CISC 233 Essential Algorithms Problem Set # 1
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Problem 1. *Give a real-world example that requires data to be sorted. (no code required)*

Problem 2. *Select two data structures that you have seen previously, and compare their strength and limitation (no code required)*

Problem 3. *Evaluate the following Summation Notation:*

A. $\sum_{i=1}^{3} i$

B. $\sum_{i=1}^{100} 3i$

C. $\sum_{i=1}^{4} \frac{i}{i+1}$

D. $\sum_{i=1}^{100} (3-2i)^2$

E. $\sum_{i=2}^{5} i$

F. $\sum_{i=1}^{3} 2^i$

when you finish answering this question, check your answers on symbolab.com

Problem 4. *Find the value of the logarithm:*

A. $log_2 1$

B. $log_2 16$

C. $log_2 20 + log_2 50$

D. $log_{19} 9$

E. $log_2 \sqrt{128}$

when you finish answering this question, check your answers on symbolab.com

Problem 5. *For each the following functions, sort the functions in increasing order of their complexity:*

A. $nlog_2 n$

B. n^2

C. 2^n

D. n

E. $n!$

F. n^3

write a small note to justify your answer.

Problem 6. *Write a Python procedure to print all the elements of the following nested list:*

$$[[1,2,3],[5,6,7],[8,9,10]]$$

Problem 7. *Write a Python program to find the factorial of a given integer using recursion.*

If n is a positive integer, n factorial denoted by $n!$ is a product of all positive integers less than or equal to n. It is defined by

$$n! = n(n-1)(n-2)\dots(2)(1)$$

As a special case: $0! = 1$

Example: $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Problem 8. *Let* $n = 10$ *Write a program to evaluate the summation of the following notation:*

$$\sum_{i=1}^{n} \frac{i}{i+1} + \frac{n+1}{n+2}$$

Problem 9. *let* $n = 10^3$, *Write a python program to evaluate the summation of the following double summation notation:*

$$\sum_{i=1}^{n} \sum_{j=1}^{n} 3ij$$

In milliseconds and seconds, how long did it take your code to run? when you finish answering this question, check your answers on symbolab.com

Listing 1: The following code will help you to get started

```
from datetime import datetime

start_time = datetime.now()
x = 0
for i in range(0, 10000000):
  x = x + i

time_elapsed = datetime.now() - start_time

print(x)
print('Time elapsed (hh:mm:ss.ms) {}'.format(time_elapsed))
```

Problem 10. *Use* ***Mathematical Induction*** *to show that*

1. *Any amount of money of at least 14¢ can be made up using 3¢ and 8¢ coins.*
2. *Any postage of at least 12¢ can be obtained using 3¢ and 7¢ stamps.*

Problem 11. *For each positive integer n, let $P(n)$ be the formula:*

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

1. *Write $P(1)$*
2. *Write $P(k)$*
3. *Write $P(k+1)$*
4. *In a proof by mathematical induction that the formula holds for all integers $n \geq 1$, what must be shown in the inductive step?*

Problem 12. *Given a while loop and a predicate. Show that if the predicate is true before entry to the loop, then it is also true after exit from the loop.*

Algorithm 1 predicate: $m + n$ is odd

```
while (m ≥ 0 and m ≤ 100) do
  m := m + 4
  n := n − 2
end while
```

Problem 13. *Design an algorithm to find all the common elements in two sorted lists of numbers. For example, for the lists 2, 5, 5, 5 and 2, 2, 3, 5, 5, 7, the output should be 2, 5. What is the maximum number of comparisons your algorithm makes if the lengths of the two given lists are m and n, respectively?*

Problem 14. *Graph the following functions (you can use a graph tool like desmos.com) :*

A. $f(x) = 2x + 3$

B. $f(x) = x^2 + 1$

C. $f(x) = 1$

D. $f(x) = x!$

E. $f(x) = log_2 x$

F. $f(x) = x\ log_2 x$

g. $f(x) = 2^x$

Which one of the previous functions has fewer number of run time steps of an algorithm and which one is the worst when the domain of these functions tends towards infinity?