Computational Astronomy '17 Spring
Apr. 12 DUE: Apr. 19 (Wed)

PROBLEM SET #3

For the problems below, you need to write programs using Python. Turn in your own source programs written independently together with a report in pdf format by email (both to the instructor at wkim@astro.snu.ac.kr and the TA at moon@astro.snu.ac.kr.)

1. Integrate the following integral to 6 significant digits

$$\int_0^4 \sin(x^4)dx, \tag{1}$$

using (a) the composite trapezoidal rule, (b) the composite Simpson's rule, and (c) the Gaussian quadrature.

2. The exact limb darkening formula normalized at $\mu = 0$ is given by

$$H(\mu) = \frac{1}{1+\mu} \exp\left[\frac{1}{\pi}\int_0^{\pi/2} \frac{\phi \arctan(\mu \tan\phi)}{1-\phi\cot\phi} d\phi\right]. \tag{2}$$

Use the composite Simpson's rule to integrate the above equation and plot $H(\mu)$ as a function of μ for $0 \le \mu \le 1$.

3. Consider the data in Table 1 in the next page.

(a) Interpolating linearly the data, calculate the expected value of the function at $x = 3.2$, 0.4, -0.128, and -2.0.

(b) Interpolate the above data using 10-th order polynomials to calculated the expected values at the x positions given in (a).

(c) Now use cubic splines to interpolate the data and plot the results together with the data points. What are the expected values at the x positions given in (a)?

4. It has been well established that all galaxies contain supermassive black holes at their centers, and that the black hole mass $M_{\rm BH}$ is related to the stellar velocity dispersion σ_e in the bulge of its host galaxy such that $\log(M_{\rm BH}/1M_\odot) = a + b\log(\sigma_e/1{\rm km\ s}^{-1})$, with a and b being constants. The `BlackHall.txt` file in the class web page contains data for 67 galaxies: the first and second columns give $M_{\rm BH}$ and its measurement error $\Delta M_{\rm BH}$ in units of $M_\odot$, respectively; the third and fourth columns give σ_e and its error $\Delta\sigma_e$ in units of km s^{-1}, respectively.

(a) Ignore $\Delta\sigma_e$ and $\Delta M_{\rm BH}$, and fit the data to find a and b and their errors.

(b) Allowing for both $\Delta\sigma_e$ and $\Delta M_{\rm BH}$, fit the data to find a and b and their errors.

x	y
-2.1	0.012155
-1.45	0.122151
-1.3	0.184520
-0.2	0.960789
0.1	0.990050
0.15	0.977751
0.8	0.527292
1.1	0.298197
1.5	0.105399
2.8	3.936690×10^{-4}
3.8	5.355348×10^{-7}

Table 1: Data points for Problem 3

(c) Make a plot of the data with errorbars together with your fits in (a) and (b).

(d) Now fit the data as $\log(\sigma_e/1\text{km s}^{-1}) = c + d\log(M_{\rm BH}/1M_\odot)$ allowing for $\Delta\sigma_e$ and $\Delta M_{\rm BH}$ to find the fitting coefficients c and d. Discuss the relation between the coefficients in (b) and (d).

5. The `hw3p5.dat` file in the class web page contains two-column data: x and y.

(a) Fit the data using a Gaussian function

$$y = p_0 + p_1 \exp\left[-\frac{(x-p_2)^2}{2p_3^2}\right], \tag{3}$$

and find the fitting coefficients and their errors, as well as χ^2.

(b) Fit the data using a Lorenzian function

$$y = q_0 + \frac{q_1}{q_2 + (x-q_3)^2}, \tag{4}$$

and find the fitting coefficients and their errors, as well as χ^2.

(c) Make a plot of the data together with your fits in (a) and (b). Which one gives a better fit to the data between (a) and (b)?