

Characterizing Measurement Data

Assignment 1

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Question 1.

a)

$$\lambda = 5 \frac{\text{session}}{\text{minute}} \rightarrow \text{mean} = \frac{1}{\lambda} = \frac{1 \text{minute}}{5 \text{session}} \times \frac{60 \text{seconds}}{1 \text{minute}} = 12 \text{seconds}$$

b)

Exponential distribution has *memoryless property*. It means that no matter how much time has elapsed since the previous arrival, the mean waiting time at time T for the next entry will still be 12 seconds.

c)

$$\text{arrivals} = \text{period} * \text{lambda} = 2 \text{minutes} * 5 \frac{\text{arrival}}{\text{minute}} = 10 \text{arrivals}$$

d)

Because interarrival times are IID, the first arrival doesn't change the mean.

$$\text{time} = 3 * 60 = 180 \text{seconds} \rightarrow \text{arrivals} = \left\lfloor \frac{180 - 18}{12 \frac{\text{arrival}}{\text{seconds}}} \right\rfloor = 13 + 1(\text{first arrival}) = 14$$

Question 2.

a)

W:

$$\text{mean} = \frac{1}{\lambda} = 12 \rightarrow \lambda = \frac{1}{12} = 0.08\bar{3}$$

X:

$$\text{mean} = n\gamma \rightarrow \gamma = \frac{12}{3} = 4$$

Y:

$$\begin{aligned} \text{mean} = e^{(\mu + \frac{\sigma^2}{2})} \rightarrow e^{(1 + \frac{\sigma^2}{2})} = 12 \rightarrow \ln(e^{(1 + \frac{\sigma^2}{2})}) = \ln(12) \rightarrow 1 + \frac{\sigma^2}{2} = \ln(12) \rightarrow \frac{\sigma^2}{2} = \ln(12) - 1 \rightarrow \sigma^2 = 2\ln(12) - 2 \\ \sigma = \pm(2\ln(12) - 2) \end{aligned}$$

```
sigma=2*log2(12) - 2
cat("sigma is: +-",sigma)
```

```
## sigma is: +- 5.169925
```

Z:

$$mean = \begin{cases} \infty & \text{for } \alpha \leq 1 \\ \frac{\alpha * k}{\alpha - 1} & \text{for } \alpha > 1 \end{cases}$$

$$\alpha = 1.25 \rightarrow mean = \frac{\alpha * k}{\alpha - 1} \rightarrow 12 = \frac{1.25k}{1.25 - 1} \rightarrow 12 = \frac{1.25k}{0.25} \rightarrow 12 = 5k \rightarrow k = \frac{12}{5} = 2.4$$

b)

W:

$$F(x) = P(x < 12) = 1 - e^{-\lambda x} = 1 - e^{-\frac{1}{12} * 12} = 1 - \frac{1}{e}$$

```
p=pexp(12, rate=1/12)
cat("Probability for exponential distribution is: ",p)
```

```
## Probability for exponential distribution is: 0.6321206
```

X:

$$F(x) = P(x < 12) = 1 - \sum_{n=0}^2 \frac{e^{-\frac{x}{\gamma}} x^n}{\gamma^n n!} = 1 - \sum_{i=0}^2 \frac{e^{-\frac{x}{4}} x^n}{4^n n!}$$

```
p=pgamma(12, shape= 3, scale= 4)
cat("Probability for erlang distribution is: ",p)
```

```
## Probability for erlang distribution is: 0.5768099
```

Y:

$$F(x) = P(x < 12) = \phi\left(\frac{(\ln x) - \mu}{\sigma}\right)$$

where ϕ is the cumulative distribution function of the standard normal distribution.

```
p=plnorm(12, meanlog = 1, sdlog = sigma)
cat("Probability for lognormal distribution is: ",p)
```

```
## Probability for lognormal distribution is: 0.6130281
```

Z:

$$F(x) = P(x < 12) = 1 - \frac{k^\alpha}{x}$$

```
library(EnvStats)
p=ppareto(12, location=2.4, shape=1.25)
cat("Probability for pareto distribution is: ",p)
```

```
## Probability for pareto distribution is: 0.8662519
```

c)

W:

X:

Y:

Z:

Question 3.

Question 4.

Question 5.

a)

b)

c)