Universidad de Guadalajara
Centro Universitario de los Valles

Ingeniería en Electrónica y Computación

Reporte del proyecto:

Tarea 2

Presentado por:

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Profesor

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1. Encuentre la transformada de Laplace de las siguientes funciones

$$f(t) = \begin{cases} e^{at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

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$$\mathcal{L}\{F(t)\} = \int_0^\infty e^{at} e^{-st}\, dt = \int_0^\infty e^{at} e^{-st}\, dt =$$

$$\int_0^\infty e^{-t(s-a)}\, dt = \int_0^\infty e^{-s't}\, dt \qquad s' = (s-a)$$

$$= \int_0^\infty e^{-s't}\, dt = \frac{1}{s'} = \boxed{\frac{1}{s-a}}$$

$$f(t) = \begin{cases} \cos\omega t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$F(t) = \begin{cases} \cos \omega t & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad \cos \omega t = \frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right)$$

$$\mathcal{L}\{\cos \omega t\} = \frac{1}{2}\int_0^\infty e^{j\omega t} e^{-st}\,dt + \frac{1}{2}\int_0^\infty e^{-j\omega t} e^{-st}\,dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t(s-j\omega)}\,dt + \frac{1}{2}\int_0^\infty e^{-t(s+j\omega)}\,dt$$

$$s' = s - j\omega$$
$$s'' = s + j\omega$$

$$= \frac{1}{2}\int_0^\infty e^{-s't}\,dt + \frac{1}{2}\int_0^\infty e^{-s''t}\,dt$$

$$\frac{1}{2}\cdot\frac{1}{s'} + \frac{1}{2}\cdot\frac{1}{s''} = \frac{1}{2}\left(\frac{1}{s'} + \frac{1}{s''}\right) =$$

$$\frac{1}{2}\left(\frac{1}{-j\omega + s} + \frac{1}{j\omega + s}\right) = \frac{1}{2}\left(\frac{j\omega + s + (-j\omega + s)}{-j^2\omega^2 - sj\omega + s^2 + sj\omega}\right) =$$

$$\cancel{\frac{1}{2}}\left(\frac{\cancel{2}s}{-j^2\omega^2 + s^2 + \cancel{s - s}\,0}\right) = \boxed{\frac{s}{\omega^2 + s^2}}$$

$$j^2 = -1$$
$$(-1)(-1) = 1$$

$$f(t) = \begin{cases} e^{-at}\cos\omega t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$F(t) \begin{cases} e^{-at}\cos wt & t \geq 0 \\ 0 & t < 0 \end{cases} =$$

$$\mathcal{L}\{F(t)\} = \int_0^\infty e^{-at} \cdot \cos wt \cdot e^{-st}\, dt =$$

$$\int_0^\infty e^{-at} \cdot e^{-st} \left(\frac{e^{jwt} + e^{-jwt}}{2} \right) dt =$$

$$\frac{1}{2} \int_0^\infty e^{-at} \cdot e^{-st} \left(e^{jwt} + e^{-jwt} \right) dt =$$

$$\frac{1}{2} \left[\int_0^\infty e^{-at} \cdot e^{-st} \cdot e^{jwt}\, dt + \int_0^\infty e^{-at} \cdot e^{-st} \cdot e^{-jwt}\, dt \right] =$$

$$\frac{1}{2}\left[\int_0^\infty e^{-(a+s-j\omega)t}\,dt + \int_0^\infty e^{-(a+s+j\omega)t}\,dt\right] =$$

$$\frac{1}{2}\left[\int_0^\infty e^{-s't}\,dt + \int_0^\infty e^{-s''t}\,dt\right] = \qquad \boxed{\begin{aligned} s' &= a+s-j\omega \\ s'' &= a+s+j\omega \end{aligned}}$$

$$\frac{1}{2}\left[\frac{1}{s'} + \frac{1}{s''}\right] = \frac{1}{2}\left[\frac{1}{a+s-j\omega} + \frac{1}{a+s+j\omega}\right] =$$

$$\frac{1}{2}\left(\frac{a+s\cancel{+j\omega}+a+s\cancel{-j\omega}}{(a+s-j\omega)(a+s+j\omega)}\right) = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}a+\cancel{2}s}{(a+s)^2+\omega^2}\right) =$$

$$\frac{a+s}{(a^2+s^2+2as+\omega^2)} = \boxed{\frac{a+s}{(a+s)^2+\omega^2}}$$

2. Hallar la trasnformada inversa de Laplace de las siguientes funciones de transferencia

$$y(s) = \frac{s+2}{s^3+5s^2+2s-8} \tag{4}$$

2.

$$y(s) = \frac{s+2}{s^3+5s^2+2s-8} = \frac{s+2}{(s+2)(s-1)(s+4)}$$

$$= \frac{A}{s+2} + \frac{B}{s+1} + \frac{C}{s+4} =$$

$$\frac{A(s-1)(s+4)+B(s+2)(s+4)+C(s+2)(s+1)}{(s+2)(s-1)(s+4)} =$$

$$As^2+3sA-4A+Bs^2+6sB+8B+Cs^2+3sC+2C$$

$$-4A+8B+2C=2$$

$$3A+6B+3C=1$$

$$A+B+C=0$$

$$A = -\frac{1-4B-C}{2}$$

$$3\left(-\frac{1-4B-C}{2}\right)+6B+3C=1$$

$$\frac{24B+9C-3}{2}=1$$

$$B \Rightarrow \frac{24B+9C-3}{2}=1 \Rightarrow B=\frac{-9C+5}{24}$$

$$\left(-\frac{1-4B-C}{2}\right)+\left(\frac{-9C+5}{24}\right)+C=0 \Rightarrow \frac{3C+1}{8}=0 \Rightarrow$$

$$C=-\frac{1}{3}$$

$$B = \frac{-9\left(-\frac{1}{3}\right)+5}{24} \Rightarrow \boxed{B = \frac{1}{3}}$$

$$A = -\frac{1 - 4 \cdot \frac{1}{3} - \left(-\frac{1}{3}\right)}{2} \Rightarrow \boxed{A = 0}$$

$A = 0;\ B = 1/3;\ C = -1/3$

$$\frac{0}{s+2} + \frac{1/3}{s-1} + \frac{-1/3}{s+4} =$$

$$\mathcal{L}^{-1}\left\{\frac{0}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{1/3}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{-1/3}{s+4}\right\}$$

$$\cancel{\mathcal{L}^{-1}\{0\}} + \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\}$$

$$= \frac{e^{t}}{3} - \frac{e^{-4t}}{3} = \frac{e^{t} - e^{-4t}}{3}$$

$$y(t) = \frac{e^{t} - e^{-4t}}{3}$$

$$y(s) = \frac{s-1}{s^3 + 7s^2 + 11s + 5}$$

$$y(s) = \frac{s-1}{s^3+7s^2+11s+5} = \frac{s-1}{(s+1)^2(s+5)} =$$

$$\frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+5} =$$

$$\frac{A(s+1)(s+5) + B(s+5) + C(s+1)^2}{(s+1)^2(s+5)} =$$

$$A(s^2+6s+5) + B(s+5) + C(s^2+2s+1) =$$

$$As^2 + 6sA + 5A + Bs + 5B + Cs^2 + 2sC + C$$

$$5A + 5B + C = -1$$

$$6A + B + 2C = 1$$

$$A + C = 0$$

$$A + C = 0 \Rightarrow A = -C$$

$$5(-C) + 5B + C = -1 \Rightarrow -4C + 5B = -1$$

$$6(-C) + B + 2C = 1 \Rightarrow -4C + B = 1$$

$$C = -\frac{-1-5B}{4} \Rightarrow -4\left(-\frac{-1-5B}{4}\right) + B = 1) \Rightarrow$$

$$-1 - 4B = 1 \Rightarrow \boxed{B = -1/2}$$

$$C = -\frac{-1-5(-1/2)}{4} \Rightarrow -3/8 = C$$

$$A = -C \Rightarrow A = -(-3/8) \Rightarrow A = 3/8$$

$$A = 3/8;\ B = -1/2;\ C = -3/8$$

$$\frac{3/8}{s+1} + \frac{(-1/2)}{(s+1)^2} + \frac{(-3/8)}{s+5} =$$

$$\mathcal{L}^{-1}\left\{\frac{3/8}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{-1/2}{(s+1)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{-3/8}{s+5}\right\} =$$

$$\frac{3}{8}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} - \frac{3}{8}\mathcal{L}^{-1}\left\{\frac{1}{s+5}\right\} =$$

$$y(t) = \frac{3}{8}e^{-t} - \frac{1}{2}e^{-t}t - \frac{3}{8}e^{-5t}$$

3. Hallar la solución de las siguientes ecuaciones diferenciales

$$\frac{d^2y(t)}{dt^2} + 14\frac{dy(t)}{dt} + 100y(t) = 100 \tag{6}$$

3.

$$\frac{d^2y(t)}{dt^2} + 14\frac{dy(t)}{dt} + 100y(t) = 100$$

$$\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\} + 14\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} + 100\mathcal{L}\left\{y(t)\right\} = \mathcal{L}\left\{100\right\} \Rightarrow$$

$$s^2Y(s) + 14sY(s) + 100Y(s) = \frac{100}{s}$$

$$Y(s)(s^2+14s+100) = \frac{100}{s} \Rightarrow$$

$$\boxed{Y(s) = \frac{100}{s(s^2+14s+100)}}$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+14s+100} \Rightarrow 100 = A(s^2+14s+100) + s(Bs+C)$$

$$\Rightarrow (As^2+14As+100A) + (Bs^2+sC) = 100$$

$$\Rightarrow A+B=0 \Rightarrow 1+B=0 \Rightarrow B=-1$$

$$14A+C=0 \Rightarrow 14(1)+C=0 \Rightarrow C=-14$$

$$100A=100 \Rightarrow A=1$$

$$(s+a)^2 = s^2 + 2as + a^2$$

$$s^2 \Rightarrow 1 = 1$$
$$s \Rightarrow 2a = 14$$
$$CTE \Rightarrow a^2 = ?$$

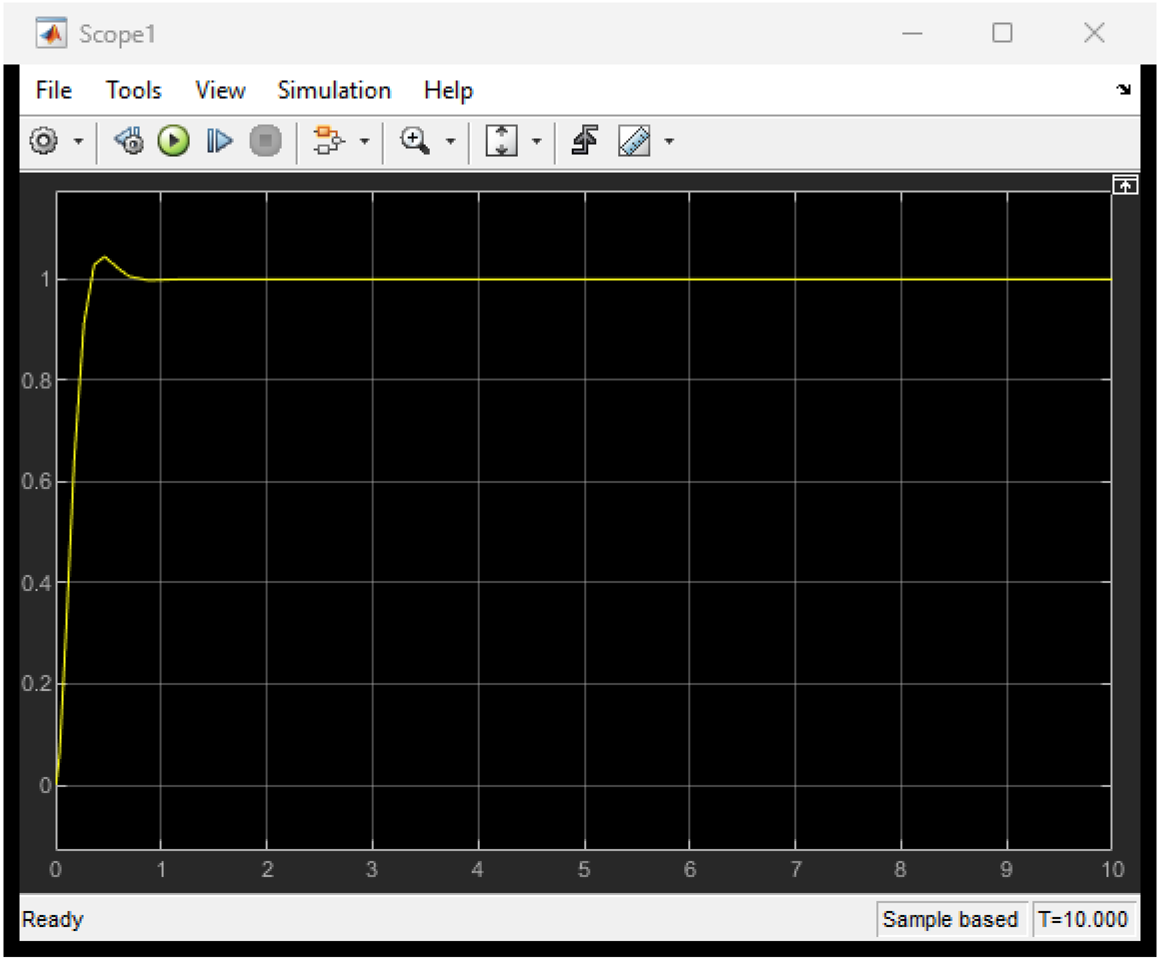
$$Si\ 2a = 14 \Rightarrow a = 7 \Rightarrow a^2 = 49$$

$$Y(s) = \frac{1}{s} - \frac{s+14}{s^2+14s+100} \Rightarrow$$

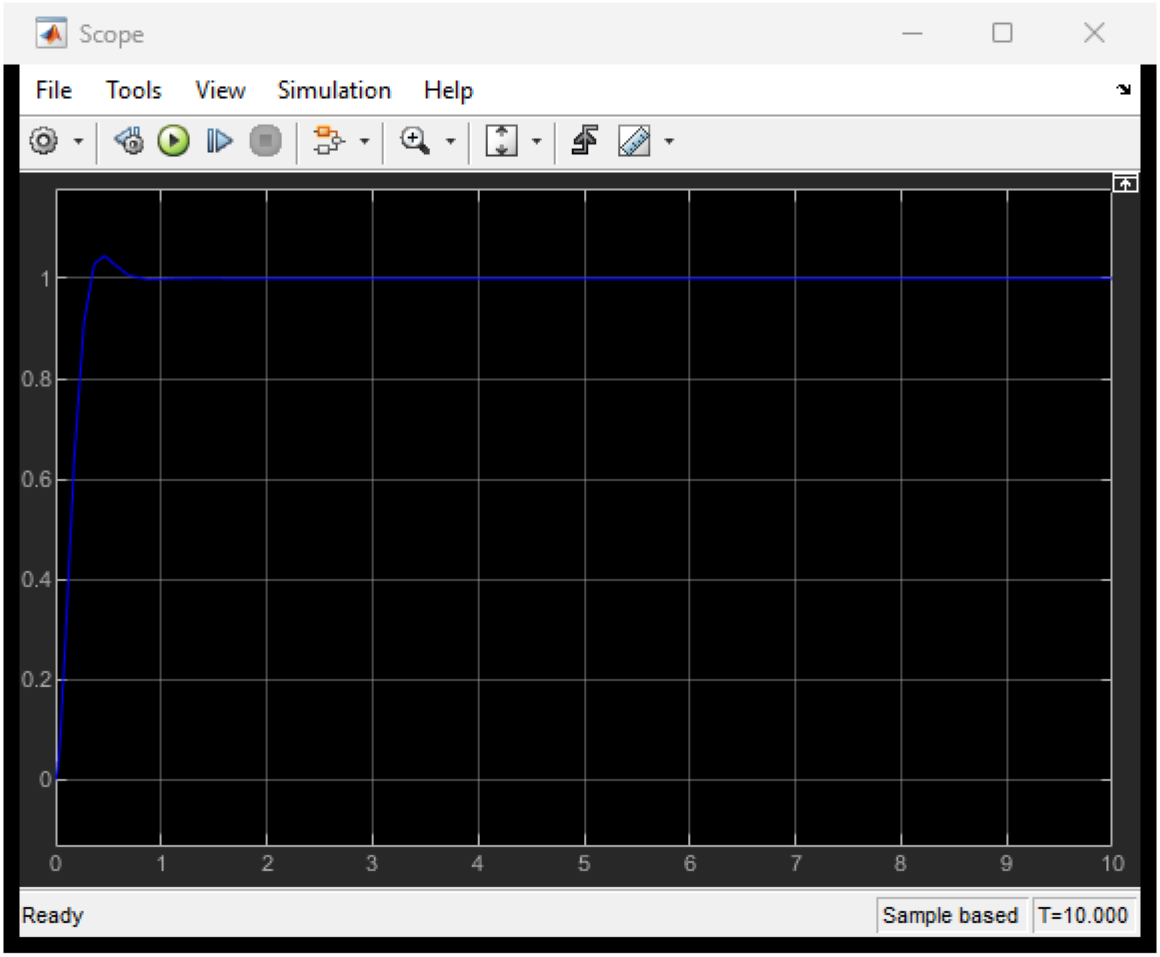
$$Y(s) = \frac{1}{s} - \frac{s+14}{s^2+14s+49-49+100} \Rightarrow$$

$$Y(s) = \frac{1}{s} - \frac{s+14}{(s+7)^2+51}$$

$$Y(s) = \frac{1}{s} - \frac{s+7}{(s+7)^2+51} - \frac{7}{(s+7)^2+51}$$



Gráfica de función de transferencia



Gráfica de ecuación diferencial

$$\frac{d^2y(t)}{dt^2} + 16\frac{dy(t)}{dt} + 64y(t) = 1 \tag{7}$$

$$\frac{d^2y(t)}{dt^2} + 16\frac{dy(t)}{dt} + 64y(t) = 1$$

$$\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\} + 16\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} + 64\mathcal{L}\{y(t)\} = \mathcal{L}\{1\} \Rightarrow$$

$$s^2Y(s) + 16sY(s) + 64Y(s) = \frac{1}{s} \Rightarrow$$

$$Y(s)(s^2+16s+64) = \frac{1}{s} \Rightarrow$$

$$Y(s) = \frac{1}{s(s^2+16s+64)} \Rightarrow Y(s) = \frac{1}{s(s+8)(s+8)}$$

$$\frac{A}{s} + \frac{B}{s+8} + \frac{C}{(s+8)^2} = \frac{As^2+16As+64A+s^2B+8sB+sC}{s(s+8)^2}$$

$$A+B=0 \Rightarrow B = -1/64$$

$$16A+8B+C=0 \Rightarrow 16(1/64)+8(-1/64)+C=0 \Rightarrow$$

$$64A=1 \Rightarrow A = 1/64$$

$$1/4 + (-1/8) + C = 0 \Rightarrow C = -1/8$$

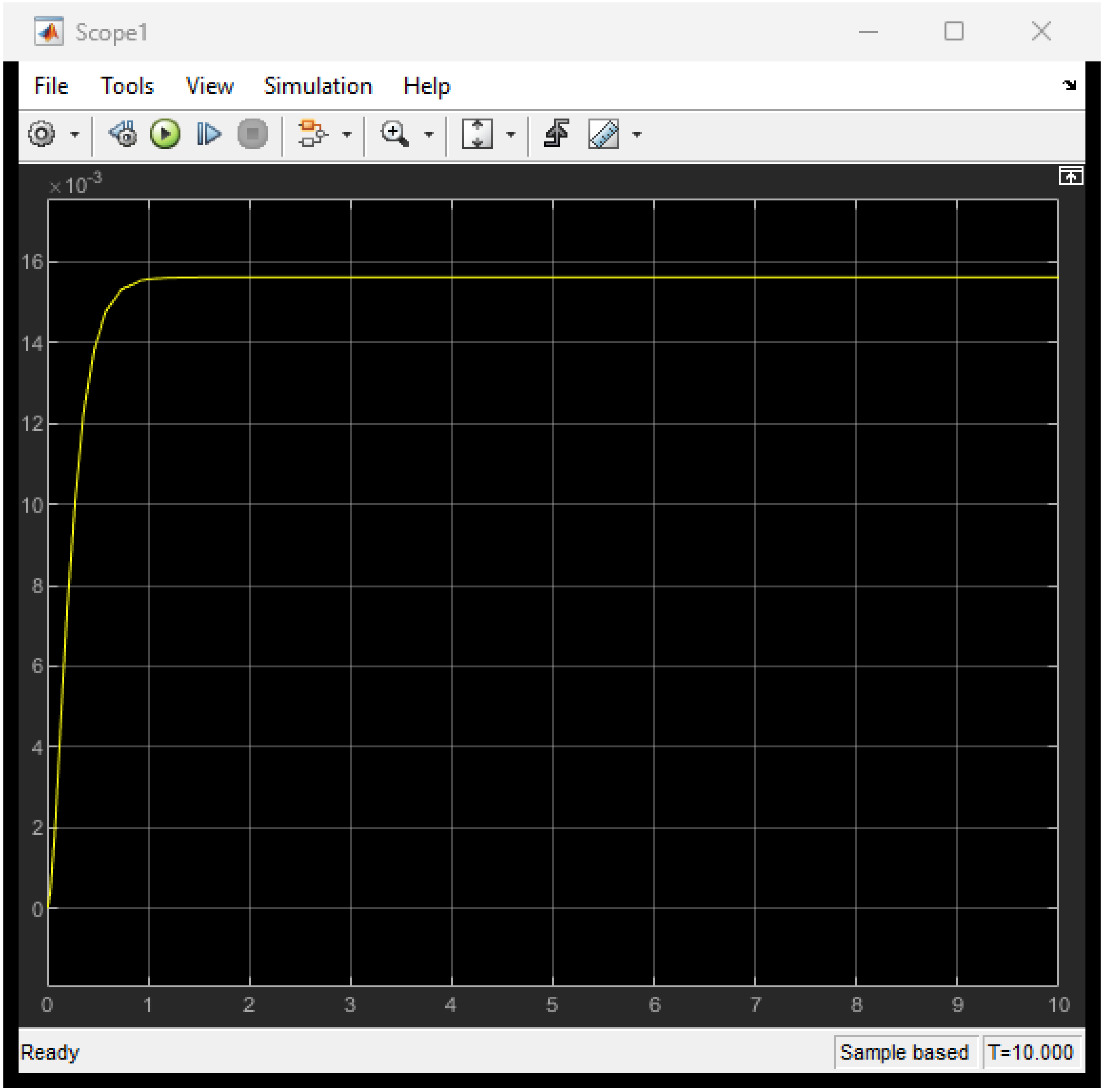
$A = 1/64;\ B = -1/64;\ C = -1/8$

$$\frac{1/64}{s} - \frac{1/64}{s+8} - \frac{1/8}{(s+8)^2} \Rightarrow$$

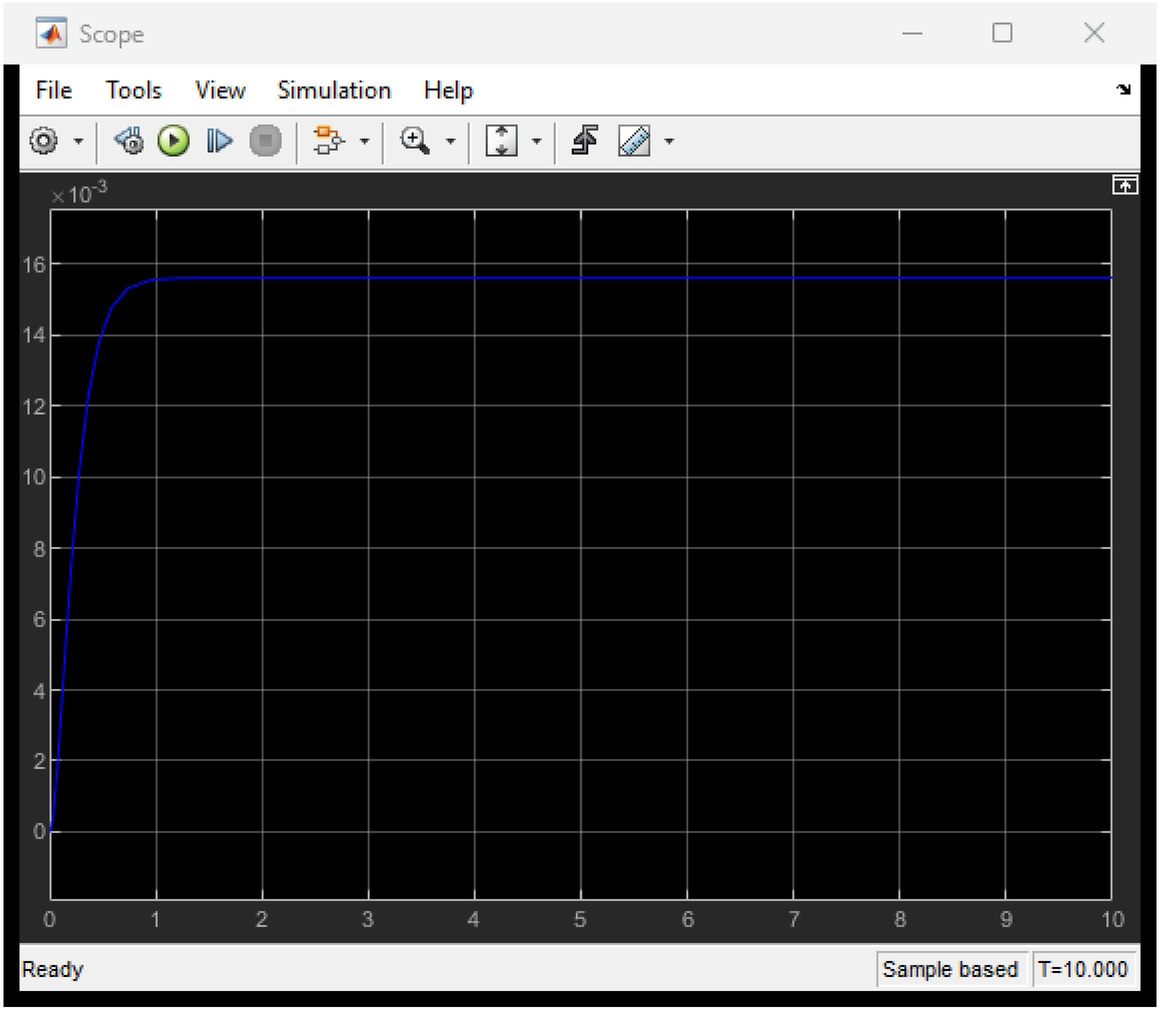
$$\mathcal{L}^{-1}\left\{\frac{1/64}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1/64}{s+8}\right\} - \mathcal{L}^{-1}\left\{\frac{1/8}{(s+8)^2}\right\} \Rightarrow$$

$$\frac{1}{64}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{64}\mathcal{L}^{-1}\left\{\frac{1}{s+8}\right\} - \frac{1}{8}\mathcal{L}^{-1}\left\{\frac{1}{(s+8)^2}\right\} \Rightarrow$$

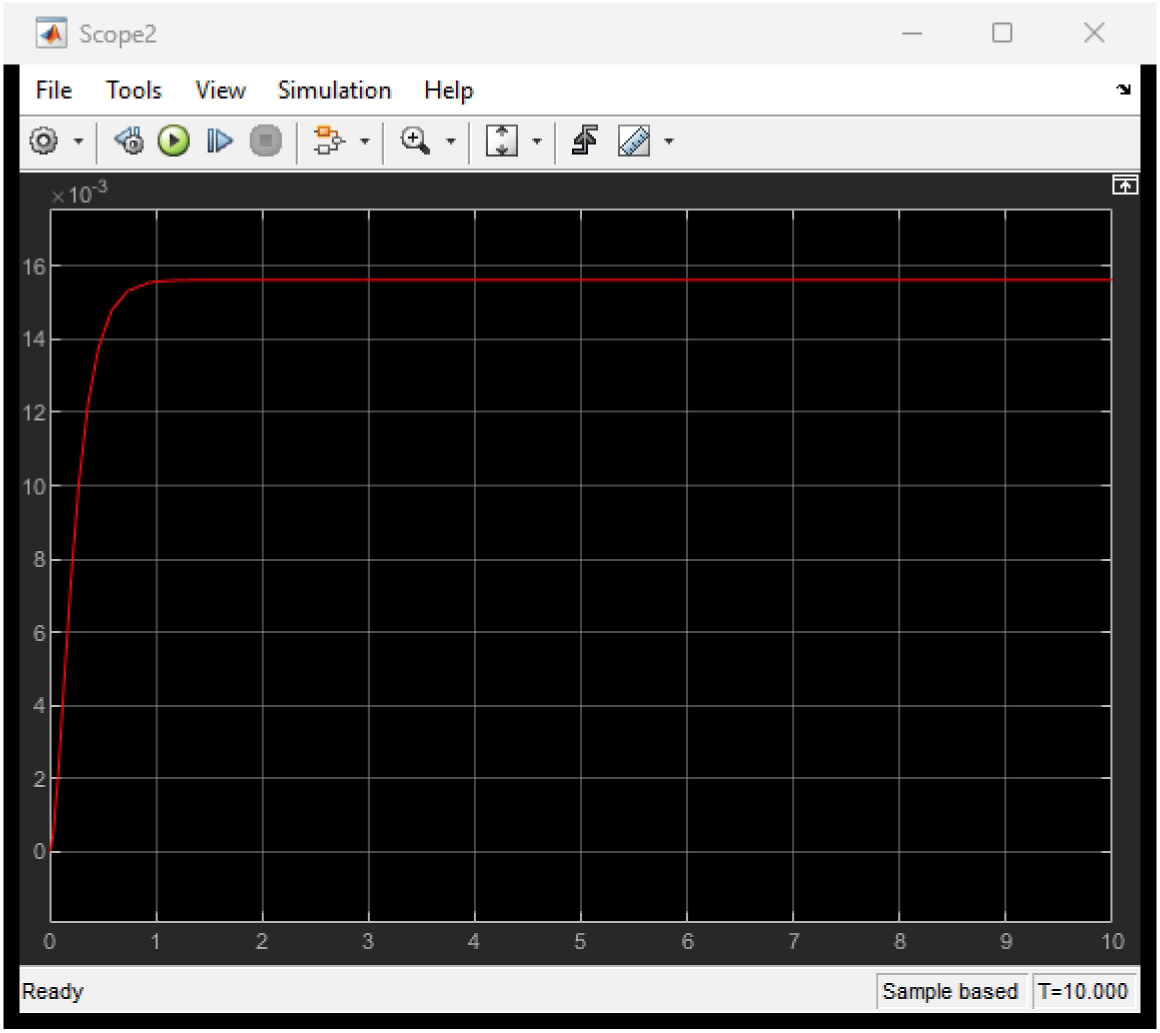
$$\frac{1}{64} - \frac{e^{-8t}}{64} - \frac{e^{-8t}t}{8}$$



Gráfica de función de transferencia



Gráfica de ecuación diferencial



Gráfica del función del tiempo

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 12y(t) = 3 \tag{8}$$

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 12y(t) = 3$$

$$\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\} + 7\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} + 12\mathcal{L}\left\{y(t)\right\} =$$

$$\mathcal{L}\{3\} \Rightarrow$$

$$s^2 Y(s) + 7s\,Y(s) + 12\,Y(s) = 3/s \Rightarrow$$

$$Y(s)(s^2+7s+12) = 3/s \Rightarrow$$

$$Y(s) = \frac{3}{s(s^2+7s+12)} \Rightarrow Y(s) = \frac{3}{s(s+3)(s+4)}$$

$$\frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4} = \frac{A(s+3)(s+4) + B(s)(s+4) + C(s)(s+3)}{s(s+3)(s+4)}$$

$$= \frac{A(s^2+7s+12) + B(s^2+4s) + C(s^2+3s)}{s(s+3)(s+4)} =$$

$$As^2 + 7As + 12A + Bs^2 + 4Bs + Cs^2 + 3Cs \qquad B = -1$$

$$A + B + C = 0 \Rightarrow 1/4 + B + C = 0 \Rightarrow B = -C - 1/4$$

$$7A + 4B + 3C = 0 \Rightarrow 7(1/4) + 4(-C - 1/4) + 3C = 0$$

$$12A = 3 \Rightarrow A = 1/4$$

$$7/4 - 4C - 1 + 3C = 0 \Rightarrow -C = -3/4 \Rightarrow C = 3/4$$

$A = 1/4$; $B = -1$; $C = 3/4$.

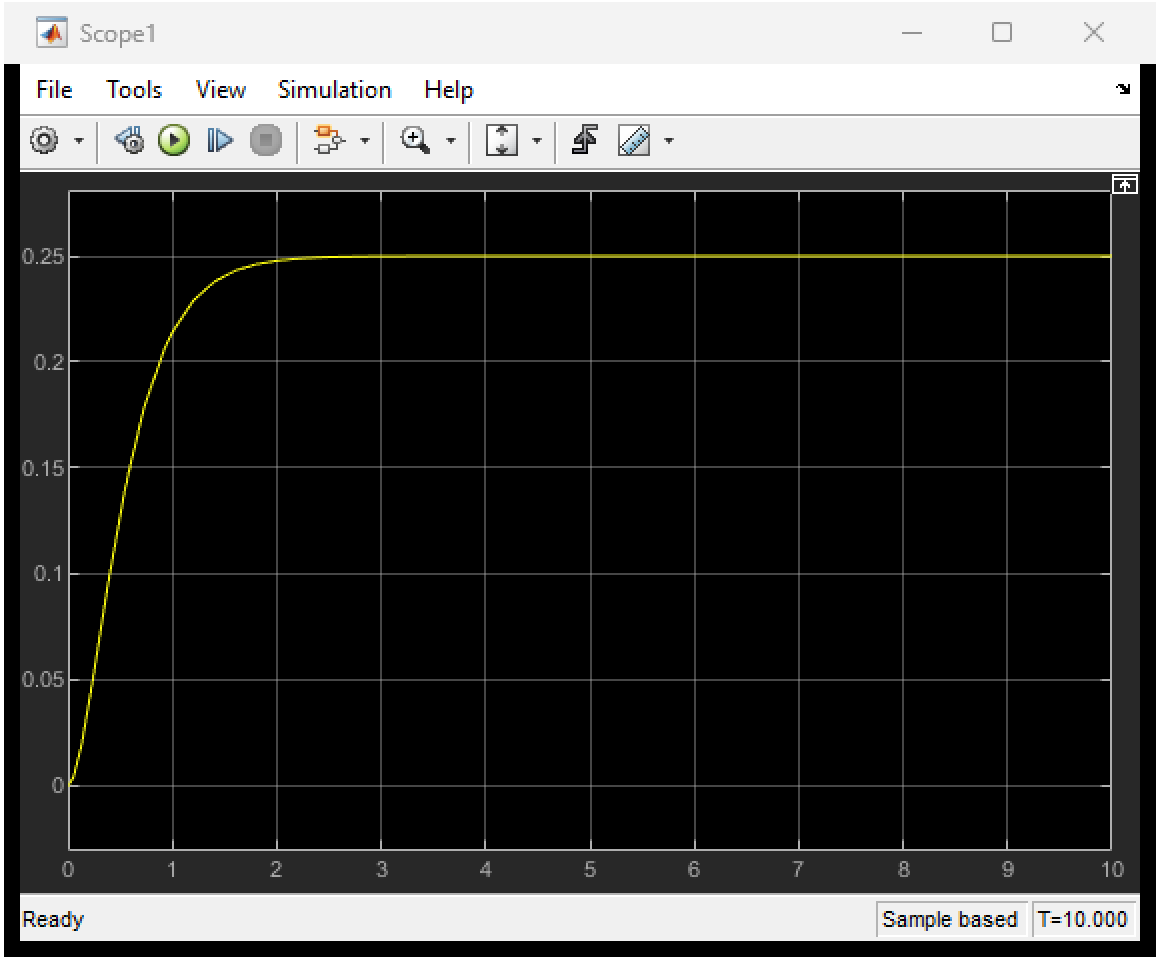
$$\frac{1/4}{s} + \frac{-1}{s+3} + \frac{3/4}{s+4} \Rightarrow \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \left(-1\,\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}\right) +$$

$$\frac{3}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\} \Rightarrow$$

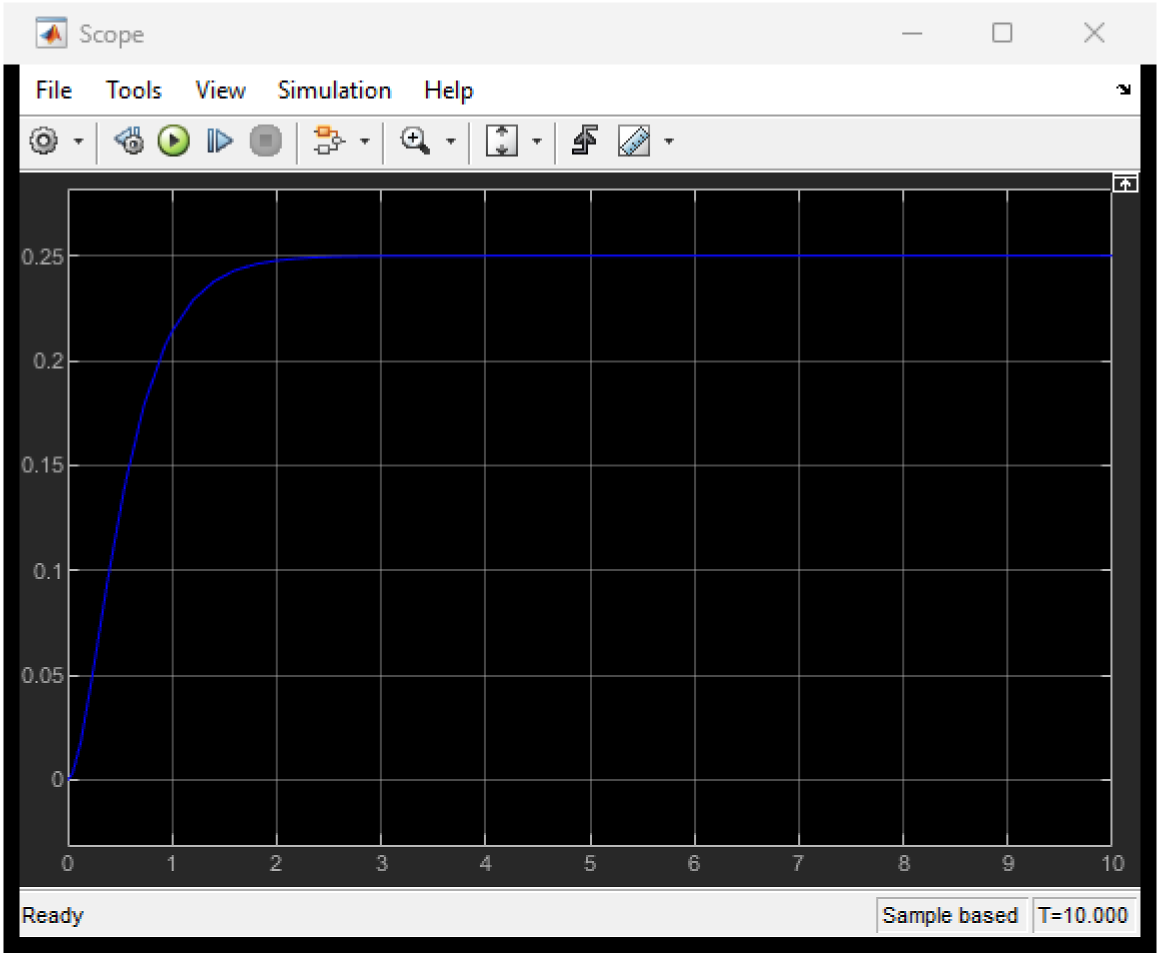
$$\frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + \frac{3}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\} \Rightarrow$$

$$\frac{1}{4} - e^{-3t} + \frac{3}{4} e^{-4t} \Rightarrow$$

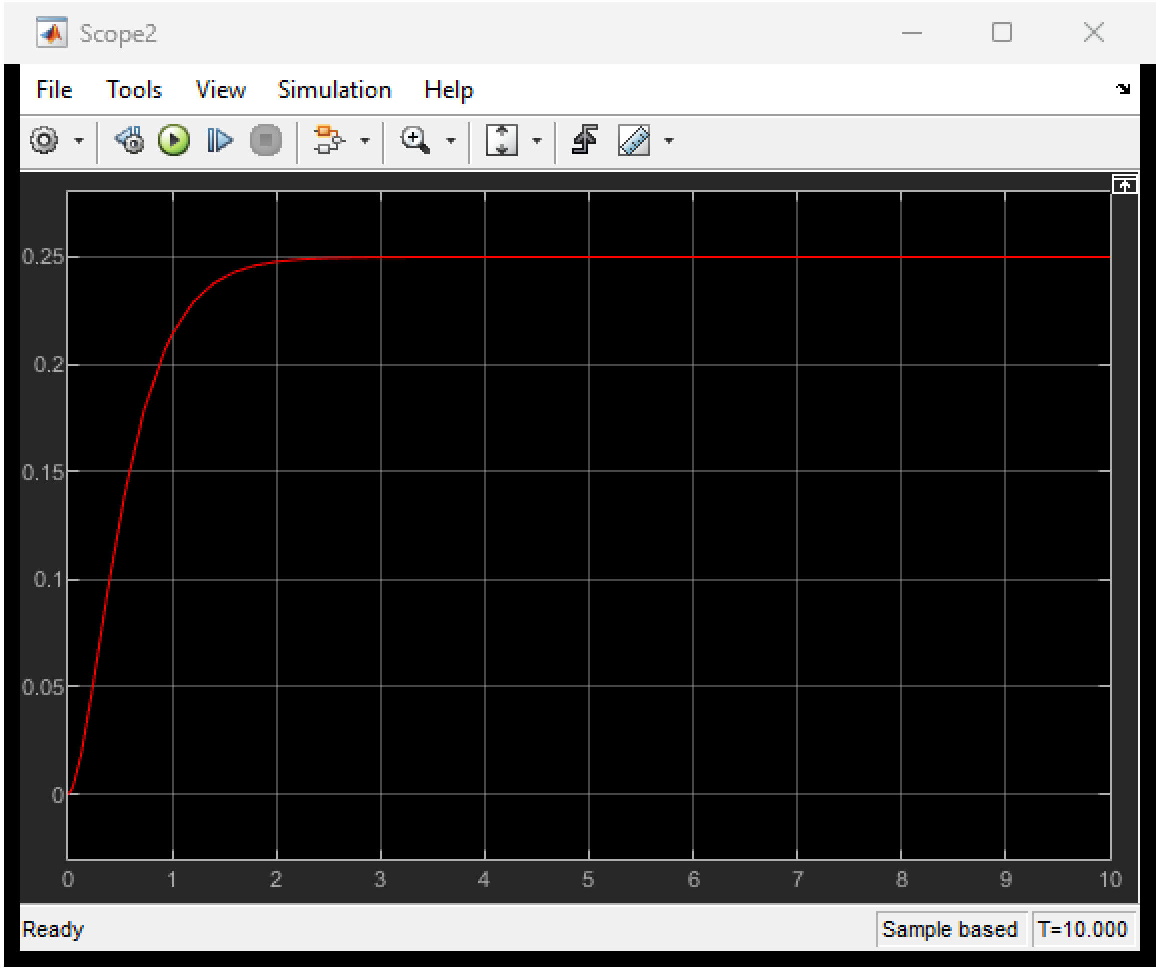
$$y(t) = \frac{1}{4} - e^{-3t} + \frac{3}{4} e^{-4t}$$



Gráfica de función de transferencia



Gráfica de ecuación diferencial



Gráfica del función del tiempo