INVERSE LAPLACE TRANSFORM

Find the inverse laplace transform of following functions:

65. $\dfrac{4s+12}{s^2+8s+12}$ **[Ans:** $e^{-4t}(4\cosh 2t - \sinh 2t)$ **]**

66. $\dfrac{s}{s^2+2s+2}$ [Ans: $e^{-t}(\cos t - \sin t)$]

67. $\dfrac{s}{(2s+1)^2}$ & $L^{-1}\left\{\frac{s+2}{s^2+4s+5}\right\}$

68. $\dfrac{s+1}{s^2-4}$ & $L^{-1}\{\frac{s+4}{s^2-8s}\}$

69. $\dfrac{s^2+2s-4}{(s^2+2s+5)(s^2+2s+2)}$ [Ans: $\dfrac{3}{2}e^{-t}\sin 2t - 2e^{t}\sin t$]

70. $\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}$ [Ans: $\dfrac{1}{a^2-b^2}(a\sin at - b\sin bt)$]

71. $\dfrac{s}{(s^2+a^2)(s^2+b^2)}$ [Ans: $\dfrac{1}{b^2-a^2}(\cos at - \cos bt)$]

72. $\dfrac{5s^2+8s-1}{(s+3)(s^2+1)}$ [Ans: $2e^{-3t}+3\cos t - \sin t$]

73. $\dfrac{2s}{s^4+4}$ [Ans: $\sin t \sinh t$]

74. $\dfrac{1}{s^3+1}$ [Ans: $\dfrac{1}{3}e^{-t} - \dfrac{e^{t/2}}{3}\cos\left(\dfrac{\sqrt{3}}{2}t\right) + \dfrac{e^{t/2}}{\sqrt{3}}\sin\left(\dfrac{\sqrt{3}}{2}t\right)$]

75. $\dfrac{1}{s^3(s-1)}$ [Ans: $1-t+\dfrac{t^2}{2}-e^{-t}$]

76. $\dfrac{s}{(s+1)^2(s^2+1)}$ [Ans: $\dfrac{1}{2}\left[\sin t - te^{-t}\right]$]

77. $\dfrac{5s^2-15s-11}{(s+1)\,(s-2)^2}$ [Ans: $e^{-t}+4e^{2t}-7te^{2t}$]

78. $\dfrac{s}{(s^2+1)\,(s^2+4)(s^2+9)}$ [Ans: $\dfrac{1}{24}\cos t - \dfrac{1}{15}\cos 2t + \dfrac{1}{40}\cos 3t$]

79. $\dfrac{s^2}{(s+1)^3}$ [Ans: $e^{-t}\left(1-2t+t^2\right)$]

80. $\dfrac{3s-2}{s^{5/2}} - \dfrac{7}{3s+2}$

81. $\left\{\frac{3}{s^2+6s+18}\right\}$ & $\left\{\frac{8}{4s^2+4s+1}\right\}$

82. $\{\frac{s}{(s^2+2-2s)(s^2+2+2s)}\}$

83. $\{\frac{s}{(s^2+1-s)(s^2+1+s)}\}$

84. $\{\frac{1}{(s^2+16)(s^2+25)}\}$

85. $\{\frac{s}{(s^2+16)(s^2+25)}\}$

86. $\{\frac{s}{(s^2-16)(s^2-25)}\}$

87. $\log\left(\dfrac{s+a}{s+b}\right)$ [Ans: $-\dfrac{1}{t}\,(e^{-at}-e^{-bt})$]

88. $2\tanh^{-1}s$ [Ans: $\dfrac{2}{t}\ \sinh t$]

89. $\tan^{-1}\left(\dfrac{2}{s^2}\right)$ [Ans: $2\sin t\sinh t$]

90. $\tan^{-1}\left(\dfrac{s+a}{b}\right)$ [Ans: $-\dfrac{1}{t}\ e^{-at}\sin bt$]

91. $\log\sqrt{\dfrac{s^2+1}{s^2}}$ [Ans: $\dfrac{1}{t}\,(1-\cos t)$]

92. $\cot^{-1}(s+1)$ [Ans: $\dfrac{1}{t}\ e^{-t}\sin t$]

93. $\log\left[s^2+4\right]$ [Ans: $-\dfrac{2}{t}\cos 2t$]

FIND THE INVERSE OF THE FOLLOWING USING CONVOLUTION THEOREM:

94. $\dfrac{s^2}{(s^2+a^2)^2}$ [Ans: $\dfrac{1}{2a}\left[\sin at+at\cos at\right]$]

95. $\dfrac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$ [Ans: $\dfrac{e^{-t}}{3}(\sin 2t+\sin t)$]

96. $\dfrac{(s+2)^2}{(s^2+4s+8)^2}$ [Ans: $\dfrac{e^{-2t}}{4}(2t\cos 2t+\sin 2t)$]

97. $\dfrac{1}{(s+3)(s^2+2s+2)}$ [Ans: $\dfrac{1}{5}\left[e^{-t}(2\sin t-\cos t)+e^{-3t}\right]$]

98. $\dfrac{1}{(s-2)^4(s+3)}$ [Ans: $\dfrac{e^{-3t}}{625}-e^{2t}\left[\dfrac{1}{625}-\dfrac{t}{125}+\dfrac{t^2}{50}-\dfrac{t^3}{30}\right]$]

99. $\dfrac{1}{s}\log\left(1+\dfrac{1}{s^2}\right)$ [Ans: $\displaystyle\int_0^t -\frac{2}{u}(\cos u-1)\,du$

100. $\dfrac{s^2+s}{(s^2+1)(s^2+2s+2)}$ [Ans: $\dfrac{1}{10}\left[e^{-t}(2\sin t-6\cos t)+(2\sin t+6\cos t)\right]$]

101. $\dfrac{s}{s^4+8s^2+16}$ [Ans: $\dfrac{1}{4}\,t\sin 2t$]

102. Find $\displaystyle\int_0^\infty \sin(tx^2)\,dx$ and hence find $\displaystyle\int_0^\infty \sin x^2\,dx$ [Ans: $\dfrac{1}{2}\sqrt{\dfrac{\pi}{2}}$]

103. Using Convolution theorem prove that $L^{-1}\left[\dfrac{1}{s}\log\left(a+\dfrac{b}{s^2}\right)\right]=\displaystyle\int_0^t \frac{2}{u}\left[1-\cos\left(\frac{b}{a}\right)u\right]du$

104. Using Convolution theorem prove that $L^{-1}\left[\dfrac{1}{s}\log\left(\dfrac{s+1}{s+2}\right)\right]=\displaystyle\int_0^t \frac{e^{-2u}-e^{-u}}{u}\,du$

Find the laplace transform of periodic function:

105. $f(t)=K\dfrac{t}{T}\ for\ 0<t<T\ and\ f(t)=f(t+T)$ [Ans: $K\left[\dfrac{1}{Ts^2}-\dfrac{e^{-st}}{s(1-e^{-st)}}\right]$]

106. $f(t)=1,\ for\ 0\le t<a\ and\ f(t)=-1,\ a<t<2a\ and\ f(t)$ is periodic with period 2a.

[Ans: $\dfrac{1}{s}\tanh\left(\dfrac{as}{2}\right)$]

107. $f(t) = |\sin pt|, \ t \geq 0$ [Ans: $\dfrac{p}{s^2+p^2} \cdot \coth\left(\dfrac{\pi s}{2p}\right)$]

108. $f(t) = t, \ for \ 0 < t < 1 \ and \ f(t) = 0, 1 < t < 2 \ and \ f(t+2) = f(t) \ for \ t > 0$

[Ans: $\dfrac{1}{s^2(1-e^{-2s})}\left(1 - e^{-s} - se^{-s}\right)$]

109. $f(t) = \dfrac{t}{a}, \ 0 < t \leq a; \ f(t) = \dfrac{1}{a}(2a - t), \ a < t < 2a \ \ and \ f(t) = f(t+2a)$

[Ans: $\dfrac{1}{as^2}\tanh\left(\dfrac{as}{2}\right)$]

HEAVISIDE'S UNIT-STEP FUNCTION & DIRAC DELTA FUNCTION

FIND THE LAPLACE TRANSFORM OF FOLLOWING FUNCTIONS:

110. $f(t)) = \begin{cases} 0 \,; 0 < t < \pi \\ sin2t \,; \pi < \ t < 2\pi \\ 0 \ ; 2\pi < t \end{cases}$

111. $\sin t \cdot H\left(t - \dfrac{\pi}{2}\right) - H\left(t - \dfrac{3\pi}{2}\right)$ [Ans: $e^{-\pi s/2} \cdot \dfrac{s}{s^2+1} - e^{-3\pi s/2} \cdot \dfrac{1}{s}$]

112. $\left(1 + 2t - 3t^2 + 4t^3\right) H(t-2)$ [Ans: $e^{-2s}\left[\dfrac{25}{s} + \dfrac{38}{s^2} + \dfrac{42}{s^3} + \dfrac{24}{s^4}\right]$]

113. $\left(e^{at}\delta(t-a)\right)$ & $sin(3t)\delta\left(t - \frac{\pi}{3}\right)$

114. Using Laplace transform evaluate

$\int_0^\infty e^{-t}\left(1 + 2t - 3t^2 + 4t^3\right) H(t-2)\, dt$ [Ans: $\dfrac{e^{-2}}{129}$]

FIND THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

115. $\dfrac{e^{-as}}{(s+b)^{5/2}}$ [Ans: $\dfrac{4}{3\sqrt{\pi}} \cdot e^{b(t-a)} \cdot (t-a)^{3/2} \cdot H(t-a)$]

116. $\dfrac{(s+1)e^{-s}}{s^2+s+1}$ [Ans: $e^{-t/2}\left[\cos\left(\sqrt{3}\,(t-1)/2\right) + \dfrac{1}{\sqrt{3}}\sin\left(\sqrt{3}\,(t-1)/2\right)\right] \cdot H(t-1)$]

117. $\dfrac{e^{-\pi s}}{s^2 - 2s + 2}$ [Ans: $e^{(t-\pi)} \cdot \sin(t-\pi) \cdot H(t-\pi)$]

118. $e^{-s}\left(\dfrac{1-\sqrt{s}}{s^2}\right)^2$ [Ans: $\left[\dfrac{(t-1)^3}{6} - \dfrac{16}{15\sqrt{\pi}}(t-1)^{5/2} + \dfrac{(t-1)^2}{2}\right] \cdot H(t-1)$]

119. $\{\dfrac{s^2}{s^2-16}\}$ & $\{\dfrac{s^2}{s^2+16}$

120. $\left(\dfrac{s+4}{s-1}\right)$ & $\left(\dfrac{s-4}{s+1}\right)$

121. $\dfrac{s^2}{s^2-9}$ & $\dfrac{3s^2}{s^2+9}$

USING LAPLACE TRANSFORM SOLVE THE FOLLOWING DIFFERENTIAL EQUATIONS WITH THE GIVEN CONDITION:

122. $\left(D^2-4\right)y=3e^t, \quad y(0)=0,\ y'(0)=3$ [Ans: $y=-e^t+\frac{3}{2}e^{2t}-\frac{1}{2}e^{-2t}$]

123. $\left(D^2+D\right)y=t^2+2t, \quad y(0)=4,\ y'(0)=-2$ [Ans: $y=2+2e^{-t}+\frac{t^3}{3}$]

124. $\left(D^2+2D+1\right)y=3t\,e^{-t}, \quad y(0)=4,\ y'(0)=-2$ [Ans: $y=e^{-t}\left(4+6t+\frac{t^3}{2}\right)$]

125. $(D^2-2D-8)y=4\ y(0)=\ y'(0)=-2$ [Ans: $y=-\frac{1}{2}+\frac{1}{6}e^{-2t}+\frac{1}{3}e^{4t}$]

126. $\frac{d^2y}{dt^2}+4y=H(t-2)\ with\ conditions\ y(0)=0,\ y'(0)=1$

[Ans: $y=\frac{1}{2}\sin 2t+\frac{1}{4}H(t-2)-\frac{1}{4}\cos 2(t-2)\,H(t-2)$]

127. $\frac{dy}{dt}+2y+\int_0^t y\,dt=\sin t,\ given\ that\ y(0)=1$ [Ans: $y=e^{-t}-\frac{3}{2}t\,e^{-t}+\frac{1}{2}\sin t$]

128. $\frac{d^2y}{dt^2}+9y=18\,t\ with\ conditions\ y(0)=0,\ y\,(\pi/2)=0$ [Ans: $y=2t+\pi\sin 3t$]

129. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=0$, $where\ \ y(0)=0,\ y'(0)=4$ [Ans: e^x-e^{-3x}]

130. $\frac{d^2y}{dt^2}+4y=f(t)\ with\ conditions\ y(0)=0,\ y'(0)=1\ and\ f(t)=1,\ when\ 0<t<1$

$=0,\ when\ t>1$

[Ans: $y=\frac{1}{2}\sin 2t+\frac{1}{4}(1-\cos 2t)-\frac{1}{4}\{1-\cos(t-1)\}\,H(t-1)$]