Investigating the relationship between the outer pipe radius and the time it takes for a magnet to fall through it.

Word count: 2974

1 Research Design

1.1 Research Question

How does changing the outer radius of an aluminum, non-magnetic pipe (10, 12, 13, 15, 17 mm) affect the time it takes for a magnet to fall through it?

1.2 Background Theory and Hypothesis

When a magnet falls through a conductive pipe, an electromotive force (EMF) is induced, and since the pipe is conductive, circular currents called Eddy currents are also produced (Britannica, 2016). These currents produce magnetic fields that oppose the motion of the magnet. If the magnet moves faster, the braking force increases. Therefore, the magnet quickly reaches terminal velocity when its opposing force equals the force of gravity, assuming that air resistance is negligible.

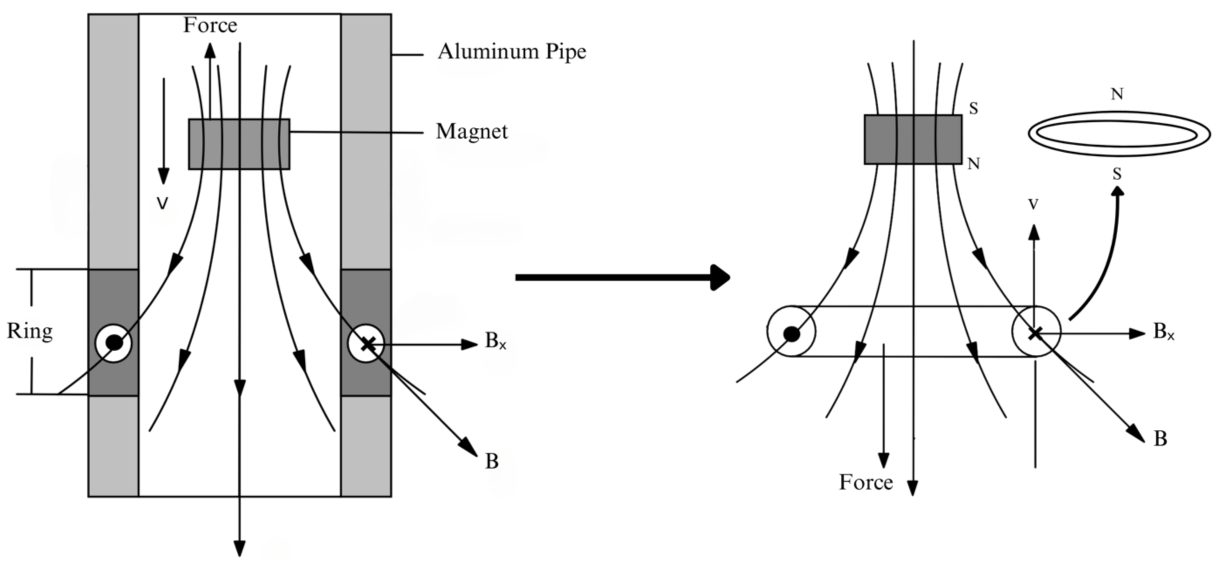

Figure 1: Diagram showing how a pipe approximates a series of rings that can be considered to be moving while the magnet remains stationary (Donoso et al., 2009).

The pipe can be considered to be comprised of many infinitesimally small rings. By Newton's third law, the force exerted on the magnet is the same as that exerted on the loop. Rather than considering the magnet to be moving downwards, the loop can be considered to be moving upwards with the same speed, v, while the magnet remains stationary.

When the magnet is above the loop, by using the third right hand rule, the current moves counterclockwise. So, the loop acts as a bar magnet with its north end facing up, therefore exerting a repulsive upward force on the magnet. When the magnet is below the loop, the direction of B_x reverses along with the direction of the current. In this case, the loop acts like a bar magnet with its North end facing down which exerts an attractive upward force on the magnet. In both cases, the loop applies an upward braking force on the magnet, thus slowing it down. Also, if the magnet fell with its south end facing down, the effect would be the same.

To see how the braking force relates to the speed, a mathematical relationship must be developed. The EMF, measured in V, induced across a wire moving in a perpendicular magnetic field with speed v, measured in ms^{-1}, is given by:

$$\varepsilon = BvL \tag{1}$$

B is the magnetic flux density, or magnetic field, measured in T and L is the length of the wire in m. The force in N on a current-carrying conductor is given by:

$$\vec{F} = L(\vec{I} \times \vec{B}) \tag{2}$$

$\vec{I}$ is the current vector whose magnitude is measured in amperes, and $\vec{B}$ is the vector of the magnetic flux density (LibreTexts, n.d.). For the loop, the equations become:

$$\varepsilon = vB_x 2\pi r_{pipe} \tag{3}$$

$$F_z = IB_x 2\pi r_{pipe} \tag{4}$$

According to equation 3, increasing the relative speed between the loop and magnet results in a greater induced EMF. By Ohm's Law, this leads to an increase in induced current, provided that the resistance of the loop remains constant. By equation 4, the action-reaction force acting on the loop increases when the current increases. Therefore, when the speed increases, the breaking force on the magnet increases. This explains why the magnet reaches terminal velocity.

To obtain an equation for the magnetic field of the magnet, it can be approximated to a magnetic dipole. A magnetic dipole is a microscopic magnet that is a flow of charge around a loop; the magnitude of its strength is called the dipole moment, μ, measured in JT^{-1} (Britannica, 2022). Using equations 3 and 4, when integrated over the entire pipe as well as the thickness of the pipe, the following expression is obtained:

$$F = \frac{15\pi^2\sigma\mu^2 v}{64}\left(\frac{1}{a^3} - \frac{1}{b^3}\right) \tag{5}$$

Where σ is the conductivity of the pipe's material measured in $\Omega^{-1}m^{-1}$, v is the speed of the magnet, and a and b are the inner and outer radii of the pipe respectively measure in m. (Donoso et al., 2009)

Replacing the constant variables with k:

$$F = kv\left(\frac{1}{a^3} - \frac{1}{b^3}\right)$$

It can be assumed that the magnet reaches terminal velocity as soon as it enters the pipe due to its small weight and large magnetic flux density. Therefore,

$$mg = F$$

$$mg = kv\left(\frac{1}{a^3} - \frac{1}{b^3}\right)$$

$$v = \frac{s}{t}$$

s, the displacement in m, is simply equal to the length of the pipe. Therefore, t, the time it takes for the magnet to fall measured in s has the following relationship with the pipe's radii.

$$t = \frac{ks}{mg}\left(\frac{1}{a^3} - \frac{1}{b^3}\right) \tag{6}$$

k, s, m, g, and a are all positive constants. It is thus hypothesized that the time it takes for the magnet to fall will have a negative, linear correlation to the reciprocal of the outer radius cubed.

1.3 Variables

Table 1: Independent and dependent variables.

Variable	Values/explanation	How it is measured
Independent Variable: The outer radius of an aluminum pipe.	Values: 10, 12, 13, 15, 17 mm The smallest value of 10 mm was chosen at it allowed for a smaller magnet to be used (explained below). Past 17 mm, the magnetic flux density of the magnets would likely be very weak. The increments were chosen based on pipe availability.	Vernier Caliper Its uncertainty is ± 0.05 mm which is the smallest increment of the instrument.
Dependent Variable: Time it takes for eleven magnets to fall through the pipes.	This is measured by subtracting the final and start times for each trial.	Camera, CapCut analysis software The uncertainty is ± 0.03 s. Since the camera recorded in 60 frames per second, the uncertainty of each time is $\frac{1}{60}$ of a second. Since two times were subtracted the uncertainty of each trial becomes $\frac{2}{60}$ of a second.

Table 2: Controlled variables.

Control variable	Reason for control	How it is controlled
Number of magnets/magnetic field of magnets	The number of magnets or their flux density affects the magnetic field strength. By equations 3 and 4, a stronger magnetic field would lead to a higher induced current and thus a higher breaking force. This can also be seen in equation 5, where a higher magnetic dipole moment, μ, leads to a larger braking force.	Eleven small magnets were dropped for each trial. For the relationship of equation 5 to remain true, the magnet must approximate a magnet dipole, therefore the magnet must be as small as possible (Kraftmakher, 2007). During preliminary trials (explained in section 1.6), it was found that using less than eleven magnets would result in less significant differences for thicker pipes and the magnets becoming horizontally stuck. Using eleven magnets ensured a greater magnetic flux density and it had a height greater than the inner diameter of the pipes preventing it from rotating out of the vertical position.

Inner radius of pipes	Changing the inner radius of the pipes influences how strong the magnetic field is at the position of the pipe walls. Also, equation 5 shows that a smaller inner radius, a, leads to a higher braking force.	A small inner radius of 8 mm was used so that magnets can be kept small to approximate the dipole while ensuring that the pipe is not too much wider than the magnet that it is not influenced by it.
Length of the pipes	Since it is assumed that the magnets fall at terminal velocity for the entire length of the pipe, a longer pipe would enhance the accuracy of this approximation by making the portion where it is not in terminal velocity insignificant to the overall length. Also, a longer pipe would lead to a longer falling time which will help determine a relationship.	All the pipes had a length of 40 cm, the longest length that could be found.
Conductivity of the pipes	A higher conductivity would lead to a higher induced current which in turn increases the force on the magnet. Conductivity, σ, is also found in equation 5.	All the pipes were made of aluminum, a good conductor. Also, all the pipes were sourced from the same manufacturer, ensuring that they are all of the same composition.
Drop height	If the magnets were dropped from too high a height, their speed would accelerate too much before entering the pipe. This would mean it would take longer for them to decelerate to terminal velocity. Therefore, the assumption of terminal velocity for the entire fall would be less accurate.	All the magnets were dropped from immediately above the pipe.
Magnetic atmosphere	Any ferromagnetic material in or near the mounting system of the pipes will influence the fall of the magnet.	Since the pipes were taped to a wall, a magnet was run across the walls ahead of time to ensure that there are no magnetic materials within the wall, sometimes the pipes had to be moved.

1.4 Materials

1. Five aluminum pipes with different outer radii (10, 12, 13, 15, 17 mm), length of 40 cm
2. Eleven identical small neodymium magnets
3. A camera that can record in 60 frames per second (fps)
4. Laptop with video analysis software, CapCut
5. Vernier caliper

1.5 Procedure and Apparatus

1. Tape the pipe of outer radius 10 mm to the wall, using a level to ensure it is vertical.
2. While recording, drop the magnets vertically into the pipe from immediately above it. Ensure that the magnets are dropped within the centre of the pipe.
3. Upload the video to the analysis software CapCut to obtain the start and end times of the fall.
4. Repeat steps 2-3 four more times.
5. Repeat steps 1-3 for the remaining pipes (12, 13, 15, 17 mm)

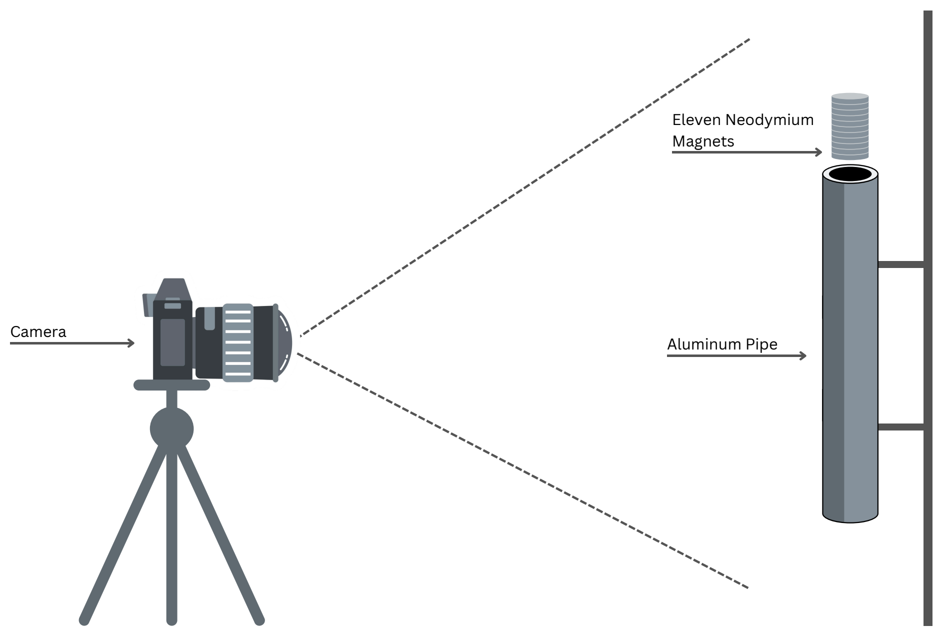

Figure 2: Diagram showing the experimental setup (created in Canva). All the retort stands available were made of iron, a ferromagnetic material, therefore the pipes were mounted to the wall instead.

1.6 Preliminary Trials

As shown in figure 3, using only six magnets did not result in an increase to fall time from the fourth to fifth pipe, 15 to 17 mm. There is actually a decrease shown but this is likely due to error. An explanation for this is that the magnets do not have a magnet field strong enough to be significant past 15 mm, therefore, more than six magnets were used in the experiment. It was also found that using a smaller number of magnets with a short height resulted in the magnets rotating and becoming horizontally stuck within the pipes. From trying different numbers of magnets, it was found that eleven provided the most significant differences while preventing the magnets from becoming horizontally stuck.

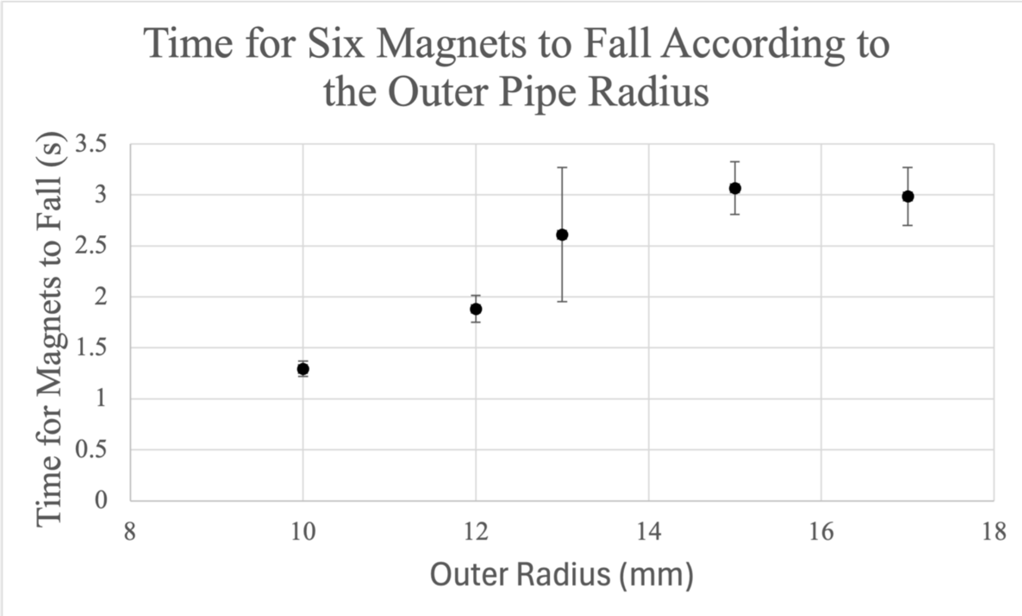

Figure 3: Graph showing the time it takes for six magnets to fall versus the outer radius of the pipe. This data was collected during preliminary trials. Error bars show uncertainty, some horizontal error bars are too small to be seen.

1.7 Risk Assessment

Table 3: Environmental, ethical, and safety considerations.

Risk	Consideration
Environmental Considerations	All the pipes and magnets used are reusable, so no waste was created during this experiment. Therefore, there are no environmental considerations.
Ethical Considerations	There are no ethical considerations associated with this experiment as no organisms or moral issues are involved.
Safety Considerations	Since the magnets used were quite powerful, care was taken when separating or combining them. Also, the magnets were caught after falling out of the pipe to ensure that they do not fall and break, or cause injury.

2 Analysis

2.1 Raw Data

Qualitative Data

The magnets could not be seen while falling. However, it was noted that all the magnets came out of the pipe in the vertical position. Also, the magnets were heard hitting the walls of the pipes as they fell.

Quantitative Data

For each trial the fall time is taken as the end time minus the start time. The uncertainty of the fall time was $\frac{2}{60}$ of a second which corresponded to the camera frame rate, as explained in the variables section. The uncertainty of the outer radius is taken as the smallest mark on a vernier caliper. These results are shown in table 4.

Table 4: Time for magnets to fall with 5 trials for each outer radius.

Outer radius, b ($\pm$ 0.05 mm)	**Time for magnets to fall ($\pm$ 0.03 s)**				
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
10.00	1.08	1.08	1.27	1.10	0.90
12.00	1.83	1.85	2.10	2.00	1.82
13.00	2.00	2.22	2.23	1.98	2.02
15.00	2.37	2.42	2.42	2.32	2.27
17.00	2.52	2.50	2.50	2.55	3.57

2.2 Data Processing

As shown with equation 6, to obtain a linear relationship the time should be graphed over the outer radius raised to the -3 power.

Table 5: Calculating the reciprocal of the outer radius cubed for the thinnest pipe.

Value	**Uncertainty**
$b^{-3} = \dfrac{1}{\left(10.00\text{ mm} * \dfrac{1\text{ m}}{1000\text{ mm}}\right)^3}$ $= 1000000\text{ m}^{-3}$	$\dfrac{\Delta(b^{-3})}{b^{-3}} = 3\left(\dfrac{0.05\text{ mm}}{10.00\text{ mm}}\right)$ $= 0.015$ $\Delta(b^{-3}) = 0.015 * 1000000\text{ m}^{-3}$ ≈ 20000
Therefore, $b^{-3} = (1.0 \pm 0.02) \times 10^6\text{ m}^{-3}$ for the pipe of outer radius 10.00 $\pm$ 0.05 mm.	

The fall times from table 4 are averaged in table 6. The uncertainty was taken as the larger between $\frac{\max - \min}{2}$ and the reading error of the trials, $\pm$ 0.03 s.

Table 6: Calculating the average fall time.

Value	**Uncertainty**
$t_{ave} = \dfrac{t_1 + t_2 + t_3 + t_4 + t_5}{5}$ $= \dfrac{1.08 + 1.08 + 1.27 + 1.10 + 0.90}{5}$ $\approx 1.1\text{ s}$	$\Delta(t_{ave}) = \dfrac{\max - \min}{2}$ $= \dfrac{1.27 - 0.90}{2}$ $\approx 0.2\text{ s}$
Therefore, the average fall time for the pipe of outer radius 10.00 $\pm$ 0.05 mm is 1.1 $\pm$ 0.2 s.	

2.3 Processed Data

Table 7: Processed data of the reciprocal of the outer radius cubed and the average fall time.

Outer radius, b ($\pm$ 0.05 mm)	**b^{-3} (m^{-3})**	**Average time for magnets to fall, t (s)**
10.00	$(1.0 \pm 0.02) \times 10^6$	1.1 ± 0.2
12.00	$(5.79 \pm 0.07) \times 10^5$	1.9 ± 0.1
13.00	$(4.55 \pm 0.05) \times 10^5$	2.1 ± 0.1
15.00	$(2.96 \pm 0.03) \times 10^5$	2.4 ± 0.1
17.00	$(2.04 \pm 0.02) \times 10^5$	2.7 ± 0.5

Then the relationship from table 7 can be graphed with a linear interpolation.

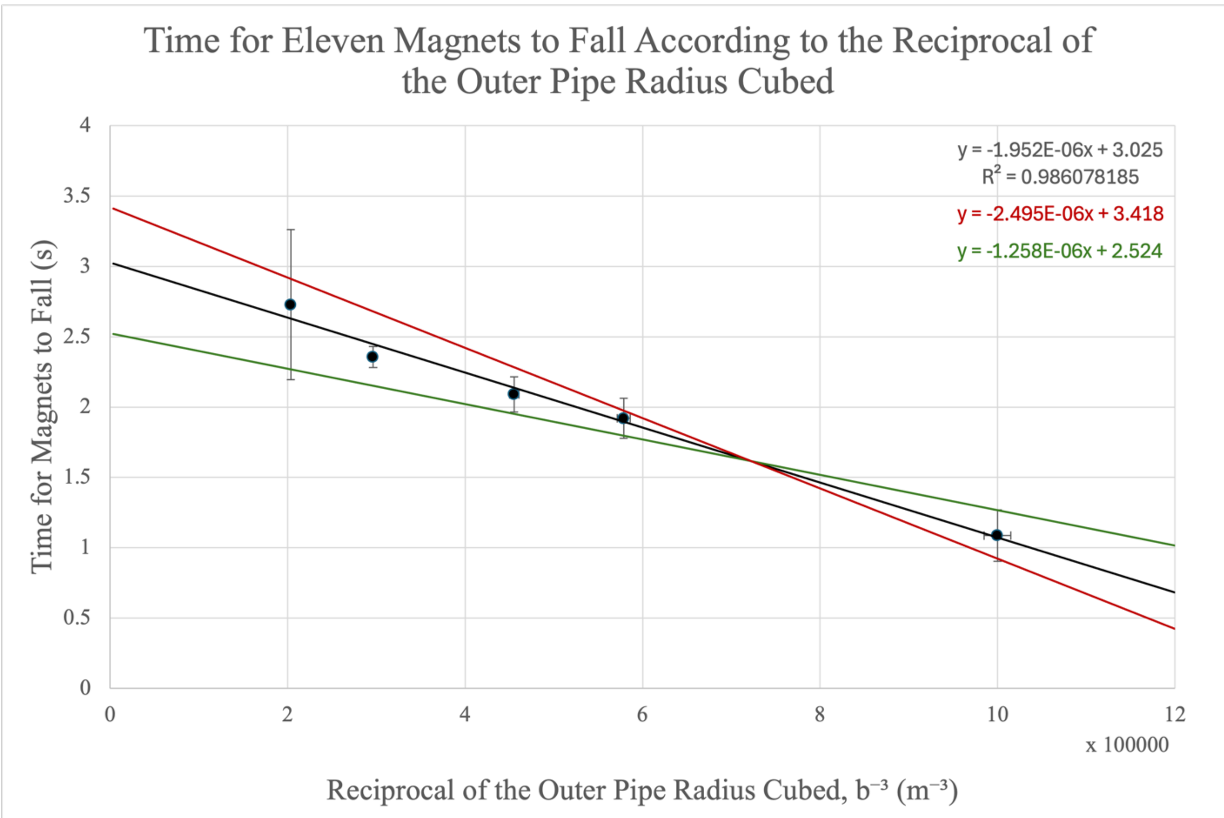

Figure 4: Graph showing the time it takes for eleven magnets to fall versus the outer radius of the pipe raised to the -3 power. Error bars show uncertainty, some are too small to be seen. Note: the first data point represents the pipe with the largest outer radius, 17.00 ± 0.05 mm.

Uncertainty in gradient and y-intercept

$$\begin{aligned}\Delta m &= \frac{\max - \min}{2} \\ &= \frac{\left(-1.258 - (-2.495)\right) \times 10^{-6}}{2} \\ &= 6.225 \times 10^{-7} \\ &\approx 6 \times 10^{-7}\ \mathrm{s \cdot m^3}\end{aligned}$$

Therefore, the gradient is $(-2.0 \pm 0.6) \times 10^{-6}\ \mathrm{s \cdot m^3}$.

$$\begin{aligned}\Delta b &= \frac{\max - \min}{2} \\ &= \frac{3.418 - 2.524}{2} \\ &= 0.447 \\ &\approx 0.4\ \mathrm{s}\end{aligned}$$

Therefore, the y-intercept is $3.0\ \mathrm{s} \pm 0.4\ \mathrm{s}$.

Calculating the inner radius

The known inner radius of all the pipes is 8 ± 0.05 mm. Using equation 6, $t = \frac{ks}{mg}\left(\frac{1}{a^3} - \frac{1}{b^3}\right)$, this value can be calculated. The calculated value will be compared to the true value of the inner radius, which will serve as a measure for the accuracy of the relationship obtained in figure 4.

Table 8: Calculating inner radius from the relationship in figure 4.

Value	**Uncertainty**
$\mathrm{slope} = -\frac{ks}{mg}, \mathrm{y\ intercept} = \frac{ks}{mga^3}$ $a^{-3} = \frac{\mathrm{y\ intercept}}{-\mathrm{slope}} = \frac{3.025}{1.952 \times 10^{-6}} = 1549692.623$ $a = 8.641413986 \times 10^{-3}$ m	$\frac{\Delta a}{a} = \frac{1}{3}\left(\frac{6.225 \times 10^{-7}}{1.952 \times 10^{-6}} + \frac{0.447}{3.025}\right) \approx 0.15556$ $\Delta a \approx 1 \times 10^{-3}$
Therefore, the inner radius is $(9 \pm 1) \times 10^{-3}$ m.	

The calculated value is in alignment with the known one.

3 Conclusion

From figure 4, the time it takes for the magnet to fall correlates linearly to the outer radius raised to the -3 power. This supports the hypothesis. The R^2 value for this relationship is 0.9861 indicating a very strong linear correlation. Furthermore, the inner radius value calculated using the gradient and y-intercept is $(9 \pm 1) \times 10^{-3}$ m which is very similar to the known value of $(8 \pm 0.05) \times 10^{-3}$ m that was measured using a vernier caliper. The known value lies within the uncertainty range of the calculated one. This supports the accuracy of the relationship obtained in figure 4.

The error bars of figure 4 are small, except for the vertical error bar of the first data point, which corresponds to the thickest pipe of outer radius 17.00 ± 0.05 mm. This one is particularly large which can be attributed to its fifth trial. Nevertheless, it still fits the trend and the R^2 value is high.

This relationship shows that increasing the outer radius, and in turn the pipe thickness, increases the fall time. However, this effect diminishes with increasing outer radius (b) since the fall time is linearly correlated to $-b^{-3}$. Thus, as b increases, its impact on the fall time becomes progressively smaller. Other literature supports this relationship; figure 5 shows the falling positions for a magnet within 4 aluminum pipes of increasing thickness. The gradients of the lines in figure 5 are the speeds of the magnet. Increasing the thickness causes the magnet to fall at slower speeds, since the gradients become shallower, but it has a diminishing effect as the lines get closer together. The third and fourth lines do not appear to be converging but the thickness increases by a larger increment (see figure 5 caption) (Ladera et al., 2011).

In addition, the fact that all the lines in figure 5 are linear supports the theory that the magnets quickly achieve and maintain a terminal speed.

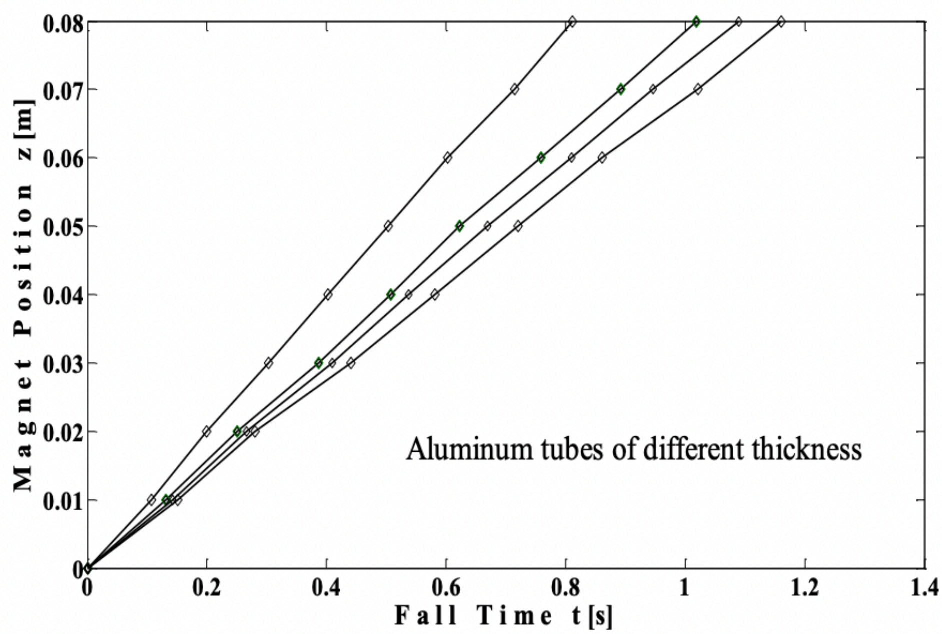

Figure 5: Graph showing the vertical position, starting at 0 m, with fall time for a magnet dropped in 4 aluminum pipes of different thicknesses. From left to right the thicknesses are, 2.10, 3.10, 4.10, and 5.80 mm (Ladera et al., 2011).

An explanation for this is that the magnetic field of a magnetic dipole, which is used to approximate the magnets, is inversely proportional to the distance from the dipole cubed (University of Victoria, n.d.). Since a magnetic dipole has a radically decreasing magnetic field at large distances, it likely has a smaller influence in inducing a current in the pipe at farther distances. Therefore, increasing the thickness of the pipe, with the same magnet, leads to a diminishing effect on the fall time.

4 Evaluation

4.1 Limitations and Weaknesses

Table 9: Experiment systematic and random errors.

Source of error	Impact	Improvement
Systematic errors		
Short pipe length	The longest pipes that could be sourced for this procedure were ones of 40 cm in length, but these are still relatively short. As a result, for the thinner pipes which exerted weaker braking forces, the magnets may have been falling at a velocity greater than its terminal velocity for a significant portion of the pipe. Also, the range of fall times was 1.6 s from the shortest to longest time which is a very small range.	Longer pipes should be used.
High camera angle relative to the bottom of the pipes	As a result of the high camera angle, seeing the magnets leave the pipes were a little delayed in the videos. This may have caused all the fall times to be slightly higher than the real times.	The camera angle should be lowered. As well, I believe that moving the camera farther from the pipe would have helped fix the coverage.
Pipe friction	From the qualitative data in section 2.1, the magnets could be heard hitting the walls of the pipe as they were falling. This friction would slow down the magnets, increasing the fall time. The friction force was omitted in the derivation of equation 6.	Pipes with larger inner radii relative to the size of the magnets can be used. However, this would require that the magnets have greater magnetic fields to be able to induce currents at farther distances. Or a frictionless, non-conductive pipe (ie. glass pipe) can be used within the metal one to guide the magnet as it falls (Ladera et al., 2011).
Random errors		
Inconsistent initial magnet velocity, dropping inconsistency	The magnets might not have all been dropped with zero velocity. Due to their very small mass, they may have gained some initial velocity as they were released (or pushed) from my hand. This error is random because it is unintentional and varies unpredictably with each trial. A higher initial velocity would take longer to decelerate to terminal velocity, therefore decreasing fall time – it would fall faster.	A release mechanism should be used.

Low camera frame rate	It was sometimes difficult to determine the frame in which the magnets exited the pipe. The time may have been overestimated or underestimated in different cases.	Using a higher camera frame rate would provide more frames making it easier to determine when the magnets fall from the pipe. Also, it would lead to lower uncertainty in the independent variable.
Magnets falling off alignment with the centre of the pipe.	It was difficult to drop the magnets exactly in the centre of the pipe, and even if this could be achieved the magnet could deviate from the centre while falling. Previous research found that as the magnet moves off the centre axis, the breaking force increases (Ladera et al., 2011).	As suggested earlier, a guiding pipe can be used to ensure the magnet maintains the centre position when falling.

4.2 Strengths

Table 10: Experiment strengths.

Strength	**Explanation**
5 pipes that differed only in outer radii	Sourcing 5 pipes of the same material, from the same manufacturer, ensured that the effect of the outer pipe radius could be isolated. This certainly contributed to the strong linear correlation of figure 4.
Large number of trials - 5 trials	Having a larger number of trials minimizes the influence of random error. All the vertical error bars in figure 4, with the exception of the first data point, are relatively small.
Preliminary trials	Using preliminary trials allowed me to determine the optimal number of magnets to use, eleven.
Supporting literature and a mathematical relationship – calculation of the inner radius aligned with the known value	The literature supported the relationship obtained in this procedure. Furthermore, the theoretical relationship given by equation 6 was consistent with the experimental relationship since the calculated inner radius agreed with the known value.

4.3 Extension

The preliminary data indicated that beyond a certain outer thickness, the linear relationship outlined by figure 4 and equation 6 no longer holds. This may be attributed to the fact that the magnetic field of the magnets is too weak past a certain distance to induce a current. This should be investigated further to determine if there is in fact a breaking point for the relationship.

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