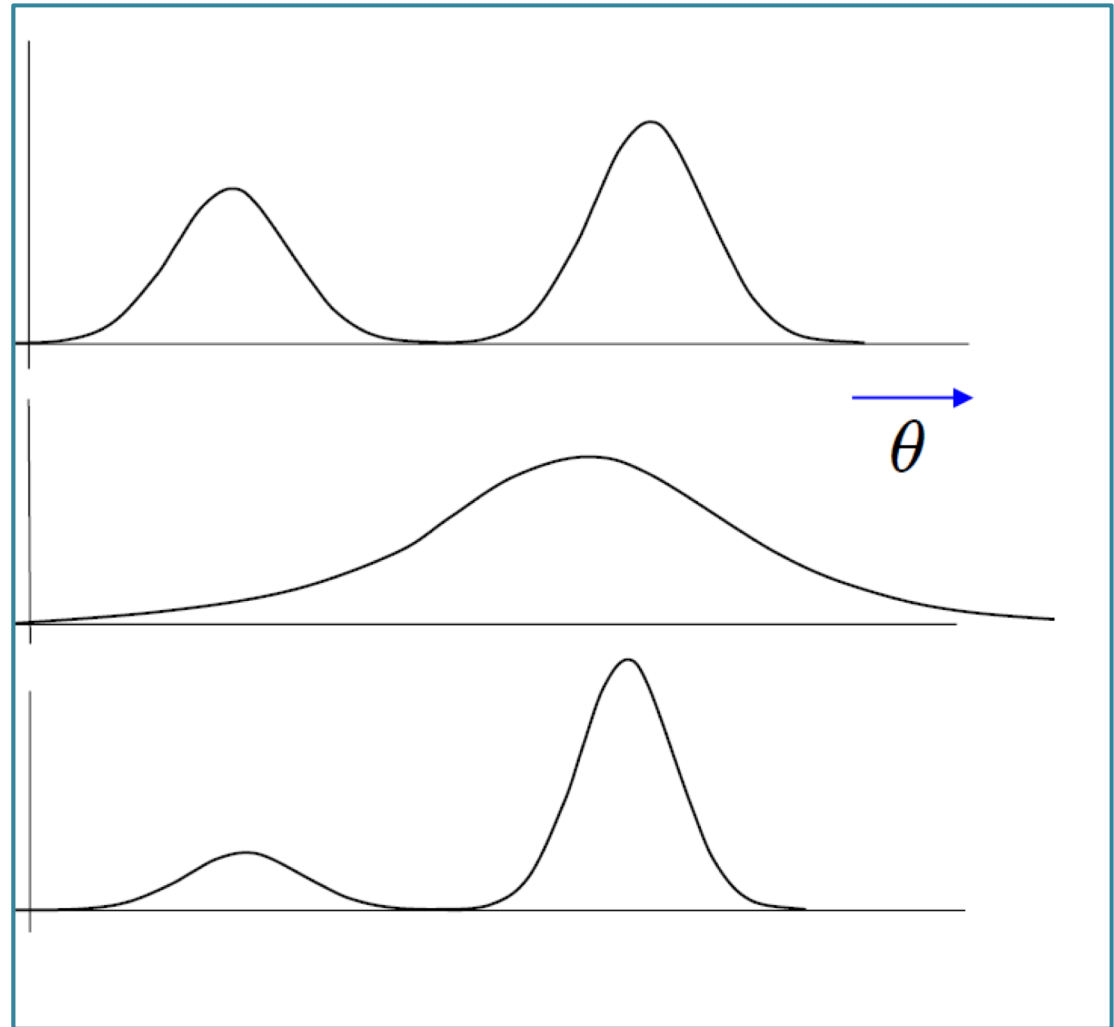
$$p(\theta|\mathbf{x}) \propto p(\mathbf{x}|\theta)p(\theta)$$

# MLE vs MAP

Likelihood  
 $p(\mathbf{x}|\theta)$

Prior  
 $p(\theta)$

Posterior  
 $p(\theta|\mathbf{x}) \propto p(\mathbf{x}|\theta)p(\theta)$

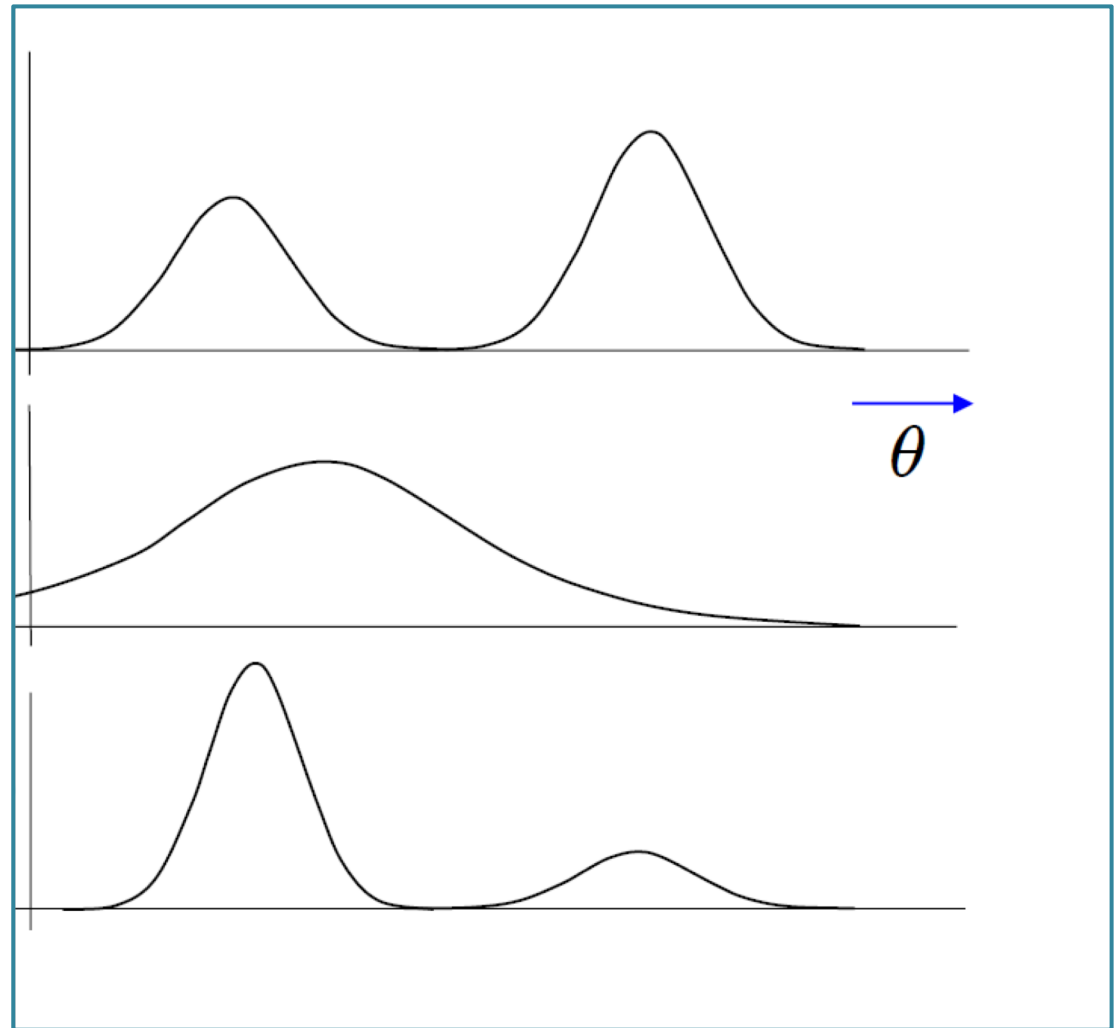


# MLE vs MAP

Likelihood  
 $p(\mathbf{x}|\theta)$

Prior  
 $p(\theta)$

Posterior  
 $p(\theta|\mathbf{x}) \propto p(\mathbf{x}|\theta)p(\theta)$





# MLE vs MAP

Likelihood  
 $p(\mathbf{x}|\theta)$

Prior  
 $p(\theta)$

Posterior  
 $p(\theta|\mathbf{x}) \propto p(\mathbf{x}|\theta)p(\theta)$

