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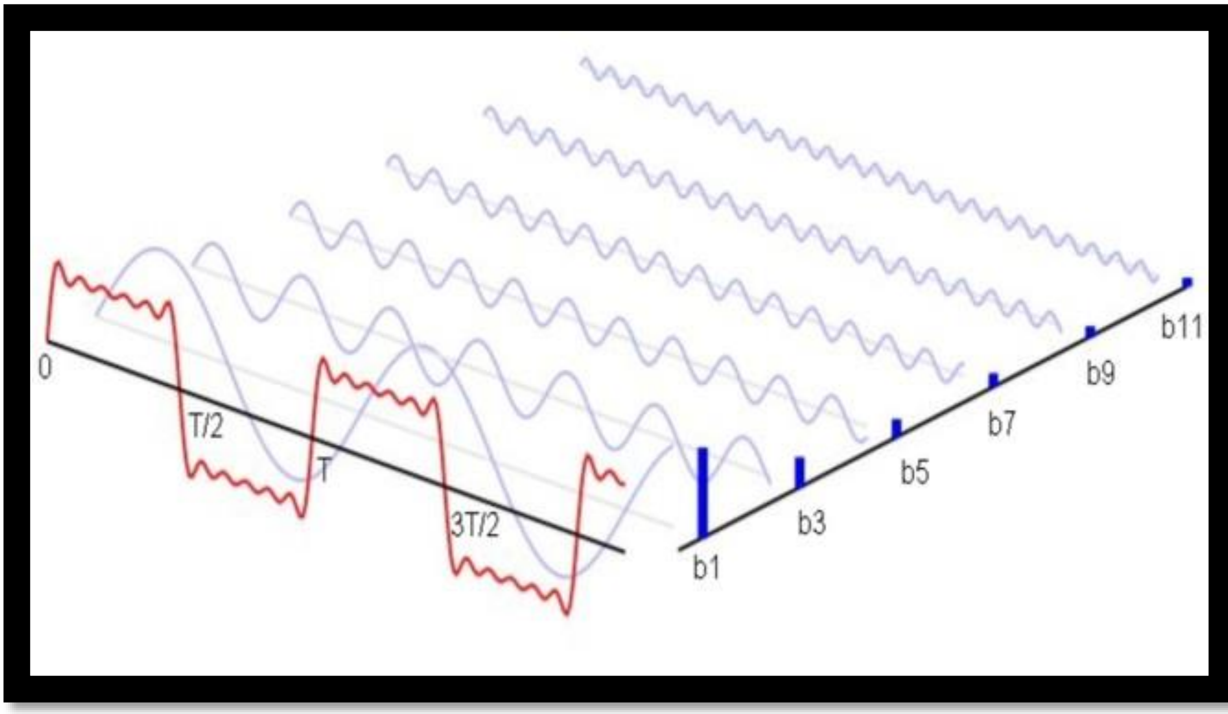

Análisis de Fourier

PRIMER EXAMEN

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1. **Encuentra la serie de Fourier de:**

$f(x) = x$ **donde** $-\pi \leq x \geq \pi$

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1er Examen 17 04 24

Analisis de Fourier

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \operatorname{sen}\left(\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} F(x)\,dx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x\,dx = \frac{1}{\pi}\,\frac{x^2}{2}\Big|_{-\pi}^{\pi} = \frac{1}{\pi}\left(\frac{\pi^2}{2} - \frac{(-\pi)}{2}\right) =$$

$$\frac{1}{\pi}\left(\frac{\pi^2}{2} - \frac{\pi^2}{2}\right) = 0$$

$$a_0 = 0$$

$$a_n = \frac{1}{L}\int_{-L}^{L} F(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos\left(\frac{n\pi x}{\pi}\right)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos(nx)\,dx$$

$$= \frac{1}{\pi}\left(\frac{1}{n}x\operatorname{sen}(nx)\Big|_{-\pi}^{\pi} - \frac{1}{n}\int_{-\pi}^{\pi}\operatorname{sen}(nx)\,dx\right.$$

$$= \frac{1}{\pi}\left(\cancelto{0}{\frac{1}{n}\pi\operatorname{sen}(n\pi)} - \cancelto{0}{\frac{1}{n}(-\pi)\operatorname{sen}(-n\pi)} + \right.$$

$$\left.\frac{1}{n^2}\cos(nx)\Big|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{1}{n^2\pi}\left(\cos(n\pi) - \cos(-n\pi)\right) =$$

$$\frac{1}{n^2\pi}\left(\cos(n\pi) - \cos(-n\pi)\right) =$$

$$\frac{1}{n^2\pi}\left(\cos(n\pi) - \cos(n\pi)\right)$$

$$a_n = 0$$

$$b_n = \frac{1}{L}\int_{-L}^{L} F(x)\,\mathrm{sen}\left(\frac{n\pi x}{L}\right)dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\,\mathrm{sen}\left(\frac{n\pi x}{\pi}\right)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\,\mathrm{sen}(nx)\,dx$$

$$= \frac{1}{\pi}\left(-\frac{1}{n}x\cos(nx)\Big|_{-\pi}^{\pi} + \frac{1}{n}\int_{-\pi}^{\pi}\cos(nx)\,dx\right)$$

$$= \frac{1}{\pi}\left(-\frac{1}{n}\pi\cos(n\pi) + \frac{1}{n}(-\pi)\cos(-n\pi) + \frac{1}{n^2}\,\mathrm{sen}(nx)\Big|_{-\pi}^{\pi}\right)$$

$$= \frac{1}{\pi}\left(-\frac{2\pi}{n}\cos(n\pi) + \cancel{\frac{1}{n^2}\,\mathrm{sen}(n\pi)} - \cancel{\frac{1}{n^2}\,\mathrm{sen}(-n\pi)}\right)$$

$\cos(n\pi) = (-1)^n$

$$b_n = -\frac{2}{n}(-1)^n = \frac{2}{n}(-1)^{n+1}$$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

$a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{2}{n}(-1)^{n+1}$

$$x = \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \, \text{Sen}(nx)$$

2. Obtén la transformada de Fourier utilizando la definición de Fourier

$$f(t) = \begin{Bmatrix} e^{-at} & t \geq 0 \\ 0 & t < 0 \end{Bmatrix}$$

$$F(t) = \begin{cases} e^{-at}, & t > 0 \qquad a > 0 \\ 0, & t < 0 \end{cases}$$

$$F[F(t)] = \int_{-\infty}^{\infty} F(t)\, e^{-iwt}\, dt$$

$$F[F(t)] = \cancelto{0}{\int_{-\infty}^{0} 0\, e^{-iwt}\, dt} + \int_{0}^{\infty} e^{-at} e^{-iwt}\, dt$$

$$= \int_{0}^{\infty} e^{-at-iwt}\, dt = \int_{0}^{\infty} e^{-(a+iw)t}\, dt$$

$$= \frac{1}{-(a+iw)} e^{-(a+iw)t} \Big|_{0}^{\infty} =$$

$$= -\frac{1}{a+iw} e^{-at} e^{-iwt} \Big|_{0}^{\infty}$$

$$\lim_{t \to \infty} e^{-at} = 0 \quad \text{si } a > 0$$

$$= -\frac{1}{a+iw} \left(\cancelto{0}{\lim_{t \to \infty} e^{-at} e^{iwt}} - \cancelto{1}{e^{0}} \right)$$

$$= -\frac{1}{a+iw} (-1)$$

$$F[F(t)] = \frac{1}{a+iw}$$