Essential Algorithms **Name:** ____________________
CISC 233 Spring 2024
Exam 01
03/06/2024
Time Limit: 90 Minutes Mina Gabriel

Short Time Questions 25 - points

1. (5 points) Give an example of $O(n \log n)$ algorithm.

2. (5 points) Identify two similarities and two differences between AVL and BST.

3. (5 points) Consider all possible scenarios, what is the best and worst case of deleting an element from a Binary Search Tree?

4. (5 points) What are the properties of Red-Black Tree? When would you use it?

5. (5 points) Determine the function $g(n)$ that describes the Big O notation for $f(n) = 2^n + 3^n$. Identify the constants $c > 0$ and $n_0 > 0$ such that $f(n) \leq c \cdot g(n)$ for all $n \geq n_0$.

Medium Time Questions 30 - points

1. (10 points) What is the upper bound runtime of each line of the below code listings? (Don't Explain the code)

Listing 1: Calculate sum and product of an array

```
sum = 0 _________________________________________(     )
product = 1 _____________________________________(     )
for i in range(1, length(A) + 1):    ____________(     )
    sum += A[i] _________________________________(     )
for i in range(1, length(A) + 1) ________________(     )
    product *= A[i] _____________________________(     )
print(sum, product) _____________________________(     )
```

Listing 2: Factorial Function

```
def factorial(n): _____________________________(     )
    if n < 0: _________________________________(     )
        return -1 _____________________________(     )
    elif n == 0: ______________________________(     )
        return 1 ______________________________(     )
    else: _____________________________________(     )
        return n * factorial(n - 1) ___________(     )
```

2. (10 points) use Min-Heap to sort the following list:

10	5	3	4	1

   (Show all steps)

3. (10 points) Consider an algorithm with a time complexity of $O(n^2 + n \log n + 25)$. If the algorithm takes 200 seconds to complete when $n = 100$, provide an **estimate** for the runtime in seconds when $n = 200$.

Long Time Questions 45 - points

1. (15 points) Consider the following AVL tree:

```
    A
   / \
  B   C
 /   / \
D   E   F
```

   Suppose we insert a new node G into the AVL tree as a child of node E, making the tree unbalanced.

```
    A
   / \
  B   C
 /   / \
D   E   F
 \
  G
```

Question: Perform the necessary rotations to balance the tree. How will the tree look after re-balancing? Show the height and balance factor of each node. Discuss the time complexity of this algorithm in detail.

2. (15 points) The same array is given to the four sorting algorithms listed below

2	9	6	4	1	7	3	0	8	5

a. Heap Sort
b. Insertion Sort
c. Merge Sort
d. Selection Sort

The following are the intermediate results produced by each of them at a certain time during the sorting process. Your task is to label each array to indicate which algorithm produces it.

(____)

1	2	4	6	9	0	3	5	7	8

(____)

2	5	3	4	1	0	6	7	8	9

(____)

1	2	4	6	7	9	3	0	8	5

(____)

9	8	7	4	5	6	3	0	2	1

3. (15 points) You are provided with two matrices: Matrix A of dimensions $m \times n$, where the elements in each row and column are sorted in ascending order (as shown below), and Matrix A', which is an $n \times n$ symmetric matrix with randomly generated elements (assume not sorted). Your task is to find a specific key k in each matrix. explain and detail an **OPTIMAL** algorithm you would use. Also, determine the upper bound of the time complexity for your algorithms. Express these bounds in terms of m and n. Use the following two matrices to show how your algorithm will step to find $k = 18$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \\ 17 & 18 & 19 & 20 \end{bmatrix} \quad A' = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$