Software Requirements Specification for: Simulating the Inverted Pendulum System

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February 11, 2023

Contents

Revision History

Date	Version	Notes
February 1, 2023	1.0	First version of document
February 3, 2023	1.1	Complete the Description part
February 4, 2023	1.2	Update Requirements part
February 6, 2023	1.3	Update Specific System Description part
February 10, 2023	1.4	Refine Solution Characteristic Specification

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

Throughout this document SI (Système International d'Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

symbol	unit	SI
kg	mass	kilogram
N	force	newton
deg	angle	degree
m	distance or length	metre
τ	torque	newton.metre

1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The symbols are listed in alphabetical order.

symbol	unit	description
b	dimensionless	Coefficient of friction for the cart
$d_p y$	m	Distance traveled by the center of mass of pendulum in y direction
$d_p x$	m	Distance traveled by the center of mass of pendulum in x direction
$F(t)$	N	Horizontal force exerted on the cart
g	$\mathrm{m\,s^{-2}}$	Acceleration due to gravity
H	N	Horizontal reaction force of the cart or the pole
I	$\mathrm{kg\,m^2}$	Moment of inertia of the pendulum
l	m	Length to pendulum center of mass
m_c	kg	Mass of the cart
m_p	kg	Mass of the pole
r	m	Radius from the axis of rotation to the location where the force is exerted for calculating torque
V	N	Vertical reaction force component of the cart or the pole
$x(t)$	m	The distance which cart transfer during time (t)
$\dot{x}$	$\mathrm{m\,s^{-1}}$	Velocity of the cart

$\ddot{x}$	$\mathrm{m\,s^{-2}}$	Acceleration of the cart
$\theta(t)$	deg	Pendulum angle from vertical
$\dot{\theta}$	deg/s	Angular velocity of the pendulum
$\ddot{\theta}$	deg/s^2	Angular acceleration of the pendulum

1.3   Abbreviations and Acronyms

symbol	description
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
IP	Inverted Pendulum
TM	Theoretical Model

1.4   Mathematical Notation

In this document, the common mathematical notation used in undergraduate-level mathematics will be used.

2 Introduction

The Inverted pendulum is a classical control problem used for illustrating non-linear control techniques. The system is motivated by applications such as control of rockets and anti-seismic control of buildings.The inverted pendulum system consists of a pendulum which is attached to a cart that can move horizontally. It is unstable because its center of mass is above its pivot point so without additional help will fall over.The cart move horizontally by an external force which is the only control input to the system. By manipulating the force, the position of pendulum is to be controlled.

Considering the force, we would like to simulate the behaviour of the system and measure the position of the cart and the angle of the pendulum in the presence of the force. The following section provides an overview of Software Requirement Specification (SRS) for a pole-cart system, discussing the scope and purpose of the work.

2.1 Purpose of Document

The main purpose of this document is to describe the mathematical model of Inverted Pendulum(IP) system. To clarify software requirements, detailed information about goals, assumptions, theoretical models, and definitions will be explained.

Therefore, this document will be the reference during producing the software. It will provide requirements of the software which will be used in planing for design stage. The following SRS document will remain abstract, exploring what is being solved rather than how it will be solved.

2.2 Scope of Requirements

The problem is restricted to two dimensions. The pendulum is attached to the cart in such a way that it is free to rotate in the pivot, and the cart can move horizontally,left or right. for more details, see the assumptions section (Section 4.2.1)

2.3 Characteristics of Intended Reader

Intended Reader(s) or Reviewer(s) should have knowledge about Physics and more specifically in equations of motion. Good understanding of solving an ordinary differential equation (ODE) in Mathematics is also needed, both of the topics are offered in the undergraduate engineering courses or high school Mathematics and Physics subjects.

2.4 Organization of Document

This document follows the SRS template as specified by Smith and Lai[1]. It will present the system's goals, theories, definitions, and assumptions. Goals are good points to start. To be familiar with mathematical model, Section 4.2.2 and then Section 4.2.2, Section 4.2.3, Section 4.2.6 can be explored. Most of the material is referential and may be read as needed.

3   General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics and lists the system constraints.

3.1   System Context

Figure 1 represents an abstract view of the software. The rectangular node represents the IP simulator tool, and the circular nodes are users who interact the software through an user interface.

Figure 1: System Context

The responsibilities of the user and the software are as follows:

- User Responsibilities:

    - Provide required inputs including cart and pole specifications.

- The Inverted Pendulum simulation software Responsibilities:

    - Ensure all inputs are in correct format and detect data type mismatch.
    - Calculate and show the expected outputs including the position of the cart and position of the pendulum.

3.2   User Characteristics

The end user of the IP simulation software should have an understanding of undergraduate Calculus and Physics.

3.3   System Constraints

There are no system constraints for this project.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

4.1 Problem Description

The inverted pendulum system inherently has two equilibrium points, one of which is stable while the other is unstable. The stable equilibrium corresponds to a state in which the pendulum is pointing downwards. In the absence of any control force, the system will naturally return to this state. The unstable equilibrium corresponds to a state in which the pendulum points strictly upwards and thus requires a control force to maintain this position.

The Inverted Pendulum simulation tool is intended to describe and simulate the behaviour of the system in the presence of different value of input. As the IP is widely used as a benchmark to demonstrate concepts and test control strategies and algorithms in nonlinear control, the final tool can show system status and be a great help as an educational material for students and engineers.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- Inverted Pendulum: An inverted pendulum is a pendulum that has its center of mass above its pivot point. It is unstable and without additional help will fall over.

- Force equilibrium: A body is in force equilibrium if the sum of all the forces acting on the body is zero.

- Friction: The force that resists the sliding or rolling of one solid object over another.

- Moment: The turning effect of a force is called the moment. The moment is the result of the force multiplied by the perpendicular distance from the line of action of the force to the pivot or point where the object will turn.

4.1.2 Physical System Description

The physical system of IP simulation Tool, as shown in Figure 2, includes the following elements:

- PS1: The cart

- PS2: The pendulum

3
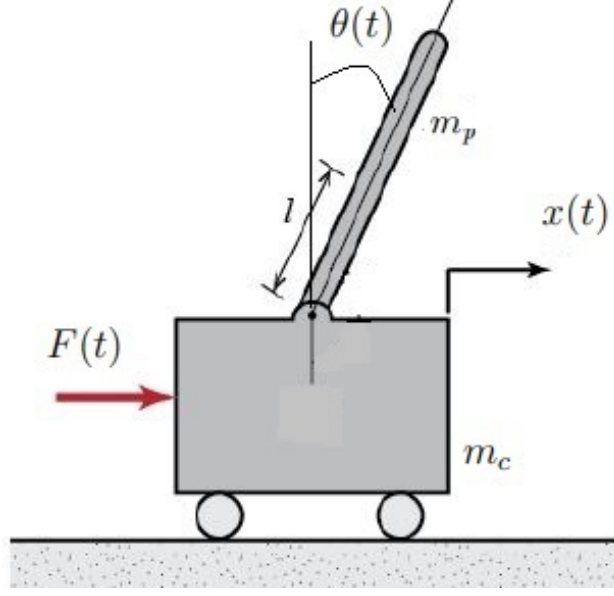
Figure 2: The physical system of IP simulation Tool

4.1.3 Goal Statements

Given the cart and the pendulum specifications and external force, the goal statements are:

GS1: Calculate the position of the cart.

GS2: Calculate the angle of the pendulum arm

4.2 Solution Characteristics Specification

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the theoretical model [T], general definition [GD], data definition [DD], instance model [IM], or likely change [LC], in which the respective assumption is used.

A1: The system has a two-degree of freedom, so the pendulum is allowed to rotate in a two-dimensional plane.

A2: The pendulum is smooth and friction-less.

A3: The pendulum is unified and its moment of inertia is constant.

4.2.2 Theoretical Models

This section focuses on the general equations and laws that the tool is based on.

Number	TM1
Label	**Newton's Second Law of Motion**
Equation	$F = m.a$
Description	Newton's second law of motion pertains to the behavior of objects for which all existing forces are not balanced. The second law states that the acceleration of an object is dependent upon two variables, the net force acting upon the object which is a and the mass of the object that is m. The acceleration of an object depends directly upon the net force acting upon the object, and inversely upon the mass of the object. As the force acting upon an object is increased, the acceleration of the object is increased. As the mass of an object is increased, the acceleration of the object is decreased.
Notes	none.
Sources	`https://www.physicsclassroom.com/class/newtlaws/Lesson-3/` `Newton-s-Second-Law`
Ref. By	GD1,GD2,GD3, IM1

Number	TM2
Label	**Newton's Second Law for Rotation**
Equation	$\sum(\tau)=I.\alpha$
Description	If more than one torque acts on a rigid body about a fixed axis, then the sum of the torques equals the moment of inertia times the angular acceleration. I is the moment of inertia for a point particle. α is angular acceleration of the object.
Notes	none.
Sources	`https://courses.lumenlearning.com/suny-osuniversityphysics/` `chapter/10-7-newtons-second-law-for-rotation/`
Ref. By	IM2

Number	TM3
Label	**Torque Definition**
Equation	$\tau = rF\sin(\theta)$
Description	Measure of the twisting action caused by a force that can cause an object to rotate about an axis.
	τ is torque,
	F is applied force,
	r is the radius from the axis of rotation to the location where the force is exerted,
	θ is the angle between F and r.
Notes	none.
Sources	`https://www.khanacademy.org/science/in-in-class11th-physics/` `in-in-system-of-particles-and-rotational-motion/` `in-in-torque-and-equilibrium-ap/a/torque-and-equilibrium`
Ref. By	GD4, T2

Number	TM4
Label	**Force of Gravity**
Equation	$W = m.g$
Description	Gravity is a force that attracts objects toward the Earth. It is an approximation of the gravitational force that attracts objects of mass toward each other at great distances.
	g is the acceleration due to gravity,
	m is the mass of the object.
Notes	none.
Sources	`https://www.school-for-champions.com/science/gravity_overview.` `htm#.Y9-9ZXbMLs0`
Ref. By	GD3

4.2.3   General Definitions

This section collects the laws and equations that will be used in building the instance models.

Number	GD1
Label	**Newton's second law of motion for the cart in horizontal direction**
SI Units	N
Equation	$m_c \ddot{x} = F - H_c - b\dot{x},$
Description	Based on Newton's second law of motion, sum of all forces applied to the cart in x direction is equal to $m_c.a$.
	F is the external force exerted to the cart in N.
	H_c is the force component in x direction of the reaction force which pendulum applies on the cart and measured in N.
	b is coefficient of friction for the cart.
	$\dot{x}$ is the velocity of cart and therefore $\ddot{x}$ will be its acceleration.
Source	`https://ctms.engin.umich.edu/CTMS/index.php?example=` `InvertedPendulum&section=SystemModeling`
Ref. By	IM1

Number	GD2
Label	**Newton's second law of motion for the pendulum in horizontal direction**
SI Units	All forces are measured in N
Equation	$m_p \ddot{x} = H_p$
Description	Based on Newton's second law of motion, sum of all forces applied to the pendulum in x direction is equal to $m_p.a$.
	H_p is the force component in x direction of the force exerted by the cart on the pendulum and measured in N.
	m_p is mass of the pendulum.
	$\ddot{x}$ is the pendulum acceleration.
Source	`https://ctms.engin.umich.edu/CTMS/index.php?example=` `InvertedPendulum&section=SystemModeling`
Ref. By	IM2

Number	GD3
Label	**Newton's second law of motion for the pendulum in vertical direction**
SI Units	All forces are measured in N
Equation	$m_p(l - l\ddot{\cos(\theta)}) = (m_p)g - V_p$
Description	Based on Newton's second law of motion, sum of all forces applied to the pendulum in y direction is equal to $m_p a$. Because the second derivation of l as a constant values is zero, therefore the equation reduces to $m_p(-l\ddot{\cos(\theta)}) = (m_p)g - V_p$.
	V_p is the force component in y direction of the force exerted by cart on the pendulum and measured in N.
	m_p is mass of the pendulum.
	$l - l\cos(\theta)$ is the Vertical displacement of the pendulum.
Source	`https://ctms.engin.umich.edu/CTMS/index.php/example=` `InvertedPendulum&section=SystemModeling`
Ref. By	IM2 , DD2

Number	GD4
Label	**Calculate angular displacement of the pendulum**
SI Units	τ
Equation	$I.\ddot{\theta} = l.(\ V.\sin(\theta) - H.\cos(\theta))$
Description	Let's take the center of mass of the pendulum as a center of rotation so the pendulum rotates about its center of mass. The torques due to forces exerted to pendulum will calculate as below: the torque of weight force of pendulum is zero since the perpendicular distance of this force uh from center of rotation is zero. For reaction forces Vp, Hp, we decompose these forces to components which are perpendicular to the pendulum which we have, $V.\sin(\theta)$ and $H.\cos(\theta)$. I is the moment of inertia of the pendulum. $\ddot{\theta}$ is pendulum angular acceleration.
Source	`https://ctms.engin.umich.edu/CTMS/index.php?example=` `InvertedPendulum&section=SystemModeling`
Ref. By	IM2, DD1, T3

4.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models. The dimension of each quantity is also given.

Number	DD1
Label	**Horizontal displacement of the pendulum**
Symbol	$d_p x$
SI Units	m
Equation	$d_p x = l \sin(\theta) + x(t)$
Description	The distance traveled by the center of mass of the pendulum is due to two reasons, first, horizontal displacement of the cart, and second, due to angular rotation of the pendulum. The distance due to angular rotation in the pendulum will be $= l \sin(\theta)$ while the half length of the pendulum is l, and the angular rotation in the pendulum is $\theta(t)$. Therefore, the total distance traveled by the center of mass of pendulum is equal to $= l \sin(\theta) + x(t)$.
Sources	`https://www.youtube.com/watch?v=c3z4eo6s0Ek`
Ref. By	GD1

Number	DD2
Label	**Vertical displacement of the pendulum**
Symbol	$d_p y$
SI Units	m
Equation	$d_p y = l - l \cos(\theta)$
Description	When the pendulum rotates, its center of mass travels a vertical displacement, and as the half length of the pendulum is l, so the vertical distance travelled by the center of mass of the pendulum will be equal to $l \cos(\theta)$, hence the total displacement is equal to $l - l \cos(\theta)$.
Sources	`https://www.youtube.com/watch?v=c3z4eo6s0Ek`
Ref. By	GD2

Number	DD3
Label	**The moment of inertia of the pendulum**
Symbol	I
SI Units	kgm^2
Equation	$I = \frac{1}{12}(m_p.l^2)$
Description	The moment of inertia of a rod is related to its mass and length.
	the mass of rod is denoted by m,
	and its length is denoted by l.
Sources	`https://www.vedantu.com/question-answer/`
	`moment-of-a-rod-of-mass-m-length-l-about-class-12-physics-cbse-5f471db855e`
Ref. By	GD4

4.2.5 Data Types

This section is not applicable for the problem.

4.2.6 Instance Models

Number	IM1
Label	**Equation of motion for the cart**
Input	F, m_c, m_p, b, l
	The input is constrained so that θ is small
Output	$x(t)$, such that $F = (m_c + m_p)\ddot{x} + b\dot{x} + m_p l \ddot{\theta} \cos\theta - m_p l (\dot{\theta})^2 \sin\theta \rightarrow$
	$\ddot{x} = \dfrac{F - b\dot{x} - m_p l \ddot{\theta} \cos\theta + m_p l (\dot{\theta})^2 \sin\theta}{(m_c + m_p)}$
Description	m_c is the mass of the cart (kg).
	m_p is the mass of the pendulum (kg).
	b is the coefficient friction of the cart (dimensionless).
	l is the length of pendulum.
	and by writing equation of motion for the cart, the relationship between inputs and cart motion clarified.
Sources	`https://www.mathworks.com/help/control/ug/` `control-of-an-inverted-pendulum-on-a-cart.html`
Ref. By	

Number	IM2
Label	**Equation of motion for the pendulum**
Input	m_c, m_p, b, l
Output	$\theta(t)$, such that $(I + m_p l^2)\ddot{\theta} + m_p l \ddot{x} \cos\theta - m_p g l \sin\theta = 0 \rightarrow$
	$\ddot{\theta} = \dfrac{m_p g l \sin\theta - m_p l \ddot{x} \cos\theta}{(I + m_p l^2)}$
Description	m_c is the mass of the cart (kg).
	m_p is the mass of the pendulum (kg).
	l is the length of pendulum.
	This equation show the relationship between motion of pendulum and the system.
Sources	`https://www.mathworks.com/help/control/ug/` `control-of-an-inverted-pendulum-on-a-cart.html`
Ref. By	

4.2.7 Derivation of Instance Models

- Finding IM1: This subsection shows how the IM1 is derived. We should start from GD4, where we have this equation, $I.\ddot{\theta} = l.(V.\sin(\theta) - H.\cos(\theta))$, we need to substitute V and H in this equation. Therefore, we have the value of H form GD2, consider that we also calculate x in the DD1, so we will have the H as $m_p\ddot{x} + m_p l\ddot{\sin}\theta$.

  For finding V, we use GD3, when we have $m_p(l - l\ddot{\cos}(\theta)) = (m_p)g - V_p$, so $V_p = (m_p)g - m_p(l - l\ddot{\cos}(\theta))$. The point is that the force that the cart applied to the pendulum is equal to the force that exerted to the cart by the pendulum. Hence, by substituting V and H in GD4, the IM1 is obtained.

- Finding IM2: This subsection shows how the IM2 is derived. In GD1, we have $m_c\ddot{x} = F - H_c - b\dot{x}$, by writing the Newton's second law of motion for the pendulum in vertical direction. As F in the input of the system and we also have the value of H form GD2, consider that we also calculate x in the DD1, so we will have the H as $m_p\ddot{x} + m_p l\ddot{\sin}\theta$.

  Therefore, by substituting H_c, we will reach $(I + m_p l^2)\ddot{\theta} + m_p l\ddot{x}\cos\theta - m_p gl\sin\theta = 0$. The point is that the force that the cart applied to the pendulum is equal to the force that exerted to the cart by the pendulum.

4.2.8 Input Data Constraints

Table 1 shows the data constraints on the input output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The column for software constraints restricts the range of inputs to reasonable values. The software constraints will be helpful in the design stage for picking suitable algorithms. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise.

The specification parameters in Table 1 are listed in Table 2.

4.2.9 Properties of a Correct Solution

Table 3shows the physical constraints on the output.

5 Requirements

This section provides the functional requirements, and the business tasks that the IP simulation software is expected to complete, and the nonfunctional requirements, the qualities

Table 1: Input Variables

Var	Physical Constraints	Software Constraints	Typical Value	Uncertainty
m_p	$m_p > 0$	$m_p > 0$	0.2 kg	–
m_c	$m_c > 0$	$m_c > 0$	0.5 kg	–
b	$b > 0$	$b > 0$	0.1(dimensionless)	–
l	$l > 0$	$l > 0$	0.3 m	–
I	$I > 0$	$I > 0$	0.006 kg m^2	–

Table 2: Specification Parameter Values

Var	Value
θ	0

Table 3: Output Variables

Var	Physical Constraints
θ	$0 \leq \theta \leq 360$
x	$0 \leq x$

that the software is expected to exhibit.

5.1 Functional Requirements

R1: Provide input values such as the cart and pole features according to Table1.

R2: Calculate horizontal position of the cart form IM1.

R3: Calculate the angular position of the pendulum from IM2.

5.2 Nonfunctional Requirements

The nonfunctional requirements are listed below:

NFR1: Accuracy: The accuracy of the computed solutions should meet the level needed for engineering or scientific application and have the properties described in Section 4.2.9.

NFR2: Usability: The properties of the software should be able to tested easily through verification and validation plan (VnV Plan).

NFR3: Maintainability: The effort required to make any of the likely changes listed for IP simulation software should be less than 50% of the original development time.

NFR4: Portability: IP simulation software can be run on all of the possible operating environments, such as Windows, Mac-OS, and Linux.

6 Likely Changes

LC1: The software may be changed to consider friction of the pendulum [A2]

7 Unlikely Changes

ULC1: The system has two-degree of freedom [A1].

8 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Table 7 shows the dependencies of theoretical models, general definitions, data definitions, and instance models with each other.Table 5 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.Table 6 shows the dependencies of instance models, requirements, and data constraints on each other.

	T1	T2	T3	T4	GD1	GD2	GD3	GD4	DD1	DD2	DD3	IM1	IM2
T1					X	X	X					X	
T2													X
T3											X		
T4								X					
GD1	X											X	
GD2	X												X
GD3	X												X
GD4					X								
DD1						X							
DD2						X							
DD3						X							
IM1	X												
IM2		X											

Table 4: Traceability Matrix Showing the Connections Between Items of Different SectionsAssumptions and Other Items

	T1	T2	T3	T4	GD1	GD2	GD3	GD4	DD1	DD2	DD3	IM1	IM2
A1												X	X
A2						X	X					X	
A3								X					X

Table 5: Traceability Matrix Showing the Connections Between Assumptions and Other Items

	IM1	IM2	4.2.8
R1			X
R2	X	X	
R3	X	X	

Table 6: Traceability Matrix Showing the Connections Between Requirements and Instance Models

9 Development Plan

This section is optional. It is used to explain the plan for developing the software. In particular, this section gives a list of the order in which the requirements will be implemented.

10 Values of Auxiliary Constants

	R1	R2	R3
T1			
T2			
T3			
T4			
GD1	X		
GD2	X		
GD3	X		
GD4	X		
DD1			
DD2			
DD3			
IM1	X	X	X
IM2	X	X	X

Table 7: Development Plan for IP

Symbol	Description	Value	Units
θ_{min}	the maximum value for the pendulum angle	0	deg
θ_{max}	the minimum value for for the pendulum angle	360	deg
g	Acceleration due to gravity	9.8	$\mathrm{m\,s^{-2}}$

Table 8: Auxiliary Constants

References

[1] W. Spencer Smith and Lei Lai. A new requirements template for scientific computing. In J. Ralyté, P. Àgerfalk, and N. Kraiem, editors, *Proceedings of the First International Workshop on Situational Requirements Engineering Processes – Methods, Techniques and Tools to Support Situation-Specific Requirements Engineering Processes, SREP'05*, pages 107–121, Paris, France, 2005. In conjunction with 13th IEEE International Requirements Engineering Conference.

Appendix — Reflection

The information in this section will be used to evaluate the team members on the graduate attribute of Lifelong Learning. Please answer the following questions:

1. What knowledge and skills will the team collectively need to acquire to successfully complete this capstone project? Examples of possible knowledge to acquire include domain specific knowledge from the domain of your application, or software engineering knowledge, mechatronics knowledge or computer science knowledge. Skills may be related to technology, or writing, or presentation, or team management, etc. You should look to identify at least one item for each team member.

2. For each of the knowledge areas and skills identified in the previous question, what are at least two approaches to acquiring the knowledge or mastering the skill? Of the identified approaches, which will each team member pursue, and why did they make this choice?