Computational Astronomy '17 Spring
Mar. 27 DUE: Apr. 3 (Mon)

PROBLEM SET #2

For the problems below, you need to write programs using Python. Turn in your own source programs written independently together with a report in pdf format by email (both to the instructor at `wkim@astro.snu.ac.kr` and the TA at `moon@astro.snu.ac.kr`.)

1. Write a (short) program to find the machine epsilon of your computer, using Python. Run your program to obtain the outputs.

2. This problem is to show that you need to be careful to avoid unstable algorithms in which roundoff errors can increase exponentially. The "golden mean", ϕ, is given by $\phi = (\sqrt{5} - 1)/2 \simeq 0.61803398875\cdots$.

(a) Write a Python program to calculate the n-th power of ϕ, using successive multiplications

$$\phi^0 = 1, \quad \text{and} \quad \phi^n = \phi \cdot \phi^{n-1} \text{ for } n = 1, 2, 3, \cdots , \tag{1}$$

and plot ϕ^n as a function of n for $0 \leq n \leq 50$. (The ordinate should be in logarithmic scale.)

(b) Another (clever) way to calculate ϕ^n is to use following recursion relation

$$\phi^{n+1} = \phi^{n-1} - \phi^n \text{ for } n = 1, 2, 3, \cdots . \tag{2}$$

Show that Equation (2) is equivalent to Equation (1). Use Equation (2) to calculate ϕ^n for $0 \leq n \leq 50$. Compare the results with those in part (a) by overploting all the results in the same Figure.

(c) Why do you think are the results in parts (a) and (b) so different for high n? (Hint: there is another solution of Equation (2) whose magnitude is greater than unity.)

3. Evaluate

$$S_n = \sum_{j=1}^{n} \frac{1}{j(j+1)}, \tag{3}$$

using your computer for arbitrary 10^2, 10^4, and 10^6 by using the method below. Comment on the answers obtained.

(a) By summing the terms from the largest terms first to the smallest terms last.

(b) By summing the terms from the smallest terms first to the largest terms last.

(c) Using the Kahan summation formula.

4. The Planck function (measured per unit wavelength) from a blackbody with temperature T is given by

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}, \tag{4}$$

where $h = 6.626 \times 10^{-34}$ J s, $k = 1.381 \times 10^{-23}$ J K^{-1}, and $c = 2.998 \times 10^8$ m s^{-1}. All of your answers should be accurate to at least four digits.

(a) Derive Wien's displacement law by solving $dB_\lambda/d\lambda = 0$.

(b) For a blackbody with $T = 10^4$ K, find two wavelengths corresponding to $B_\lambda = 10^{13}$ J s^{-1} m^{-3}.

5. A planet is orbiting around the Sun in a Kepler orbit with semi-major axis a, semi-minor axis b, and eccentricity $e = \sqrt{1 - b^2/a^2}$. The location of the planet in the (x, y) plane is given by

$$x = a \cos E, \tag{5}$$
$$y = b \sin E, \tag{6}$$

with the *eccentric anomaly* E defined as

$$E \equiv 2\pi t/P + e \sin E, \tag{7}$$

where t and P denote the time elapsed from the perihelion and the orbital period of the planet, respectively.

(a) The Earth has $P = 365.25635$ days, $a = 1.496 \times 10^8$ km, and $e = 0.0167$. Compute E, x, y for $t = 182$ days and $t = 273$ days, using the (i) bisection, (ii) Newton, and (iii) fixed-iteration methods. The fractional error in E at the end of your computation (from one iteration to the next) should be less than 10^{-10}. How many iterations does your method need, i.e., how quickly does it converge?

(b) Repeat the calculations by assuming that the eccentricity of the Earth is changed to $e = 0.99999$, while everything else remains unchanged.