QUEUING THEORY

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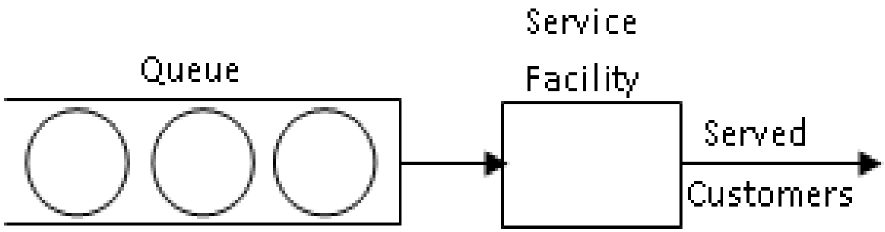
WHAT IS A QUEUE?

- A queue, in general, is formed at any place when a customer (human beings or physical entities)
- that requires services is made to wait due to the fact that
- the number of customers exceeds the number of service facilities or when
- service facilities do not work efficiently and take more time than prescribed to serve a customer

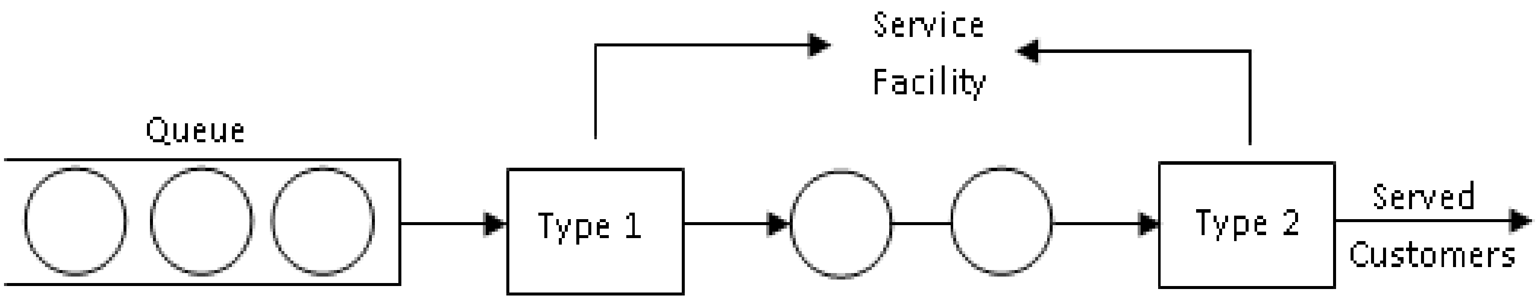
- A common situation that occurs in everyday life is that waiting in a line either at bus stop, petrol pump, Restaurants, ticket booths, doctor's clinics, bank counters, traffic lights and so on.
- Queues (waiting line) are also found in workshops where the machines wait to be repaired; at a tool crib where the mechanics wait to receive tools; in a warehouse where items wait to be used; incoming calls wait to mature in telephone exchange, trucks wait to be unloaded, airplanes wait to take off or land and so on

- The study of **queuing theory** helps to determine the balance between
- **(a)** cost of offering the service, and
- **(b)** cost incurred due to delay in offering service

ARRANGEMENT OF SERVICE FACILITIES

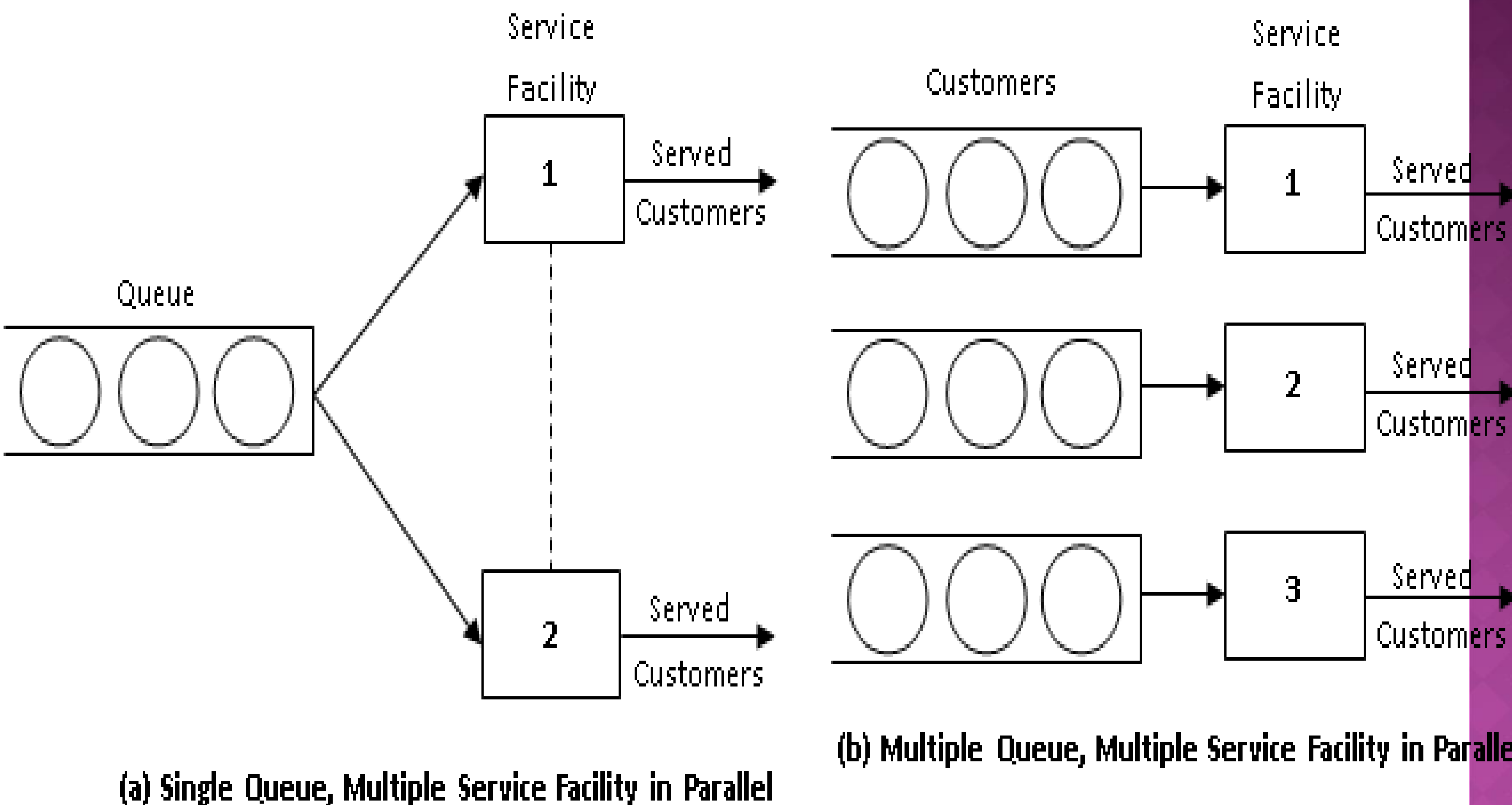


(a) Single Queue, Single Service Facility

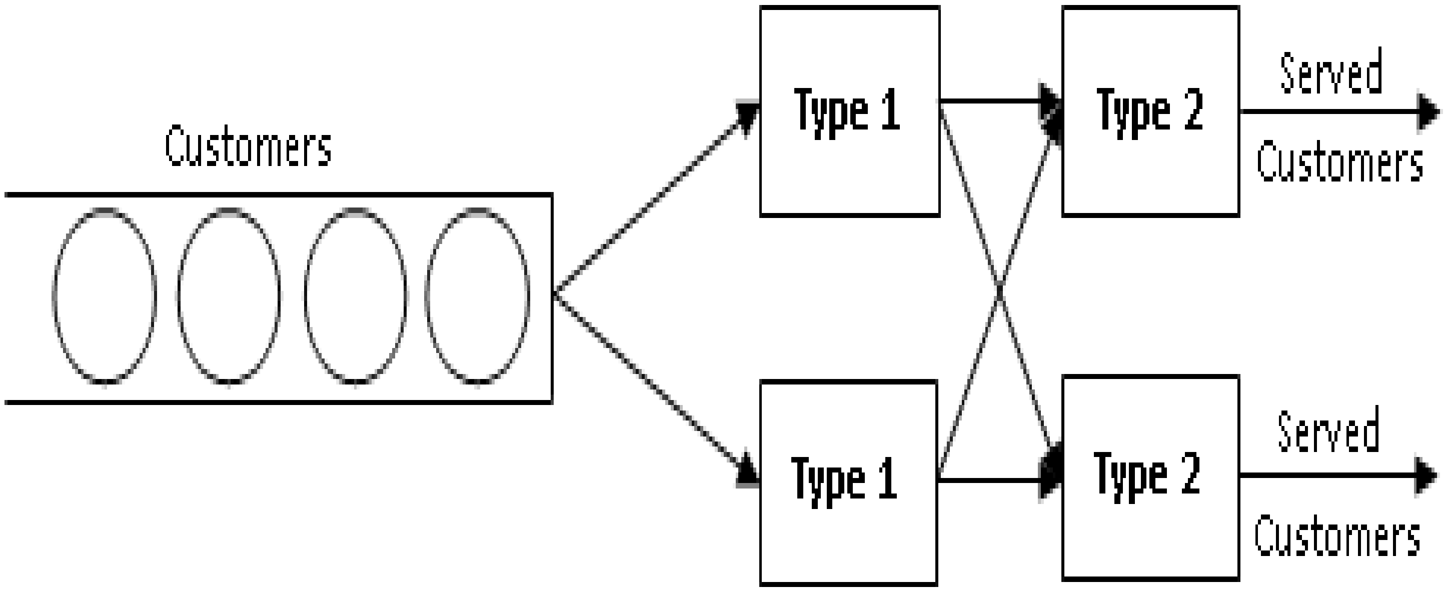


(b) Single Queue, Multiple Service Facility

ARRANGEMENTS OF SERVICE FACILITIES IN PARALLEL



SINGLE QUEUE, MULTIPLE SERVICE FACILITIES IN PARALLEL AND IN SERIES



PERFORMANCE MEASURE OF A QUEUING SYSTEM

- **Average (or expected) time spent by a customer in the queue and system**
- W_q: Average time an arriving customer has to wait in queue before being served,
- W_s: Average time an arriving customer spends in the system, including waiting and service

- **Average (expected) number of customers in the queue and system**
- L_q: Average number of customers waiting for service in the queue (queue length)
- L_s: Average number of customers in the system (either waiting for services in the queue or being served)

- **Value of time both for customers and servers**
- P_w: Probability that an arriving customer has to wait before being served (also called locking probability)
- $\rho = \frac{\lambda}{\mu}$: Percentage of time a server is busy serving customers, i.e., the system utilization
- P_n: Probability of n customers waiting for service in the queuing system
- P_d: Probability that an arriving customer is not allowed to enter in the queuing i.e., system capacity is full

NOTATIONS

- $n =$ number of customers in the system
- $P_n =$ probability of n customers in the system
- $\lambda =$ average customer arrival rate or average number of arrivals per unit of time in the queuing system
- $\mu =$ average service rate or average number of customers served per unit time at the place of service
- $\frac{\lambda}{\mu} = \rho = \frac{\text{Average service completion time } (1/\mu)}{\text{Average interarrival time } (1/\lambda)} =$ traffic intensity or server utilization factor
- $P_0 =$ probability of no customer in the system
- $s =$ number of service channels (service facilities or servers)

- $N =$ maximum number of customers allowed in the system
- L_s =average number of customers in the system (waiting and in service)
- L_q =average number of customers in the queue (queue length)
- W_s =average waiting time in the system (waiting and in service)
- $W_q =$ average waiting time in the queue
- P_w =probability that an arriving customer has to wait (system being busy), $1 - P_0 = (\lambda/\mu)$
- For achieving a steady state condition, it is necessary that, $\lambda/\mu < 1$

RELATION AMONG PERFORMANCE MEASURES

- $L_s = L_q + \text{Customer being served} = L_q + \frac{\lambda}{\mu}$
- $W_q = \frac{L_q}{\lambda}$
- $W_s = W_q + \frac{1}{\mu}$
- Probability of being in the system (waiting and being served) longer than time t is given by:
- $P(W_s > t) = e^{-(\mu-\lambda)t}$
- and $P(W_s \leq t) = 1 - P(W_s > t)$

- Probability of only waiting for service longer than time t is given by: $P\left(W_q > t\right) = \frac{\lambda}{\mu} e^{-(\mu-\lambda)t}$
- Probability of exactly n customers in the system is given by
- $P_n = P_0 \left(\frac{\lambda}{\mu}\right)^n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$
- Probability that the number of customers in the system, n exceeds a given number, r is given by
- $P(n > r) = \left(\frac{\lambda}{\mu}\right)^{r+1}$

CLASSIFICATION OF QUEUING MODELS

- Queuing theory models are classified by using special (or standard) notations described initially by D.G. Kendall in the form $(a/b/c)$ Later A.M. Lee added the symbol d and e to the Kendall's notation.
- In the literature of queuing theory, the standard format used to describe queuing models is as follows: $\{(a/b/c):(d/e)\}$

$\{(a/b/c):(d/e)\}$

- $a =$ arrivals distribution
- $b =$ service time distribution
- $c =$ number of servers (service channels)
- $d =$ capacity of the system (queue plus service)
- $e =$ queue (or service) discipline

SINGLE SERVER QUEUING THEORY

- **Model 1:** $\{(\boldsymbol{M}/\boldsymbol{M}/\mathbf{1}):(\infty/\boldsymbol{FCFS})\}$ **Exponential Service - Unlimited Queue**
- This model is based on certain assumption about the queuing system:
- **(i)** Arrivals are described by Poisson probability distribution and come from an infinite calling population
- **(ii)** Single waiting line and each arrival waits to be served regardless of the length of the queue (i.e., no limit on queue length - infinite capacity) and that there is no balking or reneging

$\{(M/M/1):(\infty/FCFS)\}$

- **(iii)** Queue discipline is 'first come, first served'
- **(iv)** Single server or channel and service time follows exponential distribution
- **(v)** Customer arrival is independent but arrival rate (average number of arrivals) does not change over time
- **(vi)** The average service rate is more than the average arrival rate

FORMULAE FOR MODEL-1

- $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \rho$ and
- $P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right) = \rho^n(1-\rho); \qquad \rho < 1,$

  $n = 0, 1, 2, \dots \dots .$
- Expected number of customers in the system (customer in the line plus the customer being served)
- $L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda};\ \rho = \frac{\lambda}{\mu}$

- Expected number of customers waiting in the queue (i.e., queue length)
- $L_q = \frac{\lambda}{\mu-\lambda} - \frac{\lambda}{\mu} = \frac{\lambda^2}{\mu(\mu-\lambda)}$
- Expected waiting time for a customer in the queue
- $W_q = \lambda\left(1 - \frac{\lambda}{\mu}\right)\frac{1}{(\mu-\lambda)^2} = \frac{\lambda}{\mu(\mu-\lambda)} \text{ or } \frac{L_q}{\lambda}$

- Expected waiting time for a customer in the system (waiting and service):
- $W_s = W_q + \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu} = \frac{1}{\mu-\lambda}$ or $\frac{L_s}{\lambda}$
- The variance (fluctuation) of queue length
- $Var(n) = \frac{\rho}{(1-\rho)^2} = \frac{\lambda\mu}{(\mu-\lambda)^2}$
- Probability that the queue is non empty
- $P(n > 1) = 1 - P_0 - P_1$
- $= 1 - \left(1 - \frac{\lambda}{\mu}\right) - \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right) = \left(\frac{\lambda}{\mu}\right)^2$

- Probability that the number of customers, n is the system exceeds a given number k
- $P(n \geq k) = \left(\frac{\lambda}{\mu}\right)^k$ and
- $P(n > k) = \left(\frac{\lambda}{\mu}\right)^{k+1}$
- Expected length of non empty queue:
- $L = \frac{expected\ length\ of\ waiting\ line}{P(n>1)}$
- $= \frac{L_q}{P(n>1)} = \frac{\frac{\lambda^2}{\mu(\mu-\lambda)}}{((\lambda)/\mu)^2} = \frac{\mu}{\mu-\lambda}$

EXAMPLES

Q.1 With the usual notations, find the average number of customers in the system and in the queue if the system is $M/M/1/\infty$ and $\lambda = 10$, $\mu=15$.

Solution:

Average number of customers in the queue

$$= L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{100}{15 \times 5} = \frac{4}{3}$$

Average number of customers in the system

$$= L_s = \frac{\lambda}{\mu-\lambda} = \frac{10}{5} = 2$$

EXAMPLES

Q.2 With the usual notation find the average waiting time per customer in the queue and in the system for M/M/1/∞ model if λ = 9, μ=15 per hour.

Solution :

Average waiting time in the queue

$$= W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{9}{15 \times 6} = 0.1\ hour$$

Average waiting time in the system

$$= W_s = \frac{1}{\mu - \lambda} = \frac{1}{6}\ hour$$

Q.3 Find the probability that a customer has to wait in an $M/M/1/\infty$ model if $\lambda = 8$, $\mu = 10$ per hour.

Solution:

P(Customer has to wait)$= P(n > 0) = \frac{\lambda}{\mu} = 0.8$

Q.4 What is the probability that in an M/M/1/∞ model with 6 persons arriving per hour and 8 persons being served per hour, there will be more than 8 persons in the system?

Solution: Given $\lambda = 6\, per\ hour, \mu = 8\, per\ hour$

P(More than 8 persons in the system) $= P(n > 8)$

$$= \left(\frac{\lambda}{\mu}\right)^9 = \left(\frac{6}{8}\right)^9 = 0.075$$

Q.5 Find the probability that a customer has to wait more than 20 minutes to be out of the service station with λ = 8 per hour and μ=11 per hour if the system is M/M/1/∞ model. Also find the probability that a customer has to wait in the queue more than 15 minutes.

Solution: Given $\lambda = 8\, per\ hour, \mu = 11\, per\ hour$

P(customer has to wait more than 20 minutes to be out of the service station) $= P\left(W_s > \frac{1}{3}\right) = e^{-(11-8)\frac{1}{3}} = e^{-1} = 0.3678$

$$\left\{P(W_s > t) = e^{-(\mu-\lambda)t}\ , here\ t = 20 \min = \frac{1}{3}\ hour\right\}$$

P(a customer has to wait in the queue more than 15 minutes)

$$= P\left(W_q > \frac{1}{4}\right) = \frac{8}{11} e^{-(11-8)\frac{1}{4}} = 0.3435$$

$$\left\{P\left(W_q > t\right) = \frac{\lambda}{\mu} e^{-(\mu-\lambda)t}\ , here\ t = 15 \min = \frac{1}{4}\ hour\right\}$$

EXAMPLE-6

- A television repairman finds that the time spent on his jobs has an exponential distribution with a mean of 30 min. If he repairs the sets in the order in which they came in, and if the arrival of sets follows a Poisson distribution with an approximate average rate of 10 per 8 hours day, what is the repairman’s expected idle time each day? How many jobs are ahead of average set just brought in?

SOLUTION

- From the data of the problem, we have:
- $\lambda = 10/8 = 5/4$ sets per hour;
- $\mu = (1/30)60 = 2$ sets per hour
- expected idle time of repairman each day:
- Since number of hours for which the repairman remains busy in an 8 - hours day (traffic intensity) is given by
- $(8)(\lambda/\mu) = (8)(5/8) = 5$ hours
- Therefore, the idle time for a repairman in an 8 - hour day will be: $(8 - 5) = 3$ hours
- Expected (or average) number of TV sets in the system
- $L_S = \frac{\lambda}{\mu - \lambda} = \frac{5/4}{2-(5/4)} = \frac{5}{3} = 2$ (approx.) TV sets

EXAMPLE-7

- In a railway yard, goods trains arrive at a rate of 30 trains per day Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 min. Calculate

(a) expected queue size (line length)

(b) probability that the queue size exceeds 10

If the input of trains increases to an average of 33 per day, what will be change in (a) and (b)?

SOLUTION

From the data of the problem, we have

$\lambda = 30$ trains per day and $\mu = 40$ trains per day

The traffic intensity then is, $\rho = \lambda/\mu = 0.75$

(a) Expected queue size (line length)

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{9}{4} = 2.25$$

(b) Probability that the queue size exceeds 10:

$$P(n > 10)$$

$$= \rho^{11} = 0.75^{11} = 0.0422$$

If the input increases to 33 trains per day, then we have

$\lambda = 33$ trains per day and $\mu = 40$ trains per day

Thus, traffic intensity, $\rho = \frac{\lambda}{\mu} = 0.825$

Hence, recalculating the values for (a) and (b)

$L_q = 3.8892$

and $P(n > 10) = \rho^{11} = (0.825)^{11} = 0.120$

EXAMPLE-8

- Arrivals at telephone booth are considered to be Poisson with an average time of 10 min. between one arrival and the next .The length of phone calls is assumed to be distributed exponentially with a mean of 3 min.
- (a) What is the probability that a person arriving at the booth will have to wait?
- (b) What is the probability that it will take a customer more than 10 min. altogether to wait for the phone and complete his call?
- (c) The telephone department will install a second booth when convinced that an arrival has to wait for at least 3 min for phone call. By how much should the flow of arrivals increase in order to justify a second booth?

SOLUTION

From the data of the problem, we have

$\lambda = 1/10 = 0.10$ person per min. and

$\mu = 1/3 = 0.33$ person per min.

(a) Probability that a person has to wait at the booth

$$P(n > 0) = 1 - P_0 = \lambda/\mu = 0.3$$

(b) Probability of waiting for 10 min. or more is given by

$$P(W_s \geq 10) = e^{-(\mu-\lambda)10} = e^{-2.3} = 0.1002$$

(b) The installation of second booth will be justified only if waiting time is more than 3.Let λ' be the increased arrival rate Then the expected waiting time in the queue will be

$$W_q = \frac{\lambda'}{\mu(\mu - \lambda')}$$

where $W_q = 3$ (given)

$$\lambda' = 0.16$$

Hence, the increases in the arrival rate is $0.16 - 0.10 = 0.06$ arrivals per min.

EXAMPLE-9

A warehouse has only one loading dock manned by a three person crew. Trucks arrive at the loading at an average rate of 4 trucks per hour and the arrival rate is Poisson distributed. The loading of truck takes 10 min. on an average and can be assumed to be exponentially distributed The operating cost of a truck is Rs 20 per hour and the members of the loading crew are paid Rs 6 each per hour. Would you advice the truck owner to add another crew of three persons?

SOLUTION

- From the data of the problem, we have $\lambda = 4$ per hour, and $\mu = 6$ per hour
- **For Existing Crew**
- $\text{Total hourly cost} = \text{loading crew cost} +$operating cost
- $= \{(\text{number of loaders}) \times (\text{Hourly wage rate})\} + \{total\ trucks\ in\ the\ system(L_s) \times (\text{hourly operating cost})\}$
- $= 6 \times 3 + \frac{1}{6-4} \times 4 \times 20 = \text{Rs } 58$ per hour

- **After Proposed Crew**
- Total hourly cost
- $= 6 \times 6 + \frac{1}{12-4} \times 4 \times 20 = \text{Rs } 46$ per hour
- Since the total hourly cost after the addition of another crew of three persons is less than the existing cost, therefore, the truck owner must add a crew of another 3 loaders

EXAMPLE-10

A road transport company has one reservation clerk on duty at a time. He handles information of bus schedule and makes reservations .Customers arrive at a rate of 8 per hour and the clerk can, on an average, service 12 customers per hour After stating your assumptions, answer the following:

(a) What is the average number of customers waiting for the service of the clerk?

(b) What is the average time a customer has to wait before being served?

(c) The management is contemplating to install a computer system for handling information and reservations This is expected to reduce the service from 5 to 3 min. The additional cost of having the new system works out to Rs 50 per day. If the cost of goodwill of having to wait is estimated to be 12 paise per min. spent waiting, before being served, should the company install the computer system? Assume an 8 hours working day.

SOLUTION

It is given that $\lambda = 8$ per hour; $\mu = 12$ per hour

(a) The average number of customers waiting for the service in the system are

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{64}{12 \times 4} = \frac{4}{3}$$

(b) The average time spent by a customer in the queue before being served is

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{8}{12 \times 4} = \frac{1}{6}\ hour = 10\ min$$

(c) With existing system

The average cost for a customer's waiting time in the system, is $0.12 \times W_q$

Also, there are 8 arrivals per hour or in 8 hours $64(= 8 \times 8)$ customers request service at a total goodwill cost of $0.12 \times W_q \times 64$

$$= 0.12 \times 10 \times 64 = 76.8\, rupees$$

By installing a computer system, the computer will increase a clerks service rate upto $\mu = 20$ customer per hour (3 customers per min.) Thus, the average time spent by a customer waiting will be

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{8}{20 \times 12} = \frac{1}{30}\ hour = 2\ min$$

and the average daily queuing (or goodwill) cost would be reduced to: $64 \times \left(0.12 \times W_q\right)$

An additional cost of having the computer would be Rs 50 per day Thus, the average total daily cost would be

$$\text{TC} = \text{Computer Cost} + \text{Goodwill Cost} =$$
$$= 50 + (64 \times 0.12 \times 2) = 65.36$$

This cost is less than existing goodwill loss cost Hence, company can install a computer

MODEL 2: $\{(M/M/1):(N/FCFS)\}$

- **Exponential Service - Finite (or Limited)Queue**
- This model is based on all assumption of Model 1, expect a limit on the capacity of the system to accommodate only N customers .
- This implies that once the line reaches its maximum length of N customers, no additional customer will be allowed to enter into the system
- A finite queue may arise due to physical constraint such as emergency room in hospital; one man barber shop with certain number of chairs for waiting customers, etc.

PERFORMANCE MEASURES FOR MODEL 2

- $$P_0 = \begin{cases} \left(\dfrac{1-\rho}{1-\rho^{N+1}}\right) & ; \dfrac{\lambda}{\mu} \neq 1 \\ \dfrac{1}{N+1} & ; \dfrac{\lambda}{\mu} = 1 \end{cases}$$

0

- $$P_n = \begin{cases} \left(\dfrac{1-\rho}{1-\rho^{N+1}}\right)\rho^n & ; n \leq N; \dfrac{\lambda}{\mu} \neq 1 \\ \dfrac{1}{N+1} & ; \dfrac{\lambda}{\mu} = 1 \end{cases}$$

- **1. Expected number of customers in the system:**
- $L_s = \sum_{n=0}^{N} nP_n$
- $$L_S = \begin{cases} \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}}; & \rho \neq 1(\lambda \neq \mu) \\ \frac{N}{2}; & \rho = 1(\lambda = \mu) \end{cases}$$
- **2. Expected number of customers waiting in the queue:**
- $L_q = L_S - (1 - P_0)$

- **3. Expected waiting time of customer in the system (waiting + service):**
- $W_s = \frac{L_s+1}{\mu}$
- **4. Expected waiting time of a customer in the queue:**
- $W_q = W_s - \frac{1}{\mu} = \frac{L_s}{\mu}$

- **Potential customer lost** $(= \textbf{time for which sustem is busy})$
- $P_N = P_0 \rho^N$
- Effective arrival rate, $\lambda_{\text{eff}} = \lambda(1 - P_N)$
- Effective traffic intensity, $\rho_{\text{eff}} = \lambda_e / \mu$

Q.1 In an M/M/1/4 Queuing system with Poisson Model $\lambda = 5$ and $\mu = 15$.

(a)Find the Probability that there is nobody in the system. Also find this probability if $\mu = \lambda$.

(b) Find the Probability that there will be 4 customers in the system. Also find this probability if $\mu = \lambda$.

SOLUTION

It is given that $\lambda = 5$; $\mu = 15$; $N = 4$

$\therefore \rho = \frac{5}{15} = \frac{1}{3}$

(a) P (nobody in the system)$= P_0 = \frac{1-\rho}{1-\rho^{N+1}} =$ 0.669

If $\lambda = \mu$ then $P_0 = \frac{1}{N+1} = 0.2$

(b) P (4 customers in the system)

$$= P_4 = \left(\frac{1-\rho}{1-\rho^{N+1}}\right)\rho^4 = 0.0082$$

If $\lambda = \mu$ then $P_4 = \frac{1}{N+1} = 0.2$

Q.2 a small Aerodrome has capacity of handling one plane at a time. The area available can accommodate only 2 planes in the waiting for take off. Aeroplanes arrive at the rate of 4 planes per day (in 8 hour shift), the aerodrome can handle on an average 4 planes per day.

Find the probabilities that there will be 1,2 or 3 planes at the aerodrome.

Find the expected number of planes at the aerodrome and in the queue.

SOLUTION

It is given that $\lambda = 4$; $\mu = 4$; $N = 3$

Here $\lambda = \mu$

$$\therefore P(1\, plane) = P_1 = \frac{1}{N+1} = 0.25$$

$$\therefore P(2\, planes) = P_2 = \frac{1}{N+1} = 0.25$$

$$\therefore P(3\, planes) = P_3 = \frac{1}{N+1} = 0.25$$

Expected number of planes at the aerodrome

$$= \; L_s = \frac{N}{2} = \frac{3}{2}$$

Expected number of planes in the queue

$$= L_q = L_s - (1 - P_0) = \frac{3}{2} - \left(1 - \frac{1}{4}\right) = \frac{3}{4}$$

Q.3 a hospital which has only one operation theatre , has the arrival rate of 3 patients per day (8 hours) and the expected time of an operation is 0.25 day. The hospital has capacity to take care of 3 patients (including the one on the operation table) at the most.

(a) Find the average number of patients in the hospital in the queue.

(b) How will the result change if the arrival rate is 4 patients per day.

SOLUTION

Case-1: It is given that $\lambda = 3$; $\mu = 4$; $N = 3$

Average number of patients in the queue

$$= L_q = L_s - (1 - P_0)$$

Now $L_s = \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}} = 1.1485$

$P_0 = \frac{1-\rho}{1-\rho^{N+1}} = 0.366$

$\therefore L_q = 0.5145$

Case-2 : It is given that $\lambda = 4$; $\mu = 4$; $N = 3$

Average number of patients in the queue

$$= L_q = L_s - (1 - P_0)$$

Now $L_s = \frac{N}{2} = \frac{3}{2}$; $P_0 = \frac{1}{N+1} = \frac{1}{4}$

$L_q = L_s - (1 - P_0) = \frac{3}{2} - \left(1 - \frac{1}{4}\right) = \frac{3}{4}$

Q. 4 A scooter mechanic can accommodate 5 scooters at a time, 4 in the queue and 1 being repaired. The number of scooters arriving at the center follows Poisson distribution with mean 6 per hour. The scooter repair time has an exponential distribution with the mean of four per hour.

(i) What percentage of time the mechanic is idle?
(ii) What percentage of the customers are turned away?
(iii) What is the effective arrival rate?
(iv) What is the expected number of scooters waiting for repair?
(v) What is the average time a customer will be at the auto center?

SOLUTION

It is given that $\lambda = 6$; $\mu = 4$; $N = 5$

(i) percentage of time the mechanic is idle

$= P_0 = \frac{1-\rho}{1-\rho^{N+1}}$ =0.0481

4.81% of time the mechanic is idle

(ii) percentage of the customers are turned away = $P_N = P_0\rho^N = 0.3654$

36.54% of the customers are turned away

(iii) Effective arrival rate, $\lambda_{\text{eff}} = \lambda(1 - P_N) =$ 3.8076

(iv) Expected number of scooters waiting for repair $= L_q = L_s - (1 - P_0)$

Now $L_s = \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}}$ =3.5774

$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$ =0.0481

$\therefore L_q = 2.6255$

(v) Average time a customer will be at auto center = $W_s = \frac{L_s+1}{\mu} = 1.14435$ hour

EXAMPLE-5

- Consider a single server queuing system with Poisson input and exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hour and the maximum permissible calling units in the system is two. Derive the steady state probability distribution of number of calling units in the system, and then calculate the expected number in the system.

SOLUTION

- From the data of the problem, we have
- $\lambda = 3$ units per hour; $\mu = 4$ units per hour, and $N = 2$
- The traffic intensity, $\rho = \lambda/\mu = 3/4 = 0.75$
- The steady state probability distribution of the number of n customers (calling units) in the system
- $P_n = \frac{(1-\rho)\rho^n}{1-\rho^{N+1}}$
- $= (0.43)(0.75)^n;\ \rho \neq 1$
- $P_0 = \frac{(1-\rho)}{1-\rho^{N+1}} =$
- 0.431

- The expected number of calling units in the system is given by:
- $L_S = \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}}$
- OR
- $L_S = \sum_{n=1}^{N} nP_n$
- $= \sum_{n=1}^{2} n(0.43)(0.75)^n$
- $= 0.81$

EXAMPLE-6

- If in a period of 2 hours, in a day (08:00 to 10:00 am), trains arrive at the yard every 20 min. but the service time continues to remain 36 min. then calculate, for this period
- **(a)** The probability that the yard is empty, and
- **(b)** The average number of trains in the system, on the assumption that the line capacity of the yard is only limited to 4 trains.

SOLUTION

- From the data of the problem, we have
- $\lambda = \frac{1}{20}$trains per minute; μ =1/36 trains per minute, and $N = 4$
- The traffic intensity, $\rho = \lambda/\mu = 36/20 = 1.8$
- (a) Probability that the yard is empty $=\ P_0$
- $= \frac{1-\rho}{1-\rho^{N+1}}$
- $= 0.045$

- (b) Average no. of trains in the system= L_s
- $= \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}}$
- $= 3.03$