

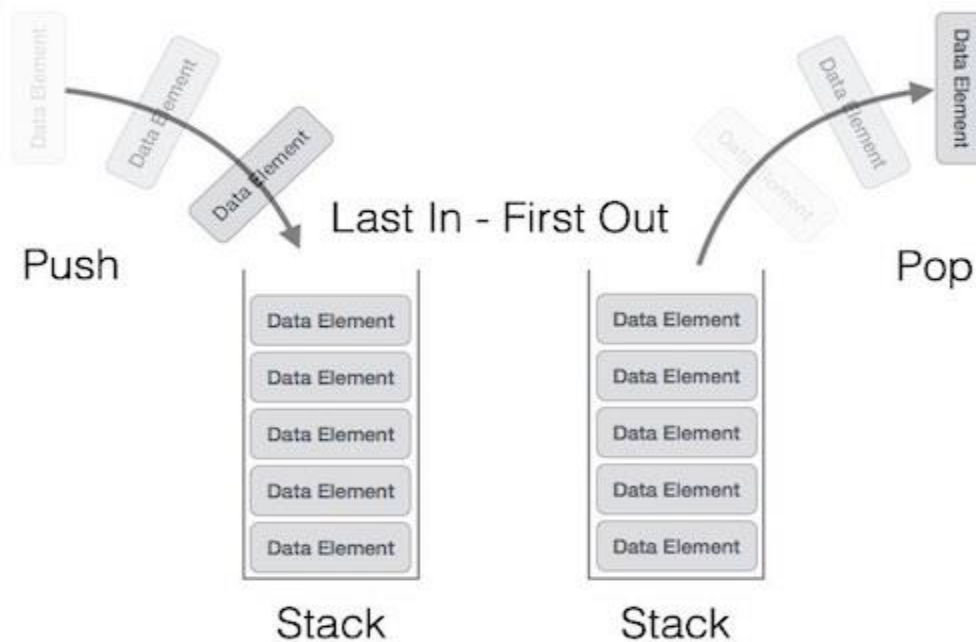


Stacks

Stacks

2

- Stack is a linear data structure which follows a particular order in which the operations are performed.
- The order may be LIFO (Last In First Out) or FILO (First In Last Out).



What is this good for ?

- To store history in a Web browser
- Undo sequence in a any application software or text editor
- Saving local variables during function calls
- Recursions
- Watchlists?

A Stack

- Definition:
 - An ordered collection of homogenous data items
 - Can be accessed at only one end (the top)
- Operations:
 - Create an empty stack
 - check if it is empty
 - Push: add an element to the top
 - Pop: remove the top element
 - Peek: retrieve the top element(Not the deletion)
 - Destroy : remove all the elements one by one and destroy the data structure

The Stack ADT: Value definition

Abstract typedef StackType(ElementType
ele)

Condition: none

Stack ADT: Operator definition

1. Abstract StackType CreateStack()

Precondition: none

Postcondition: EmptyStack is created

2. Abstract StackType PushStack(StackType Stack, ElementType Element)

Precondition: Stack not full or NotFull(Stack)= True

Postcondition: stack= stack + Element at the top

Or Stack= original stack with new Element at the top

Stack ADT: Operator definition

3. Abstract ElementType PopStack(StackType stack)

Precondition: Stack not empty or NotEmpty(Stack)= True

Postcondition: PopStack= element at the top,

Stack = stack - Element at the top

Or Stack= original stack without top Element

4. Abstract DestroyStack(StackType Stack)

Precondition: Stack not empty or NotEmpty(Stack)= True

Postcondition: Element from the stack are removed one by one starting from top to bottom.

Empty(Stack)= True

Stack ADT: Operator definition

5. Abstract Boolean NotFull(StackType stack)

Precondition: none

Postcondition: NotFull(Stack)= true if Stack is not full

NotFull(Stack)= False if Stack is full.

6. Abstract Boolean NotEmpty(StackType stack)

Precondition: none

Postcondition: NotEmpty(Stack)= true if Stack is not empty

\sim Empty(Stack)= False if Stack is empty.

Stack ADT: Operator definition

7. Abstract ElementType Peek(StackType stack)

Precondition: Stack not empty or NotEmpty(Stack)= True

Postcondition: Peek= element at the top,

Stack = original stack

Exercise: Stacks

–Push(8)

–Push(3)

–Pop()

–Push(2)

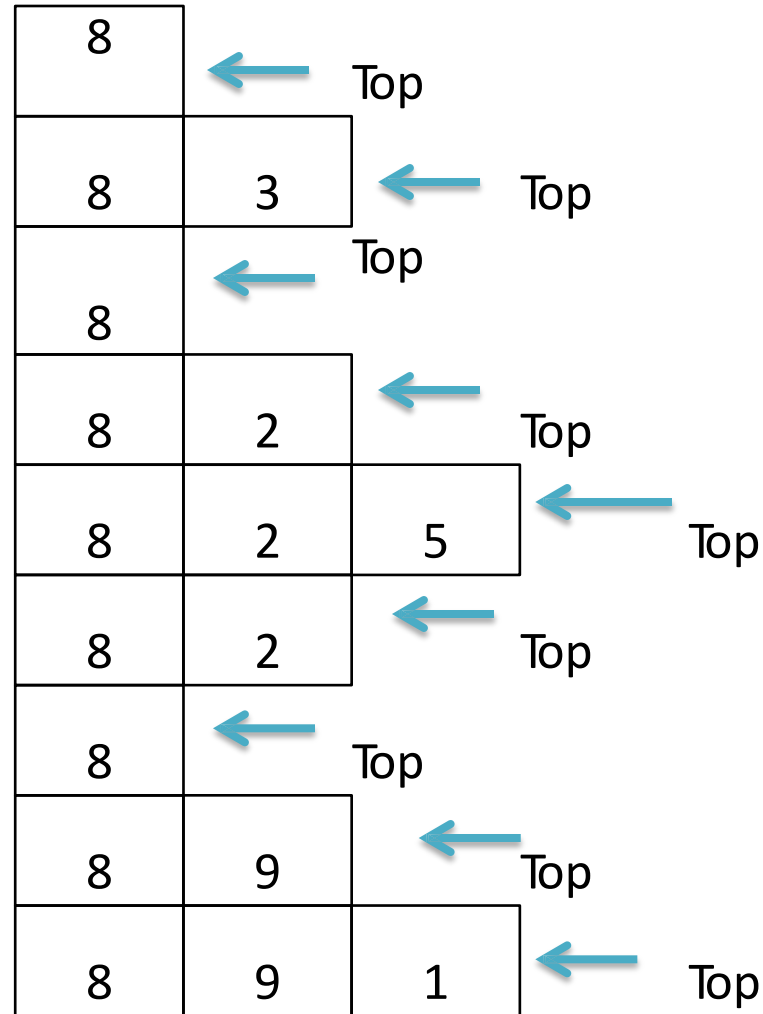
–Push(5)

–Pop()

–Pop()

–Push(9)

–Push(1)



Implementing a Stack

- At least three different ways to implement a stack
 - array
 - linked list
- Which method to use depends on the application
 - what advantages and disadvantages does each implementation have?

Implementing Stacks: Array

- Advantages -best performance
- Disadvantage - fixed size
- Basic implementation
 - initially empty array
 - field to record where the next data gets placed into
 - if array is full, push() returns false
 - otherwise adds it into the correct spot
 - if array is empty, pop() returns null
 - otherwise removes the next item in the stack

Implementing a Stack: Linked List

- Advantages:
 - always constant time to push or pop an element
 - can grow to an infinite size
- Disadvantages
 - the common case is the slowest of all the implementations
- Basic implementation
 - list is initially empty
 - *push()* method adds a new item to the head of the list
 - *pop()* method removes the head of the list

Writing an algorithm

- Specify algorithm name, list of inputs, data types of the inputs and return data types clearly
- Specify purpose of the algorithm
- Algo should produce at least one Output
- Definiteness: Each step must be clear and unambiguous.
- Should react correctly to all valid and invalid inputs
- Finiteness: If we trace the steps of an algorithm, then for all cases, the algorithm must terminate after a finite number of steps.
- Effectiveness:
- Comment Session: Comment is additional info of program for easily modification. In algorithm comment would be appear between two square bracket []. For example: [this is a comment of an algorithm].

Stack ADT: Array Implementation

1. Algorithm StackType CreateStack()

//This Algorithm returns an empty stack- stack

```
{ integer StackTop = -1;  
Return stack;  
}
```

2. Algorithm StackType PushStack(StackType Stack, ElementType Element)

// This algorithm accepts a StackType stack and ElementType Element as input and adds 'Element' at the top of 'stack'. StackTop is an integer index that holds current value of StackTop position.

```
{  
    if NotFull(Stack)= True  
        stack[++StackTop]= Element  
    Else "Error Message"  
}
```

Stack ADT: Array Implementation

3. Algorithm ElementType PopStack(StackType stack)

// This algorithm accepts a stack as input and returns 'Element' at the top of 'stack'.

```
{ if NotEmpty(Stack)= True
Return Stack[StackTop--]
Else print "Error Message"
}
```

4. Abstract DestroyStack(StackType Stack)

//This algorithm returns all the elements from Stack in LIFO order and destroys the data structure

```
{ if NotEmpty(Stack) = true
    while(NotEmpty(Stack))
        print PopStack(Stack)
    else print "Error Message"
}
```


Stack ADT: Array Implementation

5. Abstract Boolean NotFull(StackType stack)

// This algorithm returns true if the stack is not full, false otherwise.

```
{ if NotFull(Stack)
    retrun True
else
    return False
}
```

6. Abstract Boolean NotEmpty(StackType stack)

// This algorithm returns true if the stack is not empty, false otherwise.

```
{ if NotEmpty(Stack)
    retrun True
else
    return False
}
```

Stack ADT: Array Implementation

7. Abstract ElementType Peek(StackType stack)

//// This algorithm accepts a stack as input and returns
'Element' at the top of 'stack'.

{ if NotEmpty(Stack)= True

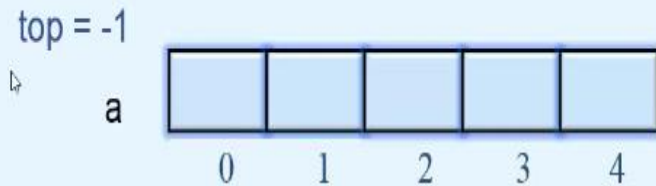
Return Stack[StackTop]

Else print "Error Message"

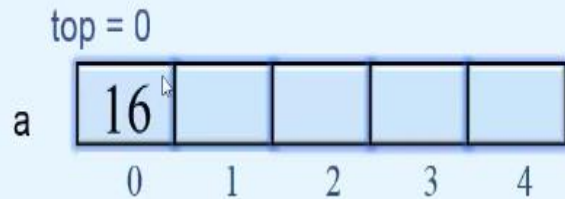
}

PUSH Operation on Stack

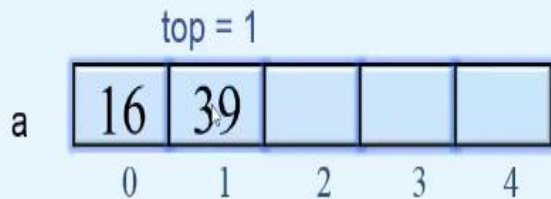
Empty stack



Push 16



Push 39



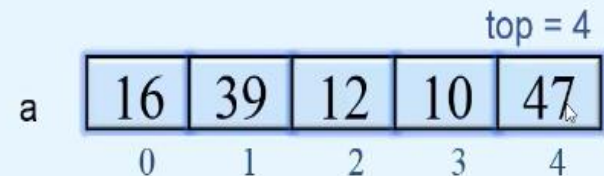
Push 12



Push 10



Push 47



POP Operation on Stack

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Empty stack

top = -1



a



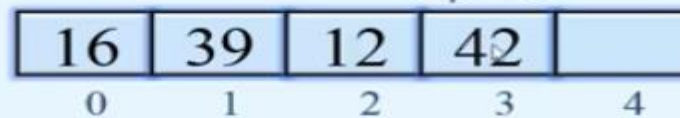
Stack Underflow
condition

If(top == -1)

Push 42

top = 3

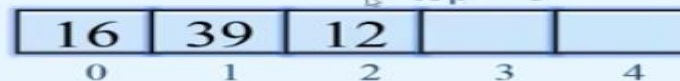
a



Pop

top = 3

a

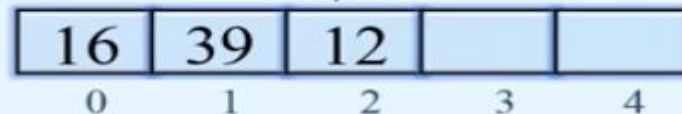


Popped item = 42

Pop

top = 2

a



Implementing Stacks: Linked List

```
Struct NodeType{  
    ElementType Element;  
    NodeType Next;  
}
```

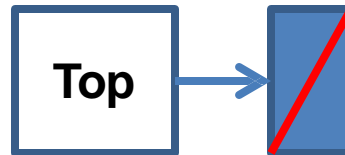
1. Algorithm StackType CreateStack()

//This Algorithm creates and returns an empty stack- pointed by a pointer-Top

{ createNode(Top);

Top =NULL;

}



Implementing Stacks: Linked List

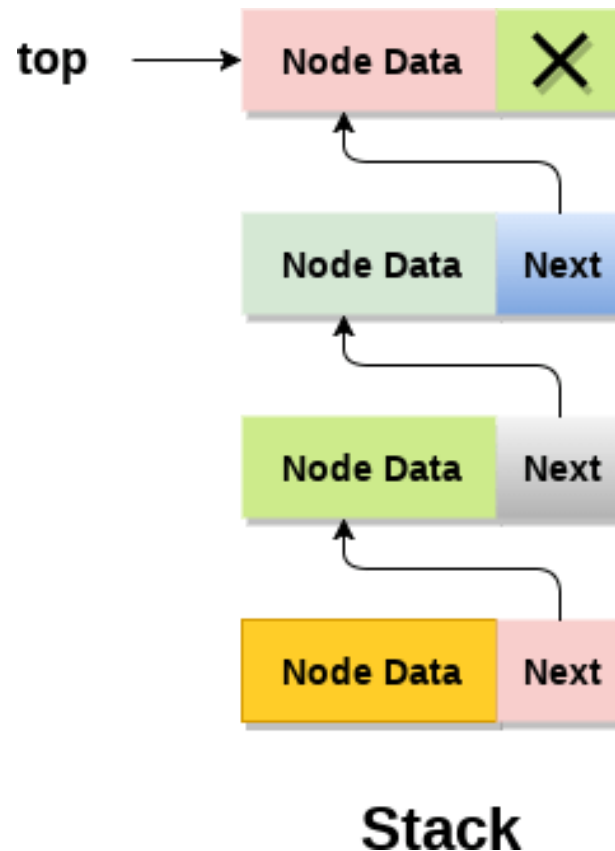


Image Courtesy: <https://www.javatpoint.com/ds-linked-list-implementation-of-stack>

Implementing Stacks: Linked List

2. StackType PushStack(StackType Stack, NodeType NewNode)

// This Algorithm adds a NewNode at the top of 'stack'. Top is an pointer that points to the topmost Stack node.

```
{ if Top ==NULL // first element in stack
    NewNode->Next = NULL;
    Top=NewNode;
Else NewNode->Next=Top;// General case
    Top=NewNode;
}
```

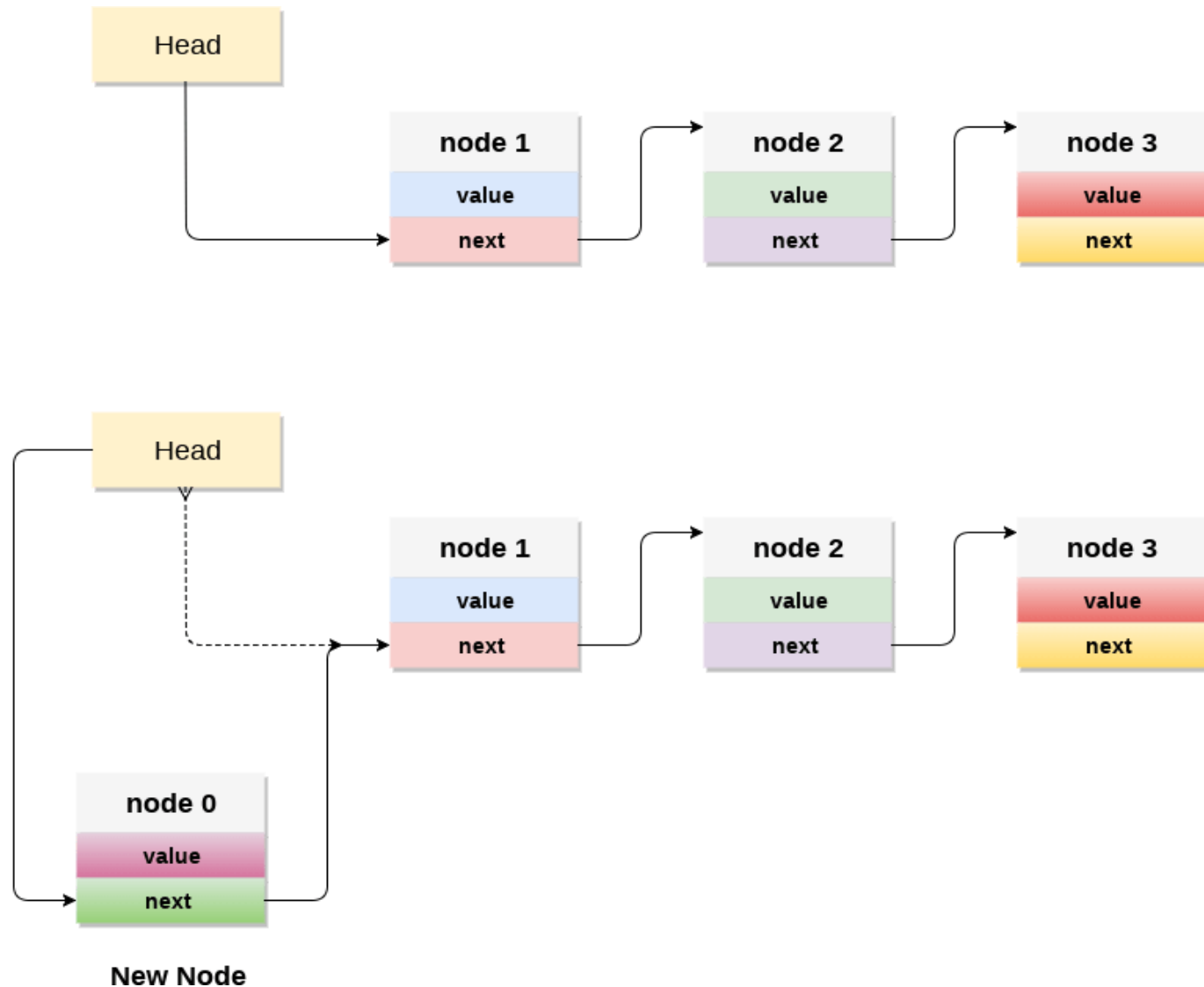


Image Courtesy: <https://www.javatpoint.com/ds-linked-list-implementation-of-stack>

Implementing Stacks: Linked List

3. Algorithm ElementType PopStack(StackType stack)

//This algorithm returns value of ElementType stored in topmost node of stack. Temp is a temporary node used in pop process.

```
{ if Top==NULL
    Print "Underflow"
Else
    {
        createNode(Temp);
        Temp=Top;
        Top= Top->next;
        Return(temp->Data);
    }
}
```

Implementing Stacks: Linked List

4. Abstract DestroyStack(StackType Stack)

//This algorithm returns values stored in data structure and free the memory used in data structure implementation.

```
{ { if Top==NULL
    Print "Underflow"
Else   createNode(Temp);
        while(NotEmpty(stack))
        {
            Temp=Top;
            Top= Top->next;
            Return(temp->Data);
        }
}
```

Implementing Stacks: Linked List

5. Abstract ElementType Peek(StackType stack)

//This algorithm returns value of ElementType stored in topmost node of stack.

```
{ if Top==NULL
    Print "Error Message"
Else
    Return(Top->Data);
}
```

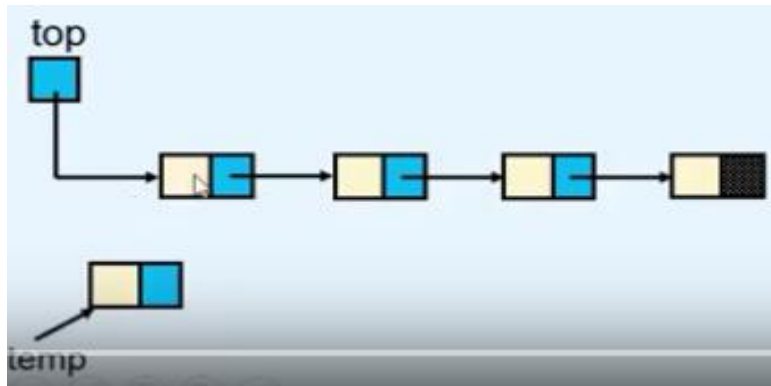
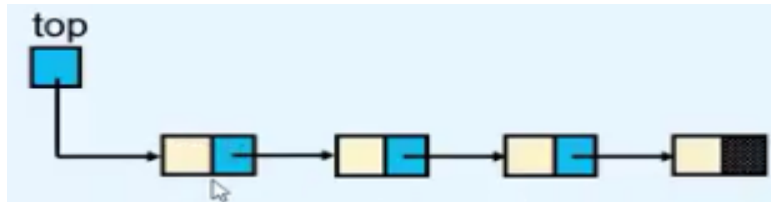
6. Abstract DisplayStack(StackType stack)

//This algorithm Prints all the Elements stored in stack. Temp purpose?

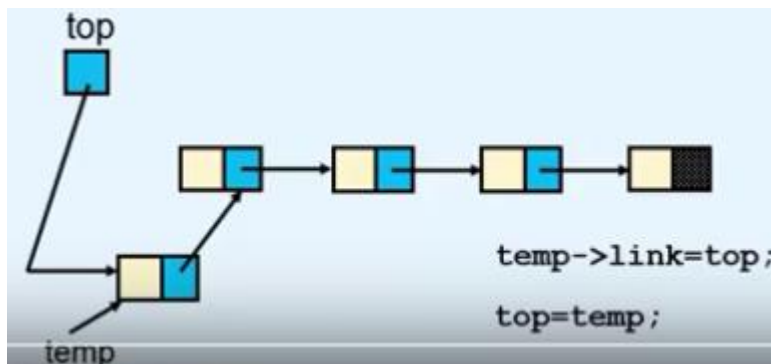
```
{ if Top==NULL
    Print "Error Message"
Else {createNode(Temp)
    Temp=Top;
    While(Temp!=Null)
        Print(Temp->Data);
        Temp= Temp->next;
    }
}
```

PUSH Operation on Stack

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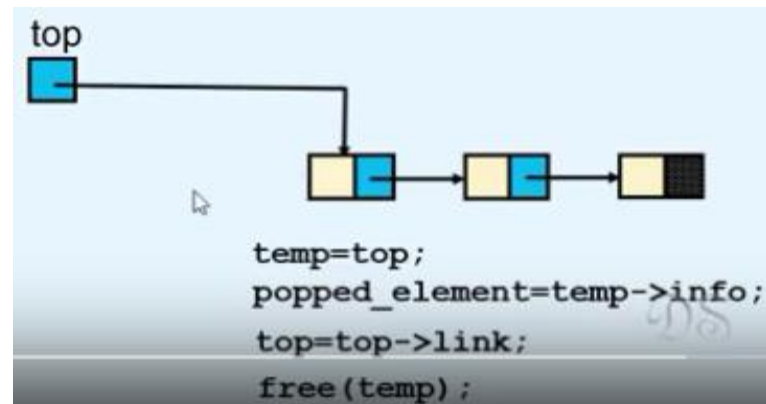
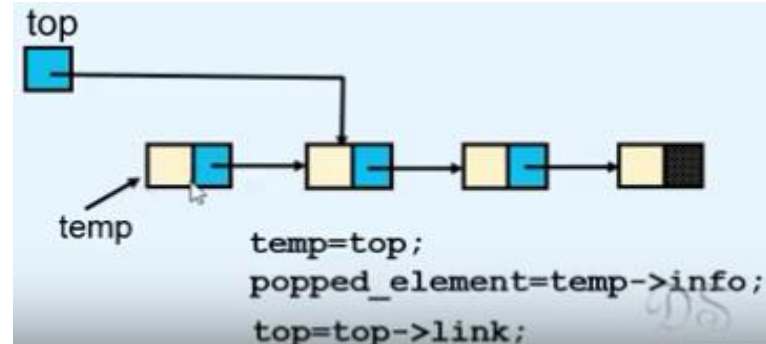
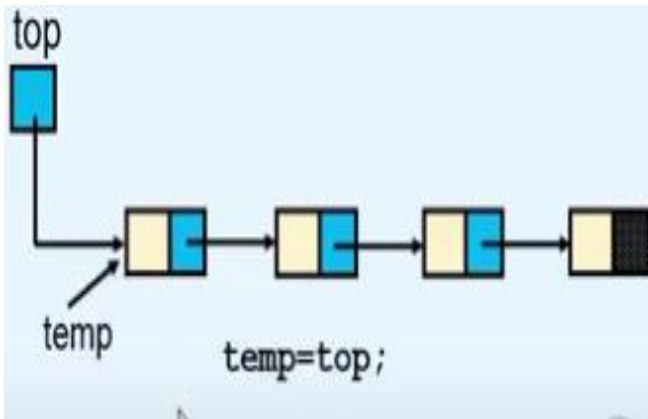
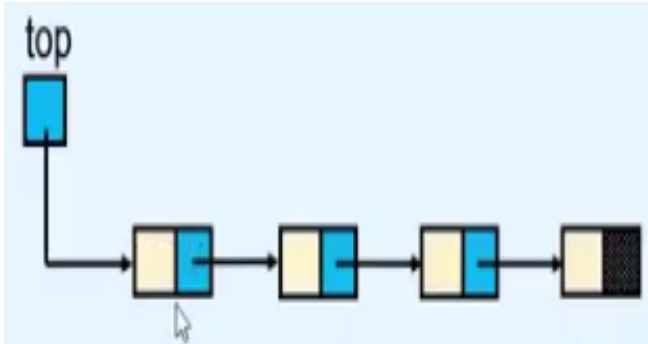
```
tmp = (struct node  
*)malloc(sizeof(struct node));
```



```
tmp->link=top;  
top=tmp;
```

POP Operation on Stack

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Application of Stack

Parentheses Matching Algorithm

Algorithm Boolean ParenMatch(X, n):

Input: An array X of n tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number

Output: **true** if and only if all the grouping symbols in X match

Let S be an empty stack

for $i=0$ to $n-1$ **do**

if $X[i]$ is an opening grouping symbol **then**

$S.\text{push}(X[i])$

else if $X[i]$ is a closing grouping symbol **then**

if $S.\text{isEmpty}()$ **then**

return false {nothing to match with}

if $S.\text{pop}()$ does not match the type of $X[i]$ **then**

return false {wrong type}

if $S.\text{isEmpty}()$ **then**

return true {every symbol matched}

else

return false {some symbols were never matched}

Parentheses Matching Algorithm

| | | | | | | | | | |
|--------------------|--|-----------------|--|---------------------------------------|--|-----------------|--|---------------------------------------|--|
| i/p string= {(())} | | | | | | | | | |
| i/p= {, push | | i/p= (, push | | i/p=), pop; ToS=(, match= true | | i/p= (, push | | i/p=), pop; ToS=(, match= true | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| { | | { | | { | | { | | { | |
| step 1 | | Step 2 | | step 3 | | Step 4 | | step 5 | |
| | | | | | | | | | |
| | | | | | | | | | |

After step 6, stack is empty. So given string of parenthesis is balanced

Parentheses Matching Algorithm

| i/p string= {()}{()} | | | | | | | |
|----------------------|----------------|---|-----------------|----------------|---|--|--|
| i/p= {, push | i/p= (push | i/p=), pop; ToS=(, match= true | i/p= {, push | i/p= (push | i/p=), pop; ToS=(, match= true | i/p= }, pop; ToS=}, match= true | i/p= }, pop; underflo w; Error |
| | | | | (| | | |
| { | ({ | { | { { | { { { | { { | | |
| step 1 | Step 2 | step 3 | Step 4 | step 5 | step 6 | step 7 | step 8 |

After step 8, stack is nonempty but there are more characters in input string. So given string of parenthesis is not balanced

Application of Stack –Reversal of a String

- Push each character of the string on the stack.
- When whole string is pushed on the stack
- Pop the characters from the stack to get the reversed string

Application of Stack –Reversal of a String

- Try Writing the Code snippet for it.....

Application of Stack –Polish Notation

- Polish notation (PN), also known as
 - simply prefix notation,

Reverse Polish Notation-Postfix Notation

- Reverse Polish notation (RPN), in which operators follow their operands.
- **The term Polish notation is sometimes taken (as the opposite of infix notation) to also include reverse Polish notation.**

Expression Representation

- An expression is defined as a number of operands or data items combined using several operators.
- 3 popular methods for representation:
 - Infix Notation
 - Prefix Notation
 - Postfix Notation

Infix Notation

- Used in General Mathematics
- Operator is written in between the operands
- Eg- $a+b$, $x+y*z$
- Called infix because of the position of the operator in expression.

Evaluation-

- Evaluated left to right but operator precedence must be taken into consideration
- **Not used inside computer, due to additional complexity of handling of precedence**

Prefix Notation

- Operator is written before the operands
- Also called Polish Notation
- Eg- $+ab$, $+x*yz$

Postfix Notation

- Operator is written after the operands
- Also called Reverse Polish or Suffix Notation
- Eg- $ab+$, xyz^*+

Polish Notation

- Prefix and Postfix are free from any precedence
- Computers use postfix form

Notation Conversion

- Scan the expression from left to right
- **Operator Precedence-**
 - Paranthesis evaluated first
 - After that evaluation is on the basis of operator precedence
- Logical Not
- Exponential Operator
- Multiplication/division/modulus
- Addition/Subtraction
- Left shift, Right Shift
- Relational
- Logical And
- Logical Or



Manual Conversion -Infix to Postfix

- $A + B * C$ Given infix form
- Left to right scan, Brackets first then Operator Precedence
- $A + \underline{B C *}$ Convert the multiplication
- $A B C * +$ Convert the addition

Manual Conversion -Infix to Postfix

- $A + [(B + C) + (D + E) * F] / G$

Manual Conversion -Infix to Postfix

- $A + [(B + C) + (D + E) * F] / G$
- Left to right scan, Brackets first then Operator Precedence
- $A + [(\underline{BC+}) + (D + E) * F] / G$
- $A + [(\underline{BC+}) + (\underline{DE+}) * F] / G$
- $A + [(\underline{BC+}) + (\underline{DE+}) F *] / G$
- $A + [(BC+) (DE+) F * +] / G$
- $A + [(BC+) (DE+) F * +] G /$
- $A [(BC+) (DE+) F * +] G / +$
- $ABC + DE + F * + G / +$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

- 1) $A + B + C + D$
- 2) $(A + B) / (C - D)$
- 3) $(A + B) * C - (D - E) * (F + G)$
- 4) $((A+B) - C * (D/E)) + F$
- 5) $((A + B) * (C - D) + E) / (F + G)$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

1) $A + B + C + D = A B + C + D +$

$= \underline{AB} + C + D$

$= \underline{AB+C} + D$

$= AB+C+D +$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

1) $A + B + C + D = A B + C + D +$

2) $(A + B) / (C - D) = A B + C D - /$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

1) $A + B + C + D = A B + C + D +$

2) $(A + B) / (C - D) = A B + C D - /$

3) $(A + B) * C - (D - E) * (F + G) = AB + C * DE - FG + * -$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

1) $A + B + C + D = A B + C + D +$

2) $(A + B) / (C - D) = A B + C D - /$

3) $(A + B) * C - (D - E) * (F + G) = AB + C * DE - FG + * -$

4) $((A + B) - C * (D / E)) + F = A B + C D E / * - F +$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

- 1) $A + B + C + D = A B + C + D +$
- 2) $(A + B) / (C - D) = A B + C D - /$
- 3) $(A + B) * C - (D - E) * (F + G) = AB + C * DE - FG + * -$
- 4) $((A + B) - C * (D / E)) + F = A B + C D E / * - F +$
- 5) $((A + B) * (C - D) + E) / (F + G) = A B + C D - * E + F G + /$

Manual Conversion -Infix to Prefix

- A/B^C+D
- Left to right scan, Brackets first then Operator Precedence
- $A/^BC+D$
- $/A^BC+D$
- $+/A^BCD$

Manual Conversion -Infix to Prefix

Convert the following expressions from Infix to Prefix;

- 1) $(P * Q \wedge R + S)$
- 2) $(A - B / C) * (D * E - F)$
- 3) $(A * B + (C / D)) - F$
- 4) $A + B * C - (D / E \wedge F) * G * H$
- 5) $(A + B) * C / D + E \wedge F / G$

Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1) $(P * Q \wedge R + S) = + * P \wedge Q R S$

Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1) $(P * Q^R + S) = + * P^R Q R S$

2) $(A - B / C) * (D * E - F) = * (- A / B C) (- * D E F)$

Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1) $(P * Q^R + S) = + * P^R QRS$

2) $(A - B / C) * (D * E - F) = * (- A / BC) (- * DEF)$

3) $(A * B + (C / D)) - F = - + * AB / CDF$

Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1) $(P * Q \wedge R + S) = + * P \wedge Q R S$

2) $(A - B / C) * (D * E - F) = * (- A / B C) (- * D E F)$

3) $(A * B + (C / D)) - F = - + * A B / C D F$

4) $A + B * C - (D / E \wedge F) * G * H$

$$= A + \underline{* B C} - (\underline{/ D \wedge E F}) * G * H$$

$$= A + * B C - \underline{* / D \wedge E F G} * H$$

$$= A + * B C - \underline{** / D \wedge E F G H}$$

$$= \underline{+ A * B C} - \underline{** / D \wedge E F G H}$$

$$= - + A * B C ** / D \wedge E F G H$$

Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1) $(P * Q \wedge R + S) = + * P \wedge Q R S$

2) $(A - B / C) * (D * E - F) = * (- A / B C) (- * D E F)$

3) $(A * B + (C / D)) - F = - + * A B / C D F$

4) $A + B * C - (D / E \wedge F) * G * H$

$$= A + * B C - (/ D \wedge E F) * G * H$$

$$= A + * B C - * / D \wedge E F G * H$$

$$= A + * B C - ** / D \wedge E F G H$$

$$= + A * B C - ** / D \wedge E F G H$$

$$= - + A * B C ** / D \wedge E F G H$$

5) $(A + B) * C / D + E \wedge F / G$

$$= + \underline{A B} * C / D + E \wedge F / G$$

$$= + \underline{A B} * C / D + \underline{\wedge E F} / G$$

$$= * + \underline{A B C} / D + \underline{\wedge E F} / G$$

$$= / * + \underline{A B C D} + \underline{\wedge E F} / G$$

$$= / * + \underline{A B C D} + / \underline{\wedge E F G}$$

$$= + / * + \underline{A B C D} / \underline{\wedge E F G}$$

Infix to postfix process with parenthesis

Let, X is an arithmetic expression written in infix notation. This algorithm finds the equivalent postfix expression Y.

1. Scan X from left to right and repeat Step 2 to 5 for each element of X until the Stack is empty.
 2. If an operand is encountered, add it to Y
 3. If a left parenthesis is encountered, push it onto Stack.
 4. If an operator is encountered ,then:
 - Repeatedly pop from Stack and add to Y each operator (on the top of Stack) which has the same precedence as or higher precedence than operator until an opening parenthesis is encountered.
 - Add operator to Stack.
 5. If a right parenthesis is encountered ,then:
 - Repeatedly pop from Stack and add to Y each operator (on the top of Stack) until a left parenthesis is encountered.
 - Remove the left Parenthesis.
2. END.

Input: input expression:(((A + B) * C) - ((D - E) * (F + G)))

| Input char | stack | Output |
|------------|--------|---------------|
| (| (| |
| (| ((| |
| (| ((| |
| A | ((| A |
| + | (((+ | A |
| B | (((+ | AB |
|) | ((| AB+ |
| * | ((* | AB+ |
| C | ((* | AB+C |
|) | (| AB+C* |
| - | (- | AB+C* |
| (| (- | AB+C* |
| (| (-((| AB+C* |
| D | (-((| AB+C*D |
| - | (-((- | AB+C*D |
| E | (-((- | AB+C*DE |
|) | (- | AB+C*DE- |
| * | (-(* | AB+C*DE- |
| (| (-(* | AB+C*DE- |
| F | (-(* | AB+C*DE-F |
| + | (-(*(+ | AB+C*DE-F |
| G | (-(*(+ | AB+C*DE-FG |
|) | (-(* | AB+C*DE-FG+ |
|) | (- | AB+C*DE-FG+* |
|) | EMPTY | AB+C*DE-FG+*- |

Infix to Postfix Conversion using stack

Eg- $P = (A + B) * C$

Infix to Postfix Conversion using stack

Eg- $P = (A+B)*C$

$P = ((A+B) * C)$

| Scan | Stack | Expression Q |
|------|-------|--------------|
| (| (| |
| (| ((| |
| A | ((| A |
| + | ((+ | A |
| B | ((+ | AB |
|) | (| AB+ |
| * | (* | AB+ |
| C | (* | AB+C |
|) | NULL | AB+C* |

Infix to Postfix Conversion using stack

Convert $P/(Q-R)*S+T$

Infix to Postfix Conversion using stack

Convert $P/(Q-R)*S+T=(P/(Q-R)*S+T)$

| Scan | Stack | Expression Q |
|------|--------|--------------|
| (| (| |
| P | (| P |
| / | (/ | P |
| (| (/ (| P |
| Q | (/ (| PQ |
| - | (/ (- | PQ |
| R | (/ (- | PQR |
|) | (/ | PQR- |
| * | (* | PQR-/ |
| S | (* | PQR-/S |
| + | (+ | PQR-/S* |
| T | (+ | PQR-/S*T |
|) | NULL | PQR-/S*T+ |

Infix to Postfix Conversion using stack

Convert $(P/(Q-R)*S+T)$

$(P/(Q-R)*S+T)$

| Symbol | Stack | Expression |
|--------|-------|------------|
| (| (| — |
| P | (| P |
| / | (/ | P |
| (| (/(| P |
| Q | (/(| PQ |
| — | (/(— | PQ |
| R | (/(— | PQR |
|) | (/ | PQR— |
| * | (* | PQR—/ |
| S | (* | PQR—/S |
| + | (+ | PQR—/S* |
| T | (+ | PQR—/S*T |
| | null | PQR—/S*T+ |

Notation Conversion

- Scan the expression from left to right
- **Operator Precedence-**
 - Paranthesis evaluated first
 - After that evaluation is on the basis of operator precedence
- Logical Not
- Exponential Operator
- Multiplication/division/modulus
- Addition/Subtraction
- Left shift, Right Shift
- Relational
- Logical And
- Logical Or



Infix to Postfix Conversion using stack

Convert $P = (A + (B * C - (D / E \wedge F) * G * H))$

Infix to Postfix Conversion using stack

Convert $P = (A + (B * C - (D / E \wedge F) * G * H))$

| Scan | Stack | Expression Q |
|------|--------------|-----------------|
| (| (| |
| A | (| A |
| + | (+ | A |
| (| (+ (| AB |
| B | (+ (| AB |
| * | (+ (* | AB |
| C | (+ (* | ABC |
| - | (+ (- | ABC* |
| (| (+ (- (| ABC* |
| D | (+ (- (| ABC*D |
| / | (+ (- (/ | ABC*D |
| E | (+ (- (/ | ABC*DE |
| ^ | (+ (- (/ ^ | ABC*DE |
| F | (+ (- (/ ^ | ABC*DEF |
|) | (+ (- | ABC*DEF^/ |
| * | (+ (-* | ABC*DEF^/ |
| G | (+ (-* | ABC*DEF^/G |
| * | (+ (-* | ABC*DEF^/G* |
| H | (+ (-* | ABC*DEF^/G*H |
|) | (+ | ABC*DEF^/G*H*- |
|) | null | ABC*DEF^/G*H*-+ |

Convert $P=(A+(B*C-(D/E^F)*G*H))$

| Scan | Stack | Expression Q |
|------|------------|------------------|
| (| (| |
| A | (| A |
| + | (+ | A |
| (| (+ (| AB |
| B | (+ (| AB |
| * | (+ (* | AB |
| C | (+ (* | ABC |
| - | (+ (- | ABC* |
| (| (+ (- (| ABC* |
| D | (+ (- (| ABC*D |
| / | (+ (- (/ | ABC*D |
| E | (+ (- (/ | ABC*DE |
| ^ | (+ (- (/^ | ABC*DE |
| F | (+ (- (/^ | ABC*DEF |
|) | (+ (- | ABC*DEFA^/ |
| * | (+ (-* | ABC*DEFA^/ |
| G | (+ (-* | ABC*DEFA^/G |
| * | (+ (-* | ABC*DEFA^/G* |
| H | (+ (-* | ABC*DEFA^/G*H |
|) | (+ | ABC*DEFA^/G*H*- |
|) | null | ABC*DEFA^/G*H*-+ |

Manual Evaluation of a Prefix notation

- Lets take an eg: $+5*32$
- Find an operator from left to right having 2 operands after it
- Multiplication of 3 and 2 is carried out
- Expression becomes $+56$
- Now $+$ has two operands so evaluated
- Exp=11

Manual Evaluation of a Postfix notation

- Lets take an eg: 532^*+
- Find first operator from left to right having 2 operands before it
- perform the operation
- Multiplication of 3 and 2 is carried out
- Expression becomes $56+$
- Now $+$ is evaluated
- $Exp=11$

Evaluation of postfix expression

- Create a stack for storing operands
- Scan the input expression from left to right
 - If the element is operand, push it onto the stack
 - If the element is operator, pop two operands, evaluate and push the result onto the stack
- If the expression is over, the stack contains the final answer

Input: input expression: $AB+C*DE-FG+*-$

e.g. $A=2, B=3, C=1, D=4, E=5, F=7, G=8$

| Input char | stack |
|------------|------------|
| 2 | 2 |
| 3 | 2, 3 |
| + | $(2+3)=5$ |
| 1 | 5, 1 |
| * | $(5*1)= 5$ |
| 4 | 5,4 |
| 5 | 5,4,5 |
| - | 5,-1 |
| 7 | 5,-1,7 |
| 8 | 5,-1,7,8 |
| + | 5,-1,15 |
| * | 5,-15 |
| - | 20 |

Algorithm for Evaluation of Postfix using stack

P=653+9*+

P=53+9*+

P=3+9*+

P=+9*+

Evaluate $5+3=8$

Push 8

P=9*+

P=*+

Evaluate

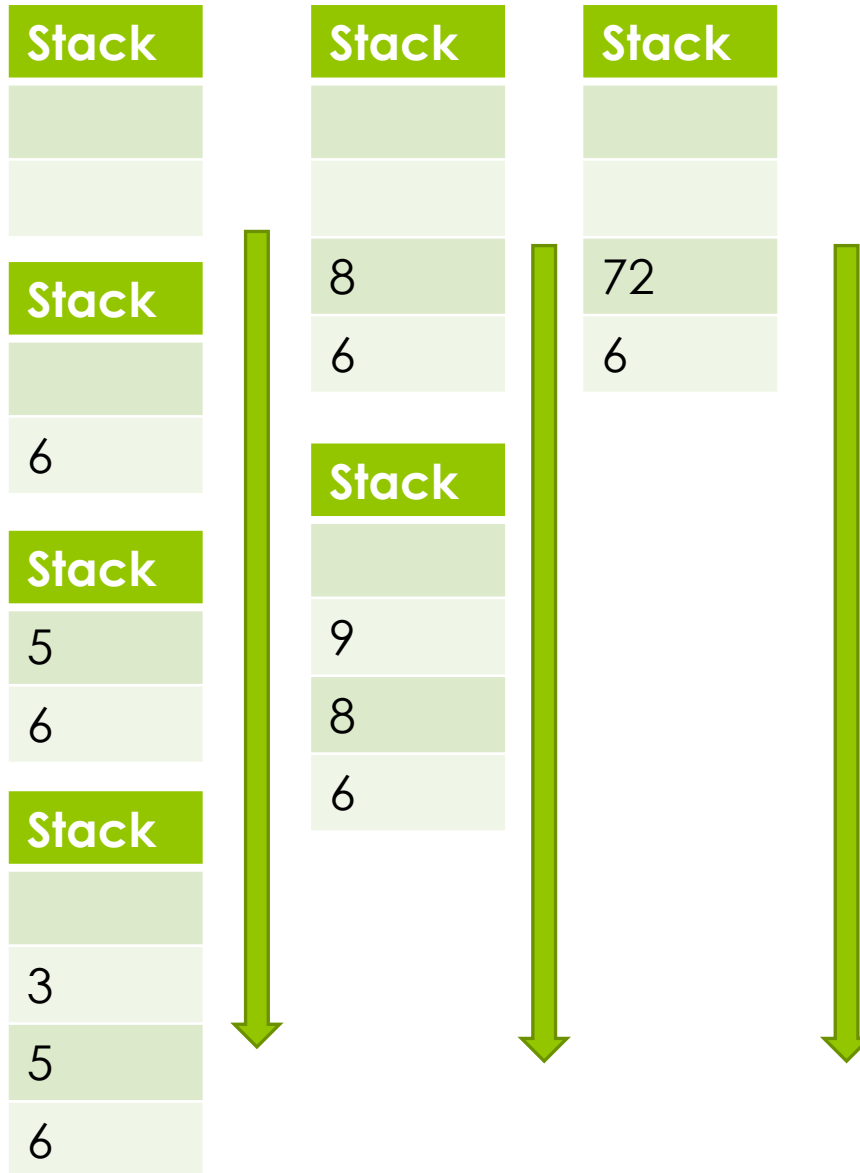
$8*9=72$

Push 72

P=+

Evaluate $6+72=78$

Ans=78



Algorithm for Evaluation of Postfix using stack

Evaluate the following expression-

- 1) 432^*+5-
- 2) 532^*+4-5+
- 3) $53+82-*$
- 4) Evaluate $562+*(12)4/-$
taking 12 as a single number

Evaluate 432^*+5-

Postfix \rightarrow a b c * + d - Let, a = 4, b = 3, c = 2, d = 5

Postfix \rightarrow 4 3 2 * + 5 -

| Operator/Operand | Action | Stack |
|------------------|---|---------|
| 4 | Push | 4 |
| 3 | Push | 4, 3 |
| 2 | Push | 4, 3, 2 |
| * | Pop (2, 3) and $3*2 = 6$ then Push 6 | 4, 6 |
| + | Pop (6, 4) and $4+6 = 10$ then Push 10 | 10 |
| 5 | Push | 10, 5 |
| - | Pop (5, 10) and $10-5 = 5$ then Push 5 | 5 |

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Example

Evaluate the postfix expression
 $5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$

(a) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(b) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(c) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(d) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(e) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(f) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(g) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(h) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



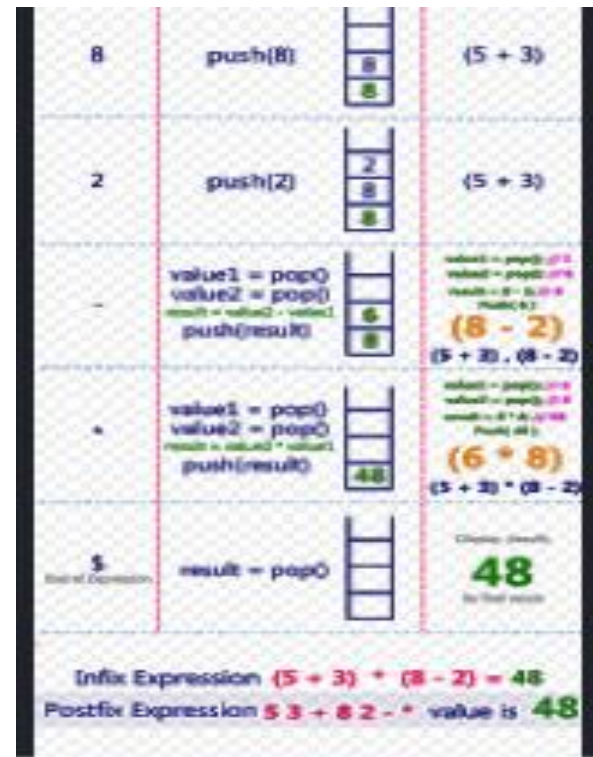
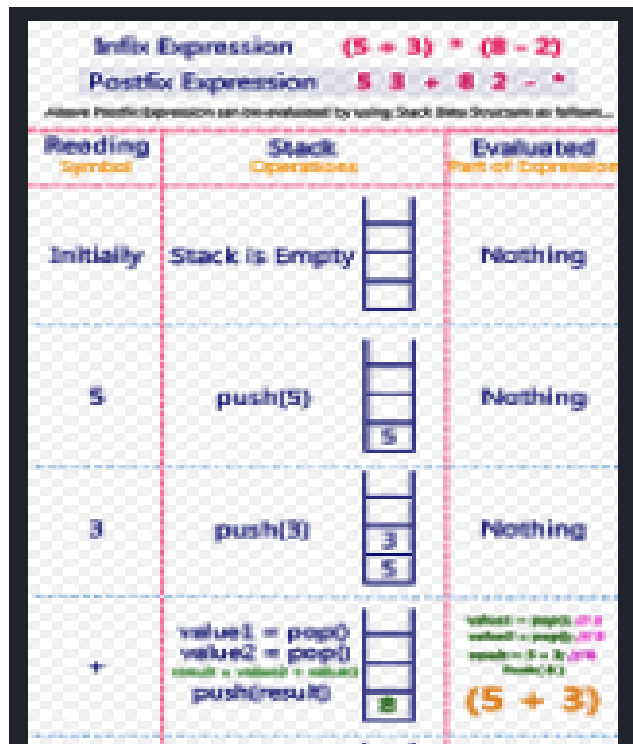
(i) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



The result of the computation is 12.

Postfix Expression 5 3 + 8 2 - *



Courtesy: http://btechsmartclass.com/data_structures/postfix-evaluation.html

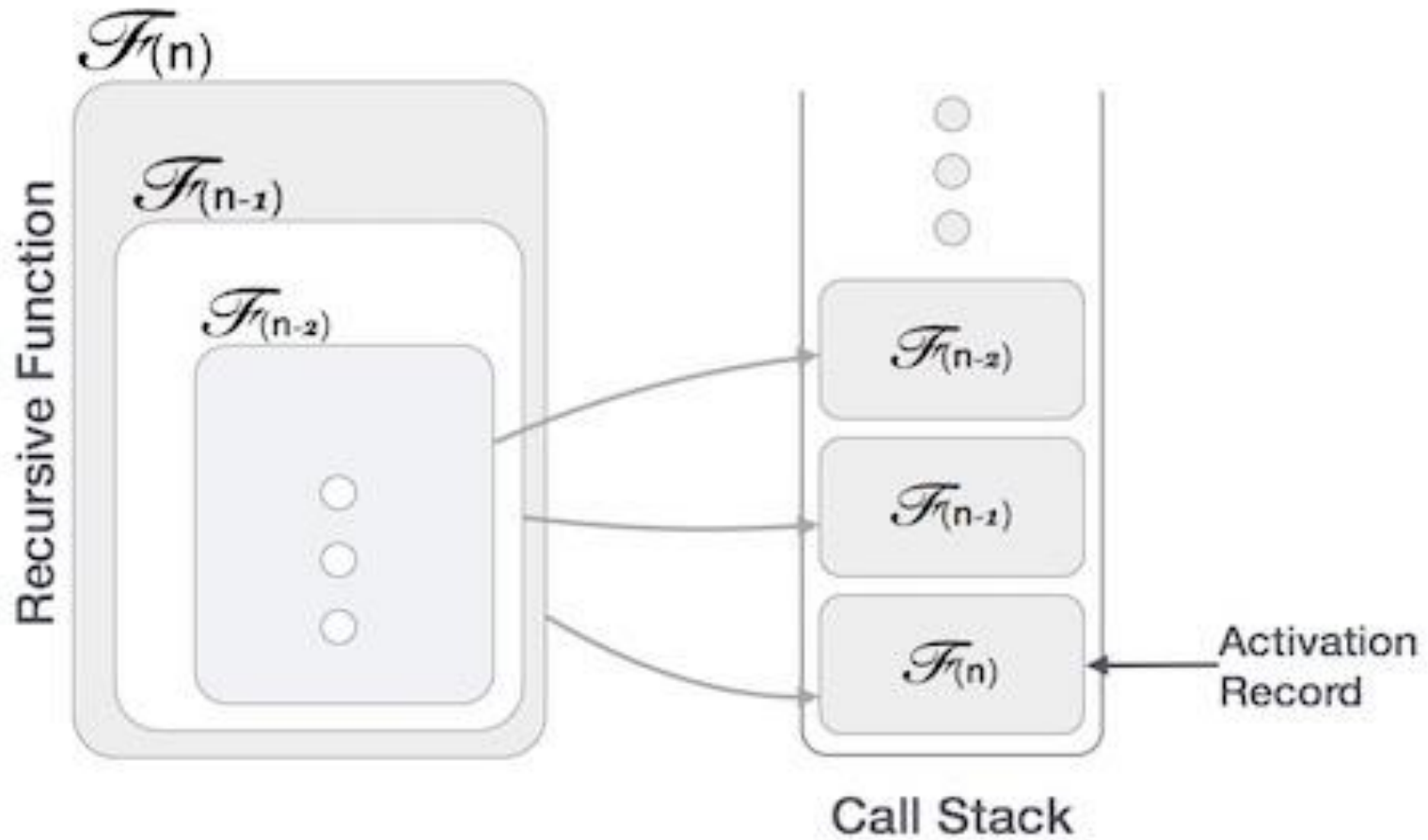
Evaluate $562+*(12)4/-$
taking 12 as a single number

$562+*124/-$

| Step | Input Symbol/Element | Stack | Intermediate Calculations Output |
|------|-------------------------------|-----------|----------------------------------|
| 1 | 5 Push | 5 | |
| 2 | 6 Push | 5, 6 | |
| 3 | 2 Push | 5, 6, 2 | |
| 4 | + Pop 2 elements and evaluate | 5 | $6 + 2 = 8$ |
| 5 | Push result 8 | 5, 8 | |
| 6 | * Pop 2 elements and evaluate | # empty | $5 \times 8 = 40$ |
| 7 | Push result 40 | 40 | |
| 8 | 12 Push | 40, 12 | |
| 9 | 4 Push | 40, 12, 4 | |
| 10 | / Pop 2 elements and evaluate | 40 | $12 / 4 = 3$ |
| 11 | Push result 3 | 40, 3 | |
| 12 | - Pop 2 elements and evaluate | # empty | $40 - 3 = 37$ |
| 13 | Push result 37 | 37 | |
| 14 | No more elements | | 37 |

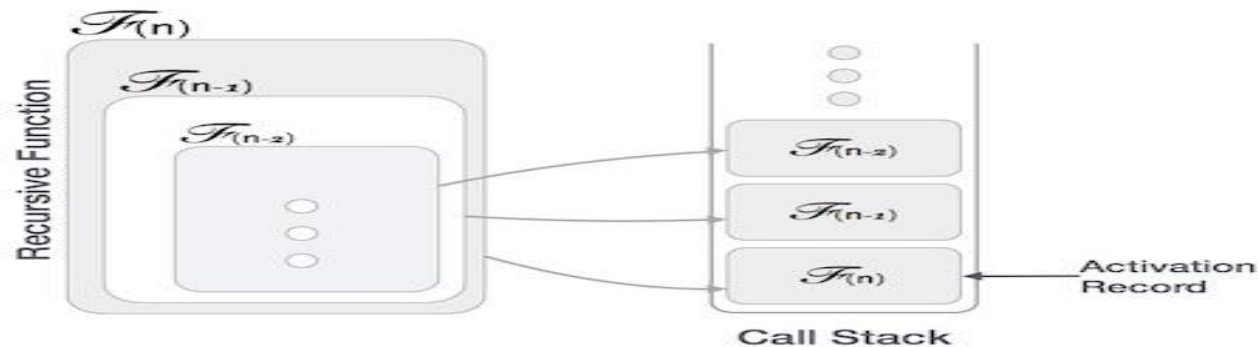
Courtesy: <https://unacademy.com/lesson/evaluation-of-a-postfix-expression-in-tabular-form/8Z5PDFL5>

Application of Stack – Recursion



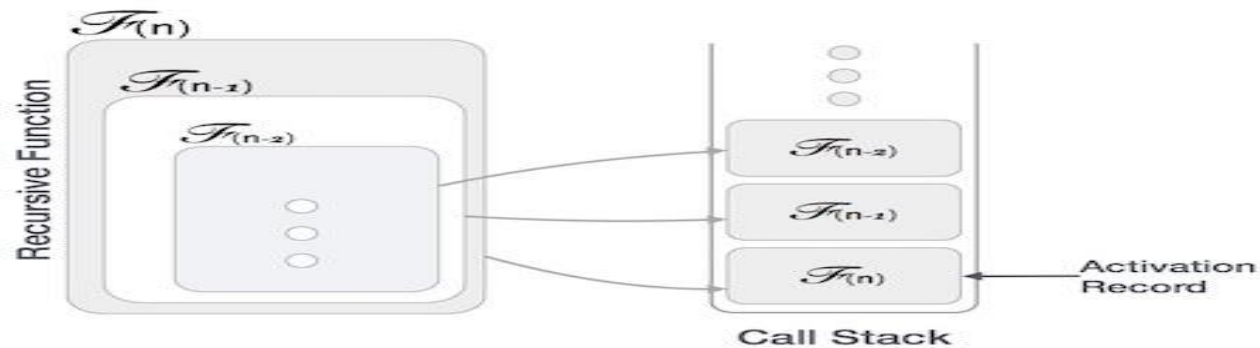
Application of Stack – Recursion

- Many programming languages implement recursion by means of **stacks**.
- A function (**caller**) calls another function (**callee**) or itself as callee,
 - The caller function transfers execution control to the callee.
 - This transfer process may also involve some data to be passed from the caller to the callee.



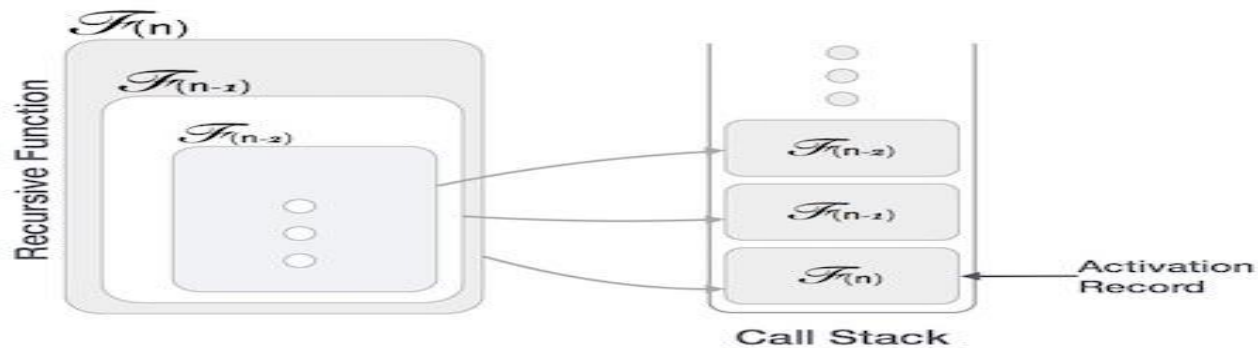
Application of Stack – Recursion

- The caller function has
 - to suspend its execution temporarily and
 - resume later
 - when the execution control returns from the callee function.



Application of Stack – Recursion

- Here, the caller function needs to start exactly from the point of execution where it puts itself on hold.
- It also needs the exact same data values it was working on.
- So, an activation record (or stack frame) is created for the caller function.
- Activation record keeps the information about
 - local variables,
 - formal parameters,
 - return address and
 - all information passed to the callee function.



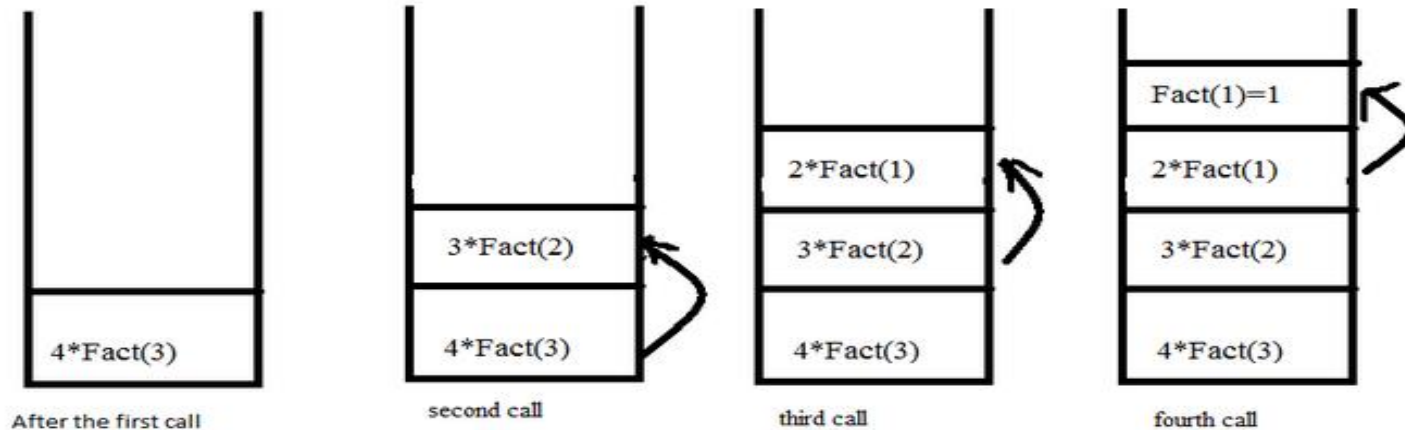
Application of Stack – Recursion

A recursive function to find the factorial of a positive whole number

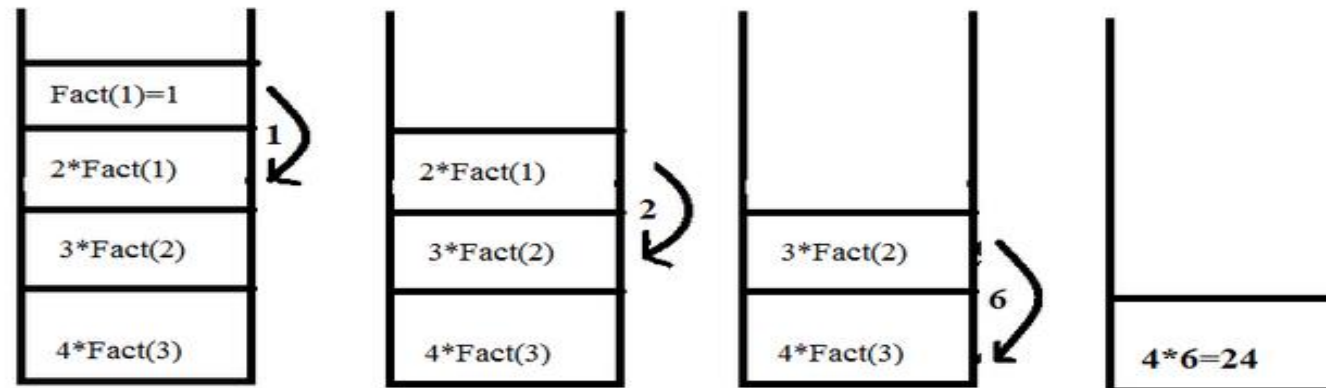
```
int factorial(n)
{
    if (n == 1)
    {
        // base case
        return 1;
    }
    else
    {
        return n * factorial(n - 1);
        // function calls itself
    }
}
```

Application of Stack – Recursion

When function call happens previous variables gets stored in stack



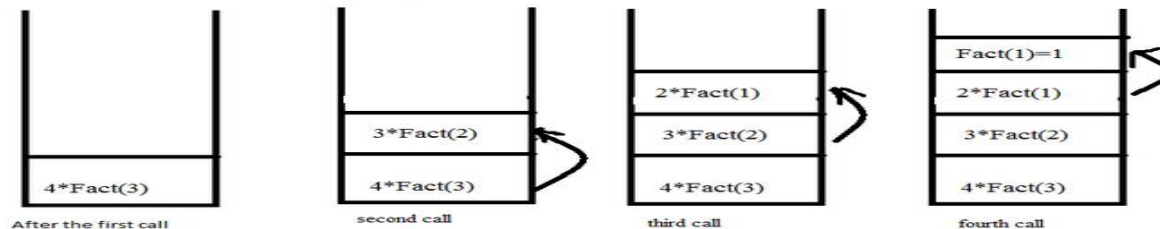
Returning values from base case to caller function



Application of Stack – Recursion

- Functions calls are 'stacked' one on top of the other,
- This is called the call stack (or execution stack)
- The call stack operates on a “Last In, First Out” basis. An item is “pushed” onto a stack on function call, and an item is “popped” off the stack when that function returns a value.

When function call happens previous variables gets stored in stack



Returning values from base case to caller function

