40

STUDENT'S NAME: Elissar Fadel

STUDENT'S NUMBER: 2134398

DAWSON COLLEGE - DEPARTMENT OF MATHEMATICS	
LINEAR ALGEBRA COMPUTER SCIENCE SECTION 11	
EXAM-3B	
NOVEMBER 30, 2022 (from 1:10 to 2:20)	INSTRUCTOR: A. JIMENEZ

NOTE: This exam has 6 questions for a total of 42 marks and worth 21% of the final mark.

(12 Marks)

1.) Given points A(1,0,1), B(0,1,0), C(1,1,1) and P(4,0,3)

12

a) Find the closest point to C on the line passing through A and B.

b) Find the closest point to P on the plane passing through À, B and C.

c) Find the distance from P to the line passing through A and B

a) Line =

direc vector = B-A = (-1, 1, -1)

initial point = A = (1,0,1)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$ ✓

$v = C - A = \langle 0, 1, 0 \rangle$

$\text{proj}_d V = \dfrac{v \cdot d}{d \cdot d} = \dfrac{1}{3}$

d = direc vector

V = C-A vector

then

$x = 1 + \frac{1}{3} \cdot -1$

$y = \frac{1}{3} \cdot 1$

$z = 1 + \frac{1}{3} \cdot -1$

closest Point

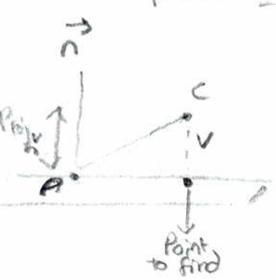
$\rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

b) plane =

AB = $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

AC = no time to write = $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

plane = $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ ✓



n = h

formula = n. (C-P) - proj(C

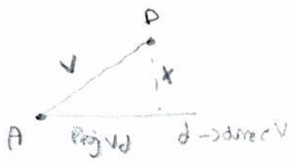
c. $d(P, \text{line})$

$P(4, 0, 3)$

line: $A(1,0,1)$ $B(0,1,0)$

found the equation in a

$$\text{line} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$



$v = PA = \langle 3, 0, 2 \rangle$

$$\text{Proj}_v d = \frac{v \cdot d}{d \cdot d} \cdot d = -\frac{5}{3}\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} \\ -\frac{5}{3} \\ \frac{5}{3} \end{bmatrix}$$

$V - \text{Proj}\,V = X$ in drawing

$V - \text{Proj}\,V = \langle \frac{4}{3}, \frac{5}{3}, \frac{1}{3} \rangle$

$$\| V - \text{Proj}_v d \| = \boxed{\sqrt{\frac{42}{9}}}$$

8

(8 Marks)

2.) Calculate the distance between the following lines. Based on your results are these lines skew-lines?

l_1 : x= 4 + t, y= -8 – 2t, z= 12t

l_2 : x= 3+ 2s, y= -1 + s, z= -3 -3s

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 12 \end{bmatrix} t$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -3 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} s$$

are they parralled =

$\frac{1}{2} = \frac{-2}{1} = \frac{12}{-3}$ nope ✓

one point on the line = (4, -8, 0)

d (P, line)

P

v

Initial P

$v = P - \text{initial}P = \langle 1, -7, 3 \rangle$

$\text{proj}_v d = \dfrac{v \cdot \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}}{\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}} \cdot \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} = \dfrac{-14}{14} \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ -3 \end{bmatrix}$

$\| v - \text{proj}_v d \| = \| \langle 3, -6, 6 \rangle \| = \boxed{9}$

✓

lines are skew-lines because they are not parralled and do not intersect so distance is not equal to 0 at any point

no intersection? = x = x y = y z = z

x ✓ $4 + t = 3 + 2s$

y $-8 - 2t = -1 + s$

z $12t = -3 - 3s$

$t = 2s - 1$

$-8 - (2s - 1) = 6 + s$

$s = -2$

$12t = -3 + 6$

$t = \frac{1}{4}$

x = 4.25 ≠ x = 3.5

no intersection

5

(5 Marks)

3.)

Calculate the intersection of the following planes

p1: 2x+y-z -10 =0
p2: x-2y+z-5=0

put everything $x =$

$p1 = 2x = 10 + z - y$

$x = 5 + \frac{z}{2} - \frac{y}{2}$

$p2 = \quad x = 5 - z + 2y$

intersection

must

$\cancel{5} + \frac{z}{2} - \frac{y}{2} = \cancel{5} - z + 2y$

$z - y = -2z + 4y$

$3z = 5y$

$z = \frac{5}{3}y$

$y = t$ → we assume

so

$\boxed{z = \frac{5}{3}t \quad y = t}$

$x = 5 - \frac{5}{3}t + 2t$

$\boxed{x = 5 + \frac{1}{3}t}$

equation of lin

$x = 5 + \frac{1}{3}t$
$y = t$
$z = \frac{5}{3}t$

$\longrightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} \frac{1}{3} \\ 1 \\ \frac{5}{3} \end{bmatrix}$

8

(8 Marks)

4.)

Maximize $P = 5x_1 + 4x_2 + 7x_3 - x_4$

Subject to the conditions $\begin{Bmatrix} x_1 + 2x_2 + x_3 + 2x_4 \le 7 \\ x_1 - x_3 + 3x_4 \le 3 \\ 3x_2 + x_3 \le 4 \\ x_1, x_2, x_3, x_4 \ge 0 \end{Bmatrix}$

place in a matrice

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 7 \\ 1 & 0 & -1 & 3 & 3 \\ 0 & 3 & 1 & 0 & 4 \\ \hline 5 & 4 & 7 & -1 & 0 \end{array}\right] \quad \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 7 \\ 0 & -2 & -2 & 1 & -4 \\ 0 & 3 & 1 & 0 & 4 \\ \hline 0 & -6 & 2 & -11 & -35 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 7 \\ 0 & 6 & -2 & 11 & 35 \\ 0 & 3 & 1 & 0 & 4 \\ \hline 0 & -2 & -2 & 1 & -4 \end{array}\right] \to$$

reduce row echelon

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 7 \\ 0 & 0 & -4 & 11 & 27 \\ 0 & 3 & 1 & 0 & 4 \\ 0 & 2 & 2 & -1 & 4 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 7 \\ 0 & 0 & -4 & 11 & 27 \\ 0 & 1 & \frac{1}{3} & 0 & \frac{4}{3} \\ 0 & 2 & 2 & -1 & 4 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 7 \\ 0 & 0 & -4 & 11 & 27 \\ 0 & 1 & \frac{1}{3} & 0 & \frac{4}{3} \\ 0 & 0 & \frac{4}{3} & -1 & \frac{4}{3} \end{array}\right] \to$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 7 \\ 0 & 3 & 1 & 0 & 4 \\ 0 & 0 & -4 & 11 & 27 \\ 0 & 0 & 4 & -3 & 4 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 7 \\ 0 & 3 & 1 & 0 & 4 \\ 0 & 0 & -4 & 11 & 27 \\ 0 & 0 & 0 & 8 & 31 \end{array}\right]$$

$x_1 + 2\cancel{x_2} + 2x_3 + 2x_4 = 7$

$3x_2 + x_3 = 4$

$-4x_3 + 11x_4 = 27$

$8x_4 = 31$

$x_4 = \frac{31}{8}$ $\qquad x_1 =$

$x_3 = \frac{125}{32}$

$x_2 = \frac{3}{96}$

(9 Marks)

5.) Minimize $C=20x_1+60x_2+10x_3$

7

Subject to the conditions: $\left| \begin{array}{c} x_1+x_2+2x_3 \geq 6 \\ x_1+2x_2+3x_3 \geq 4 \\ x_1+x_2+x_3 \geq 8 \\ x_1, x_2, x_3 \geq 0 \end{array} \right|$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 8 \\ 20 & 60 & 10 & 0 \end{array}\right]^T = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 1 & 2 & 1 & 60 \\ 2 & 3 & 1 & 10 \\ 6 & 4 & 8 & 0 \end{array}\right]$$

maximize

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 1 & -1 & -30 \\ 0 & -2 & 2 & -120 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & -1 & -70 \\ 0 & 0 & 2 & -40 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 70 \\ 0 & 0 & 0 & -180 \end{array}\right]$$

×

$x + y + z = 20$

$y = 40$

$z = 70$

$x = -90 \rightarrow$ does not respect

I made an error somewhere

$$\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \\ 6 & 4 & 8 \end{array}$$

$20.(-90) + 60(40) + 70(10) \overset{?}{=} -180$

$-1800 + 2400 + 700 \overset{\text{should}}{=} -180$