Characterizing Measurement Data

Assignment 1

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2019-10-01 03:32:13

Question 1.

a)

$$\lambda = 5\frac{session}{minute} \rightarrow mean = \frac{1}{\lambda} = \frac{1minute}{5session} \times \frac{60seconds}{1minute} = 12seconds$$

b)

Exponential distribution has *memoryless property*. It means that no matter how much time has elapsed since the previous arrival, the mean waiting time at time T for the next entry will still be 12 seconds.

c)

$$arrivals = period * lambda = 2minutes * 5\frac{arrival}{minute} = 10arrivals$$

d)

Because interarrival times are IID, the first arrival doesn't change the mean.

$$time = 3 * 60 = 180seconds \rightarrow arrivals = \left\lfloor \frac{180 - 18}{12\frac{arrival}{seconds}} \right\rfloor = 13 + 1(firstarrival) = 14$$

Question 2.

a)

W:

$$mean = \frac{1}{\lambda} = 12 \rightarrow \lambda = \frac{1}{12} = 0.08\overline{3}$$

X:

$$mean = n\gamma \rightarrow \gamma = \frac{12}{3} = 4$$

Y:

$$mean = e^{(\mu+\frac{\sigma^2}{2})} \rightarrow e^{(1+\frac{\sigma^2}{2})} = 12 \rightarrow ln(e^{(1+\frac{\sigma^2}{2})}) = ln(12) \rightarrow 1+\frac{\sigma^2}{2} = ln(12) \rightarrow \frac{\sigma^2}{2} = ln(12)-1 \rightarrow \sigma^2 = 2ln(12)-2$$

$$\sigma = \pm(2ln(12) - 2)$$

```
sigma=sqrt(2*log(12) - 2)
cat("sigma is: +-",sigma)
```

```
## sigma is: +- 1.723315
```

Z:

$$mean = \begin{cases} \infty & for \quad \alpha \le 1 \\ \frac{\alpha * k}{\alpha - 1} & for \quad \alpha > 1 \end{cases}$$

$$\alpha = 1.25 \rightarrow mean = \frac{\alpha * k}{\alpha - 1} \rightarrow 12 = \frac{1.25k}{1.25 - 1} \rightarrow 12 = \frac{1.25k}{0.25} \rightarrow 12 = 5k \rightarrow k = \frac{12}{5} = 2.4$$

b)

W:

$$F(x) = P(x < 12) = 1 - e^{-\lambda x} = 1 - e^{-\frac{1}{12}*12} = 1 - \frac{1}{e}$$

```
p=pexp(12, rate=1/12)
cat("Probability for exponential distribution is: ",p)
```

```
## Probability for exponential distribution is:  0.6321206
```

X:

$$F(x) = P(x < 12) = 1 - \sum_{n=0}^{2} \frac{e^{-\frac{x}{\gamma}} x^n}{\gamma^n n!} = 1 - \sum_{i=0}^{2} \frac{e^{-\frac{x}{4}} x^n}{4^n n!}$$

```
p=pgamma(12, shape= 3, scale= 4)
cat("Probability for erlang distribution is: ",p)
```

```
## Probability for erlang distribution is:  0.5768099
```

Y:

$$F(x) = P(x < 12) = \phi(\frac{(lnx) - \mu}{\sigma})$$

where ϕ is the cumulative distribution function of the standard normal distribution.

```
p=plnorm(12, meanlog = 1, sdlog = sigma)
cat("Probability for lognormal distribution is: ",p)
```

```
## Probability for lognormal distribution is:  0.8055619
```

Z:

$$F(x) = P(x < 12) = 1 - \frac{k}{x}^{\alpha}$$

```
library(EnvStats)
p=ppareto(12, location=2.4, shape=1.25)
cat("Probability for pareto distribution is: ",p)
```

```
## Probability for pareto distribution is:  0.8662519
```

c)

W:

$$1 - P(x < 240) = 1 - (1 - e^{-\lambda x}) = e^{-\frac{1}{12}*240} = \frac{1}{e^{20}}$$

```
p= 1 - pexp(240, rate=1/12)
cat("Probability for exponential distribution is: ",p)
```

```
## Probability for exponential distribution is:  2.061154e-09
```

X:

$$1 - P(x < 240) = 1 - (1 - \sum_{n=0}^{2} \frac{e^{-\frac{x}{\gamma}} x^n}{\gamma^n n!}) = \sum_{i=0}^{2} \frac{e^{-\frac{x}{4}} x^n}{4^n n!}$$

```
p= 1 - pgamma(240, shape= 3, scale= 4)
cat("Probability for erlang distribution is: ",p)
```

```
## Probability for erlang distribution is:  0
```

Y:

$$1 - P(x < 240) = 1 - \phi(\frac{(lnx) - \mu}{\sigma})$$

where ϕ is the cumulative distribution function of the standard normal distribution.

```
p= 1 - plnorm(240, meanlog = 1, sdlog = sigma)
cat("Probability for lognormal distribution is: ",p)
```

```
## Probability for lognormal distribution is:  0.004661023
```

Z:

$$1 - P(x < 240) = 1 - (1 - \frac{k}{x}^{\alpha}) = \frac{k}{x}^{\alpha}$$

```
library(EnvStats)
p= 1 - ppareto(240, location=2.4, shape=1.25)
cat("Probability for pareto distribution is: ",p)
```

```
## Probability for pareto distribution is:  0.003162278
```

Question 3.

$$P_R(n) = \frac{\frac{1}{n^\alpha}}{\sum_{m=1}^{k} \frac{1}{m^\alpha}} \to i^{-\alpha} < \frac{1}{day} = \frac{1}{60 * 60 * 24 seconds} = \frac{1}{86400} \to i^\alpha > 86400 \to i^{1.25} > 86400$$

```
library(dplyr)

powers <- data.frame("i"=0:10000000, "pow"=rep(0,10000001), stringsAsFactors = FALSE)
powers$pow <- powers$i^1.25
is <- powers %>% filter(pow > 86400)
head(is, 1)
```

```
##      i      pow
## 1 8897 86408.11
```

```r
cat("i: ", is$i[1])
```

```
## i:  8897
```

```r
powers$pow <- powers$i^0.85
is <- powers %>% filter(pow > 86400)
head(is, 1)
```

```
##        i      pow
## 1 642190 86400.05
```

```r
cat("i: ", is$i[1])
```

```
## i:  642190
```

Question 4.

Question 5.

a)

b)

c)