Elin_Ahlstrand_Exercises_5

1A

```
include("standard_start.jl");
include("animate_matrix.jl");
##=====1a
x = [x1 x2 x3]; # each column of x will be one of x1, x2, or x3
      x1 = [0.5; 1];
      x2 = [0.8; 0.2];
      x3 = [0.7; 0.3];


      M = [2 0; 0 2];

      y1 = M*x1;
      y2 = M*x2;
      y3 = M*x3;

      println(y1, y2, y3);

figure(1)
      animate_matrix(M);
      clf()
      animate_matrix(M, seed = x); # seed=x tells it which specific
points to show
##=====
```

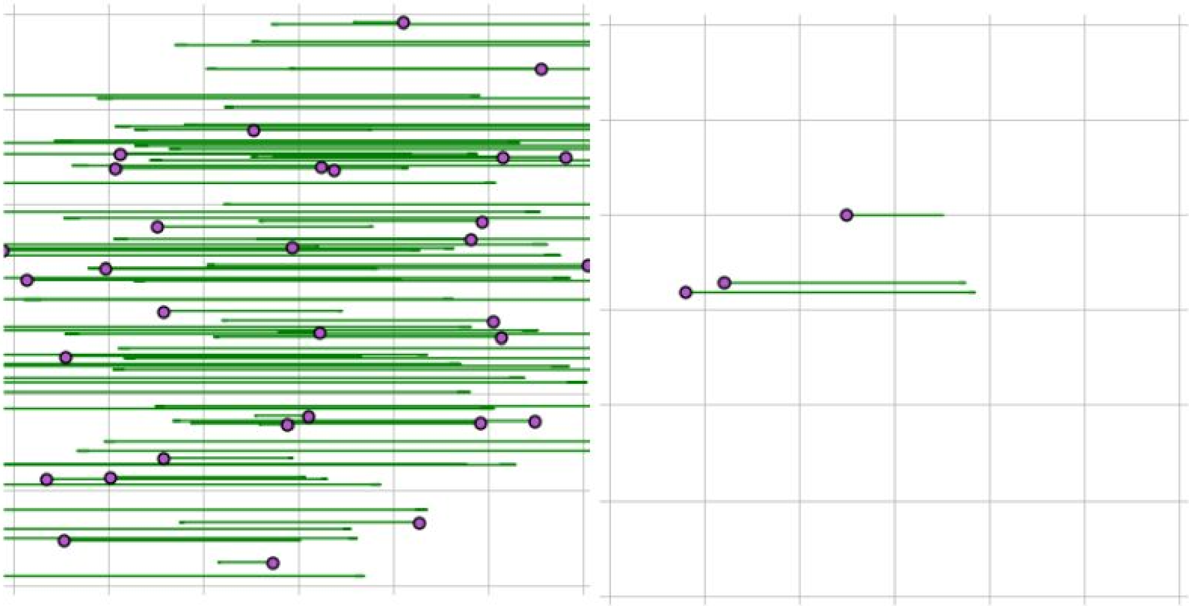
```
julia>
[1.0, 2.0][1.6, 0.4][1.4, 0.6]
```

1B

```
##===== 1b
M2 = [-3 1; 0 1];

      z1 = M2*x1
      z2 = M2*x2
      z3 = M2*x3

      println(z1, z2, z3);
##==== 1b_figure
```

```
figure(1)
	animate_matrix(M2);
	clf()
	animate_matrix(M2,seed = x);
##=====
```

```
julia>
[-0.5, 1.0][-2.2, 0.2][-1.8, 0.3]
```

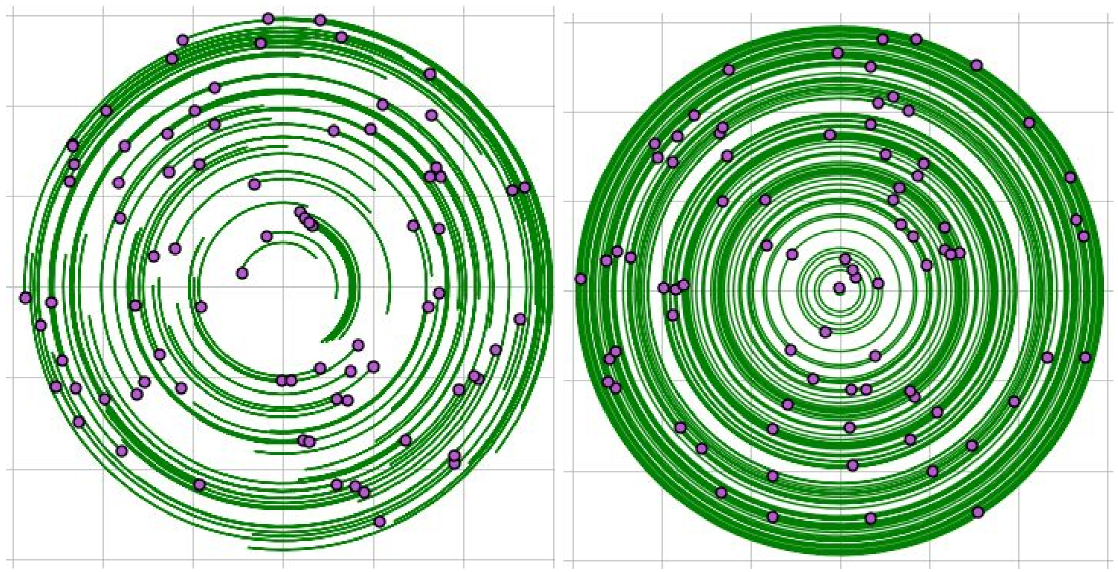


1C/D

```
##====Part II _ 1c
theta = 2*pi/3;
	R = [cos(theta) sin(theta); -sin(theta) cos(theta)];

	y1 = R*x1
	r2 = R*x2
	r3 = R*x3
	r = [r1 r2 r3];

	R_i = inv(R);

	RR = R*R;

println(r1, r2, r3);
```

```
println("R = ", R, " & R_i = ", R_i, " RR = ", RR);

##===== 1c_ animate rotations by 120 degrees
      animate_matrix(R);
      animate_matrix(R, seed=x);
##=====
##===== 1d _ undo rotations
      animate_matrix((inv(R),R));
      animate_matrix((R,R,R));
##=====
```

```
julia>
[0.616025, -0.933013][-0.226795, -0.79282][-0.0901924, -0.756218]
R = [-0.5 0.866025; -0.866025 -0.5] & R_i = [-0.5 -0.866025; 0.866025
-0.5] RR = [-0.5 -0.866025; 0.866025 -0.5]
```

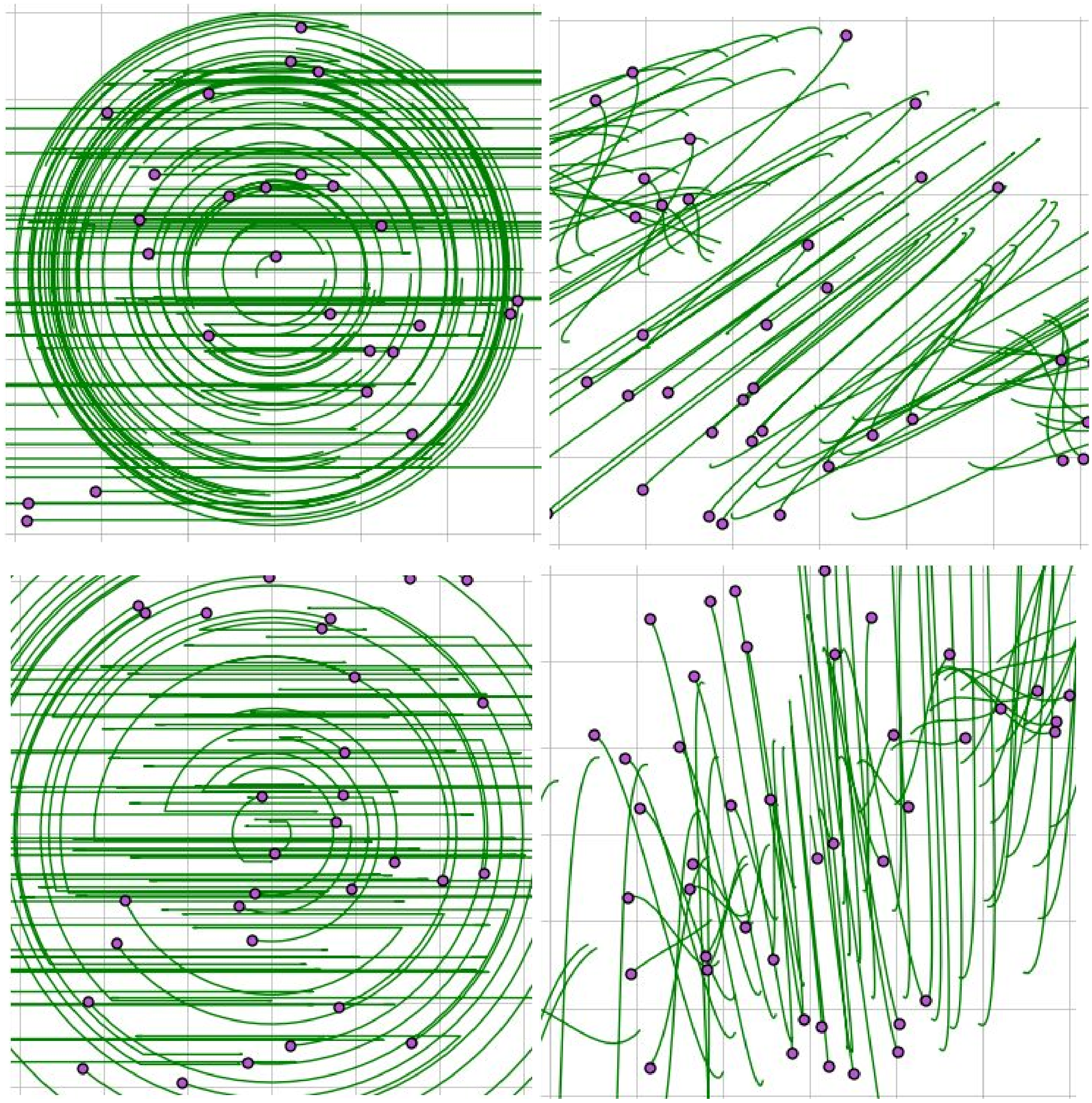


1E

```
##==== 1e
A = M2*R
      B = R*M2
      println(A,B);

animate_matrix((M2,R));
```

```
animate_matrix(A);
animate_matrix((R,M2));
animate_matrix(B);
##====
```

```
julia>
[0.633975 -3.09808; -0.866025 -0.5][1.5 0.366025; 2.59808 -1.36603]
```



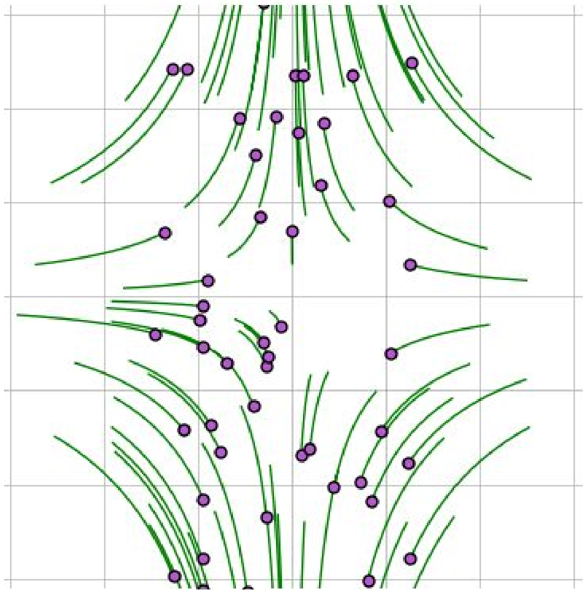
(1-4 ordered left to right, top to bottom)

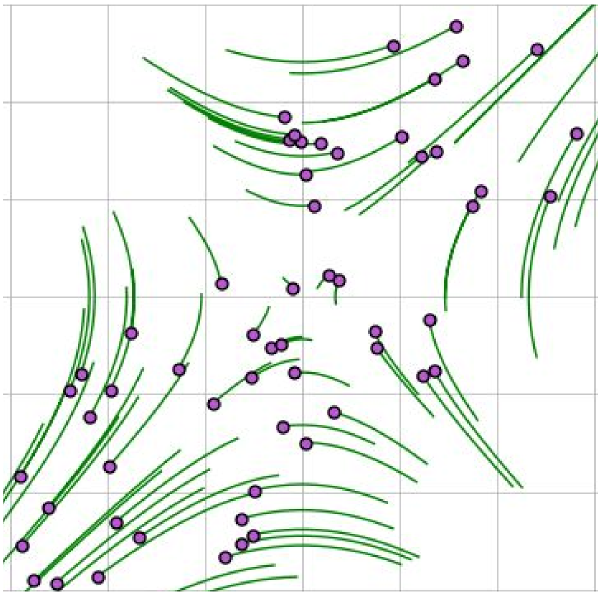
1 - Rotation by 120 degrees, then horizontal displacement
2 - Swift and smooth complex path
3 - Horizontal displacement, then a rotation by 120 degrees
4 - Swift and smooth complex path

Final results for A & B are not the same

1F

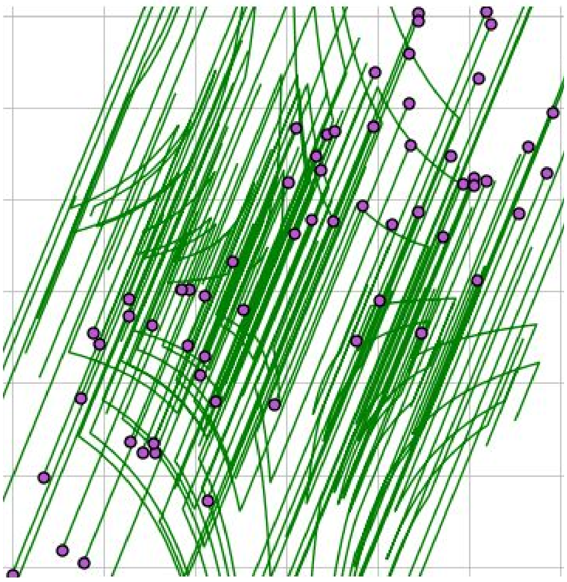
```
##====1f
M3 = [0.5 0; 0 2];
M4 = [1.25 0.75; 0.75 1.25];
animate_matrix(M3)
animate_matrix(M4)
#= EXPLANATION
M3 squashes the vectors along the x-axis by .5, and stretches in y-axis
by factor of 2
M4 aligns the vectors along the diagonal line x=y by 1.25 in x &
squashes by 0.75 in y
Both matrices diagonalize the vectors, just in different orientations
=#
```

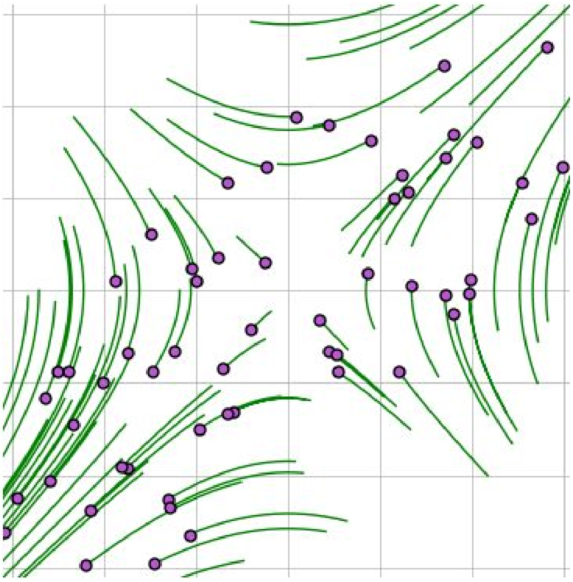




1G

```
##===== 1g
V3 = [1/sqrt(2) -1/sqrt(2); -1/sqrt(2) -1/sqrt(2)];
animate_matrix((V3,M3,V3));
animate_matrix(M4);
##=====
```





2B

```
##=====2b
A = [4 -2; 1 1]
      x = [2;1]

      using LinearAlgebra

      println(eigvecs(A) * norm(x));
##======
```

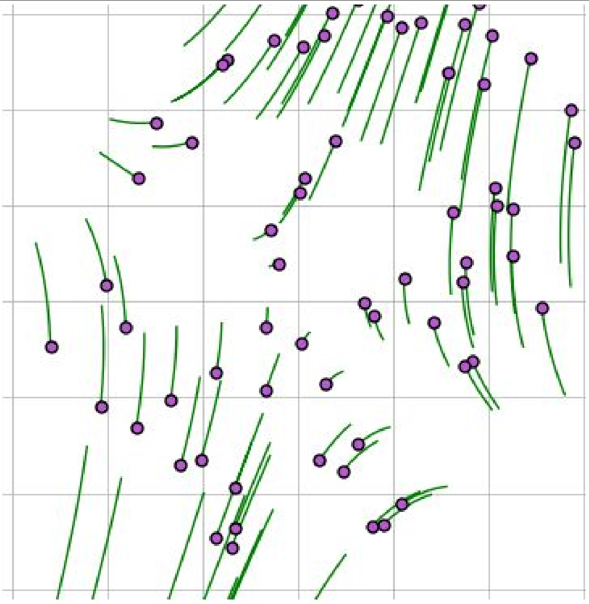
```
julia>
[2.0 1.58114; 1.0 1.58114]
```

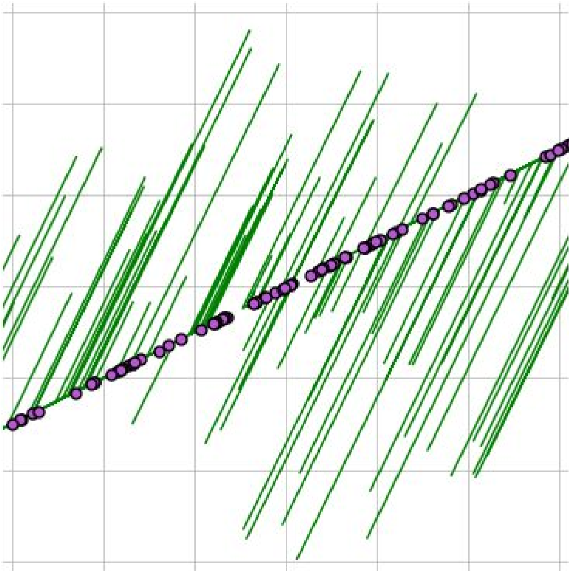
3A

```
##====== 3a
      A = [1 0.25; 0.5 1.5];
      B = [1 -.5; .5 -.25];

      animate_matrix(A)
      animate_matrix(B)

      println("det(A) = ",det(A));
```

```
      println("det(B) = ",det(B));

# B is invertible because det(B) = 0
##=====
```

```
julia>
det(A) = 1.375
det(B) = 0.0
```





3B

```
##===== 3b
A = [1 2; -1 .5];
B = [-2 -4; -3 1];
println("det(AB) = ",det(A*B));
println("det(BA) = ",det(B*A));
println("det(AB) = det(A) * det(B) true or false?   ==", det(A)*det(B)
== det(A*B));

 #= volume does not change along multiple transformations
Determinants are COMMUTATIVE =#
##======
```

```
julia>
det(AB) = -35.0
det(BA) = -35.0
det(AB) = det(A) * det(B) true or false?   ==true
```

1e

$$M_2 = \begin{bmatrix} -3 & 1 \\ 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A = M_2 R \qquad B = R M_2$$

$$A = \begin{bmatrix} -3 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} -3\cos\theta - \sin\theta & -3\sin\theta + \cos\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3\cos\theta & \cos\theta + \sin\theta \\ 3\sin\theta & -\sin\theta + \cos\theta \end{bmatrix}$$

$$A \neq B$$

1g

$$M_3 = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$C = M_3 V_3$$

$$C = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/2\sqrt{2} & -1/2\sqrt{2} \\ -2/\sqrt{2} & -2/\sqrt{2} \end{bmatrix}$$

$$D = V_3 C$$

$$D = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/2\sqrt{2} & -1/2\sqrt{2} \\ -2/\sqrt{2} & -2/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/4 + 1 & -1/2 + 1 \\ -1/2 + 1 & 1/4 + 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1.25 & 0.75 \\ 0.75 & 1.25 \end{bmatrix} = M_4$$

$$H = V_3 M_3$$

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1/2\sqrt{2} & -2/\sqrt{2} \\ -1/2\sqrt{2} & -2/\sqrt{2} \end{bmatrix}$$

c I is halved

c II is doubled

2a

$$A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} \qquad x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$Ax = \lambda x$$

$$A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

① $4x_1 - 2x_2 = \lambda x_1$

② $x_1 + x_2 = \lambda x_2$

$r = \frac{x_2}{x_1}$

$$\left.\begin{aligned} 4r - 2 &= \lambda r \\ r + 1 &= \lambda \end{aligned}\right\}$$ where $r \equiv \frac{x_1}{x_2}$

$$\frac{4r-2}{r} = r + 1$$

$$4r - 2 = r^2 + r$$

$$0 = r^2 - 3r + 2$$

$$0 = (r-1)(r-2)$$

$r = 1$ or $r = 2$

$1 + 1 = \lambda \qquad 2 + 1 = \lambda$

eigen value $\lambda = 2$ or $\lambda = 3$

$8 - 2 = 2 \times 2$

$8 - 2 = \lambda 2$

$\lambda = 3$ eigenvalue

$\lambda \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

$$\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 2e_1 \\ 2e_2 \end{bmatrix}$$

$4e_1 - 2e_2 = 2e_1$

$e_1 + e_2 = 2e_2$

$e_1 = e_2$

$\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ eigenvector for $\lambda = 2$

↳ yeah this is not great

3a

$A = \begin{bmatrix} -1 & .25 \\ .5 & 1.5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -.5 \\ .5 & -.25 \end{bmatrix}$

$|A| = -3/2 - 1/8$

$= \frac{-24}{16} - \frac{-2}{16} = \frac{-11}{8} = -1.375$

$|B| = -1/4 + 1/4$

$= 0$

$A = \begin{bmatrix} 1 & 2 \\ -1 & .5 \end{bmatrix}$ $B = \begin{bmatrix} -2 & -4 \\ -3 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} -8 & -2 \\ .5 & 4.5 \end{bmatrix}$ $|AB| = -36 + 1 = -35$

$BA = \begin{bmatrix} 1 & -6 \\ -4 & -5.5 \end{bmatrix}$

$|AB| = |BA|$

$$\begin{array}{cc} & A \\ & \begin{bmatrix} 1 & 2 \\ -1 & .5 \end{bmatrix} \\ B & \\ \begin{bmatrix} -2 & -4 \\ -3 & 1 \end{bmatrix} & \begin{bmatrix} (-2+4) & (-4-2) \\ (-3-1) & (-6+.5) \end{bmatrix} \end{array}$$

$= \begin{bmatrix} 2 & -6 \\ -4 & -5.5 \end{bmatrix} = -11 - 24 = -35$