Relativistic DWARF Field Theory: Lagrangian Formulation with Dissipation and Couplings

Tyler Nagel & Caelum

2025

Abstract

We present a relativistic field-theoretic formulation of the DWARF theory (Dynamic Wake Accretion in Relativistic Fluids), extending the core hydrodynamic model with a covariant Lagrangian, dissipative terms, and couplings to scalar and vector fields. This forms the mathematical backbone necessary for simulating particle-like excitations and interactions in a structured flow medium.

1 Step 1: Classical DWARF Flow Equation

The classical (non-relativistic) DWARF evolution equation resembles a modified Navier–Stokes equation:

$$\frac{\partial \vec{v}}{\partial t} = -\nabla\Phi + \nu\nabla^2\vec{v} - \beta\vec{v} \tag{1}$$

where $\vec{v}$ is the velocity field, Φ is the emergent potential, ν is the viscosity coefficient, and β is a damping factor.

2 Step 2: Relativistic Generalization

To make DWARF compatible with relativistic frameworks, we define:

- $u^\mu = \frac{dx^\mu}{d\tau}$: the 4-velocity field
- τ: proper time
- Φ: emergent potential in spacetime

The relativistic evolution equation becomes:

$$\boxed{u^\nu\nabla_\nu u^\mu = -\nabla^\mu\Phi + \nu\nabla^2 u^\mu - \beta u^\mu} \tag{2}$$

with the covariant continuity equation:

$$\boxed{\nabla_\mu(\rho u^\mu) = 0} \tag{3}$$

This formulation preserves local Lorentz invariance and is suitable for both flat and curved spacetimes.

3 Step 3: Field Potential and Equation of State

The potential Φ is understood as a function of density or divergence:

$$\Phi = f(\rho) + \gamma \nabla_\mu u^\mu \tag{4}$$

leading to an effective pressure:

$$P = \rho \frac{d\Phi}{d\rho} - \Phi(\rho) \tag{5}$$

which appears in the energy–momentum conservation:

$$\nabla_\nu T^{\mu\nu} = 0 \Rightarrow u^\nu \nabla_\nu u^\mu = -\frac{1}{\rho} \nabla^\mu P \tag{6}$$

This recovers a relativistic Euler-like equation driven by an emergent fluid potential.

4 Core Lagrangian Formulation

We define the fundamental fields:

- $u^\mu(x)$: 4-velocity field
- $\rho(x)$: Proper density field
- $\Phi(\rho)$: Emergent self-potential

The relativistic DWARF Lagrangian is:

$$\mathcal{L}_{\text{DWARF}} = -\frac{1}{2} \rho u^\mu u_\mu - \rho \Phi(\rho) \tag{7}$$

Variation with respect to u^μ yields the continuity equation:

$$\nabla_\mu (\rho u^\mu) = 0 \tag{8}$$

Variation with respect to ρ yields the Euler-like equation:

$$\frac{\delta \mathcal{L}}{\delta \rho} = -\frac{1}{2} u^\mu u_\mu - \Phi(\rho) - \rho \frac{d\Phi}{d\rho} \tag{9}$$

The energy-momentum tensor is:

$$T^{\mu\nu} = \rho u^\mu u^\nu + g^{\mu\nu}(-\rho\Phi) \tag{10}$$

And the dynamical equation:

$$\nabla_\nu T^{\mu\nu} = 0 \quad \Rightarrow \quad u^\nu \nabla_\nu u^\mu = -\frac{1}{\rho} \nabla^\mu P \tag{11}$$

5 Phenomenological Dissipation Terms

To incorporate diffusion and damping:

$$\boxed{u^\nu \nabla_\nu u^\mu = -\nabla^\mu \Phi + \nu \nabla^2 u^\mu - \beta u^\mu} \tag{12}$$

where:

- ν is the viscosity-like coefficient
- β is a damping or drag coefficient

These can be added via non-Hermitian or Langevin terms external to the core Lagrangian.

6 Scalar Field Coupling

Let $\phi(x)$ be a scalar field coupled to the DWARF fluid:

$$\mathcal{L}_\phi = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \alpha_\phi \rho \phi \tag{13}$$

where α_ϕ is the coupling constant.

7 Vector Field Coupling

Introduce a vector field $A_\mu(x)$, similar to electromagnetism:

$$\mathcal{L}_A = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \alpha_A \rho u^\mu A_\mu \tag{14}$$

where:

- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- α_A is the coupling strength to the DWARF current ρu^μ

8 Total Lagrangian

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{DWARF}} + \mathcal{L}_\phi + \mathcal{L}_A \tag{15}$$

This total Lagrangian allows for quantization and particle-field interactions within the DWARF paradigm.