Análisis de Fourier

SERIE TRIGONOMETRICA

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$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{T}\right) t + b_n \, sen\left(\frac{2\pi n}{T}\right) t$$

4 Terminos

a_0

a_n

b_n

$$F(t) \begin{cases} 2 & si \;\; \underset{a}{-2 < t < 1} \\[6pt] 4 & si \;\; \underset{b}{1 < t < 4} \end{cases}$$

$\frac{1}{T} = 6 \rightarrow$ período fundamental

$$a_0 = \frac{2}{T} \int_{a}^{a+T} F(t)\, dt$$

$T = |a - b| = |1-6| = \boxed{6}$

$-2 - 4 = |1-6|$

$$a_0 = \frac{2}{6} \int_{-2}^{1} 2\, dt + \int_{1}^{4} 4\, dt$$

$$a_0 = \frac{2}{6} \; 2t \Big|_{-2}^{1} + 4t \Big|_{1}^{4}$$

$$a_0 = \frac{2}{6}\left((2 - (-4)) + (16 - 4) \right) =$$

$$\frac{2}{6}(6 + 12) = \frac{2}{6}(18) = \frac{36}{6} = \boxed{6}$$

$$a_n = \frac{2}{T} \int_a^{a+t} F(t) \cos\left(\frac{2\pi n t}{T}\right) dt$$

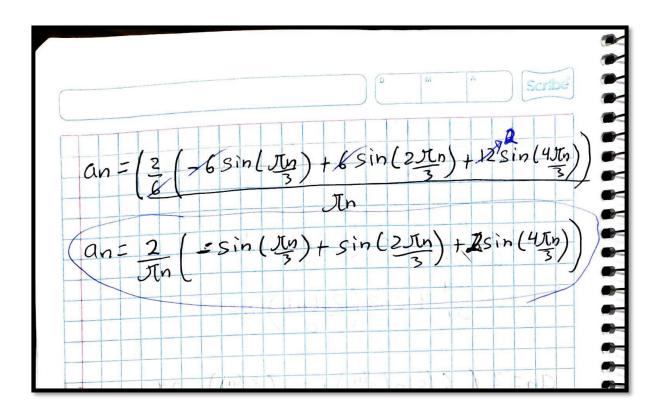$$a_n = \frac{2}{6} \int_{-2}^{1} 2 \cos\left(\frac{2\pi n t}{6}\right) dt +$$

$$\int_1^4 4 \cos\left(\frac{2\pi n t}{6}\right) dt$$

$$a_n = \frac{2}{6} \left(\frac{6\left(\sin\left(\frac{\pi n}{3}\right) + \sin\left(\frac{2\pi n}{3}\right)\right)}{\pi n} + \int_1^4 4 \cos\left(\frac{2\pi n t}{6}\right) dt \right)$$

$$a_n = \frac{2}{6} \left(\frac{6\left(\sin\left(\frac{\pi n}{3}\right) + \sin\left(\frac{2\pi n}{3}\right)\right)}{\pi n} + \frac{12\left(\sin\left(\frac{4\pi n}{3}\right) - \sin\left(\frac{\pi n}{3}\right)\right)}{\pi n} \right)$$

$$a_n = \frac{2}{6} \left(\frac{6\sin\left(\frac{\pi n}{3}\right) + 6\sin\left(\frac{2\pi n}{3}\right) + 12\sin\left(\frac{4\pi n}{3}\right)}{} \right.$$

$$\left. \frac{- 12\sin\left(\frac{\pi n}{3}\right)}{\pi n} \right)$$

$$a_n = \left(\frac{2}{6} \cdot \frac{\left(-6 \sin\left(\frac{\pi n}{3} \right) + 6 \sin\left(\frac{2\pi n}{3} \right) + 12 \sin\left(\frac{4\pi n}{3} \right) \right)}{\pi n} \right)$$

$$a_n = \frac{2}{\pi n} \left(-\sin\left(\frac{\pi n}{3} \right) + \sin\left(\frac{2\pi n}{3} \right) + 2\sin\left(\frac{4\pi n}{3} \right) \right)$$

$$b_n = \frac{2}{T} \int_a^{a+6} F(t) \, \text{sen}\left(2 \frac{\pi n t}{T} \right) dt$$

$$b_n = \frac{2}{6} \int_2^1 2 \, \text{sen}\left(\frac{2\pi n t}{6} \right) dt +$$

$$\int_1^4 4 \cos\left(\frac{2\pi n t}{6} \right) dt$$

$$b_n = \frac{2}{6} \left(\frac{6\left(-\cos\left(\frac{\pi n}{3}\right) + \cos\left(\frac{2\pi n}{3}\right)\right)}{\pi n} + \right.$$

$$\frac{12\left(-\cos\left(\frac{4\pi n}{3}\right) + \cos\left(\frac{\pi n}{3}\right)\right)}{\pi n}$$

$$b_n = \frac{2}{6} \left(\frac{-6\cos\left(\frac{\pi n}{3}\right) + 6\cos\left(\frac{2\pi n}{3}\right) - 12\cos\left(\frac{4\pi n}{3}\right)}{} \right.$$

$$\left. \frac{+ 12\cos\left(\frac{\pi n}{3}\right)}{\pi n} \right)$$

$$\boxed{b_n = \frac{2}{\pi n} \left[\cos\left(\frac{2\pi n}{3} \right) - 2\cos\left(\frac{4\pi n}{3} \right) + \cos\left(\frac{\pi n}{3}\right) \right]}$$