The Range of a Baseball with Backspin

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In physics problems, we must make a lot of approximations. A common approximation that is often justified is to neglect air drag, but this is not acceptable for the modeling of a baseball. Not only does drag impede the ball's forward motion, but it also generates lift through an effect known as the Magnus force, which applies to balls hit with backspin. This report will analyze how the effect of the Magnus lift will extend the final range.

I. INTRODUCTION

Physically, baseball seems fairly simple on the surface. A spherical projectile that stays near the surface of the earth seems like a picture-perfect Physics 121 problem. In reality, there are many complicated factors that can very seriously alter the trajectory of the ball. Altitude, variable air density, drag, and lift all play parts that are not by any means negligible. In this report, we will be including the effects of quadratic air drag, and we will examine the difference between taking into account the Magnus force and neglecting it.

II. SETUP OF PROBLEM

Being a fluid, air behaves in complicated ways. The quadratic drag coefficient B_2 depends on velocity and is given by

$$\frac{B_2}{m} = 0.0039 + \frac{0.0058}{1 + exp[(v - v_d)/\Delta]} \tag{1}$$

And the acceleration due to the Magnus force is

$$-\frac{S_0}{m}\omega \times v \tag{2}$$

where $\frac{S_0}{m} \approx 4.1 \times 10^{-4}$.[1] These are the main expressions we need in order to take the air's effects into account. When we put together all the pieces, we can get the equations of motion in two dimensions. The z dimension is not necessary because if we define the velocity as having x (horizontal) and y (vertical) components, the Magnus force in this case deflects the ball in the +y direction and z remains constant. Therefore we will not examine z.

But how do we know what direction the Magnus force acts in? The direction can be obtained by crossing the rotation axis of the ball with the translational velocity. For backspin, the ball comes off the bat spinning backwards towards home plate. Assuming a right-handed coordinate system, the angular velocity vector initially points up and slightly back. The Magnus force continues to act in the x-y plane in a direction perpendicular to the instantaneous velocity.

III. CODE

We can use the drag term, Magnus force term, and the vertical force of gravity to set up an Euler's method simulation in Python to show how the trajectories differ with and without the backspin.

```
from numpy import pi, sin, cos, exp, sqrt, cross
from matplotlib import pyplot as plt

#Initialize variables with typical values for a baseball
g = 9.80
S0_over_m = 4.1e-4
omega = [0, 0, 2000 * 2 * pi / 60]
theta = 45 * pi/180
v0 = 110 * 0.44704 #mph to m/s
vx = v0*cos(theta)
vy = v0*sin(theta)
vz = 0
v = [v0*cos(theta), v0*sin(theta), 0]
ve = sqrt(v[0]**2 + v[1]**2)
dt = 0.01

#This function returns the drag coefficient as a function
     of velocity
def B2(velocity):
    return (0.0039 + 0.0058/(1 + exp((velocity-35)/5)))

'''This function calculates the trajectory arrays
     necessary for plotting
    The parameter apply_magnus should be a value 0 or 1
     to turn the Magnus on or off
    The function performs Euler's method with all the
     necessary acceleration terms
'''
def getTrajectory(x, y, v, apply_magnus):
    while (y[-1] > 0):
        magnus_acceleration = S0_over_m * cross(omega, v)
        ve = sqrt(v[0]**2 + v[1]**2)
        x.append(x[-1] + v[0]*dt)
        y.append(y[-1] + v[1]*dt)
        v[0] += (-B2(ve) * ve * v[0] +
     magnus_acceleration[0] * apply_magnus) * dt
        v[1] += (-g + magnus_acceleration[1] *
     apply_magnus - B2(ve) * ve * v[1]) * dt
    return x, y

xs = [0]
ys = [1.016]
```

```
xs, ys = getTrajectory(xs,ys,v,1)
range_with_spin = xs[-1]

v = [v0*cos(theta), v0*sin(theta), 0]

x = [0]
y = [1.016]

x,y = getTrajectory(x,y,v,0)
range_without_spin = x[-1]

plt.axes().set_facecolor('k')
plt.plot(xs, ys, label = "Range With Backspin", color = "
     springgreen")
plt.plot(x,y, label = "Range Without Spin", color = "
     fuchsia")
plt.legend()
plt.show()

print(range_with_spin, range_without_spin)

print("Backspin extends the range by ", (range_with_spin
     - range_without_spin), "m")
```

Listing 1. Euler's Method Simulation

IV. RESULTS

The Euler's Method simulation runs very quickly. Given an angle of 45° and an inital speed of 110 mph, this produced the plot below. The range is extended by about 3.2 m.

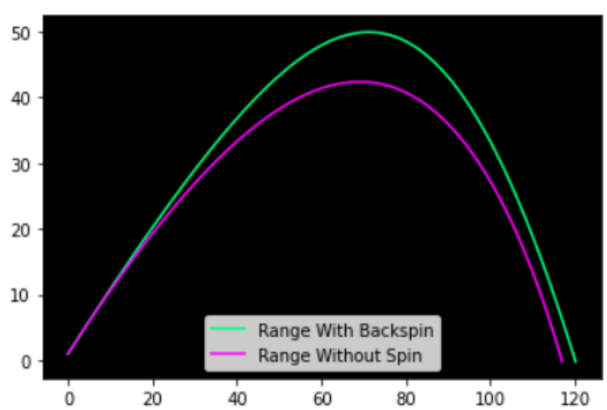

FIG. 1. Trajectory of a baseball with and without backspin (which generates the Magnus Force). The initial speed of the baseball was 110 mph and the launch angle was 45°.

At about 50°, the range with backspin is nearly the same as without. Any angles higher than this give shorter range when the Magnus Force acts on the ball.

However, these steep angles are not always typical for a baseball. One of the most common angles from the horizontal of a ball after leaving the bat is 14°.[2] The following plot shows that for such a moderate angle, the backspin makes a large difference on the range, about 23m, which certainly cannot be neglected.

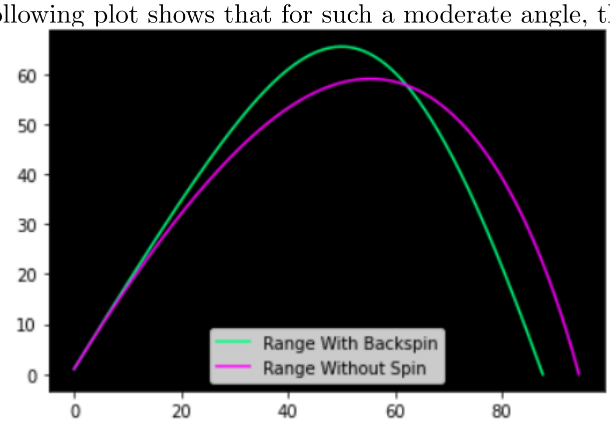

FIG. 2. Trajectory of a baseball launched at a steep angle (60°) with and without backspin. Initial speed is 110 mph. In this case the range is shorter with backspin, by about 7m.

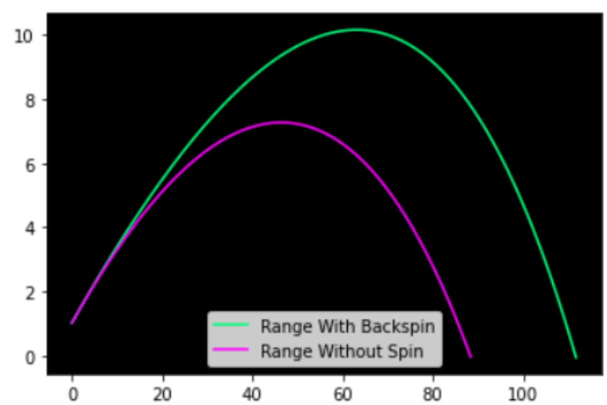

FIG. 3. Trajectory of a baseball launched at a moderate angle (14°) with and without backspin. Initial speed is 110 mph. Backspin extends the range by a large amount in this case.

V. CONCLUSIONS

From the output given by our simulation, we can clearly see the effect on the ball by the Magnus Force. This effect cannot safely be ignored, especially for shallower angles.

[1] N. J. Giordano, *Computational Physics*. Pearson Prentice Hall, 2006.

[2] B. Savant, "Hit probabilities broken down by exit velocity and launch angle." https://baseballsavant.mlb.com/

statcast_hit_probability?year=2020&type=la, 2020.