COMPUTATIONAL ASTRONOMY '17 SPRING
MAY 31 DUE: JUNE 7 (WED)

PROBLEM SET #6

For the problems below, you need to write programs using Python. Turn in your own source programs written independently together with a report in pdf format by email (both to the instructor at wkim@astro.snu.ac.kr and the TA at moon@astro.snu.ac.kr.)

1. Write a program to find a minimum of the following function

$$f(x,y) = 2x^2 + y^2 - 2xy + |x-3| + |y-2|, \tag{1}$$

using each of the following methods:

(a) Newton
(b) Steepest descent
(c) Powell
(d) Conjugate gradient

You should try each method from each of the three starting points $(-1,1)$, $(0,0)$, and $(2,0)$. Take $\epsilon = 10^{-6}$ for the tolerance value. Plot the paths from the three starting points in a single figure for each method. Give the number of iterations to arrive at the minimum point.

2. Start from $(x,y,z) = (0,2,1)$ to minimize the following banana function in three dimensions

$$f(x,y,z) = 100(y-x^2)^2 + (1-x)^2 + 100(z-y^2)^2 + (1-z)^2, \tag{2}$$

using

(a) Powell method, and
(b) Conjugate gradient method.

Where is the minimum point? Give the number of iterations to arrive at the minimum point for $\epsilon = 10^{-6}$.

3. The `sol_vel.dat` file in the class web page contains three-column ascii data for line-of-sight velocities of gas in a quiet region of the Sun. The first column is time, t, in units of minutes, while the second and third columns denote the velocities, $v_{\mathrm{H}\alpha}$ and v_{CaII}, measured from Hα lines and Ca II lines, respectively, in units of km s^{-1}.

(a) Perform Fourier transforms of $v_{\mathrm{H}\alpha}$ and v_{CaII} and plot the power spectra as functions of frequency f. Make sure to indicate the units of f in the plots.
(b) From the results of (a), what are the oscillation periods of $v_{\mathrm{H}\alpha}$ and v_{CaII}?

(c) The cross-correlation of $v_{\mathrm{H}\alpha}$ and v_{CaII} is defined by

$$\mathrm{Corr}(t) = \int_{-\infty}^{\infty} v_{\mathrm{H}\alpha}(\tau + t) v_{\mathrm{CaII}}(\tau) d\tau. \tag{3}$$

Plot Corr(t) as a function of t (*Hint*: Corr = `iFFT[FFT(`$v_{\mathrm{H}\alpha}$`)×FFT(`v_{CaII}`)]`).

4. Download `M51_hw.jpg` from the class web page, and read it using `plt.imread` into RGB channels.

(a) Perform FFT of each of the RGB channels and plot the respective angle-averaged power spectra in a single figure.

(b) Cut the power of the high-frequency modes with $k = \sqrt{k_x^2 + k_y^2} > 30$ to zero, and plot the resulting image after inverse FFT.

(c) Cut the power of the low-frequency modes with $k \leq 30$ to zero, and plot the resulting image after inverse FFT.