Analytic Models

Assignment 3

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Question 1.

For *small packets* we have:

$S_{small} = 0.01ms$, $P_{small} = 0.1$.
$\lambda_{small} = P_{small} * \lambda = (0.1) * (8000\frac{packet}{second} * \frac{1second}{1000ms}) = 0.8\frac{packet}{ms}$
So based on Little's law:
$U_{small} = \lambda_{small} * S_{small} = (0.8\frac{packets}{ms}) * (0.01ms) = 0.008$
$Q_{small} = \lambda_{small} * R_{small} = 0.8R_{small}$
Since we have a deterministic service time distribution for each class, then we have:
$S_{rem_{small}} = \frac{S_{small}}{2} = \frac{0.01ms}{2} = 0.005ms$

For *big packets* we have:

$S_{big} = 0.11ms$, $P_{big} = 0.9$.
$\lambda_{big} = P_{big} * \lambda = (0.9) * (8000\frac{packet}{second} * \frac{1second}{1000ms}) = 7.2\frac{packet}{ms}$
So based on Little's law:
$U_{big} = \lambda_{big} * S_{big} = (7.2\frac{packets}{ms}) * (0.11ms) = 0.792$
$Q_{big} = \lambda_{big} * R_{big} = 7.2R_{big}$
Since we have a deterministic service time distribution for each class, then we have:
$S_{rem_{big}} = \frac{S_{big}}{2} = \frac{0.11ms}{2} = 0.055ms$

Finally for each class based on small class non-preemptive priority we have:

$$R_{small} = S_{small} + S_{small} * (Q_{small} - U_{small}) + U_{small} * S_{rem_{small}} + U_{big} * S_{rem_{big}}$$

$$R_{small} = 0.01 + 0.01 * (0.8R_{small} - 0.008) + 0.008 * 0.005 + 0.792 * 0.055$$

```
Rs = (0.01 - (0.01*0.008) + (0.008*0.005) + (0.792*0.055))/(1 - 0.008)
cat("R small: ", Rs, 'milliseconds')
```

```
## R small:  0.05395161 milliseconds
```

$$R_{big} = S_{big} + S_{small}*(Q_{small}-U_{small}) + S_{big}*(Q_{big}-U_{big}) + U_{small}*S_{rem_{small}} + U_{big}*S_{rem_{big}} + (R_{big}-S_{big})\lambda_{small}S_{small}$$

$$R_{big} = 0.11 + 0.01*(0.8*R_{small}-0.008) + 0.11*(7.2R_{big}-0.792) + 0.008*0.005 + 0.792*0.055 + (R_{big}-0.11)*0.8*0.01$$

```
Rb = (0.11+(0.01*(0.8*Rs-0.008))-(0.11*0.792)+(0.008*0.005)+(0.792*0.055)-(0.11*0.8*0.01))/(1-0.8)
cat("R big: ", Rb, 'milliseconds')
```

```
## R big:  0.3297581 milliseconds
```

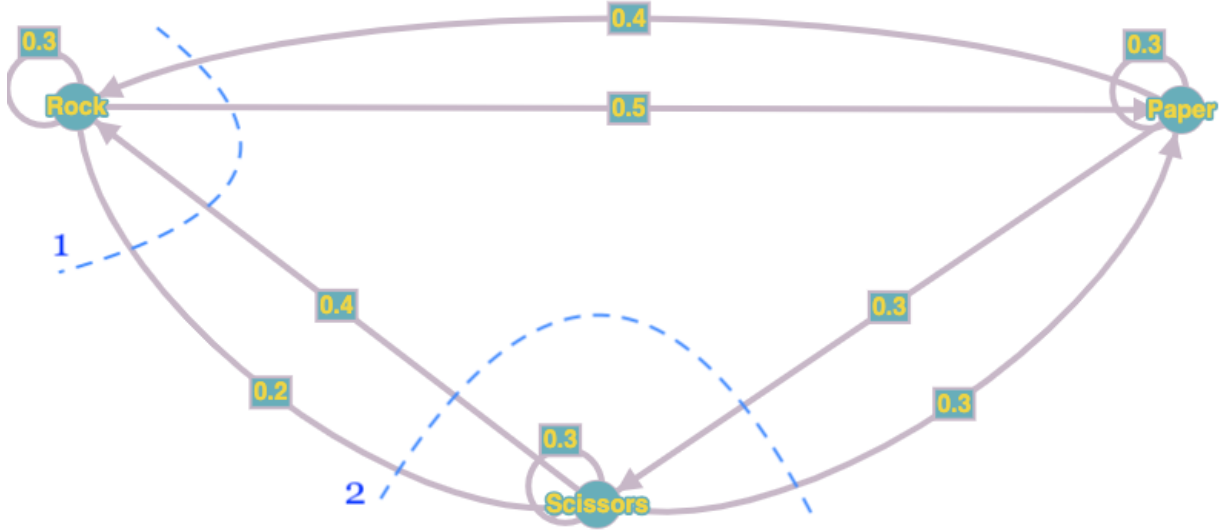Figure 1: rps state-transition model

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Question 2.

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```
N = 1:30
for(i in N){
  # find max troughput based on MVA

}
```

Question 3.

a)

This model is provided in figure 1.

b)

We can obtain the following equations by dividing the graph as in figure 1.

$$
\begin{cases}
P_{rock} + P_{paper} + P_{scissors} = 1 \\
1)0.4P_{paper} + 0.4P_{scissors} = 0.5P_{rock} + 0.2P_{rock} \\
2)0.2P_{rock} + 0.3P_{paper} = 0.4P_{scissors} + 0.3P_{scissors}
\end{cases}
$$

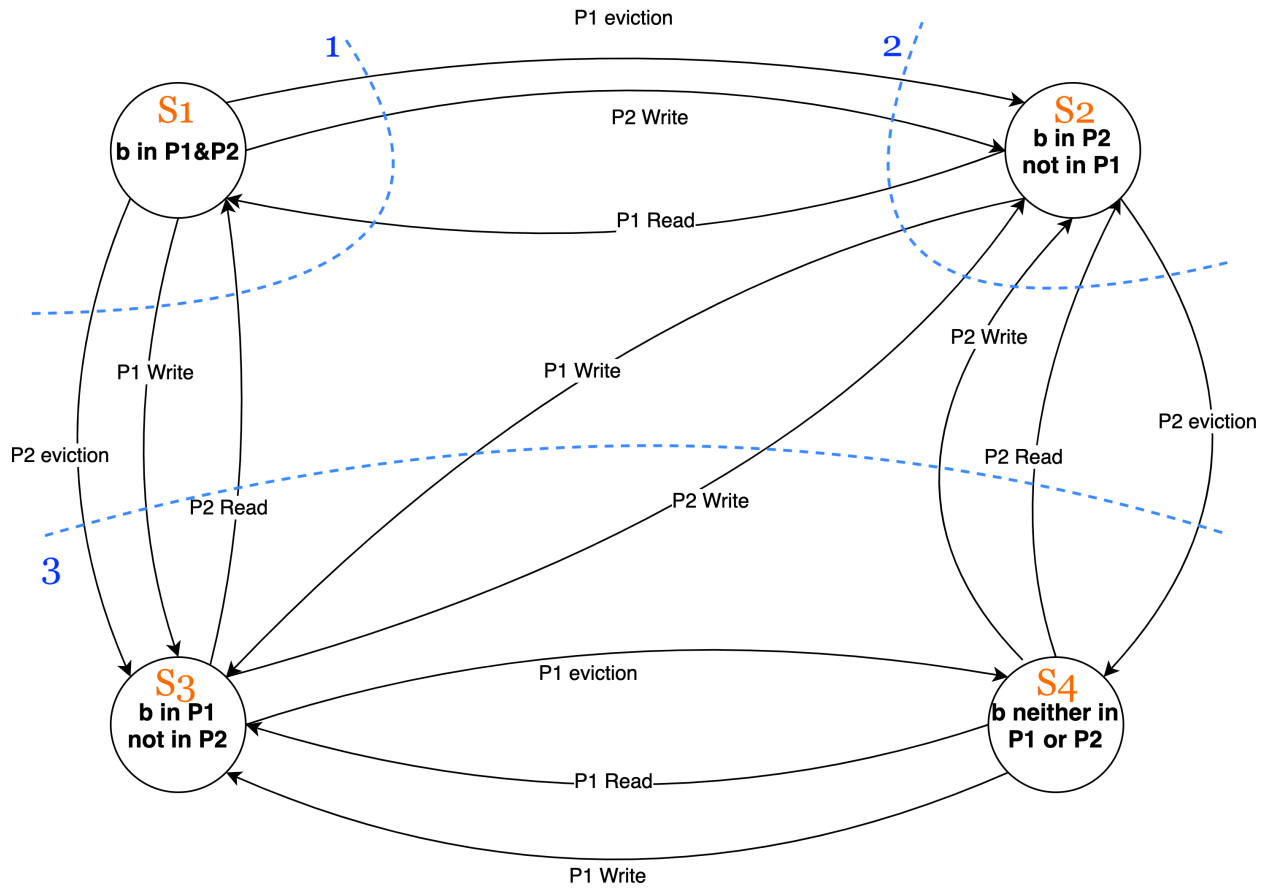The results are $P_{rock} = \frac{4}{11}$, $P_{paper} = \frac{41}{110}$, $P_{scissors} = \frac{29}{110}$

Figure 2: system state-transition model

Question 4.

a)

This model is provided in figure 2.

b)

We can obtain the following equantions by dividing the graph as in figure 2. Read rate for each processor is $\frac{1}{R}$ and Write rate for each processor is $\frac{1}{W}$.

$$
\begin{cases}
S_1 + S_2 + S_3 + S_4 = 1 \\[2mm]
1) P_1 Eviction * S_1 + P_2 Eviction * S_1 + P_1 Write * S_1 + P_2 Write * S_1 = P_1 Read * S_2 + P_2 Read * S_3 \\
\rightarrow \lambda * S_1 + \lambda * S_1 + \frac{1}{W} * S_1 + \frac{1}{W} * S_1 = \frac{1}{R} * S_2 + \frac{1}{R} * S_3 \\[2mm]
2) P_1 Read * S_2 + P_2 Write * S_2 + P_2 Eviction * S_2 = \\
P_1 Eviction * S_1 + P_2 Write * S_1 + P_2 Write * S_3 + P_2 Write * S_4 + P_2 Read * S_4 \\
\rightarrow \frac{1}{R} * S_2 + \frac{1}{W} * S_2 + \lambda * S_2 = \lambda * S_1 + \frac{1}{W} * S_1 + \frac{1}{W} * S_3 + \frac{1}{W} * S_4 + \frac{1}{R} * S_4 \\[2mm]
3) P_2 Eviction * S_1 + P_1 Write * S_1 + P_1 Write * S_2 + P_2 Eviction * S_2 = \\
P_2 Read * S_3 + P_2 Write * S_3 + P_2 Write * S_4 + P_2 Read * S_4 \\
\rightarrow \lambda * S_3 + \frac{1}{W} * S_1 + \frac{1}{W} * S_2 + \lambda * S_2 = \frac{1}{R} * S_3 + \frac{1}{W} * S_3 + \frac{1}{W} * S_4 + \frac{1}{R} * S_4
\end{cases}
$$

Finally:

$$
\begin{cases}
S_1 + S_2 + S_3 + S_4 = 1 \\[2mm]
1) 2S_1\lambda + \frac{2S_1}{W} = \frac{S_2 + S_3}{R} \\[2mm]
2) \frac{S_2}{R} + \frac{S_2}{W} + S_2\lambda = S_1\lambda + \frac{S_1 + S_3 + S_4}{W} + \frac{S_4}{R} \\[2mm]
3) S_3\lambda + S_2\lambda + \frac{S_1 + S_2}{W} = \frac{S_3 + S_4}{W} + \frac{S_3 + S_4}{R}
\end{cases}
$$

Question 5.

a)

We define states as a set of two values, which shows requests in server 1(consider m) & server 2(consider n). Starting at (0,0) which means there is no work in both servers.

Then by rate of $\frac{\lambda}{2}$ we will have a transition to (1,0) or (0,1). Then by rate of $\frac{1}{S}$ will have a transition to (0,0) from (1,0) or (0,1). From (1,0), we might go to (2,0), (1,1) by rate of $\frac{\lambda}{2}$. From (0,1), we might go to (0,2), (1,1) by rate of $\frac{\lambda}{2}$.

So the formulation can be as following:

- State(m,n) $\rightarrow$ State(m+1,n) by rate $\frac{\lambda}{2}$
- State(m,n) $\rightarrow$ State(m,n+1) by rate $\frac{\lambda}{2}$
- State(m,n) $\rightarrow$ State(m-1,n) by rate $\frac{1}{S}$
- State(m,n) $\rightarrow$ State(m,n-1) by rate $\frac{1}{S}$
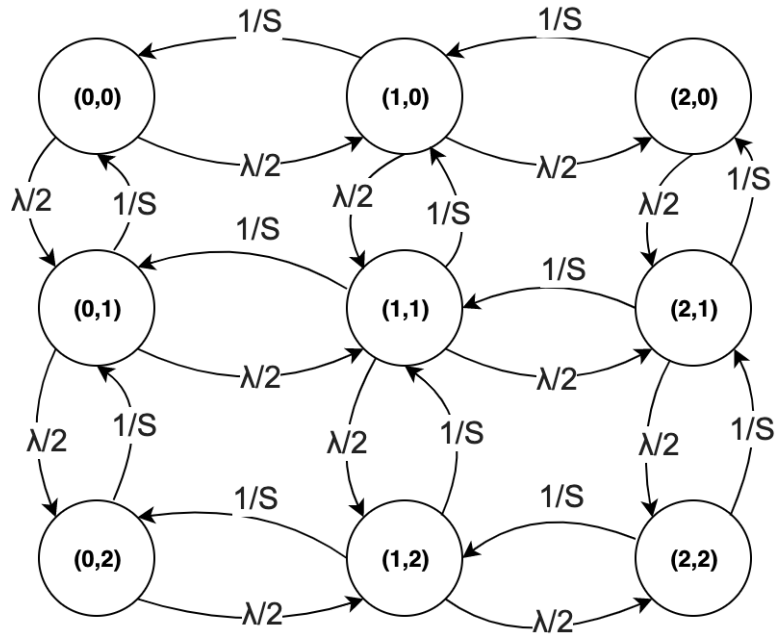
Sample diagram is shown in figure 3.

b)

Figure 3: two servers state-transition model

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c)
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d)
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