# CMPT 423/820

## Assignment 1 Question 8 ¶


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## Task

Derive $P(y|\mathbf{X}_1, \mathbf{X}_2) = \frac{m_1+m_2+a}{N_1+N_2+a+b}$

$$P(y|\mathbf{X}_1, \mathbf{X}_2) = E[\mu|\mathbf{X}_1, \mathbf{X}_2] = \int_\mu \mu \frac{P(\mathbf{X}_1, \mathbf{X}_2|\mu)P(\mu)}{P(\mathbf{X}_1, \mathbf{X}_2)} d\mu$$

Because the joggler is same in both days, $\mu$ is unchanged. Additionally, yesterday and today are independent, so we can use *Indepence* and *Conditional Indepence* formula.

*Independence*: $P(\mathbf{X}_1, \mathbf{X}_2) = P(\mathbf{X}_1)P(\mathbf{X}_2)$

*Conditional Independence*: $P(\mathbf{X}_1, \mathbf{X}_2|\mu) = P(\mathbf{X}_1|\mu)P(\mathbf{X}_2|\mu)$

$$\Rightarrow P(y|\mathbf{X}_1, \mathbf{X}_2) = \int_\mu \mu \frac{P(\mathbf{X}_1|\mu)P(\mathbf{X}_2|\mu)P(\mu)}{P(\mathbf{X}_1)P(\mathbf{X}_2)} d\mu$$

Considering $P(\mathbf{X}|\mu) = Bin(m|N, \mu) = \binom{N}{m}\mu^m(1-\mu)^{N-m}$ for $\mathbf{X}_1$ and $\mathbf{X}_2$.

$$\Rightarrow P(y|\mathbf{X}_1, \mathbf{X}_2) = \int_\mu \mu \frac{\binom{N_1}{m_1}\mu^{m_1}(1-\mu)^{N_1-m_1}\binom{N_2}{m_2}\mu^{m_2}(1-\mu)^{N_2-m_2}P(\mu)}{P(\mathbf{X}_1)P(\mathbf{X}_2)} d\mu$$

$$\Rightarrow P(y|\mathbf{X}_1, \mathbf{X}_2) = \int_\mu \mu \frac{\binom{N_1}{m_1}\binom{N_2}{m_2}\mu^{m_1+m_2}(1-\mu)^{N_1-m_1+N_2-m_2}P(\mu)}{P(\mathbf{X}_1)P(\mathbf{X}_2)} d\mu$$

$$\Rightarrow P(y|\mathbf{X}_1, \mathbf{X}_2) = \int_\mu \mu \frac{\binom{N_1}{m_1}\binom{N_2}{m_2}\mu^{(m_1+m_2)}(1-\mu)^{(N_1+N_2)-(m_1+m_2)}P(\mu)}{P(\mathbf{X}_1)P(\mathbf{X}_2)} d\mu$$

So, based on $P(y|\mathbf{X}) = \int_\mu \mu \frac{\binom{N}{m}\mu^m(1-\mu)^{N-m}P(\mu)}{P(\mathbf{X})} d\mu = \frac{m+a}{N+a+b}$:

We can suppose that our above integral will obtain the same result too. Because $\binom{N_1}{m_1}\binom{N_2}{m_2}$ are constant and have no effect on the integral, and we can substitute following variables $m_1 + m_2 = m$ and $N_1 + N_2 = N$. Consequently:

$$\Rightarrow P(y|\mathbf{X}_1, \mathbf{X}_2) = \frac{m_1 + m_2 + a}{N_1 + N_2 + a + b}$$