

Universidad de Guadalajara

Centro Universitario de los Valles



Ingeniería en Electrónica y Computación

Reporte del proyecto:

Tarea 2

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1. Encuentre la transformada de Laplace de las siguientes funciones

$$f(t) = \begin{cases} e^{at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$\begin{aligned} F(t) &= \begin{cases} e^{at} & t \geq 0 \\ 0 & t < 0 \end{cases} \\ \mathcal{L}\{F(t)\} &= \int_0^{\infty} e^{at} e^{-st} dt = \int_0^{\infty} e^{at} e^{-st} dt = \\ &= \int_0^{\infty} e^{-t(s-a)} dt = \int_0^{\infty} e^{-s't} dt \quad s' = (s-a) \\ &= \int_0^{\infty} e^{-s't} dt = \frac{1}{s'} = \boxed{\frac{1}{s-a}} \end{aligned}$$

$$f(t) = \begin{cases} \cos \omega t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$f(t) = \begin{cases} \cos \omega t & t \geq 0 \\ 0 & t < 0 \end{cases} \quad \cos \omega t = \frac{1}{2} (e^{j\omega t} + e^{-j\omega t})$$

$$\begin{aligned} \mathcal{L}\{\cos \omega t\} &= \frac{1}{2} \int_0^{\infty} e^{j\omega t} e^{-st} dt + \frac{1}{2} \int_0^{\infty} e^{-j\omega t} e^{-st} dt \\ &= \frac{1}{2} \int_0^{\infty} e^{-t(s-j\omega)} dt + \frac{1}{2} \int_0^{\infty} e^{-t(s+j\omega)} dt \\ &= \frac{1}{2} \int_0^{\infty} e^{-s't} dt + \frac{1}{2} \int_0^{\infty} e^{-s''t} dt \end{aligned} \quad \begin{cases} s' = s - j\omega \\ s'' = s + j\omega \end{cases}$$

$$\frac{1}{2} \cdot \frac{1}{s'} + \frac{1}{2} \cdot \frac{1}{s''} = \frac{1}{2} \left(\frac{1}{s'} + \frac{1}{s''} \right) =$$

$$\frac{1}{2} \left(\frac{1}{-j\omega + s} + \frac{1}{j\omega + s} \right) = \frac{1}{2} \left(\frac{j\omega + s + (-j\omega + s)}{-j^2\omega^2 - s_j\omega + s^2 + s_j\omega} \right) =$$

$$\frac{1}{2} \left(\frac{2s}{-j^2\omega^2 + s^2 + s - s} \right) = \frac{s}{\omega^2 + s^2} \quad \begin{cases} j^2 = -1 \\ (-1)(-1) = 1 \end{cases}$$

$$f(t) = \begin{cases} e^{-at} \cos \omega t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$F(t) \begin{cases} e^{-at} \cos \omega t & t \geq 0 \\ 0 & t < 0 \end{cases} =$$

$$\mathcal{L}\{F(t)\} = \int_0^{\infty} e^{-at} \cos \omega t \cdot e^{-st} dt =$$

$$\int_0^{\infty} e^{-at} \cdot e^{-st} \left(\frac{e^{j\omega t} + e^{-j\omega t}}{2} \right) dt =$$

$$\frac{1}{2} \int_0^{\infty} e^{-at} \cdot e^{-st} (e^{j\omega t} + e^{-j\omega t}) dt =$$

$$\frac{1}{2} \left[\int_0^{\infty} e^{-at} \cdot e^{-st} \cdot e^{j\omega t} dt + \int_0^{\infty} e^{-at} \cdot e^{-st} \cdot e^{-j\omega t} dt \right] =$$

$$\frac{1}{2} \left[\int_0^{\infty} e^{-(a+s-j\omega)t} dt + \int_0^{\infty} e^{-(a+s+j\omega)t} dt \right] =$$

$$\frac{1}{2} \left[\int_0^{\infty} e^{-s' t} dt + \int_0^{\infty} e^{-s'' t} dt \right] = \begin{cases} s' = a+s-j\omega \\ s'' = a+s+j\omega \end{cases}$$

$$\frac{1}{2} \left[\frac{1}{s'} + \frac{1}{s''} \right] = \frac{1}{2} \left[\frac{1}{a+s-j\omega} + \frac{1}{a+s+j\omega} \right] =$$

$$\frac{1}{2} \left(\frac{a+s+j\omega + a+s-j\omega}{(a+s-j\omega)(a+s+j\omega)} \right) = \frac{1}{2} \left(\frac{2a+2s}{(a+s)^2 + \omega^2} \right) =$$

$$\frac{a+s}{(a^2+s^2+2as+\omega^2)} = \frac{a+s}{(a+s)^2 + \omega^2}$$

2. Hallar la transformada inversa de Laplace de las siguientes funciones de transferencia

$$y(s) = \frac{s+2}{s^3 + 5s^2 + 2s - 8} \quad (4)$$

$$2. \quad y(s) = \frac{s+2}{s^3 + 5s^2 + 2s - 8} = \frac{s+2}{(s+2)(s-1)(s+4)}$$

$$= \frac{A}{s+2} + \frac{B}{s-1} + \frac{C}{s+4} =$$

$$\frac{A(s-1)(s+4) + B(s+2)(s+4) + C(s+2)(s+1)}{(s+2)(s-1)(s+4)} =$$

$$As^2 + 3sA - 4A + Bs^2 + 6sB + 8B + Cs^2 + 3sC + 2C$$

$$-4A + 8B + 2C = 2$$

$$3A + 6B + 3C = 1$$

$$A + B + C = 0$$

$$A = -\frac{1-4B-C}{2}$$

$$3\left(-\frac{1-4B-C}{2}\right) + 6B + 3C = 1$$

$$\frac{24B + 9C - 3}{2} = 1$$

$$B \Rightarrow \frac{24B + 9C - 3}{2} = 1 \Rightarrow B = \frac{-9C + 5}{24}$$

$$\left(-\frac{1-4B-C}{2}\right) + \left(\frac{-9C+5}{24}\right) + C = 0 \Rightarrow \frac{3C+1}{8} = 0 \Rightarrow$$

$$C = -\frac{1}{3}$$

$$B = \frac{-9(-\frac{1}{3}) + 5}{24} \Rightarrow B = \frac{1}{3}$$

$$A = \frac{-1 - 4 \cdot \frac{1}{3} - (-\frac{1}{3})}{2} \Rightarrow A = 0$$

$$A = 0; B = \frac{1}{3}; C = -\frac{1}{3}$$

$$\frac{0}{s+2} + \frac{\frac{1}{3}}{s-1} + \frac{-\frac{1}{3}}{s+4} =$$

$$\mathcal{L}^{-1}\left\{\frac{0}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{\frac{1}{3}}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{-\frac{1}{3}}{s+4}\right\}$$

$$\mathcal{L}^{-1}\left\{0\right\} + \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\}$$

$$= \frac{e^t}{3} - \frac{e^{-4t}}{3} = \frac{e^t - e^{-4t}}{3}$$

$$y(t) = \frac{e^t - e^{-4t}}{3}$$

$$y(s) = \frac{s-1}{s^3 + 7s^2 + 11s + 5}$$

$$y(s) = \frac{s-1}{s^3 + 7s^2 + 11s + 5} = \frac{s-1}{(s+1)^2(s+5)}$$

$$\frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+5} =$$

$$\frac{A(s+1)(s+5) + B(s+5) + C(s+1)^2}{(s+1)^2(s+5)} =$$

$$A(s^2 + 6s + 5) + B(s+5) + C(s^2 + 2s + 1) =$$

$$As^2 + 6sA + 5A + Bs + 5B + Cs^2 + 2sC + C$$

$$5A + 5B + C = -1$$

$$6A + B + 2C = 1$$

$$A + C = 0$$

$$A+C=0 \Rightarrow A=-C$$

$$5(-C)+5B+C=-1 \Rightarrow -4C+5B=-1$$

$$6(-C)+B+2C=1 \Rightarrow -4C+B=1$$

$$C = -\frac{-1-5B}{4} \Rightarrow -4\left(-\frac{-1-5B}{4}\right) + B = 1 \Rightarrow$$

$$-1-4B=1 \Rightarrow B = -\frac{1}{2}$$

$$C = -\frac{-1-5(-\frac{1}{2})}{4} \Rightarrow -\frac{3}{8} = C$$

$$A = -C \Rightarrow A = -(-\frac{3}{8}) \Rightarrow A = \frac{3}{8}$$

$$A = \frac{3}{8}, B = -\frac{1}{2}, C = -\frac{3}{8}$$

$$\frac{\frac{3}{8}}{s+1} + \frac{(-\frac{1}{2})}{(s+1)^2} + \frac{(-\frac{3}{8})}{s+5} =$$

$$\mathcal{L}^{-1}\left\{\frac{\frac{3}{8}}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{-\frac{1}{2}}{(s+1)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{-\frac{3}{8}}{s+5}\right\} =$$

$$\frac{3}{8} \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{2} \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} - \frac{3}{8} \mathcal{L}^{-1}\left\{\frac{1}{s+5}\right\} =$$

$$y(t) = \frac{3}{8}e^{-t} - \frac{1}{2}e^{-t}t - \frac{3}{8}e^{-5t}$$

3. Hallar la solución de las siguientes ecuaciones diferenciales

$$\frac{d^2y(t)}{dt^2} + 14\frac{dy(t)}{dt} + 100y(t) = 100 \quad (6)$$

3. $\ddot{y} + 14\dot{y} + 100y = 100 \Rightarrow Y(s) = \frac{100}{s(s^2 + 14s + 100)}$

Tiene raíces complejas conjugadas

$$\mathcal{L}^{-1}\left\{\frac{A}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{Bs+C}{s^2+14s+100}\right\} \Rightarrow$$

$$\frac{A(s^2+14s+100) + Bs^2 + Cs}{s(s^2+14s+100)} \Rightarrow$$

$$\frac{As^2 + 14As + 100A + Bs^2 + Cs}{s(s^2+14s+100)} \Rightarrow$$

$$A+B=0 \Rightarrow B=-1$$

$$14A+C=0 \Rightarrow C=-14$$

$$100A=100 \Rightarrow A=1$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{-s-14}{s^2+14s+100}\right\} \Rightarrow$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{s+14}{s^2+14s+100}\right\} \Rightarrow$$

$$1 - \mathcal{L}^{-1}\left\{\frac{s+14+7-7}{s^2+14s+100+49-49}\right\} \Rightarrow$$

si $2a = 14$
 $a = 7$
 $a^2 = 49$

$$1 - \mathcal{L}^{-1} \left\{ \frac{s+7+7}{s^2+14s+49+51} \right\} \Rightarrow$$

$$\omega^2 = 51$$

$$\omega = \sqrt{51}$$

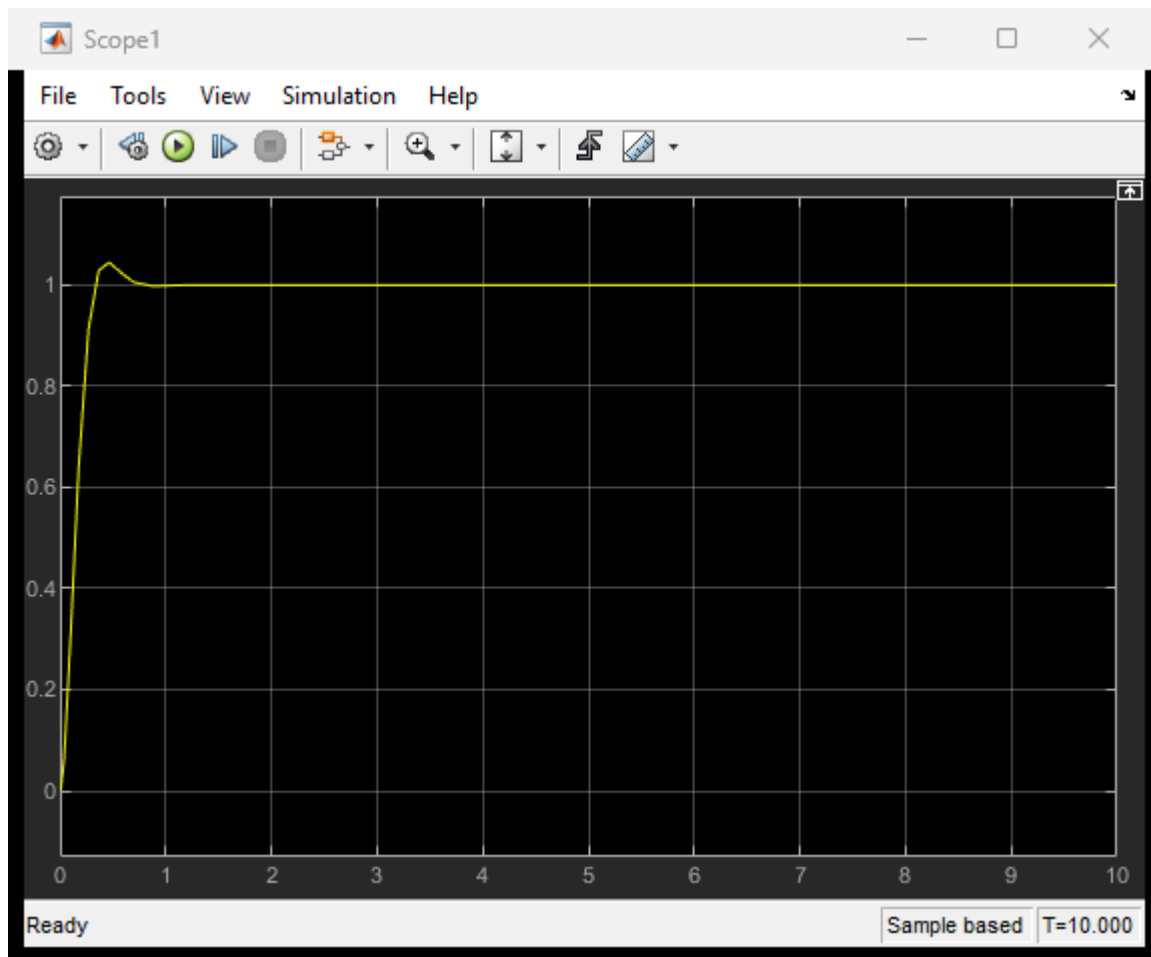
$$1 - \mathcal{L}^{-1} \left\{ \frac{s+7}{(s+7)^2+51} \right\} -$$

$$\mathcal{L}^{-1} \left\{ \frac{7}{(s+7)^2+51} \right\} \Rightarrow$$

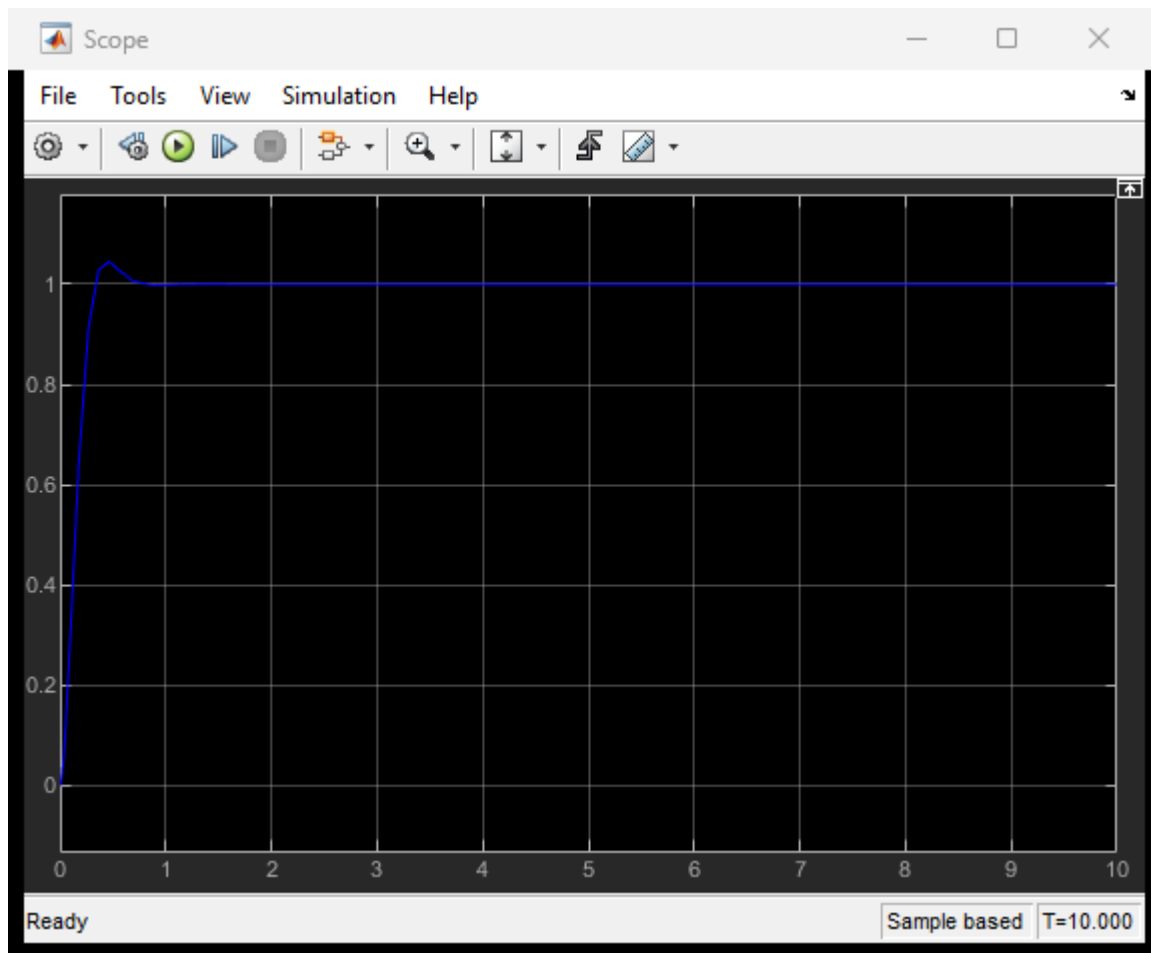
$$1 - e^{-7t} \cos(\sqrt{51})t - 7 \mathcal{L}^{-1} \left\{ \frac{\frac{7}{\sqrt{51}}}{(s+7)^2+51} \right\} \Rightarrow$$

$$1 - e^{-7t} \cos(\sqrt{51})t - 7 \cdot \frac{1}{\sqrt{51}} \mathcal{L}^{-1} \left\{ \frac{\sqrt{51}}{(s+7)^2+51} \right\} \Rightarrow$$

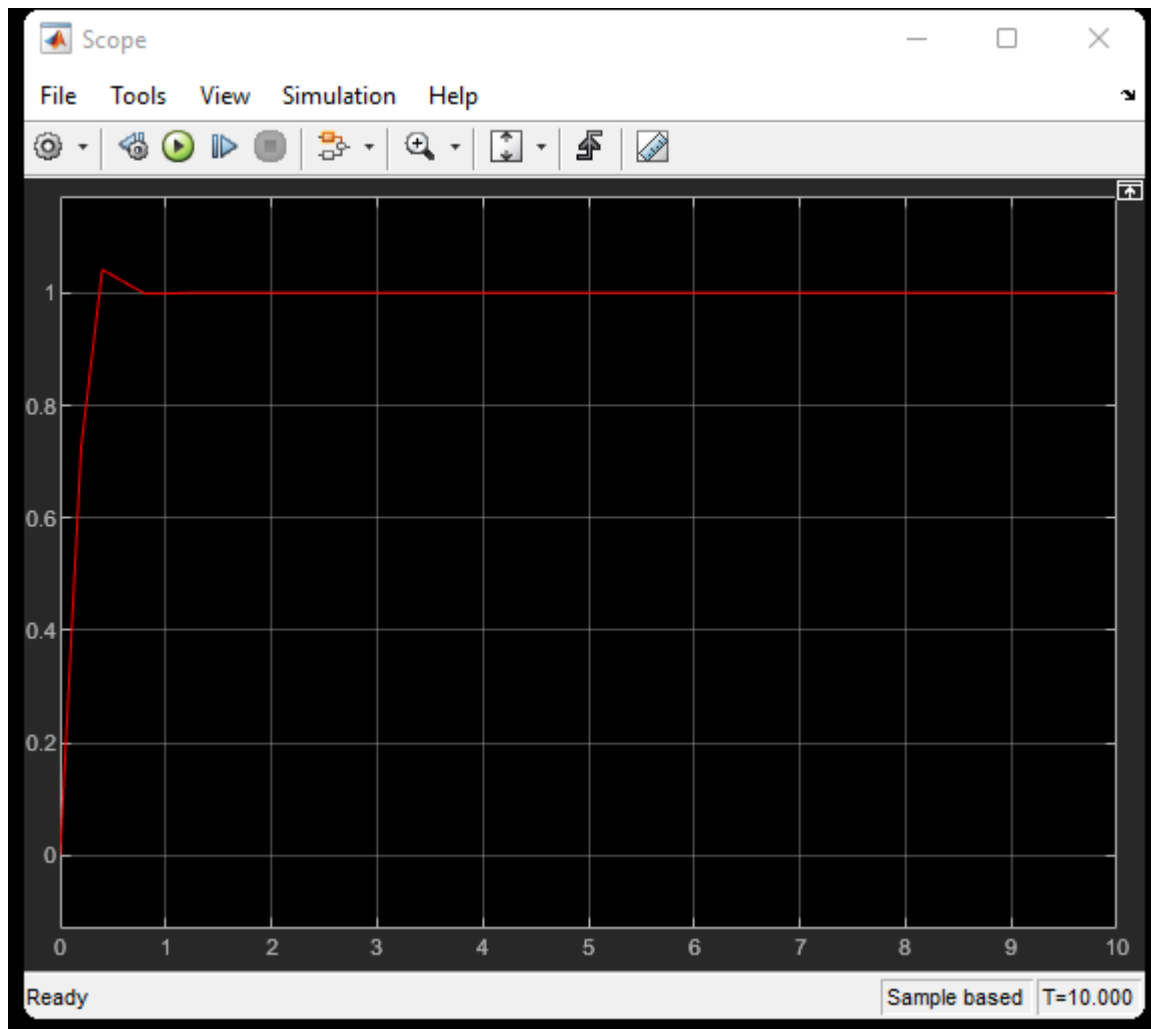
$$1 - e^{-7t} \cos(\sqrt{51})t - \frac{7}{\sqrt{51}} e^{-7t} \sin(\sqrt{51})t$$



Gráfica de función de transferencia



Gráfica de ecuación diferencial



Gráfica del función del tiempo

$$\frac{d^2 y(t)}{dt^2} + 16 \frac{dy(t)}{dt} + 64 y(t) = 1 \quad (7)$$

$$\frac{d^2 y(t)}{dt^2} + 16 \frac{dy(t)}{dt} + 64 y(t) = 1$$

$$\mathcal{L}\left\{\frac{d^2 y(t)}{dt^2}\right\} + 16 \mathcal{L}\left\{\frac{dy(t)}{dt}\right\} + 64 \mathcal{L}\{y(t)\} =$$

$$\mathcal{L}\{1\} \Rightarrow$$

$$s^2 Y(s) + 16s Y(s) + 64 Y(s) = \frac{1}{s} \Rightarrow$$

$$Y(s) (s^2 + 16s + 64) = \frac{1}{s} \Rightarrow$$

$$Y(s) = \frac{1}{s(s^2 + 16s + 64)} \Rightarrow Y(s) = \frac{1}{s(s+8)(s+8)}$$

$$\frac{A}{s} + \frac{B}{s+8} + \frac{C}{(s+8)^2} = \frac{As^2 + 16As + 64A + s^2B + 8sB + sC}{s(s+8)^2}$$

$$A + B = 0 \Rightarrow B = -\frac{1}{64}$$

$$16A + 8B + C = 0 \Rightarrow 16\left(\frac{1}{64}\right) + 8\left(-\frac{1}{64}\right) + C = 0 \Rightarrow$$

$$64A = 1 \Rightarrow A = \frac{1}{64}$$

$$\frac{1}{4} + \left(-\frac{1}{8}\right) + C = 0 \Rightarrow C = -\frac{1}{8}$$

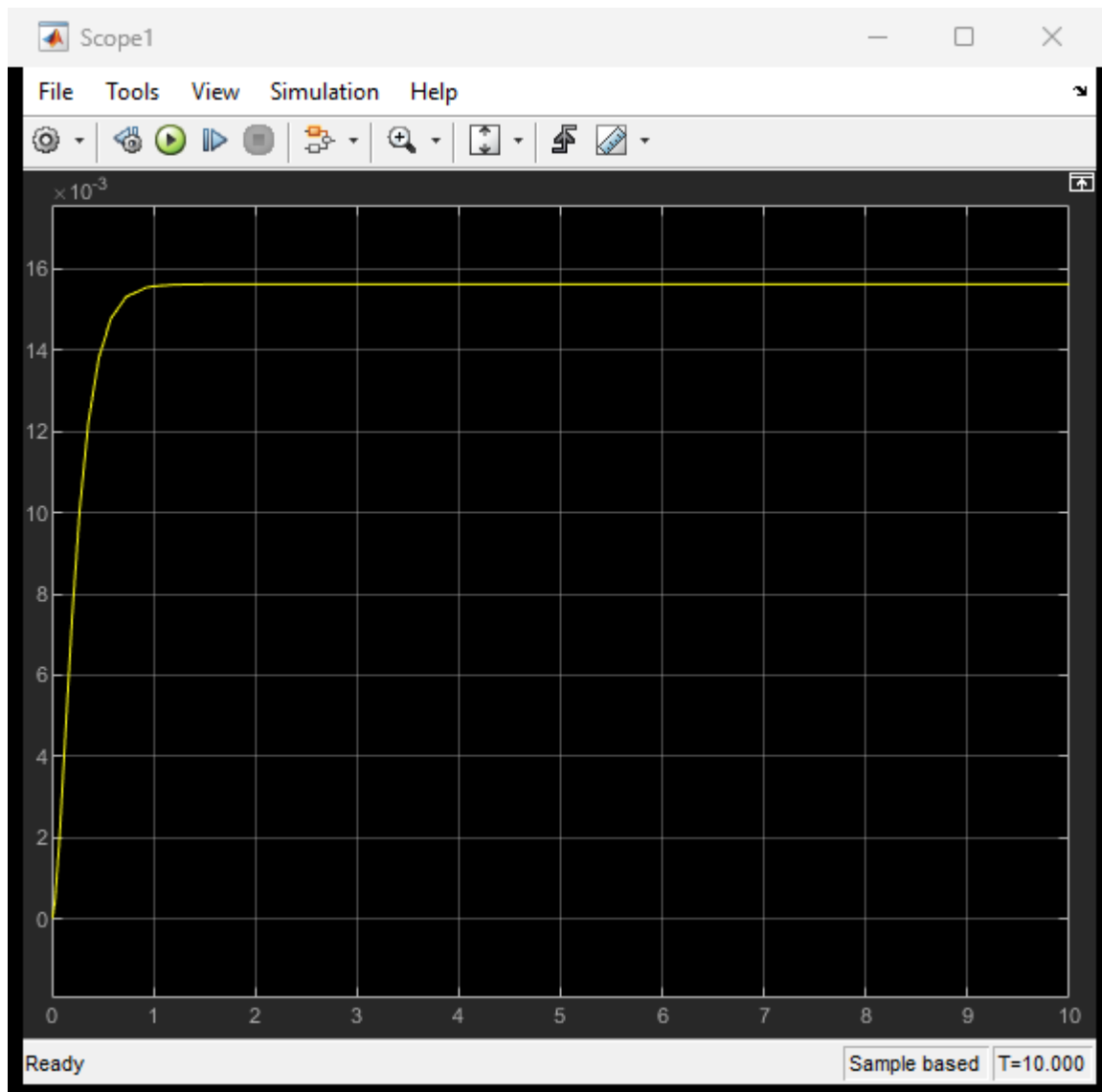
$$A = \frac{1}{64}; B = -\frac{1}{64}; C = -\frac{1}{8}$$

$$\frac{\frac{1}{64}}{s} = \frac{\frac{1}{64}}{s+8} - \frac{\frac{1}{8}}{(s+8)^2} \Rightarrow$$

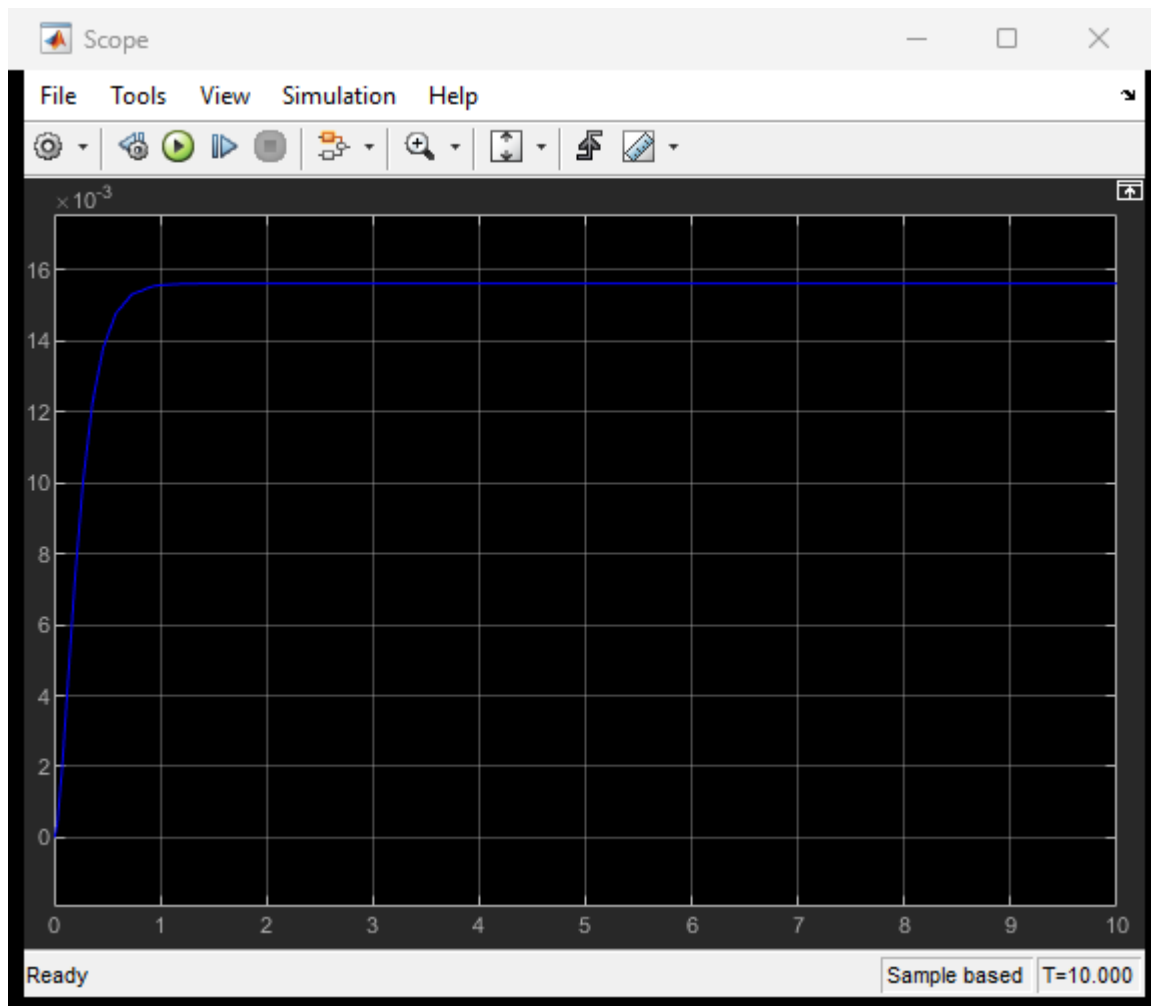
$$\mathcal{L}^{-1}\left\{\frac{1}{64s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{64(s+8)}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{8(s+8)^2}\right\} \Rightarrow$$

$$\frac{1}{64}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{64}\mathcal{L}^{-1}\left\{\frac{1}{s+8}\right\} - \frac{1}{8}\mathcal{L}^{-1}\left\{\frac{1}{(s+8)^2}\right\} \Rightarrow$$

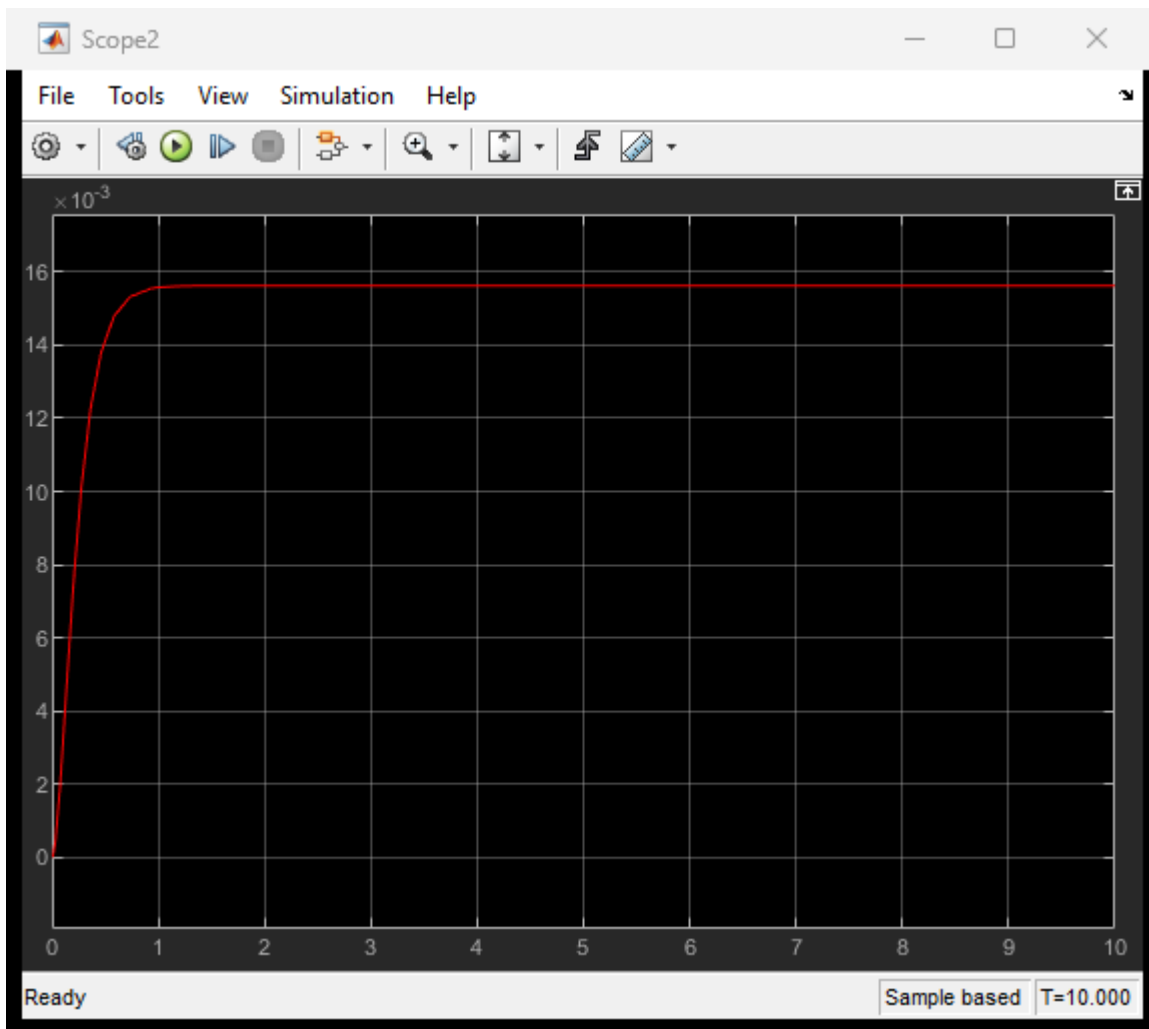
$$\frac{1}{64} - \frac{e^{-8t}}{64} - \frac{e^{-8t}t}{8}$$



Gráfica de función de transferencia



Gráfica de ecuación diferencial



Gráfica del función del tiempo

$$\frac{d^2 y(t)}{dt^2} + 7 \frac{dy(t)}{dt} + 12y(t) = 3 \quad (8)$$

$$\frac{d^2 y(t)}{dt^2} + 7 \frac{dy(t)}{dt} + 12y(t) = 3$$

$$\mathcal{L}\left\{\frac{d^2 y(t)}{dt^2}\right\} + 7\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} + 12\mathcal{L}\{y(t)\} = \mathcal{L}\{3\}$$

$$\mathcal{L}\{3\} \Rightarrow$$

$$s^2 Y(s) + 7s Y(s) + 12 Y(s) = 3/s \Rightarrow$$

$$Y(s)(s^2 + 7s + 12) = 3/s \Rightarrow$$

$$Y(s) = \frac{3}{s(s^2 + 7s + 12)} \Rightarrow Y(s) = \frac{3}{s(s+3)(s+4)}$$

$$\frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4} = \frac{A(s+3)(s+4) + B(s)(s+4) + C(s)(s+3)}{s(s+3)(s+4)}$$

$$= \frac{A(s^2 + 7s + 12) + B(s^2 + 4s) + C(s^2 + 3s)}{s(s+3)(s+4)} =$$

$$As^2 + 7As + 12A + Bs^2 + 4Bs + Cs^2 + 3Cs$$

$$B = -1$$

$$A + B + C = 0 \Rightarrow \frac{1}{4} + B + C = 0 \Rightarrow B = -C - \frac{1}{4}$$

$$7A + 4B + 3C = 0 \Rightarrow 7(\frac{1}{4}) + 4(-C - \frac{1}{4}) + 3C = 0$$

$$12A = 3$$

$$\Rightarrow A = \frac{1}{4}$$

$$\frac{7}{4} - 4C - 1 + 3C = 0 \Rightarrow -C = -\frac{3}{4} \Rightarrow C = \frac{3}{4}$$

$$A = 1/4; B = -1; C = 3/4.$$

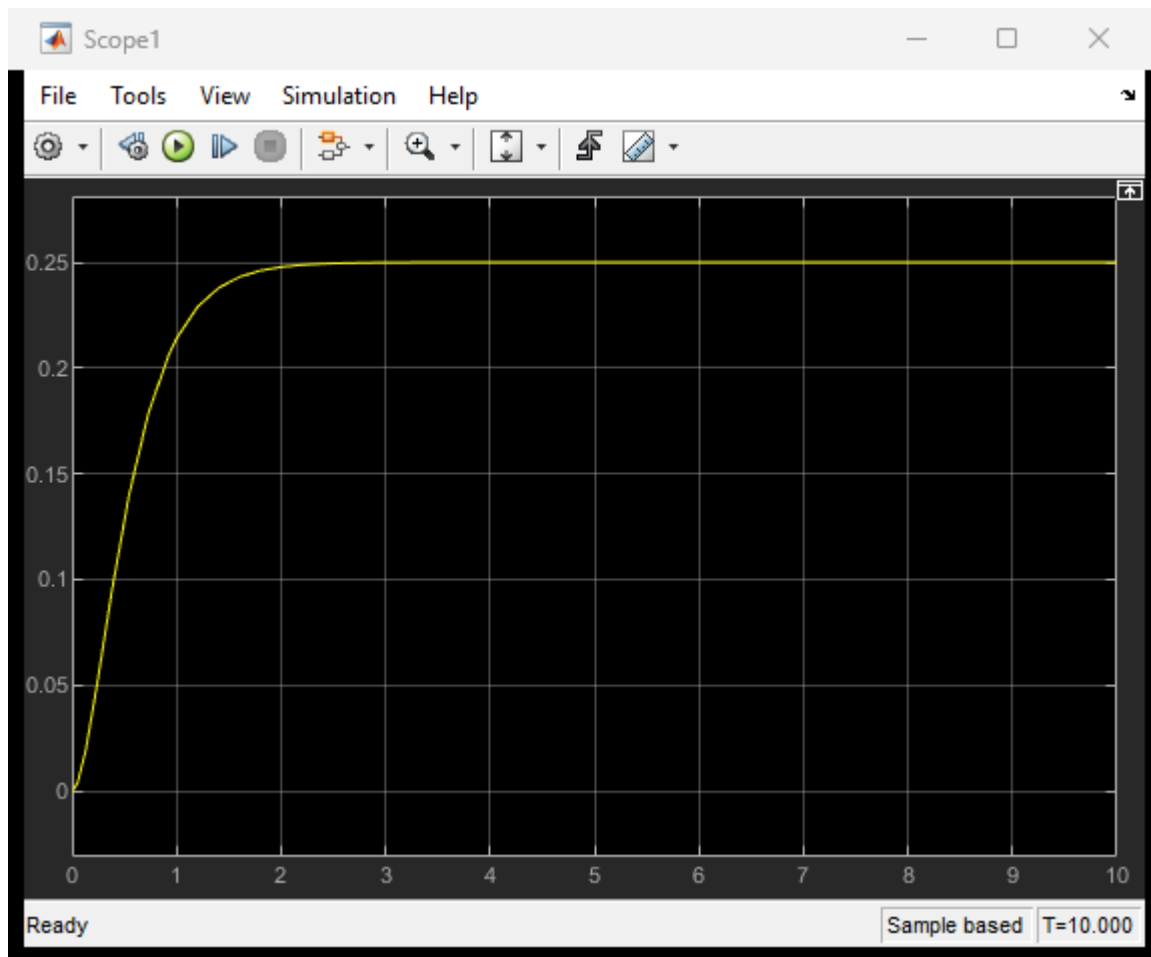
$$\frac{1/4}{s} + \frac{-1}{s+3} + \frac{3/4}{s+4} \Rightarrow \frac{1}{4} \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} + (-1) \mathcal{L}^{-1} \left\{ \frac{1}{s+3} \right\} +$$

$$\frac{3}{4} \mathcal{L}^{-1} \left\{ \frac{1}{s+4} \right\} \Rightarrow$$

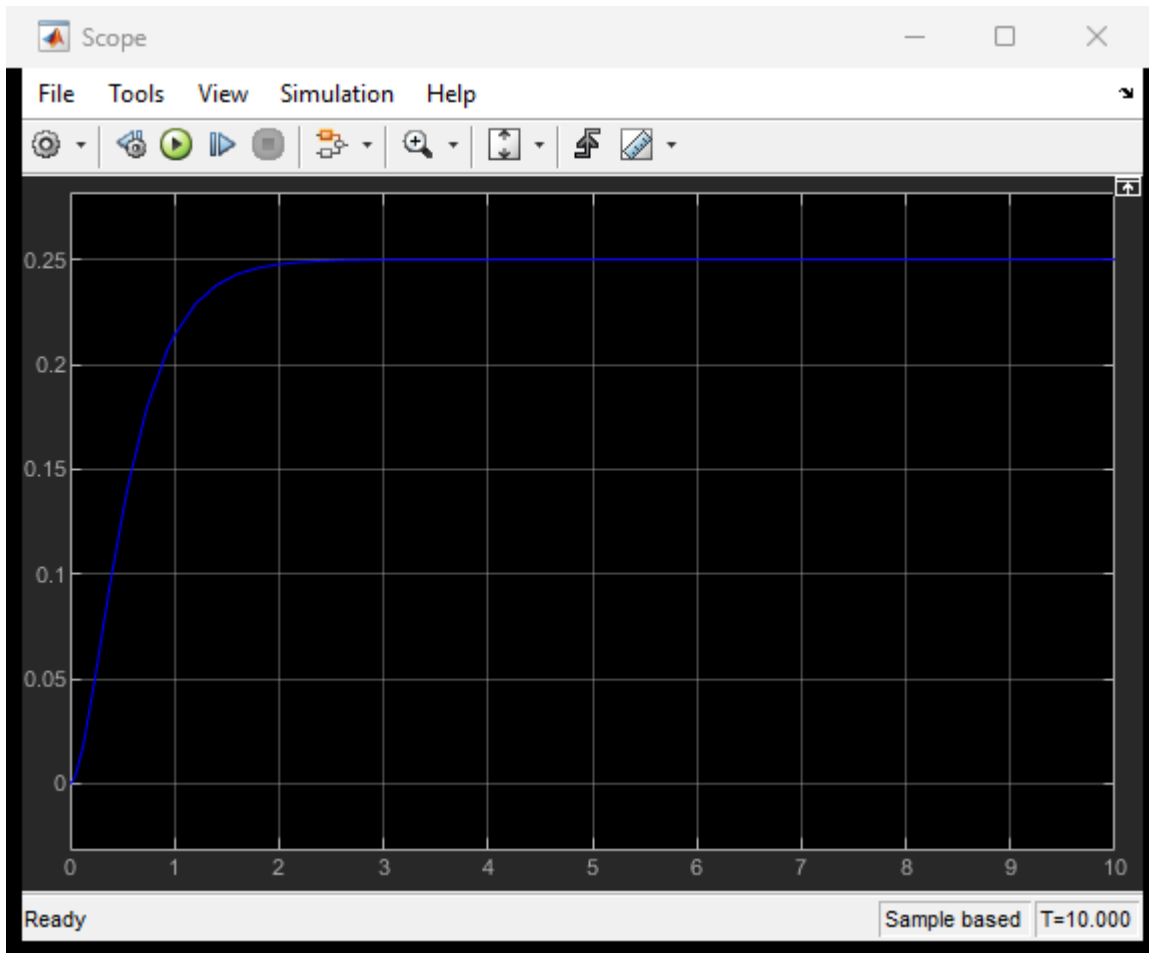
$$\frac{1}{4} \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{s+3} \right\} + \frac{3}{4} \mathcal{L}^{-1} \left\{ \frac{1}{s+4} \right\} \Rightarrow$$

$$\frac{1}{4} - e^{-3t} + \frac{3}{4} e^{-4t} \Rightarrow$$

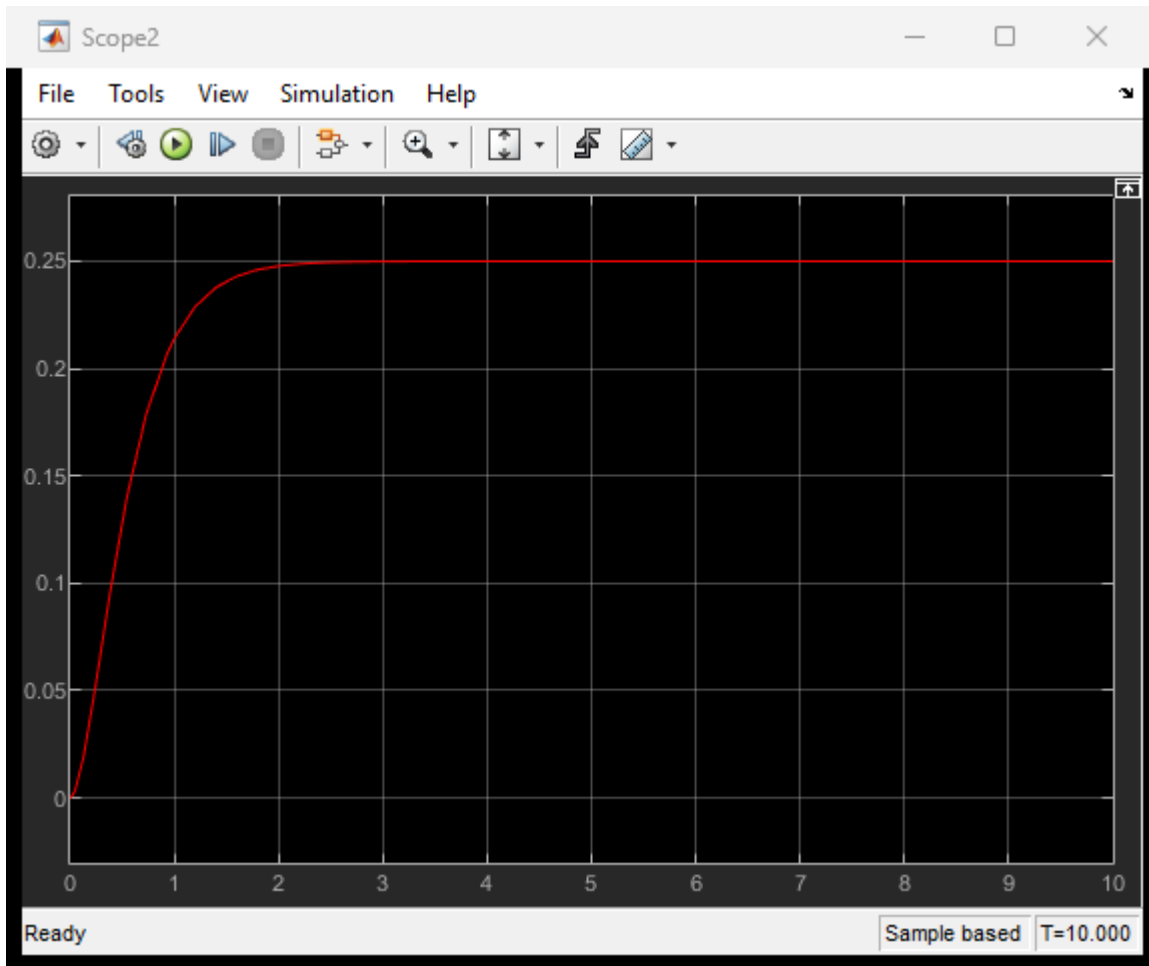
$$y(t) = \frac{1}{4} - e^{-3t} + \frac{3}{4} e^{-4t}$$



Gráfica de función de transferencia



Gráfica de ecuación diferencial



Gráfica del función del tiempo