By trial and error

$\therefore \quad \mu = 0{\cdot}493$ approximately.

$$\therefore \quad \text{Rate of issuing tickets} = \frac{1}{\mu} = \frac{1}{0{\cdot}493} = 2{\cdot}03 \text{ minutes / ticket.}$$

Example 8 : A scooter mechanic finds that the time required to repair a scooter is exponentially distributed with mean 30 minutes. The arrival rate of scooters for repairy is approximately Poisson distribution with the average of 12 scooters per 8 hour day.

Find the expected time for which the mechanic is idle in a day. How many scooters on an average are there in the system ? Also find the probability that a customer has to wait.

Solution : Arrival rate $= \lambda = \dfrac{12}{8 \times 60} = \dfrac{1}{40}$ scooters / minute

Service rate $\mu = \dfrac{1}{30}$ scooters / minute $\qquad \therefore \quad \rho = \dfrac{\lambda}{\mu} = \dfrac{1}{40} \cdot \dfrac{30}{1} = \dfrac{3}{4}$

Average number of scooters in the system $= E(N_S)$ [By (11)]

$$= \frac{\rho}{1-\rho} = \frac{3/4}{1-(3/4)} = \frac{3}{4} \cdot \frac{4}{1} = 3$$

Idle time of the system $= 1 - \rho = 1 - \dfrac{3}{4} = \dfrac{1}{4}$

Number of hours for which the mechanic is idle in 8 hours $= \dfrac{1}{4} \cdot 8 = 2$ hours

Probability that a customer has to wait $= \rho = \dfrac{3}{4} = 0{\cdot}75.$

Example 9 : Customers arriving at a public telephone booth follow a Poisson distribution with average time of 10 minutes between one customer and the next. The length of a phone call is found to be exponentially distributed with mean 4 minutes.

(i) Find the average number of customers waiting in the system.

(ii) Find the probability that a person arriving at the booth will have to wait.

(iii) What is the probability that a person arriving at the booth will require more than 15 minutes altogether to wait in the queue and complete his call ?

(iv) Find the fraction of the day in which the phone is in use.

(v) Find the average waiting time for a customer.

(vi) Another booth will be installed if a customer has to wait atleast 4 min. in the queue. By how much the mean arrival rate should increase for this ?

Solution : We have Mean Inter – Arrival Time = 10 Min.

$\therefore \quad$ Mean Arrival Rate $= \lambda = \dfrac{1}{10} = 0{\cdot}1$ customer per minute

$\therefore \quad$ Mean Service Time = 4 minutes.

$\therefore \quad$ Mean Service Rate $= \mu = \dfrac{1}{4} = 0{\cdot}25$ customer per minute

$$\therefore \quad \rho = \frac{\lambda}{\mu} = \frac{0{\cdot}1}{0{\cdot}25} = \frac{2}{5}$$

(i) Average number of customers waiting in the system =

$$E(N_S) = \frac{\rho}{1-\rho} \qquad \text{[By (11)]}$$

$$= \frac{2/5}{1-(2/5)} = \frac{2}{5} \cdot \frac{5}{3} = \frac{2}{3} \text{ customers}$$

(ii) Probability that a person arriving at the booth will have to wait for some time *i.e.*

$$P(W > 0) = 1 - P(W = 0)$$

$$= 1 - P(\text{No customer in the system})$$

$$= 1 - P_0$$

$$= 1 - [1 - \rho] \qquad \text{[By (8)]}$$

$$= \rho = 2/5.$$

(iii) Probability that it will take more than 15 minutes for a person to wait in the queue and to complete his call means

$$P(W_S > 15) = e^{-\mu(1-\rho)t} \qquad \text{[By (19)]}$$

$$= e^{-0 \cdot 25(1-0 \cdot 4)15}$$

$$= e^{-2 \cdot 25} = 0 \cdot 1054$$

(iv) Probability that the phone will remain idle

$$= P(N = 0) = P_0$$

$$= 1 - \rho \qquad \text{[By (8)]}$$

$$= 1 - \frac{2}{5} = \frac{3}{5}$$

$$\therefore \quad P(\text{Phone will be in use}) = 1 - P_0 = 1 - \frac{3}{5} = \frac{2}{5}.$$

$\therefore$ The fraction of the day the phone will be in use = 2 / 5.

(v) Average Waiting Time For A Customer

$$E(W_Q) = \frac{\rho}{\mu(1-\rho)} \qquad \text{[By (22)]}$$

$$= \frac{2/5}{0 \cdot 25(1-0 \cdot 4)} = \frac{2}{5} \cdot \frac{4}{1} \cdot \frac{10}{6}$$

$$= \frac{8}{3} = 2 \cdot 67 \text{ min.}$$

(vi) The second phone will be installed if $E(W_Q) > 4$

$$\therefore \quad \frac{\rho}{\mu(1-\rho)} > 4 \qquad \text{[By (22)]}$$

$$\text{But } \mu = \frac{1}{4} \quad \therefore \quad \frac{4}{1} \cdot \frac{\rho}{1-\rho} > 4 \quad \therefore \quad \frac{\rho}{1-\rho} > 1$$

$$\therefore \quad \rho > 1 \ \rho \quad \therefore \ 2\rho > 1 \quad \therefore \ \rho > \frac{1}{2}$$

$$\text{But } \rho = \frac{\lambda}{\mu} = \frac{\lambda}{1/4} = 4\lambda \qquad \therefore \quad 4\lambda > \frac{1}{2} \quad \therefore \ \lambda > \frac{1}{8}$$

This is the required arrival rate. The existing arrival rate is 1/10.

$\therefore$ The mean arrival rate should increase by

$$\frac{1}{8}-\frac{1}{10}=\frac{2}{80}=\frac{1}{40} \text{ customer per minute.}$$

Example 10 : Customers arrive at a clinic according to a Poisson process with a mean interval of 25 minutes. The physician needs on an average 20 minutes for a patient to examine.

(i) Find the expected number of patients at the clinic and in the queue.

(ii) Find the percentage of patients who are not required to wait.

(iii) On an average how much time is spent by a patient in the clinic ?

(iv) The physician will appoint another physician (equally competent) if the patient's time in the clinic exceeds 2 hours. How much must the rate of arrivals increase so that another physician is appointed ?

(v) Find the average time a patient has to be in queue before the physician examines him.

(vi) What is the probability that the total waiting time of a patient in the system is greater than 1 hour ?

(vii) Find the percentage of patients who have to wait before they are called by the physician for examination.

(viii) What is the probability that there are more than 4 patients in the queue ?

(ix) It is desired that fewer than 5 patients are in the queue for 99 percent of the time. How fast the service rate should be ?

Solution : We have

Mean Inter-Arrival Time = 25 minutes

$\therefore$ Mean Arrival Rate $=\lambda=\frac{1}{25}$ patients per minute

Mean Service Time = 20 minutes

Mean Service Rate $=\mu=\frac{1}{20}$ patients per minute.

And $\rho=\frac{\lambda}{\mu}=\frac{20}{25}=\frac{4}{5}$.

(i) Expected Number At the Clinic = Expected Number In the System

$$E(N_S)=\frac{\rho}{1-\rho} \qquad \text{[By (11)]}$$

$$=\frac{4/5}{1-(4/5)}=4 \text{ customers}$$

(ii) P (A patient is not required to wait)

$= P$ (There is no customer in the system)

$= P_0 = 1-\rho$ [By (8)]

$=1-\frac{4}{5}=\frac{1}{5}=0{\cdot}2$

$\therefore$ Percentage of Patients who donot have to wait = 20%.

(iii) Percentage of patient who have to wait = 80%.

(iv) Average Time Spent By A patient At The Clinic

= Average Waiting Time In The System

$$E(W_S) = \frac{1}{\mu(1-\rho)} \qquad \text{[By (18)]}$$

$$= \frac{1}{1/20} \cdot \frac{1}{1/5} = 100 \text{ minutes.}$$

(v) Another physician will be appointed if the time in the clinic (waiting time + examination time) exceeds 120 minutes.

$$\therefore \quad E(W_S) > 120 \quad \therefore \frac{1}{\mu(1-\rho)} > 120 \quad \therefore \quad \frac{1}{1-\rho} > 120 \times \frac{1}{20} = 6$$

$$\therefore \quad 1-\rho < \frac{1}{6} \quad \therefore \quad 1-\frac{1}{6} < \rho \quad \therefore \quad \frac{5}{6} < \rho \quad \therefore \quad \rho > \frac{5}{6}$$

But $\rho = \frac{\lambda}{\mu} = \lambda 20$

$$\therefore \quad 20\lambda > \frac{5}{6} \quad \therefore \quad \lambda > \frac{1}{24}$$

Hence, for appointment of another physician the average arrival rate should increase by

$$= \frac{1}{24} - \frac{1}{25} = \frac{1}{600} \text{ per minute.}$$

(vi) The Average Waiting Time In The Queue

$$E(W_Q) = \frac{\rho}{\mu(1-\rho)} \qquad \text{[By (22)]}$$

$$= \frac{4/5}{(1/20)[1-(4/5)]} = 20 \cdot \frac{4}{5} \cdot 5 = 80 \text{ minutes.}$$

(vii) P (Total waiting Time Greater Than 60 minutes) is obtained from,

$$P(W_S > t) = e^{-\mu(1-\rho)t} \qquad \text{[By (19)]}$$

$$\therefore P(W > 60) = e^{-\frac{1}{20}\left(1-\frac{4}{5}\right)60} = e^{-0.6} = 0.5488$$

(viii) Probability That There Are More Than Four Patients In The Queue

$$= P(N_Q > 4) = \rho^5 \qquad \text{[By (12)]}$$

$$= \left(\frac{4}{5}\right)^5 = 0.3277.$$

(ix) P (5 or more patients) $= \rho^5$ as above.

We want this probability to be less than 0·01.

$$\therefore \quad \rho^5 \le 0.01 \quad \therefore \quad \rho \le \left(\frac{1}{100}\right)^{1/5} \quad \therefore \frac{1}{\rho} \ge (100)^{1/5}$$

But $\rho = \frac{\lambda}{\mu} \quad \therefore \frac{1}{\rho} = \frac{\mu}{\lambda}$

$$\therefore \frac{\mu}{\lambda} \ge (100)^{1/5} \quad \therefore \quad \mu \ge \lambda(100)^{1/5}$$

$$\therefore \mu \ge \frac{(100)^{1/5}}{25} \ge 0.1005 \quad \therefore \quad \mu \ge 0.1005 \times 60 = 6.03$$

The doctor should speed up to examine atleast 7 patients per hour.

Example 11 : An airport has a capacity for landing 80 planes in good weather and 40 planes in bad weather per hour. The mean arrival rate is 30 planes per hour which is a Poisson process. When there is congestion the planes are forced to fly over the field at different altitudes awaiting the landing of the planes that had arrived earlier.

(i) Find the number of planes that would be flying over the field on an average in good weather and in bad weather.

(ii) Find the interval for which a plane would be flying over the field and in the process of landing in good weather and in bad weather.

(iii) How much flying-over time and landing time can be allowed if the priority to land out of order will have to be requested only in 1 in 25 times.

Landing time is to be taken as service time and the time of flying over the field is to be taken as the waiting time in the queue.

Solution : We have, Mean Arrived Rate $= \lambda = 30$ per hour

$$\text{Mean Service Rate } = \mu = \begin{cases} 80 & \text{per hour in good weather} \\ 40 & \text{per hour in bad weather} \end{cases}$$

$$\therefore \rho = \frac{\lambda}{\mu} = \begin{cases} 3/8 & \text{In Good Weather} \\ 3/4 & \text{In Bad Weather} \end{cases}$$

(i) Average Number of Planes Flying Over The Field *i.e.* Waiting In The Queue =

(a) $$E(N_Q) = \frac{\rho^2}{1-\rho} \qquad \text{[By (16)]}$$

$$= \frac{9/64}{1-(3/8)} = \frac{9}{64} \cdot \frac{8}{5} = \frac{9}{40} \text{ In Good Weather}$$

(b) $$E(N_Q) = \frac{9/16}{1-(3/4)} = \frac{9}{16} \cdot \frac{4}{3} = \frac{9}{4} \text{ In Bad Weather.}$$

(ii) Average Time of Flying-over The Field (*i.e.* Waiting In The Queue) and For Landing (*i.e.* For Service) = Average Waiting Time In The System =

$$E(W_S) = \frac{1}{\mu(1-\rho)} \qquad \text{[By (18)]}$$

$$= \begin{cases} \dfrac{1}{80[1-(3/8)]} = \dfrac{1}{50} = 1 \cdot 2 \text{ minutes in good weather} \\ \dfrac{1}{40[1-(3/4)]} = \dfrac{1}{10} = 6 \text{ minutes in bad weather} \end{cases}$$

(iii) Let t_R be the required time of flying over the field and landing, beyond which priority out of order is requested by the pilot. We want the probability of this to be 1 in 25.

$$\therefore \quad P(W_S > t_R) = \frac{1}{25} = 0 \cdot 04$$

$$\therefore \quad e^{-\mu(1-\rho)t} = 0 \cdot 04 \qquad \text{[By (19)]}$$

(a) $$e^{-80(1-3/8)t} = 0 \cdot 04 \quad \therefore \quad e^{-50t} = 0 \cdot 04$$

$$\therefore \quad -50t = \log 0 \cdot 04 \quad \therefore \quad -t = \frac{\log 0 \cdot 04}{50} = -0 \cdot 064$$

$$\therefore \quad t = 0 \cdot 064 \text{ hours} = 3 \cdot 84 \text{ Min. in good weather}$$

(b) $e^{-40(1-3/4)t} = 0{\cdot}04 \quad \therefore \; e^{-10t} = 0{\cdot}04$

$\therefore \; -10t = \log 0{\cdot}04 \quad \therefore \; -t = \dfrac{\log 0{\cdot}04}{10} = -0{\cdot}3219$

$\therefore \; t = 0{\cdot}3219$ hours $= 19{\cdot}3$ minutes in bad weater.

Example 12 : People arrive to purchase railway tickets at the rate of 5 per minute. On an average it takes 10 seconds to issue the ticket. A person arrives 5 minutes before the train starts. It takes 4 minutes for him to get in the train after purchasing the ticket.

(i) Can he be expected in the train before the train starts ?

(ii) What is the probability that he will be in the train before the train starts ?

(iii) How early must he arrive at the railway station so that the probability of his being in the train before it starts is 99 % ?

Solution : We have, Mean Arrival Rate $= \lambda = 5$ Customers per minute

Mean Service Rate $= \mu = \dfrac{60}{10} = 6$ Customers per minute.

$$\rho = \frac{\lambda}{\mu} = \frac{5}{6}$$

(i) Average Waiting Time In the System

$$E(W_S) = \frac{1}{\mu(1-\rho)} \qquad [\text{ By } (18)\,]$$

$$= \frac{1}{6[1-(5/6)]} = 1 \text{ minute}$$

E (Total Time Required To Purchase The Ticket And To Get In The Train) = 1 + 4 = 5 minutes.

Since he has come 5 minutes before the train starts and since expected time also is 5 minutes, he can just be in the train before the train starts.

(ii) The probability that he will be in the train means the total time will be less than 5 minutes.

$$P(\text{Total time} < 5 \text{ minutes}) = P(W_S < 1 \text{ minute})$$

$$= 1 - P(W_S > 1) = 1 - e^{-\mu(1-\rho)t} \qquad [\text{ By } (19)\,]$$

$$= 1 - e^{-6[1-(5/6)] \cdot 1}$$

$$= 1 - e^{-1} = 0{\cdot}63$$

(iii) The probability that he will be in the train is 0·99 means

$$P(W_S < t) = 0{\cdot}99 \quad \therefore \quad P(W_S > t) = 0{\cdot}01$$

$$\therefore \; e^{-\mu(1-\rho)t} = 0{\cdot}01 \qquad [\text{ By } (19)\,]$$

$$\therefore \; e^{-t} = 0{\cdot}01 \quad \therefore \; -t = \log 0{\cdot}01$$

$$\therefore \; -t = -4{\cdot}6 \quad \therefore \; t = 4{\cdot}6$$

$\therefore$ Ticket Purchasing Time = 4·6

$\therefore$ P (Total Time = Time To Get the Ticket And To Be In The Train < 4.6 + 4) = 0.99

$\therefore$ The person has to arrive in the station 8·6 minutes before the departure of the train.

Example 13 : At what rate a retail shop owner having no assistant, must work in order to ensure that 90% of the customers will not have to wait more than 12 minutes ? It is assumed that the customers arrive in Poisson process at an average rate of 18 per hour and the length of the service by the owner has an exponential distribution ?

Solution : We have, Mean Arrival Rate = $\lambda = 18$ per hour.

Let the Required Mean Service Rate = μ_R

We want μ_R such that the average waiting time of the customer in the queue W_Q is less than 12 minutes and the probability of this must be 0·9.

$$\therefore \quad P\left(W_Q < \frac{1}{5}\right) = 0{\cdot}9 \quad \therefore \quad P\left(W_Q > \frac{1}{5}\right) = 1 - 0{\cdot}9 = 0{\cdot}1$$

But $\qquad P(W_Q > t) = \rho\, e^{-\mu(1-\rho)t}$ [By 21]

$$\therefore \quad \rho\, e^{-\mu(1-\rho)t} = 0{\cdot}1$$

But $\lambda = 18$, and $\mu = \mu_R$ the required mean service rate μ.

$$\therefore \quad \frac{18}{\mu_R} e^{-(\mu_R - 18)\times 0{\cdot}2} = 0{\cdot}1 \quad \therefore \quad e^{-(\mu_R - 18)\times 0{\cdot}2} = \frac{\mu_R \times 0{\cdot}1}{18}$$

Taking logarithm of both sides,

$$-(\mu_R - 18)\, 0{\cdot}2 = \log \mu_R + \log 0{\cdot}1 - \log 18$$

$$\therefore \quad -0{\cdot}2\, \mu_R + 3{\cdot}6 = \log \mu_R + \log 0{\cdot}1 - \log 18$$

$$\therefore \quad \log \mu_R + 0{\cdot}2\, \mu_R = 3{\cdot}6 + \log(18/0{\cdot}1)$$

$$= 3{\cdot}6 + \log 180 = 3{\cdot}6 + 5{\cdot}19 = 8{\cdot}79$$

By trial and error, $\therefore\ \mu_R = 27$ approximately.

$\therefore$ The retail shop owner has to serve 27 customer per hour.

Example 14 : A xerox machine owner earns by giving xeroxing service. The time required to complete xeroxing of one customer has an exponential distribution with the mean of 5 minutes. The arrival of customers is a Poisson process with mean rate of 6 customers an hour. If the machine owner works 8 hours a day, find

(i) the percentage idle time,

(ii) the average time a customer has to remain in the shop,

(iii) the average number of customers in the queue.

(iv) the probability that there will be more than 4 customers in the shop.

Solution : We have, Mean Arrival Rate = $\lambda = 6$ customers per hour

$$\text{Mean Service Rate} = \mu = \frac{60}{5} = 12 \text{ customers per hour}$$

$$\rho = \frac{\lambda}{\mu} = \frac{6}{12} = \frac{1}{2}$$

(i) P (Fraction of Time The Machine Is Idle)

$$= P(N = 0) = P_0$$

$$= 1 - \rho$$ [By (8), page 14-8]

$$= 1 - (1/2) = 0{\cdot}5$$

$\therefore$ Percentage of Idle Time = 50 %.

(ii) The Average Time A customer Has To Be In The Shop = The Average Waiting Time + Average Service Time =

$$E(W_S) = \frac{1}{\mu(1-\rho)} \qquad \text{[By (18)]}$$

$$= \frac{1}{12[1-(1/2)]} = \frac{1}{6} \text{ hours} = 10 \text{ minutes.}$$

(iii) The Average Number of Customers In The Queue =

$$E(N_Q) = \frac{\rho^2}{1-\rho} = \frac{1/4}{1-(1/2)} = \frac{1}{2}. \qquad \text{[By (16)]}$$

(iv) Probability That There Will Be More Than Four Customers In The Shop =

$$P(N_S > n) = \rho^{n+1} \qquad \text{[By (12)]}$$

$$\therefore \quad P(N_S > 4) = \left(\frac{1}{2}\right)^5 = 0{\cdot}03.$$

Exercise - I

1. People arrive at telephone booth at the rate of 20 per hour in a Poisson process. The time of a call per person is an exponential random variable with the mean of 2 minutes. Find the probability that a person on arrival at the booth will find atleast 4 persons in the system. **[Ans. : 0·1975]**

2. The customers arrive at a reservation counter of a railway station according to a Poisson process at the rate of 8 persons per hour. The time required to issue the reservation ticket follows an exponential distribution with the mean of 6 minutes per person. Find the probability that a customer (i) has to wait on arrival, (ii) finds four persons in the queue, (iii) has to spend less than 15 minutes in the railway station, (iv) average waiting time in the railway station. (v) average number of persons in the railway station. **[Ans. : (i) 0·8, (ii) 0·01892, (iii) 0·3955, (iv) 0·5, (v) 4]**

3. The rate of arrival of customers following a Poisson process at a retail shop is one person for 10 minutes and the service time is an exponential random variable with the mean of 8 minutes.
   (a) Find (i) the average number of customers L. (ii) the average time a customer spends in the shop. (iii) the average time a customer spends in waiting in the queue W_Q.
   (b) If the arrival rate of customers increases by 10 percent, find the corresponding changes in L, W and W_Q. **[Ans. : (a)** $L = 4$, $W = 40$ Min. $W_Q = 32$ Min. **(b)** $L = 8$, $W = 72$ Min. $W_Q = 64$ Min. **]**

4. Customers arrive at the rate of 2 per minute at a window in a bank which is a $M \mid M \mid 1 \mid \infty$ queuing system. The bank authorities desire to increase the rate of service, so as to ensure that fewer than 5 customers are in the queue for 99% of time. What should be the increased service rate ? **[Ans. : More than 5·024 customer per minute.]**

5. Travellers arrive at a railway counter for tickets at the rate of 15 per hour. The cleark at the counter can issue tickets at the rate of 45 per hour. Assuming $M \mid M \mid 1 \mid \infty$ system, find
   (a) the probability that there is no customer at the counter.
   (b) the probability that there are more than 3 customers at the counter.
   (c) the probability that a customer has to wait. **[Ans. : (a) 2/3, (b) 0·012, (c) 1/3]**

6. In the above problem find out the rate at which the clerk at the counter should issue tickets, so that the probability is 0·9 that a person will not have to wait for more than 12 minutes in the system.

[**Ans.** : 2·5 minute / customer]

7. Passengers arrive in a Poisson process at the rate of 15 per hour for reservation of railway tickets and the length of the service time per passenger given by the reservation clerk has an exponential distribution. If the probability that a passenger will not be required to wait for more than 12 minutes is to be 0·90, what should be the rate of service of the clerk ? [**Ans.** : 24 passengers / hour]

8. A man earns Rs. 5 per hour on a typewriter. The time to type a job varies according to an exponential distribution with mean of 6 minutes. Jobs arrive according to a Poisson distribution at the average rate of 5 per hour.

The typist works for 8 hours a day. Find :

(i) the percentage of time for which the typist is idle.

(ii) average earning of the typist per day.

(iii) average time a job is in the system. [**Ans.** : **(i)** 50 %, **(ii)** Rs. 40, **(iii)** 12 Minutes]

9. Passengers arrive at local station at the rate of 6 per minute and a passenger needs 7·5 seconds to get the ticket. Mr. X arrives 2 minutes before the local starts and it takes exactly 1·5 minutes for him to get into the local after purchasing the ticket.

(i) Can he be in the local before the local starts ?

(ii) What is the probability that he will be in the local before it starts ?

(iii) If the probability that he will get into the local is to be 0·99, how early must be arrive ?

[**Ans.** : **(i)** Yes, **(ii)** 0·63, **(iii)** 3·8 Minutes]

10. The mean arrival rate of planes at an airport is 20 per hour is a Poisson process. The airport can give landing service to 50 planes per hour in good weather and 30 planes per hour in bad weather which again is a Poisson process. When the runways are busy the arriving planes are asked to fly over the field and are given landing facility in a queue.

(i) How many planes would be flying over the field in good weather and in the bad weather ?

(ii) How long a plane would be in air flying over the field and in the process of landing in good weather and bad weather ?

(iii) How much flying time (over the field) and landing time be assumed so that the probability would be 1 / 20 of a pilot requesting priority landing out of queue ?

[**Ans.** : **(i)** 16 planes / hour in good weather ; 80 planes / hour in bad weather.
(ii) 2 minutes in good weather ; 6 minutes in bad weather.
(iii) 6 minutes in good weather ; 18 minutes in bad weather.]

11. Customers arrive at a petrol pump at the rate of 5 persons per hour. It takes on an average 4 minutes to serve a customer. Assuming this to be $M \mid M \mid 1 \mid \infty$ system.

(i) Find the average number of persons waiting at the petrol pump *i.e.* in the system.

(ii) What is the probability that a customer arriving at the petrol pump will have to wait in the queue ?

(iii) What is the probability that a customer will have to spare more than 10 minutes at the petrol pump in queue and for filling the petrol.

(iv) Find the fraction of the day the pump is idle.

(v) The owner has decided to install another petrol pump at the same place if the customer is required to wait atleast 3 minutes in the queue. By how much the arrival rate must increase for this ?

[**Ans.** : **(i)** 1 / 2, **(ii)** 1 / 3, **(iii)** 0·1889, **(iv)** 2 / 3, **(v)** 1 / 42]

12. Customers arrive at a barber's shop for an hair-cut in accordance with a Poisson distribution mean inter-arrival time of 15 minutes. The barber requires on an average 12 minutes per person
   (i) Find the expected number of customers in the shop.
   (ii) Find the percentage of time a customer can straight way go to the barber's chair without waiting.
   (iii) How much time a person has to spend in the barbers shop ?
   (iv) It is decided to provide another chair and another barber if the waiting of a customer exceeds 1·25 hour. What should be the increase in the arrival rate of the customers to ensure this ?
   (v) What is the average time a customer spends in the queue ?
   (vi) What is the probability that a customer has to wait more than half an hour ?
   (vii) What is the percentage of customers who have to wait ?
   (viii) Find the probability that there are more than 3 persons in the system.

[Ans. : (i) 4, (ii) 20%, (iii) 1 hour, (iv) 0·2 per hour, (v) 48 min., (vi) 0·6065, (vii) 80%, (viii) 0·4096]

13. Customers arriving at a public telephone booth follow a Poisson distribution at an average rate of 12 per hour. The average time of each call is an exponential distribution with the mean of 2 minutes.
   (a) Find the probability that a customer arriving at the booth will find the booth occupied.
   (b) It is decided to install an additional booth at the same place if a customer has to wait for 3 or more minutes. Find the average arrival rate required to justify the installation of the second booth. [Ans. : (a) 0·4, (b) 18]

14. People arrive at a paying counter in a bank at the rate of 12 per hour according to a Poisson process and the average time a person is at the counter is 2 minutes with exponential distribution.
   (i) What is the probability that a person on arrival will find that the clerk at counter is busy ?
   (ii) The bank has decided to open another counter if a customer waits for 3 or more minutes at the counter. Find the average arrival rate so that the bank will open the second center.

[Ans. : (i) 0·4, (ii) 18 per hour]

7. $M|M|s|\infty$ System

In the $M \mid M \mid s \mid \infty$ queuing system the arrival process with mean arrival rate λ and the service time with mean service rate μ are Poisson processes. There are s servers and there can be any number of customers.

1. Values of P_0 and P_n

Since there are s servers if the number of customers n is less than the number of servers ($n < s$) then, only n servers will be busy and others will remain idle and hence, the mean service rate will be $n\mu$.

If the number of customers n is greater than the number of servers s ($n > s$) all the s servers will be busy and hence, the mean service rate will be $s\mu$.

$$\therefore \quad \mu_n = \begin{cases} n\mu & \text{if } 0 \le n \le s \\ s\mu & \text{if } n \ge s \end{cases}$$

Hence, by (5), page 14-7,

Exercise - III

1. A diesel pump operating on a highway for trucks only, has room for two trucks in the queue and one at pump receiving the diesel. Trucks arrive at the rate of 4 per hour and each truck needs 10 minutes on an average to get the diesel.

   Assuming that the trucks arrive in a Poisson process and the service follows an exponential distribution,

   (i) find the average total time a truck is in the serving center,

   (ii) find the average waiting time of a truck,

   (iii) what percentage of trucks are turned away ?

   [**Ans. : (i)** 20·15 min., **(ii)** 10·14 min., **(iii)** 12·3 %]

2. There are two petrol-pumps at a centre which can serve two trucks, and there is sufficient space to accommodate 5 more trucks in the waiting queue. Trucks arrive at the rate of 4 per hour and 12 minutes are required for fuelling.

   Find the probability that there will be no truck at the serving centre, one truck at the centre and seven trucks at the centre.

   Find also the average number of trucks in the queue at the center.

   [**Ans. : (i)** 0·4287, **(ii)** 0·343, **(iii)** 0·0014, (iv) 0·15]

3. At a certain yard goods trains arrive at the rate of 36 trains per day where interarrival times follow an exponential distribution. The service time for each train which follows an exponential distribution is on an average 30 minutes. If the yard can accommodate 8 trains at a time (excluding the line reserved for shunting), calculate the probability that the yard is empty and the length of queue.

   [**Ans. : (i)** $\lambda = \frac{36}{60 \times 24} = \frac{1}{40}$, $\mu = \frac{1}{30}$ $\therefore P_0 = 0.28$; **(ii)** $E(N_S) = 2{\cdot}27$, $E(N_Q) = 1{\cdot}55$]

4. A diesel pump has capacity to accommodate only 4 trucks including the one at the pump. On an average trucks arrive at the rate of 5 per hour and each truck takes 10 minutes for service. Assume that the arrival process is Poisson and the service time is an exponential random variable.

   (i) What is the average time for which a truck is at the pump ?

   (ii) What is the average waiting time for a truck ?

   (iii) Find the percentage of trucks that will turn away.

   [**Ans. : (i)** 26·4 minutes, **(ii)** 16·4 minutes, **(iii)** 13·43%]

5. In a bank-office which is a special branch sanctioning loans, there are two windows and a room for 5 persons to wait. Customers come at the average rate of 4 per hour and each customer need on an average 12 minutes at the window.

   Calculate P_0, P_1. [**Ans. : (i)** 0·43, **(ii)** 0·34]

Exercise - IV

Theory - **Write short notes on**

1. Queuing System
2. $M \mid M \mid 1 \mid \infty$ system
3. $M \mid M \mid s \mid \infty$ System
4. $M \mid M \mid 1 \mid k$ system
5. $M \mid M \mid S \mid k$ system.