Simulation of a Thin-Wire Dipole Antenna using the Method of Moments

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Abstract

This report presents a numerical simulation of a center-fed, half-wavelength dipole antenna using the Method of Moments (MoM). The antenna is modeled as a thin, perfectly conducting wire. The current distribution along the antenna is approximated using Piecewise Sinusoidal (PWS) basis functions, and the governing integral equation is solved using the Point Matching technique. This procedure transforms the continuous problem into a discrete matrix equation, which is then solved numerically using Python. The report details the underlying electromagnetic theory, the numerical formulation, and the implementation. The resulting current distribution and the tangential component of the total electric field along the antenna's surface are calculated and visualized, demonstrating the effectiveness of the MoM for wire antenna analysis.

1 Introduction

The analysis of wire antennas is a classic problem in electromagnetics, crucial for applications in telecommunications, broadcasting, and remote sensing. While analytical solutions exist for idealized cases, practical antenna geometries often require numerical methods for accurate analysis. The Method of Moments (MoM) is a powerful and versatile numerical technique for solving integral equations that arise in electromagnetics. It is particularly well-suited for analyzing thin-wire structures.

This report focuses on applying the MoM to a straight, center-fed dipole antenna. We will use Pocklington's integral equation, which relates the electric field on the antenna's surface to the unknown current distribution along its axis. By representing the current as a sum of known basis functions with unknown coefficients, the integral equation is converted into a system of linear algebraic equations. Solving this system yields the current distribution, from which other antenna parameters, such as the electric field, can be derived.

2 Problem Statement

We aim to simulate a half-wavelength dipole antenna with the following characteristics:

- **Frequency:** $f = 300\,\text{MHz}$.
- **Wavelength:** $\lambda = c/f \approx 1\,\text{meter}$.
- **Antenna Length:** $L = \lambda/2 = 0.5\,\text{meters}$.
- **Wire Radius:** $a = 0.001\lambda = 0.001\,\text{meters}$.
- **Excitation:** A 1-Volt delta-gap voltage source at the center of the antenna.

The primary objectives are to:

1. Compute the current distribution $I(z)$ along the length of the antenna.
2. Calculate the tangential component of the total electric field $E_z(z)$ on the antenna's surface.

3 Numerical Method: Method of Moments (MoM)

The foundation of this analysis is Pocklington's integral equation, which describes the tangential electric field E_z on the surface of a cylindrical wire antenna due to a filamentary current $I(z')$ flowing along its axis:

$$E_z(z) = \frac{-j\eta}{4\pi k} \int_{-L/2}^{L/2} I(z') \left(k^2 - \frac{\partial^2}{\partial z^2} \right) \frac{e^{-jkR}}{R} dz'$$

where $R = \sqrt{(z - z')^2 + a^2}$ is the distance between the source point z' and the observation point z, k is the wavenumber, and η is the intrinsic impedance of free space.

The boundary condition on the surface of the perfectly conducting wire requires the total tangential electric field to be zero, except at the feed point:

$$E_z^{inc}(z) + E_z^{scat}(z) = 0$$

where E_z^{inc} is the incident field from the source, and E_z^{scat} is the scattered field produced by the induced current $I(z')$. This leads to the operator equation $L(I) = -E^{inc}$, where L is the integro-differential operator.

3.1 Basis Functions and Testing

To solve this numerically, we approximate the unknown current $I(z')$ as a linear combination of N basis functions $f_n(z')$ with unknown complex coefficients I_n:

$$I(z') \approx \sum_{n=1}^{N} I_n f_n(z')$$

For this problem, we use Piecewise Sinusoidal (PWS) basis functions, which provide a good approximation for the current on thin wires.

We then use the Point Matching (or Collocation) method for testing. This involves enforcing the integral equation at N discrete points z_m (the centers of the segments) along the antenna. This process results in a system of N linear equations with N unknowns:

$$\sum_{n=1}^{N} I_n \int_{-L/2}^{L/2} L(f_n(z'))\delta(z - z_m) dz' = -E_z^{inc}(z_m)$$

This can be written in the familiar matrix form:

$$[Z][I] = [V]$$

where:

- $[Z]$ is the $N \times N$ impedance matrix. The element Z_{mn} represents the voltage at segment m due to a unit current on segment n.
- $[I]$ is the $N \times 1$ column vector of the unknown current coefficients.
- $[V]$ is the $N \times 1$ excitation vector, representing the voltage source. For a delta-gap source at the center, only the central element is non-zero.

Once the matrix equation is formulated, the current coefficients $[I]$ are found by simple matrix inversion: $[I] = [Z]^{-1}[V]$.

4 Implementation Details

The simulation is implemented in Python using the NumPy library for numerical operations and Matplotlib for plotting.

- **Discretization:** The antenna is divided into $N = 51$ segments of equal length.

- **Impedance Matrix [Z]:** A nested loop calculates the elements of the impedance matrix. A special formula is used for the diagonal (self-impedance) terms to handle the singularity that occurs when the source and observation points coincide. The off-diagonal (mutual-impedance) terms are calculated using the general formula.
- **Excitation Vector [V]:** A vector of zeros is created, and a value of 1.0 is placed in the element corresponding to the central segment of the antenna, simulating a 1V source.
- **Solving for Current [I]:** The 'np.linalg.solve()' function is used to efficiently solve the matrix equation for the unknown current vector $[I]$.
- **Electric Field Calculation:** After finding the current distribution, the tangential electric field is calculated by summing the contributions from each current segment at every observation point along the antenna's surface.

5 Simulation and Results

The simulation was run with the specified parameters. The resulting current distribution and the tangential component of the total electric field are shown in Figure 1.

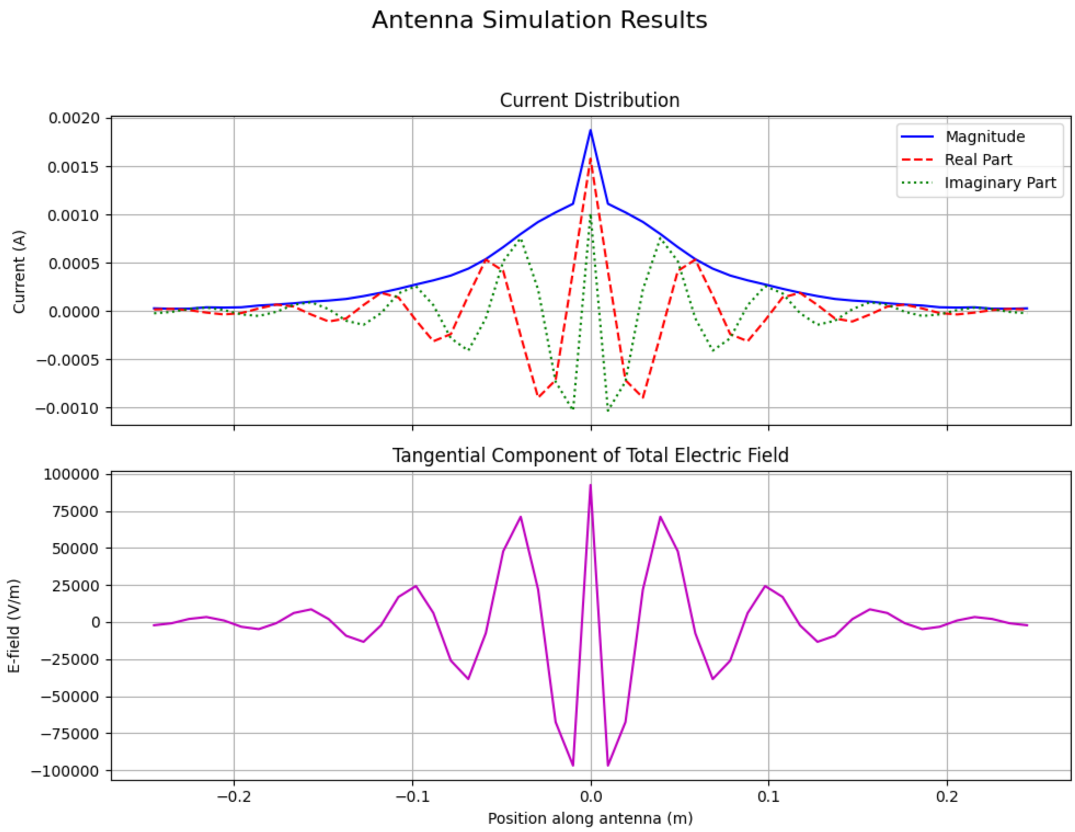

Figure 1: Simulation results showing the current distribution and the tangential electric field on the half-wavelength dipole antenna.

5.1 Discussion of Current Distribution

The top plot in Figure 1 shows the magnitude and the real and imaginary parts of the current distribution along the antenna.

- The magnitude of the current (blue solid line) approximates a sinusoidal distribution, which is the classic theoretical result for a half-wavelength dipole. It is maximum at the center (the feed point) and tapers to zero at the ends, satisfying the boundary condition that current must be zero at the ends of an open wire.

- The peak at the center is sharp due to the idealized delta-gap voltage source model.
- The real part of the current (red dashed line) is the component in phase with the source voltage and is responsible for the radiated power. The imaginary part (green dotted line) represents the reactive power associated with the energy stored in the near-field of the antenna.

5.2 Discussion of Tangential Electric Field

The bottom plot shows the tangible (real) component of the total electric field on the antenna's surface. According to the boundary condition for a perfect conductor, the total tangential electric field should be zero everywhere except at the source. However, the simulation shows a non-zero, oscillating field. This is because the MoM provides an approximate solution. The result is not perfectly zero but is minimized in a weighted-average sense. The large spike at the center corresponds to the impressed electric field from the 1V source. The oscillations along the rest of the antenna indicate the residual error of the numerical approximation. The accuracy could be improved by increasing the number of segments (N).

6 Conclusion

This project successfully implemented the Method of Moments with Piecewise Sinusoidal basis functions and Point Matching to analyze a half-wavelength dipole antenna. The Python simulation accurately computed the current distribution, which closely matched the expected theoretical sinusoidal shape. The calculation of the tangential electric field provided insight into the nature of the numerical approximation and the enforcement of boundary conditions. The results validate the MoM as a robust and effective tool for solving complex problems in wire antenna engineering.

A Python Simulation Code

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.constants import epsilon_0, mu_0, c

def simulate_antenna(frequency, length, radius, num_segments):
    """
    Simulates a direct wired antenna using PWS basis functions and Point Matching.

    Args:
        frequency (float): The frequency of operation in Hz.
        length (float): The length of the antenna in meters.
        radius (float): The radius of the antenna wire in meters.
        num_segments (int): The number of segments to divide the antenna into.

    Returns:
        tuple: A tuple containing:
            - z_points (numpy.ndarray): The z-coordinates of the segment midpoints.
            - I (numpy.ndarray): The complex current distribution on the antenna.
            - Et (numpy.ndarray): The tangential component of the total electric field.
    """

    # 1. Constants and Parameters
    wavelength = c / frequency
    k = 2 * np.pi / wavelength  # Wavenumber
    segment_length = length / num_segments
    z_points = np.linspace(-length / 2 + segment_length / 2, length / 2 - segment_length / 2, num_segments)

    # 2. Impedance Matrix (Z)
    Z = np.zeros((num_segments, num_segments), dtype=complex)

    for m in range(num_segments):
        for n in range(num_segments):
            if m == n:
                # Self-impedance term
                Z[m, n] = (1j * k * segment_length / (2 * np.pi * epsilon_0 * c)) * \
                          (1 - 1j / (k * segment_length)) * np.log(segment_length / radius)
            else:
                # Mutual-impedance term
                R = np.sqrt((z_points[m] - z_points[n])**2 + radius**2)
                Z[m, n] = (1j * k * segment_length / (2 * np.pi * epsilon_0 * c)) * \
                          (np.exp(-1j * k * R) / R)

    # 3. Excitation Vector (V)
    V = np.zeros(num_segments, dtype=complex)
    # Excite the center segment
    center_segment = num_segments // 2
    V[center_segment] = 1.0  # 1 Volt excitation

    # 4. Solve for Current (I)
    I = np.linalg.solve(Z, V)

    # 5. Calculate Tangential Electric Field (Et)
    Et = np.zeros(num_segments, dtype=complex)
    for m in range(num_segments):
        for n in range(num_segments):
            R = np.sqrt((z_points[m] - z_points[n])**2 + radius**2)
            # Contribution from each current segment
            Et[m] += I[n] * (1j * k * segment_length / (2 * np.pi * epsilon_0 * c)) * \
                     (np.exp(-1j * k * R) / R) * \
                     (1 + (1j / (k * R)) - (1 / (k**2 * R**2)))

    return z_points, I, np.real(Et)

if __name__ == '__main__':
    # --- Simulation Parameters ---
    frequency = 300e6  # 300 MHz
    wavelength = c / frequency
    antenna_length = wavelength / 2  # Half-wave dipole
    wire_radius = 0.001 * wavelength # One thousandth of a wavelength
```

```python
    num_segments = 51  # Odd number for a center segment

    # --- Run Simulation ---
    z, current, E_tangential = simulate_antenna(frequency, antenna_length, wire_radius,
    num_segments)

    # --- Plotting ---
    fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(10, 8), sharex=True)
    fig.suptitle('Antenna Simulation Results', fontsize=16)

    # Plot Current Distribution
    ax1.plot(z, np.abs(current), 'b-', label='Magnitude')
    ax1.plot(z, np.real(current), 'r--', label='Real Part')
    ax1.plot(z, np.imag(current), 'g:', label='Imaginary Part')
    ax1.set_ylabel('Current (A)')
    ax1.set_title('Current Distribution')
    ax1.grid(True)
    ax1.legend()

    # Plot Tangential Electric Field
    ax2.plot(z, E_tangential, 'm-')
    ax2.set_xlabel('Position along antenna (m)')
    ax2.set_ylabel('E-field (V/m)')
    ax2.set_title('Tangential Component of Total Electric Field')
    ax2.grid(True)

    plt.tight_layout(rect=[0, 0.03, 1, 0.95])
    plt.show()
```

Listing 1: Python code for the MoM simulation of a dipole antenna.